Particle Astrophysics and Cosmology (SS 08)
Homework no. 11 (July 9, 2008)

Tutorials: Wednesday, 17:15 to 18:45, AVZ, room 116 (first floor)

1 Inflation and slow-roll parameters

In order to check whether the slow-roll approximation is valid for a specific potential $V(\phi)$ it is useful to define the two parameters ϵ and η, as follows:

$$\epsilon(\phi) \equiv \frac{M_{Pl}^2}{2}\left(\frac{V'}{V}\right)^2 , \tag{1}$$

$$\eta(\phi) \equiv M_{Pl}^2 \frac{V''}{V} , \tag{2}$$

where $M_{Pl} = m_{Pl}/\sqrt{8\pi}$ is the reduced Planck mass. Show that the slow–roll approximations (i.e. $|\ddot{\phi}| \ll |V'|$ and $V(\phi) \gg \frac{1}{2}\dot{\phi}^2$) are necessary in order to have $\epsilon \ll 1$ and $|\eta| \ll 1$. The conditions on the slow–roll parameters ensure that an inflationary phase is occuring in which the expansion of the universe is accelerating.

2 Chaotic Inflation

As an example for an inflationary model consider the potential $V = \lambda\phi^4$, where λ is the self–coupling.

(a) Calculate the value of ϕ where each of the slow–roll conditions breaks down first.

(b) The number of $e-$foldings of inflation at time t is defined by

$$N(t) \equiv \ln \frac{a(t_{end})}{a(t)}, \tag{3}$$

Rewrite this formula via the slow–roll approximation in order to obtain:

$$N(\phi) \simeq -\frac{1}{M_{Pl}^2} \int_{\phi_{end}}^{\phi} \frac{V}{V'}\, d\phi, \tag{4}$$

where ϕ_{end} is defined by $\epsilon(\phi_{end}) = 1$ if inflation ends through violation of slow–roll conditions.

(c) Calculate the number of $e-$foldings that occur for an initial value ϕ_i by assuming that inflation ends if $\epsilon = 1$.

(d) Determine and solve the equation of motion for $\phi(t)$ and $a(t)$ for given initial conditions at $t = t_i$.

(e) Expand the solution for the FRW scale factor a at small $t - t_i$ to demonstrate that the inflation is approximately exponential at the initial stage with $a \propto \exp(\kappa\, t)$, where κ is a constant.

(f) Calculate the time constant κ and show that it equals the (slow-roll) Hubble parameter during inflation.