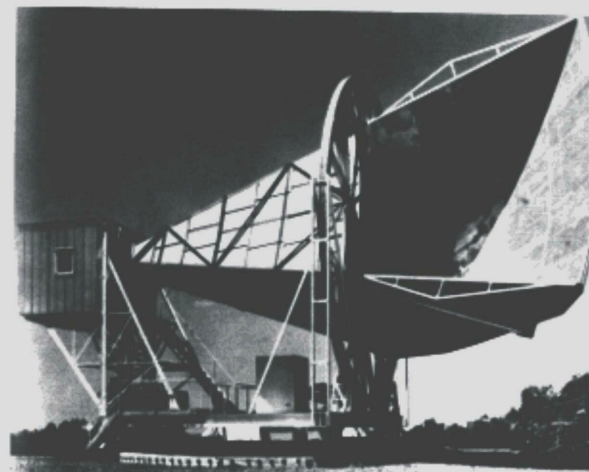
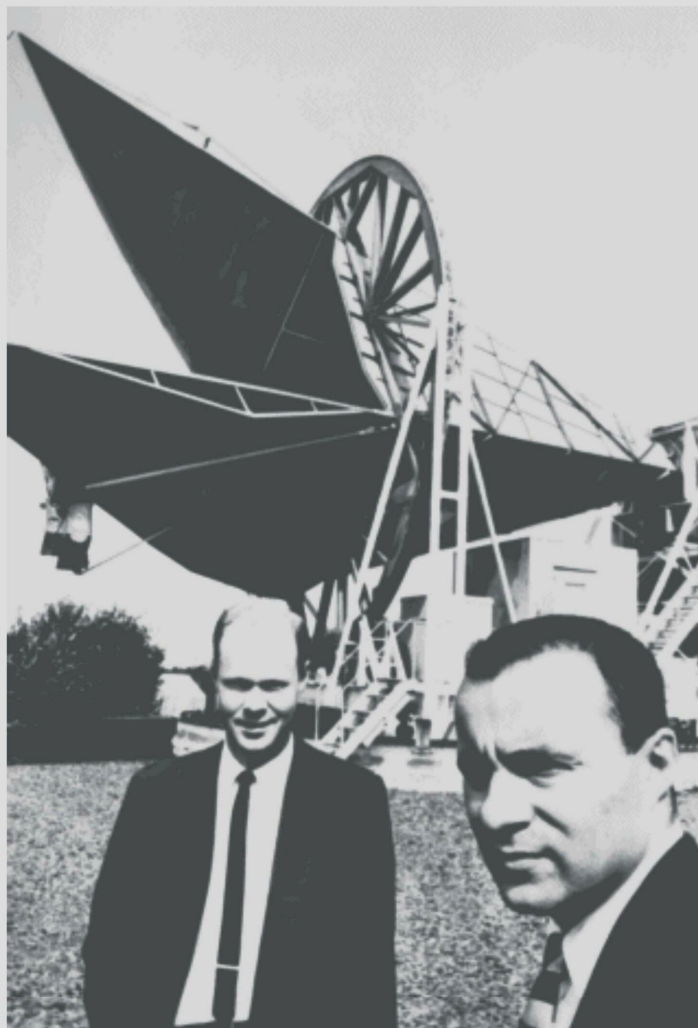
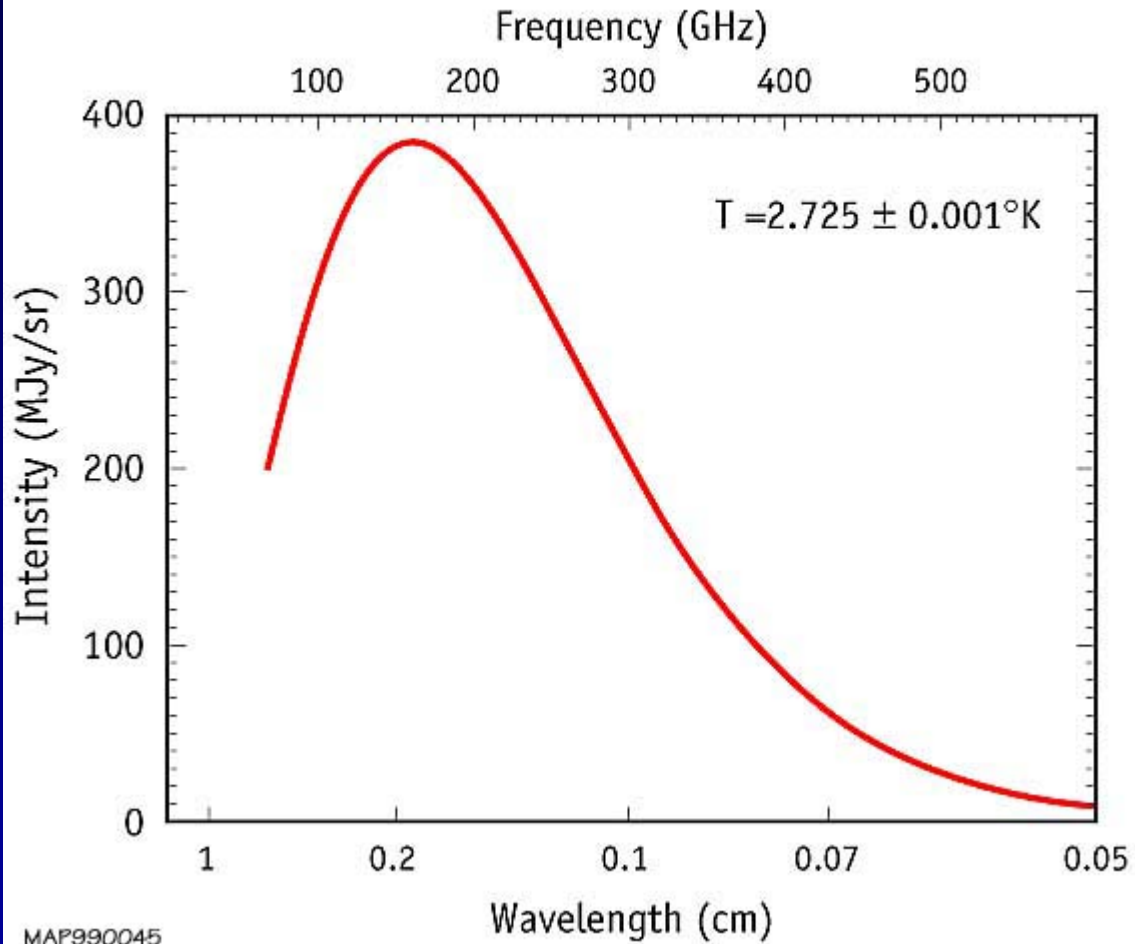


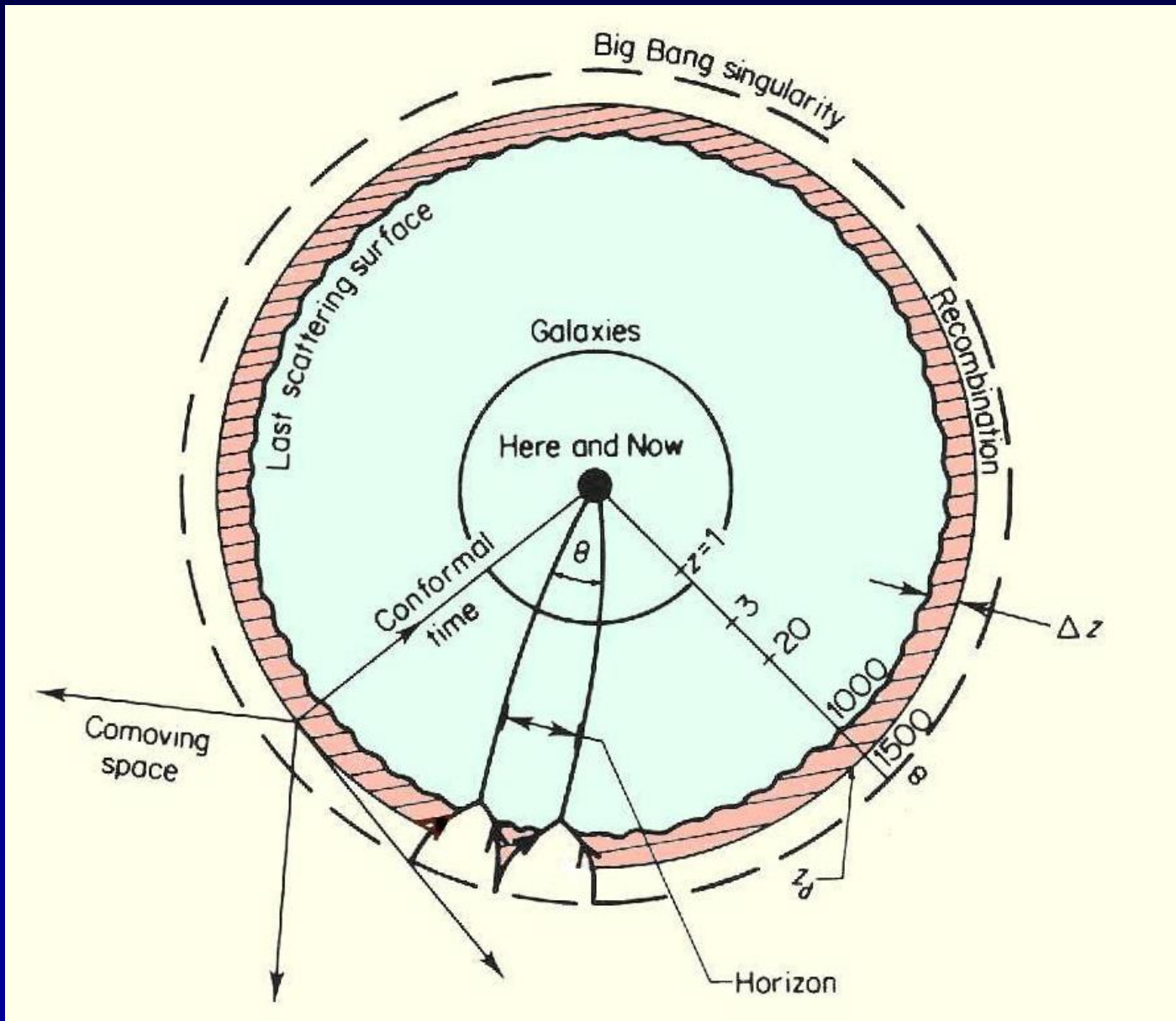
Chapter 6

Cosmic Microwave Background

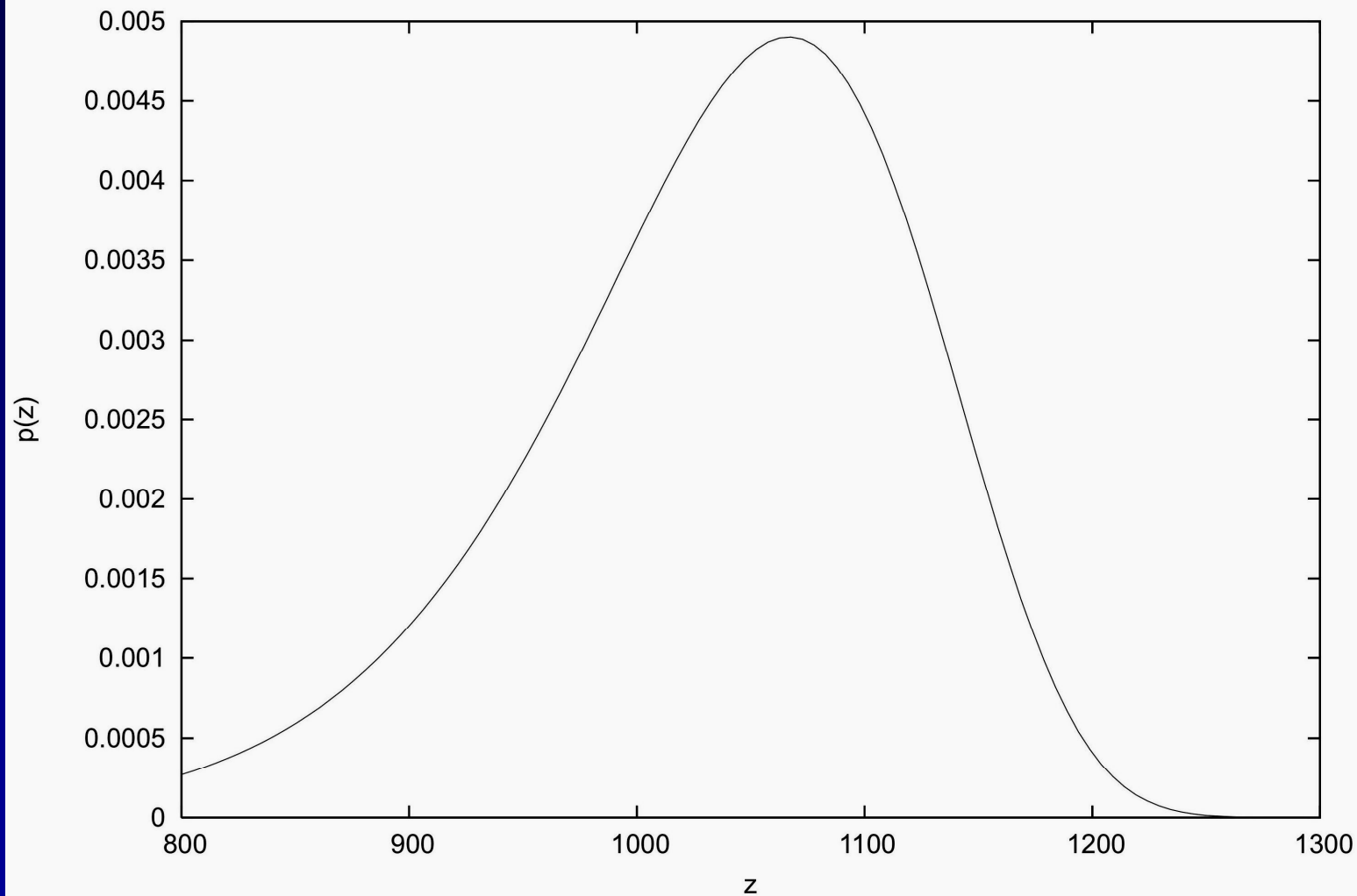


SPECTRUM OF THE COSMIC MICROWAVE BACKGROUND

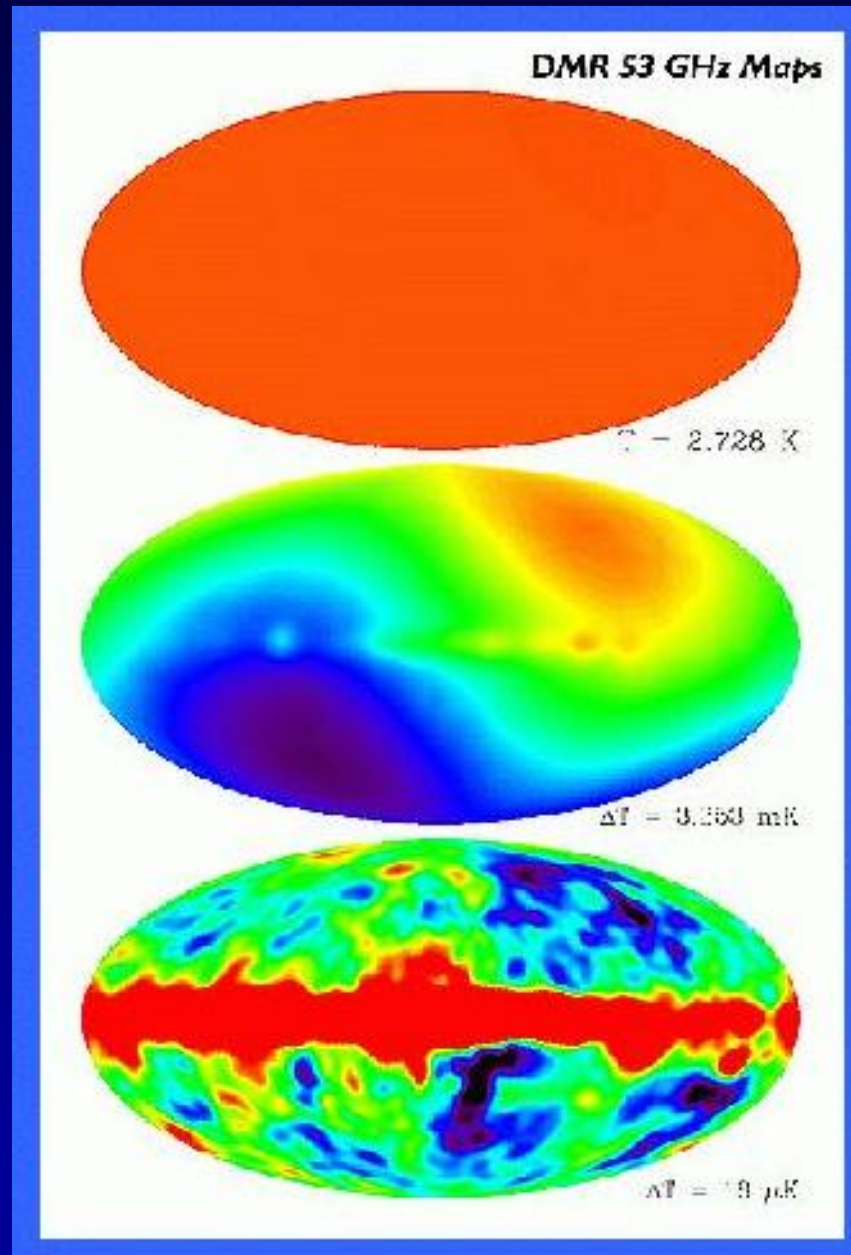




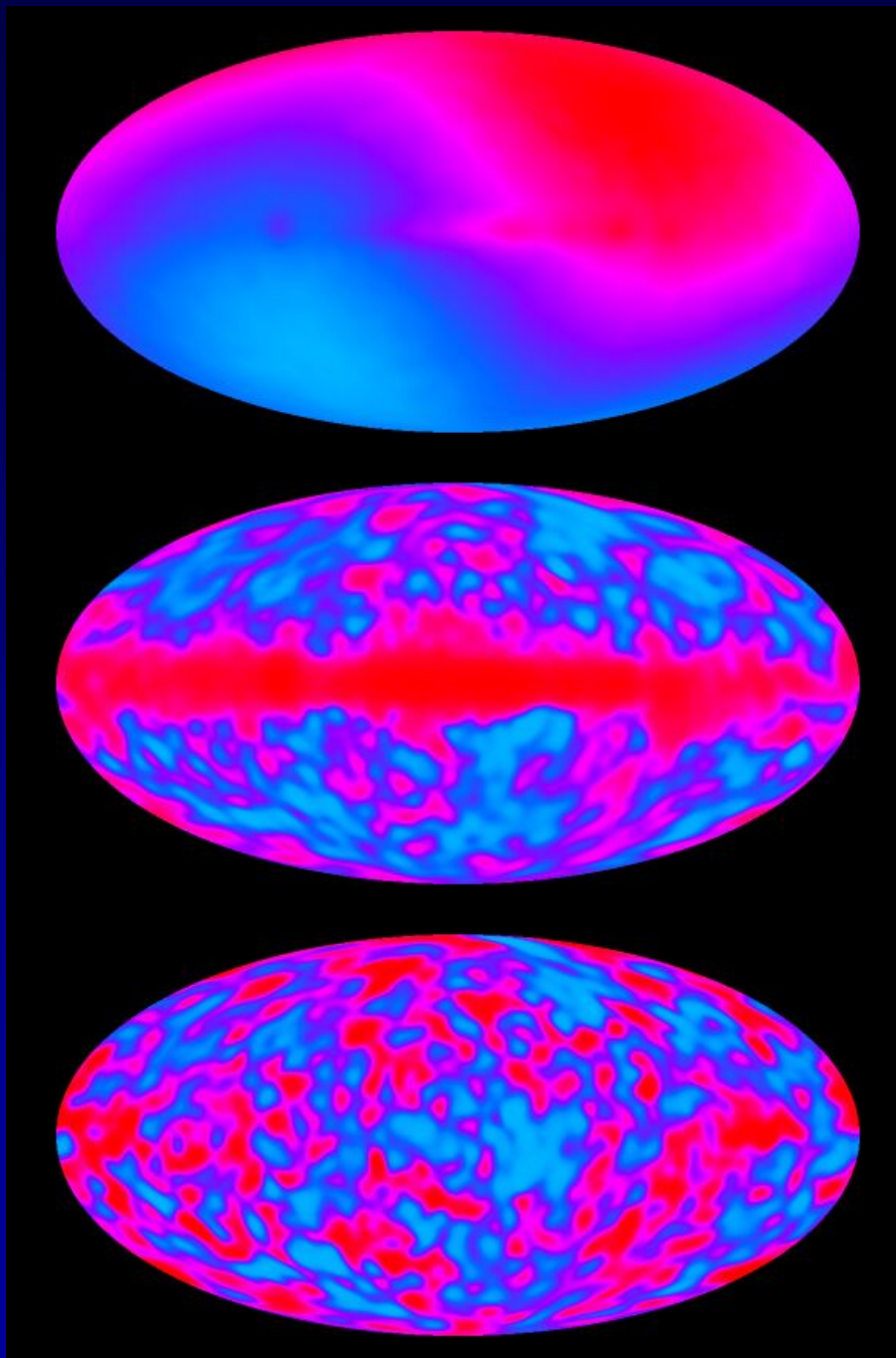
Distribution function for last scattering



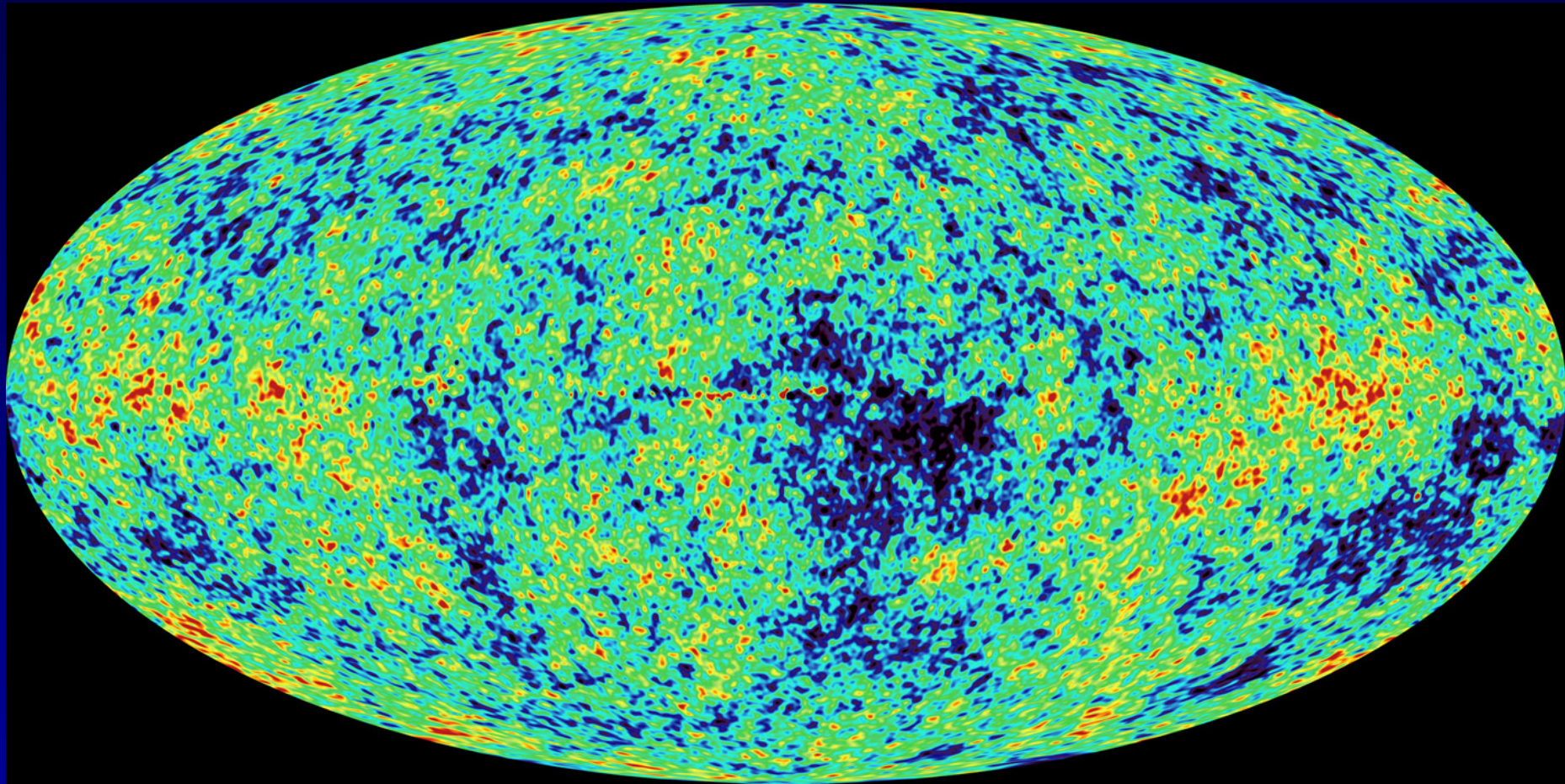
COBE



COBE

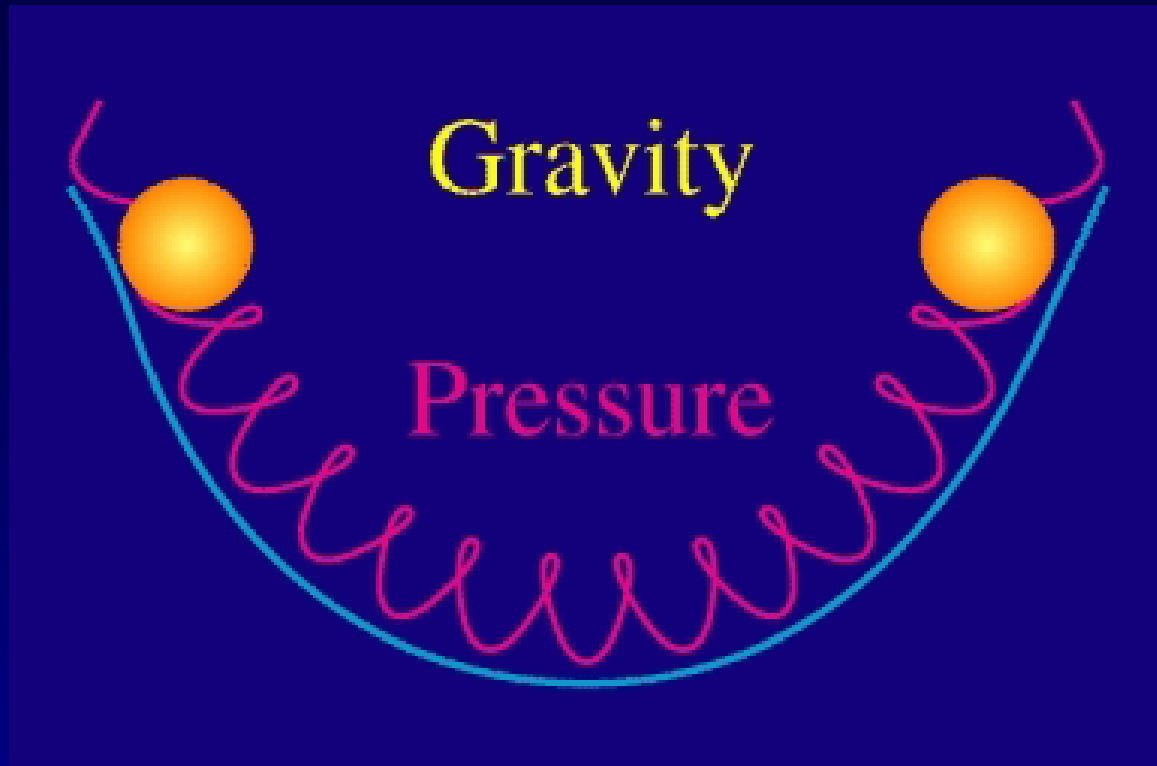


WMAP CMB anisotropy



CMB anisotropy: a toy tutorial †

† : mainly from Wayne Hu (<http://background.uchicago.edu/~whu>)



Gravity : attracting force
Photon pressure : driving force

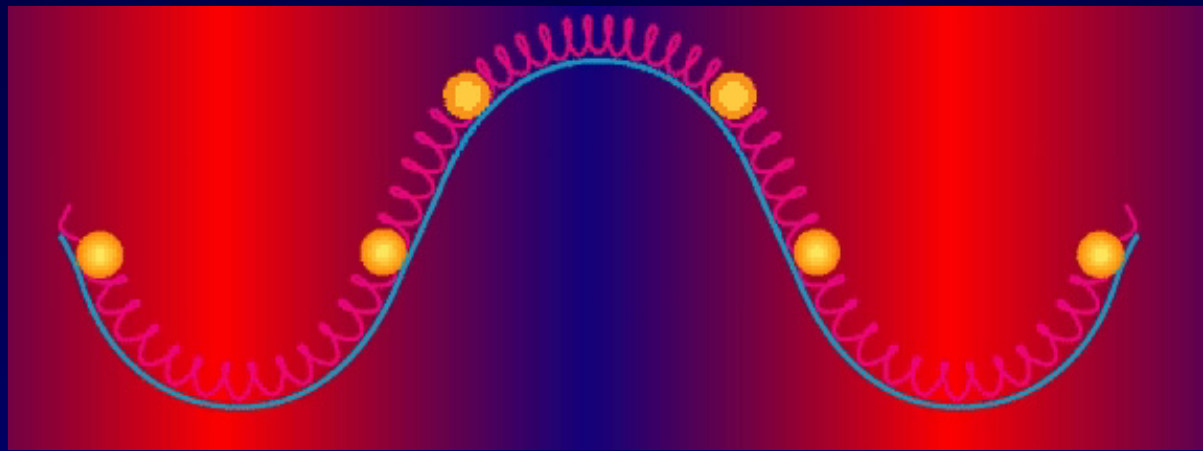
baryonic matter coupled to photons \Rightarrow photon pressure prevents collapse prior to recombination (while DM is already forming structure much earlier!)

Oscillations produce

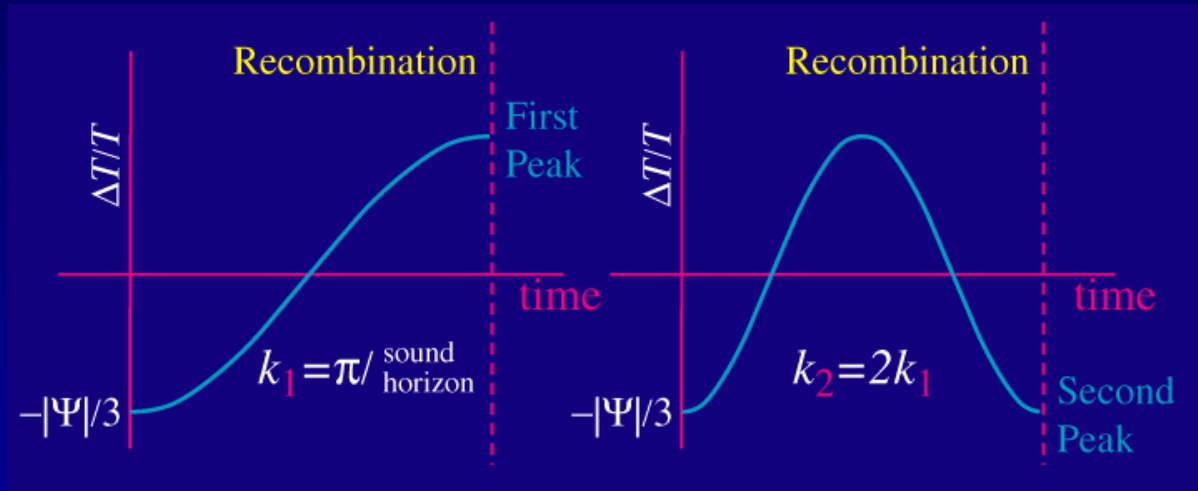
$$\Delta T > 0 \text{ or } \Delta T < 0$$

blue

red



Lowest **mode** (or wave number) corresponds to acoustic waves that managed to contract (or expand) once until recombination



2nd mode managed to contract and expand once until recombination, a.s.o.

$$v_n = n \cdot c_s \cdot \lambda_{s^*}^{-1} = n \cdot c/\sqrt{3} \cdot \lambda_s^{-1} = n \cdot 10^{13} \text{ Hz} \Rightarrow \text{not "audible" ...}$$

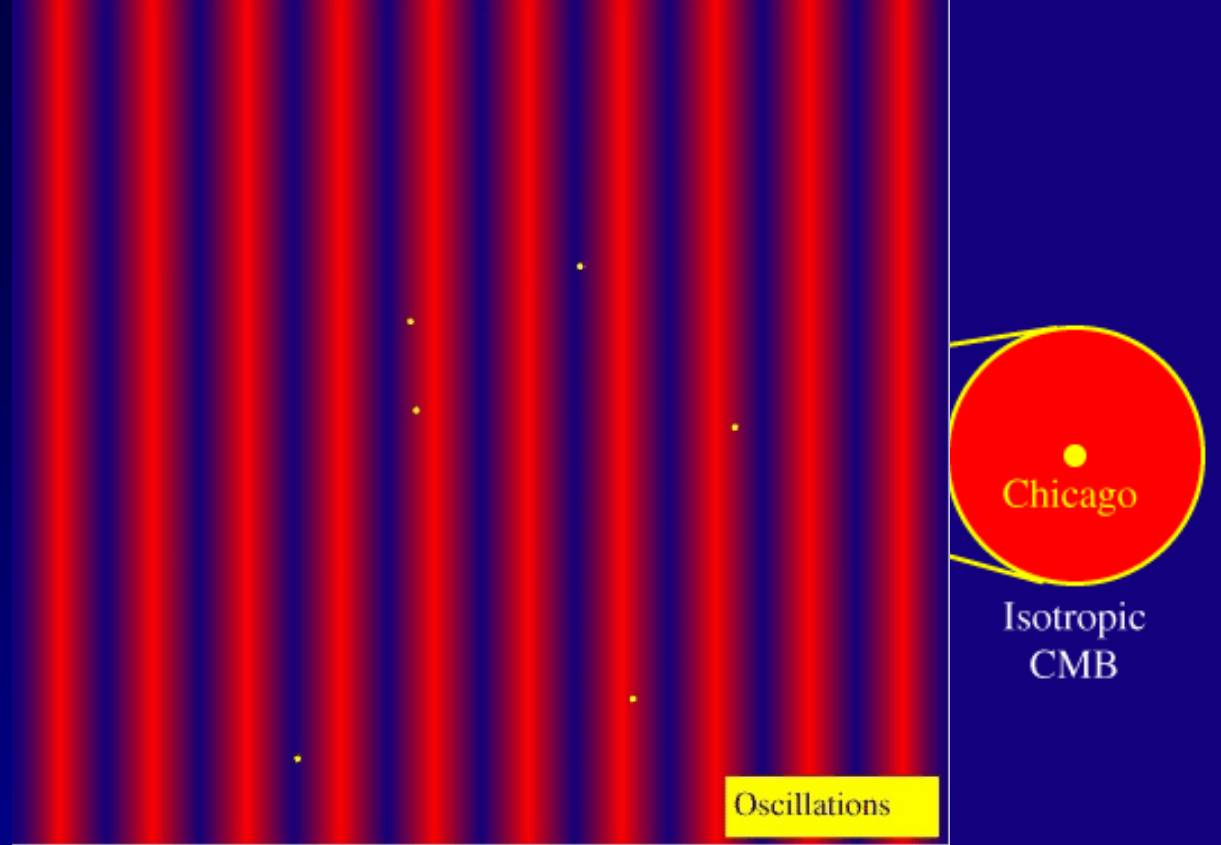
$$T_n = t_{\text{rec}}/n = n^{-1} \cdot 300000 \text{ yr}$$

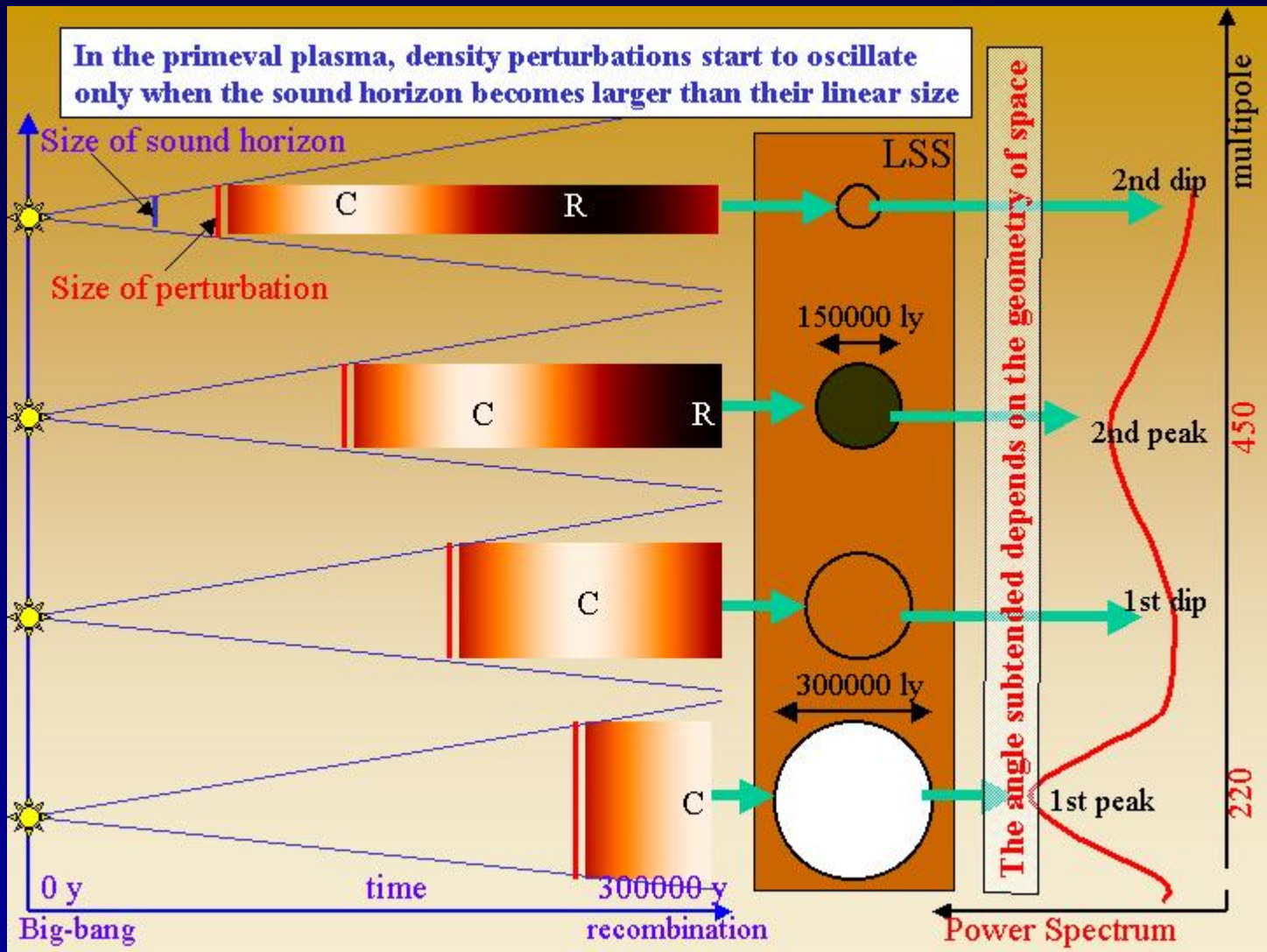
Angular distribution in the sky:

prior to recombination,
photons correspond to
higher and lower temperature

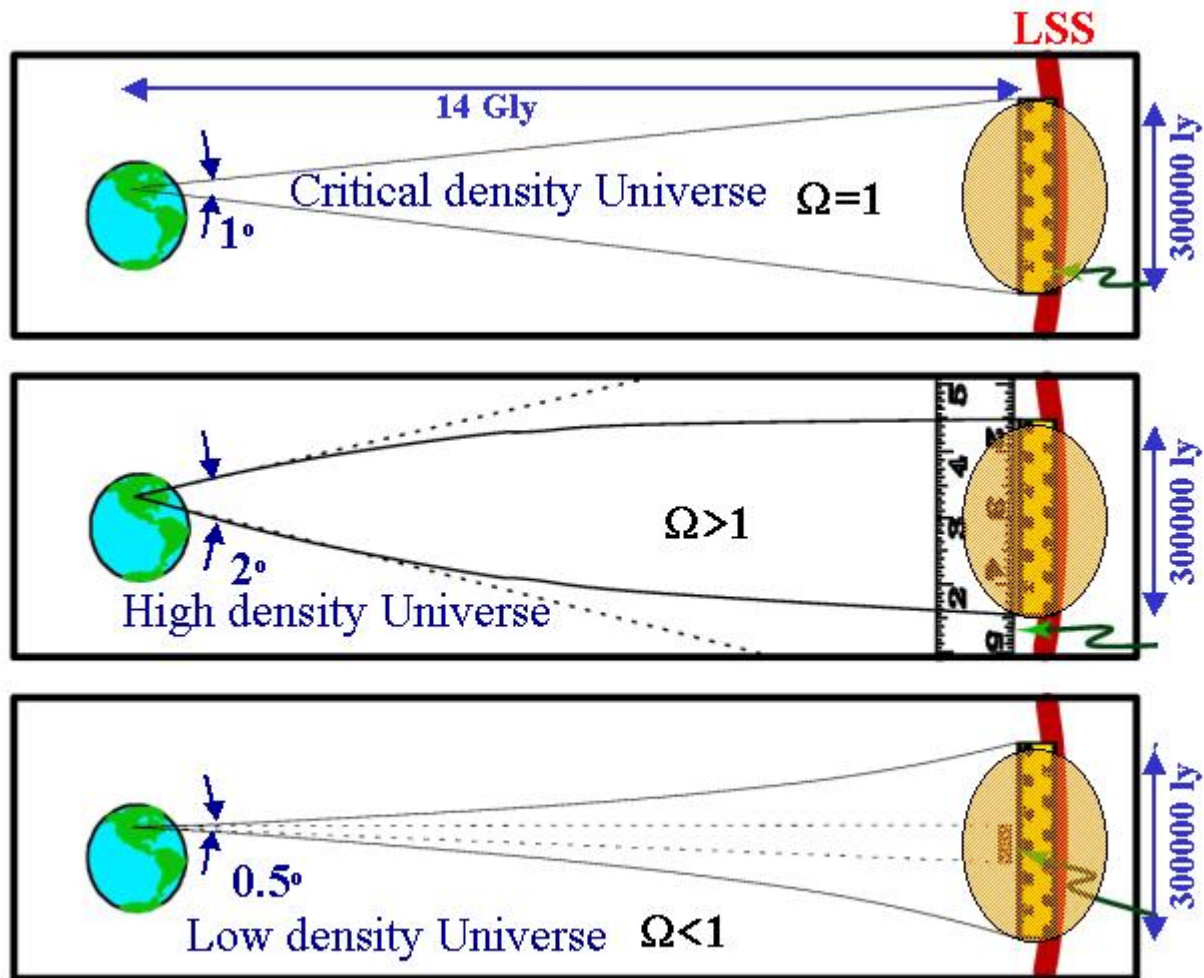
After recombination,
photons travel freely
and convey their last information - hot or cold, as a function of
angular position - to the observer; spatial inhomogeneity is converted into an angular anisotropy

The larger the distance they arrive from, the more complex the
angular pattern → higher multipoles

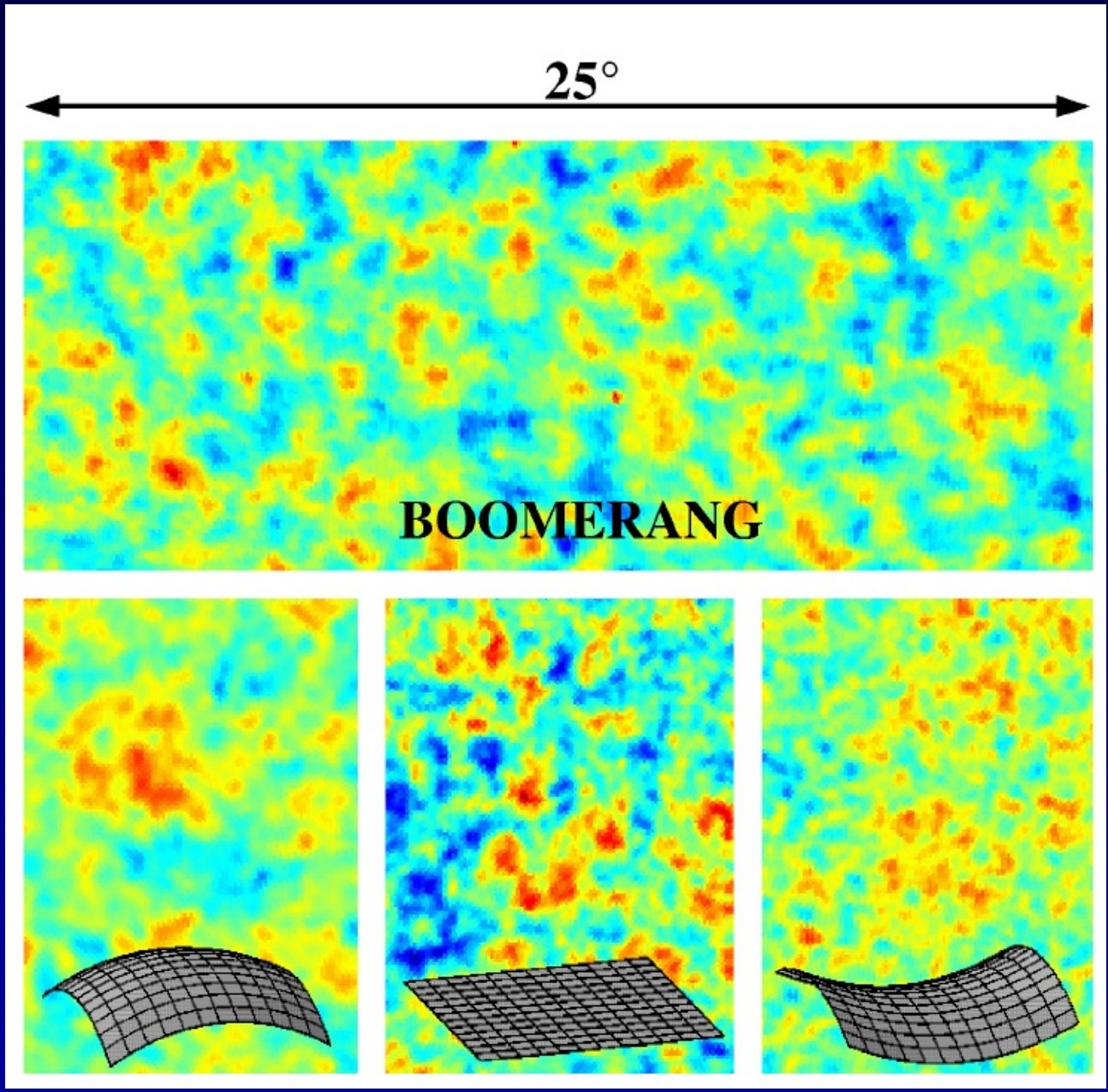




taken from de Bernardis



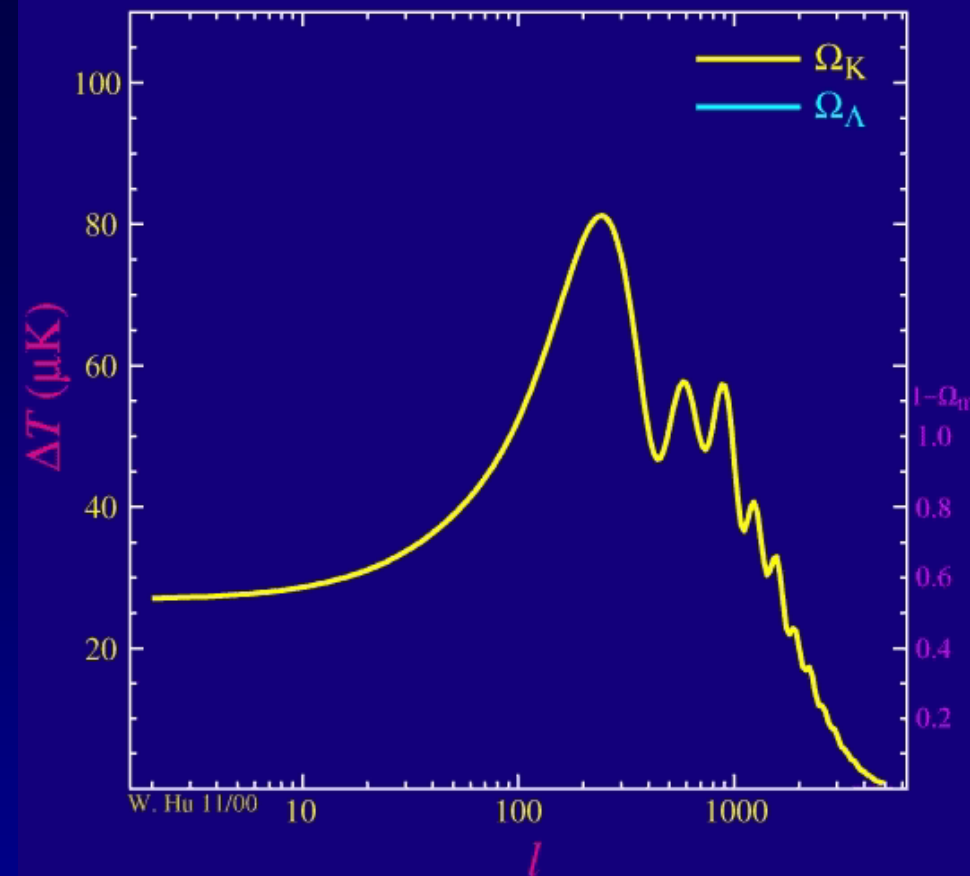
taken from de Bernardis



The first peak: spatial curvature

Fundamental scale at recombination (the distance that sound could travel) is converted into a fundamental angular scale on the sky today

$$\theta_s \approx 0.8^\circ \cdot \Omega_0^{\frac{1}{2}}$$
$$l_1 \approx \frac{220}{(\Omega_m + \Omega_\Lambda)^{\frac{1}{2}}}$$



The first (and strongest) peak measures the geometry of the universe

caveat: change in Ω_Λ produces slight shift, too (Λ causes slight change in distance that light travels from recombination to the observer)

1st peak, current score:

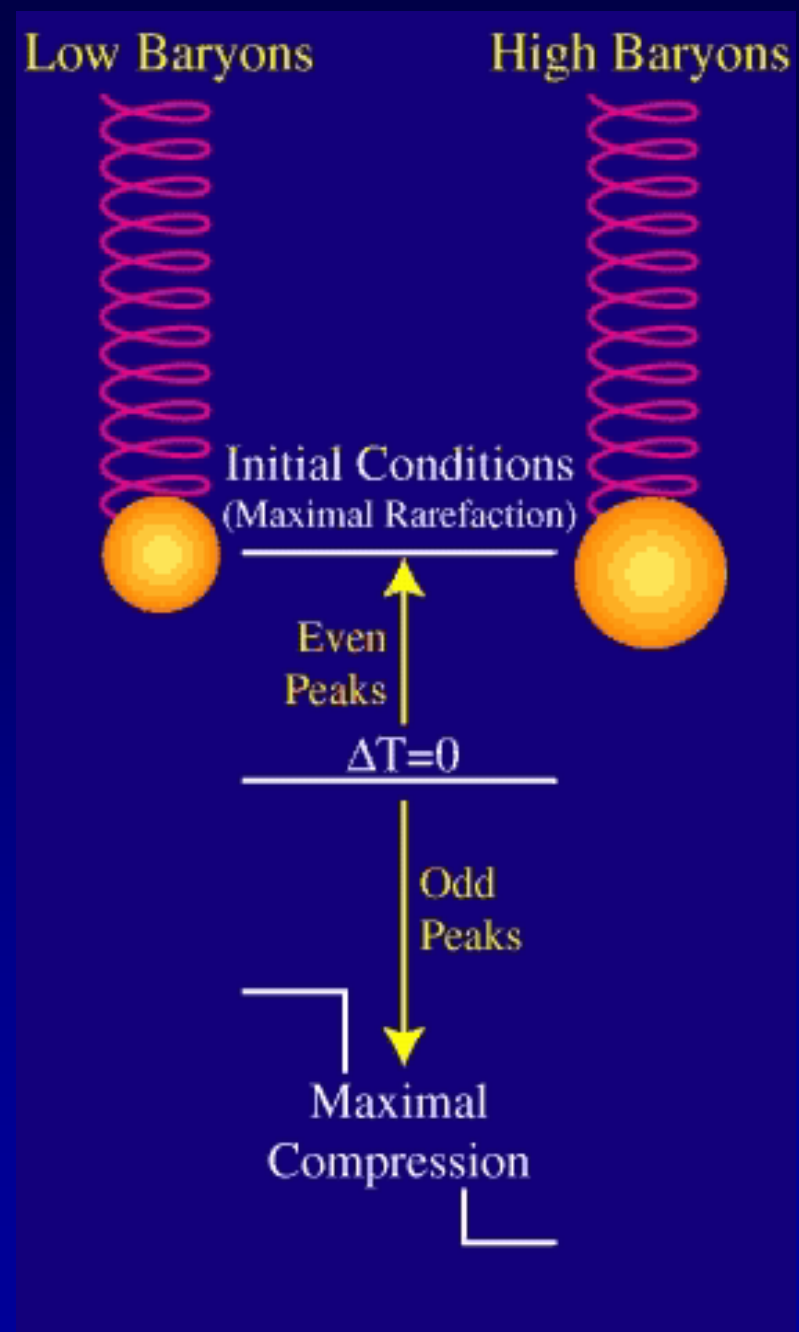
$$\Omega_0 = 1.02 \pm 0.02$$

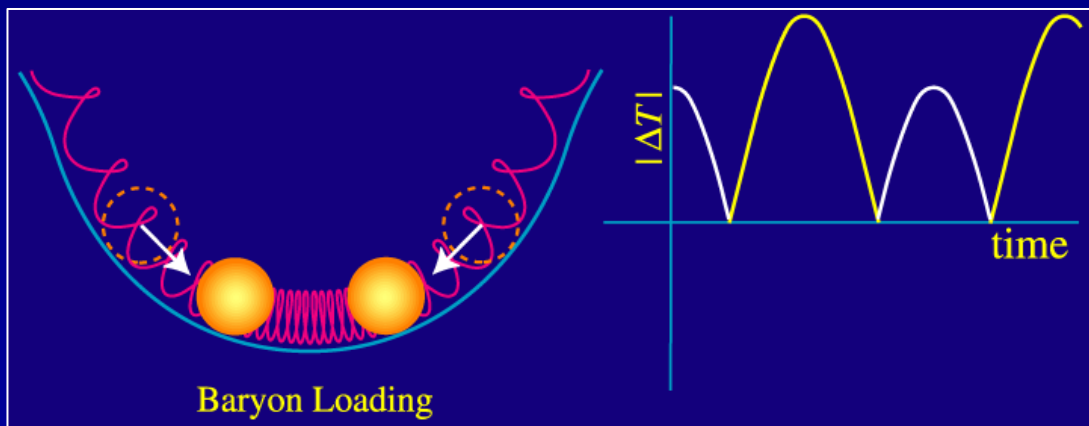
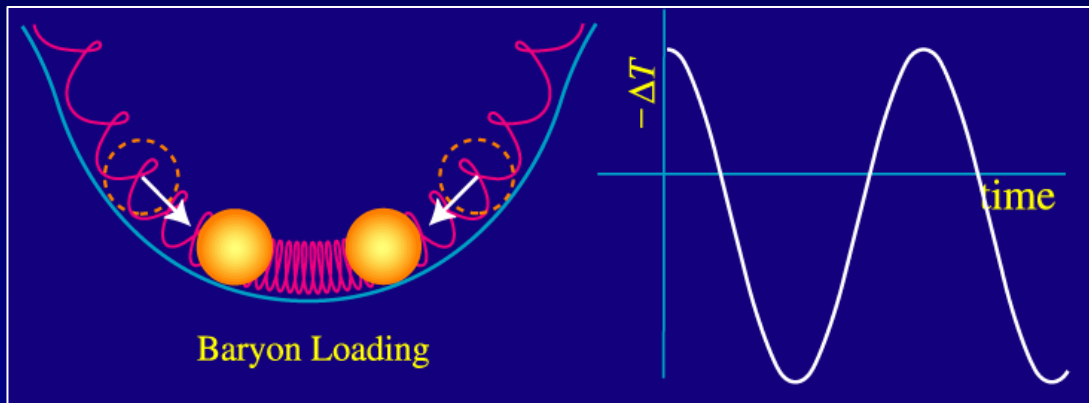
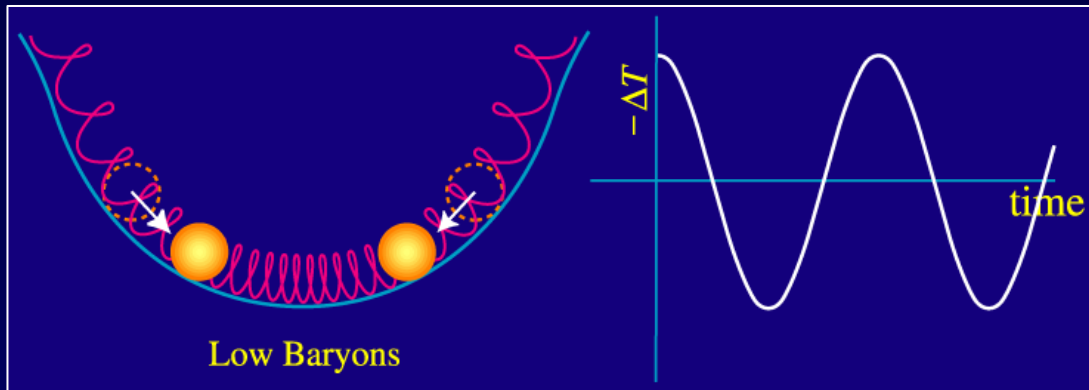
The second peak: **baryons** and inertia:

baryons add inertia to the plasma
⇒ contraction goes stronger, while the rarefaction remains the same!

compression corresponds to odd peaks
rarefaction corresponds to even peaks

⇒ higher baryon loading enhances odd over even peaks





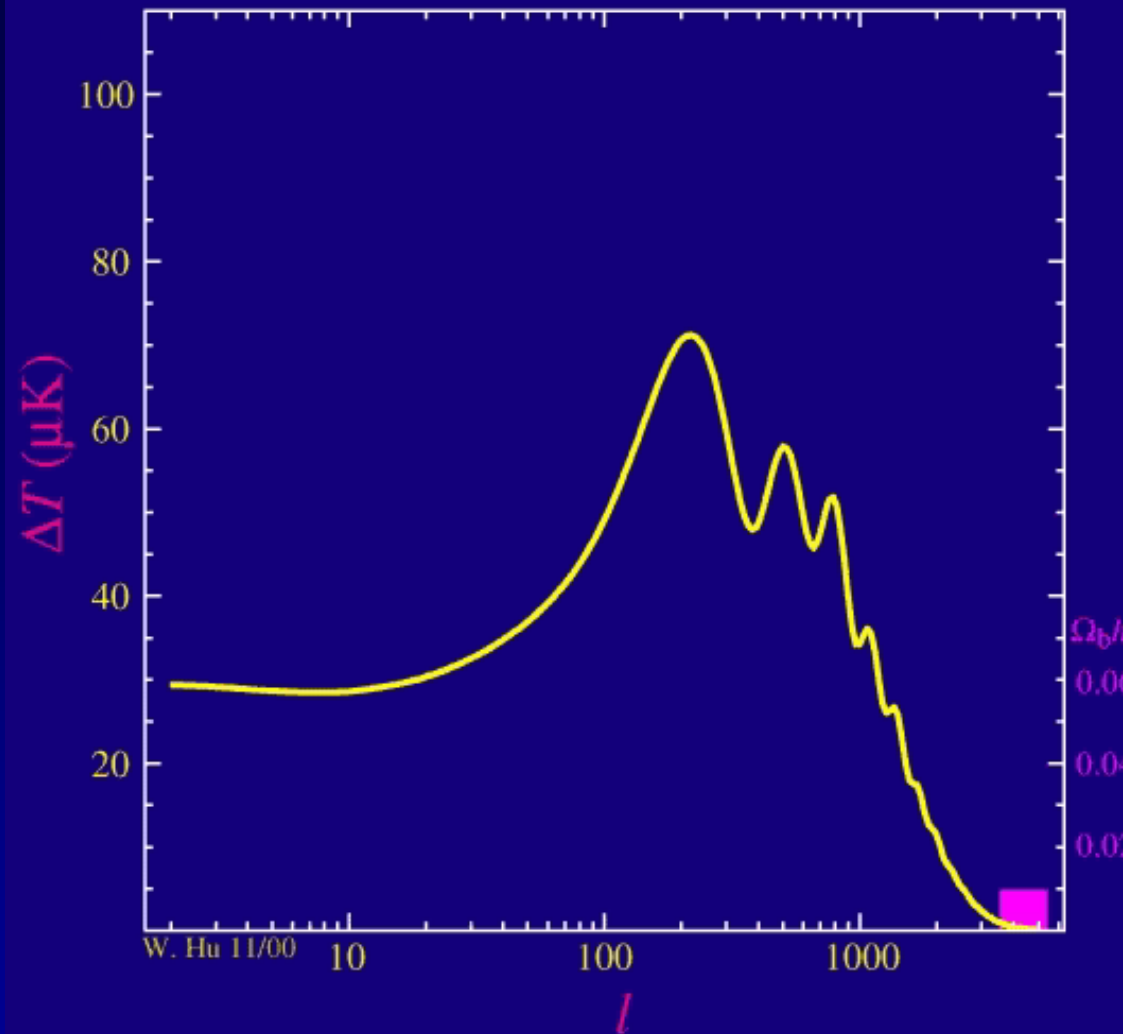
2nd peak, current score:

$$\Omega_b \cdot h^2 = 0.0224 \pm 0.0009$$

in nice agreement with Ω_b from deuterium abundance (QSO absorption lines)

Two more effects related to Ω_b :

- (i) increasing baryon load slows oscillations down
 - \Rightarrow long waves don't have enough time to build up
 - \Rightarrow larger k preferred if Ω_b increases
 - \Rightarrow power spectrum pushed to slightly higher l
- (ii) increasing Ω_b leads to more efficient damping of waves (s.b.), which is stronger for shorter wavelengths \Rightarrow spectrum falls off more rapidly towards large l
- (iii) decreasing $\Omega_m \Rightarrow$ smaller baryon-loading effect



The third peak: decay of potentials

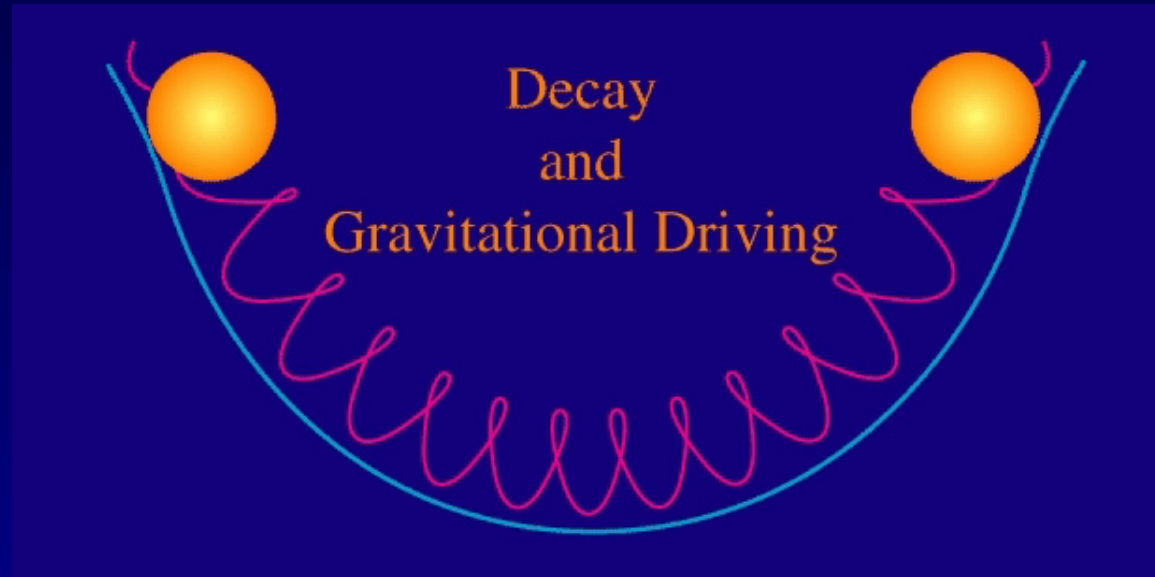
Poisson equation

$$\Delta\Phi = 4\pi \cdot G \cdot \rho$$

Since $\rho_r \propto R^{-4}$ and $\rho_m \propto R^{-3}$, Φ is governed by ρ_r in the state of highest compression

However, rapid expansion of the universe leads to instantaneous decay of Φ !

The fluid now sees no gravitation to fight against
 \Rightarrow amplitude of oscillations goes way up: **driving force!**



Driving force obviously more important in smaller (younger) universe, i.e. for $\rho_r \gg \rho_m$

Since modes with small wavelengths started first, it is the higher acoustic peaks that are more prone to this effect.

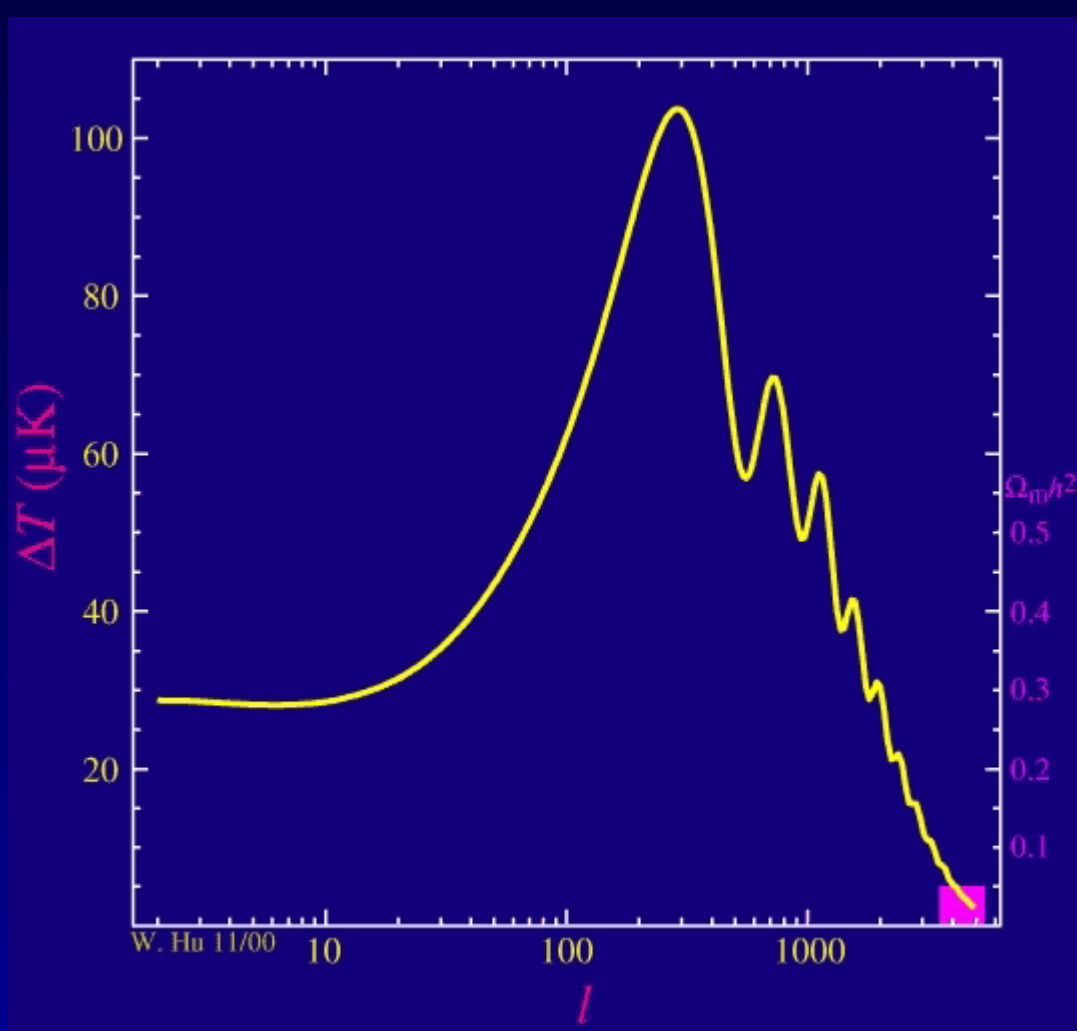
Increasing $\Omega_m \cdot h^2$ decreases the driving force \Rightarrow amplitudes of waves decrease

Influence of Ω_m only separable from that of Ω_b by measuring at least the first three peaks

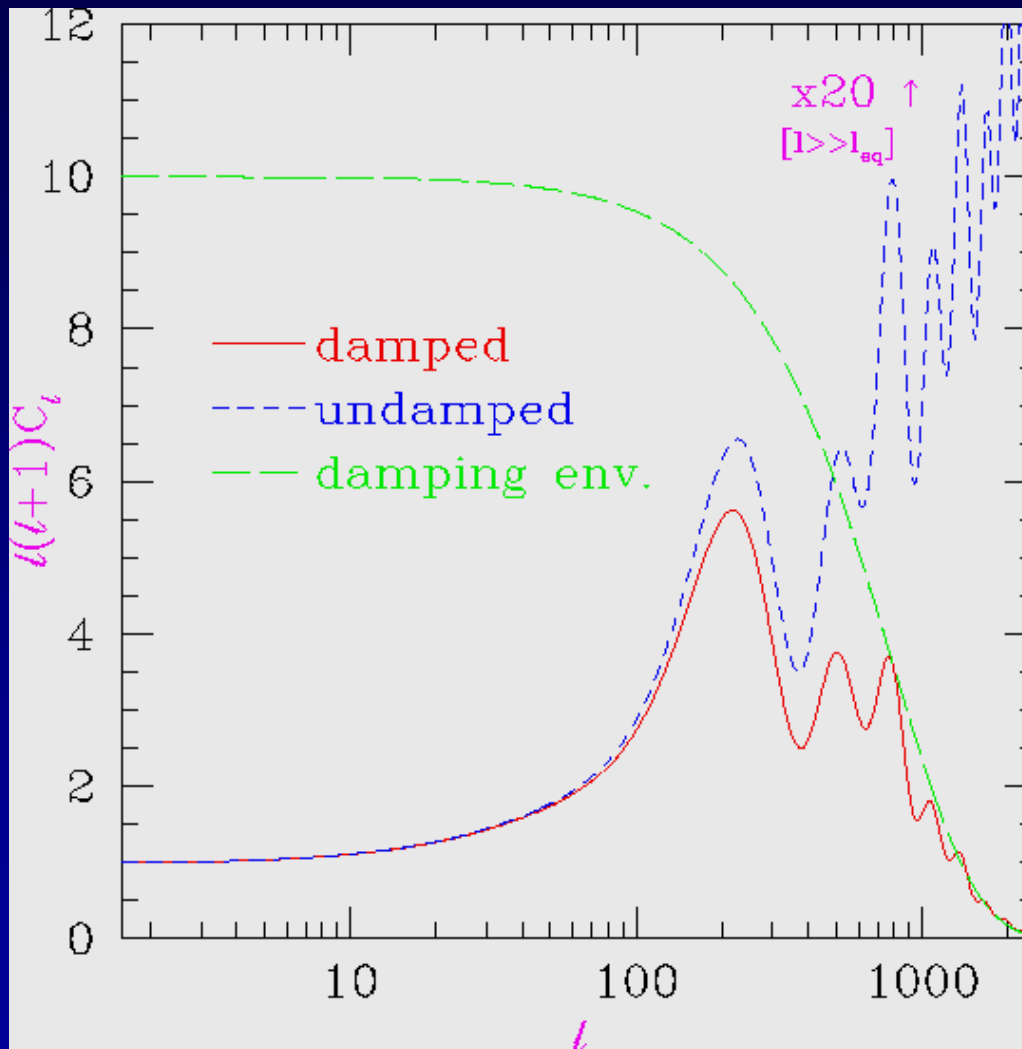
3rd peak, current score:

$$\Omega_m = 0.27 \pm 0.04 \quad \Omega_\Lambda = 0.73 \pm 0.04$$

in good agreement with other, independent methods (galaxy clusters, SNe)



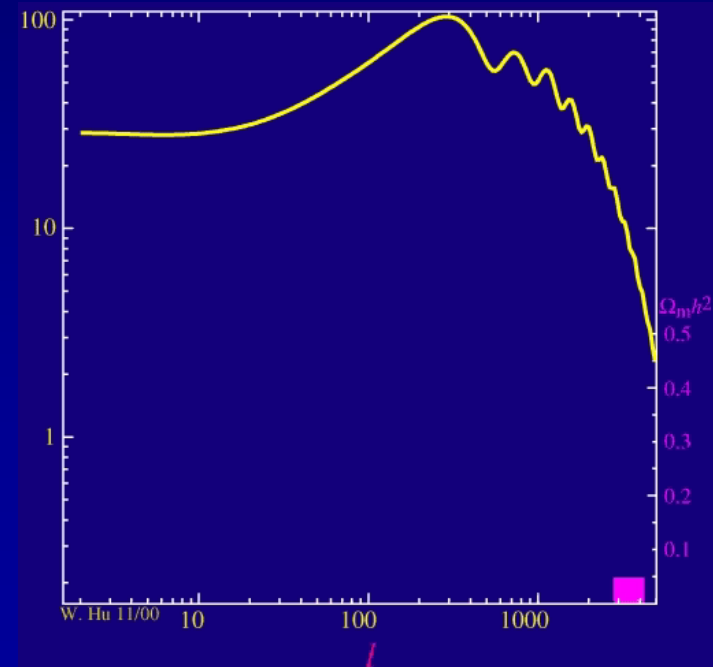
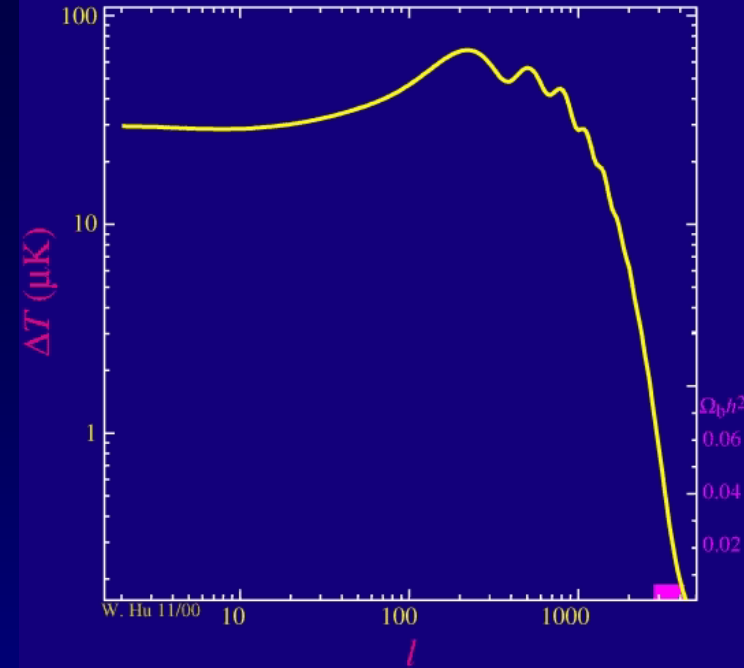
effect of damping



Damping depends on both, Ω_b and Ω_m

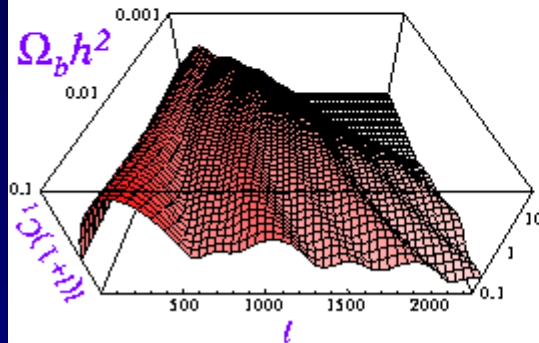
Ω_b : increasing baryon density couples the photon-baryon fluid more tightly, hence shifts the damping tail to smaller angular scales, i.e. higher l

Ω_m : increasing total matter density increases relative age of the universe, hence the angular scale of the damping is increased, shifting the damping tail to lower l

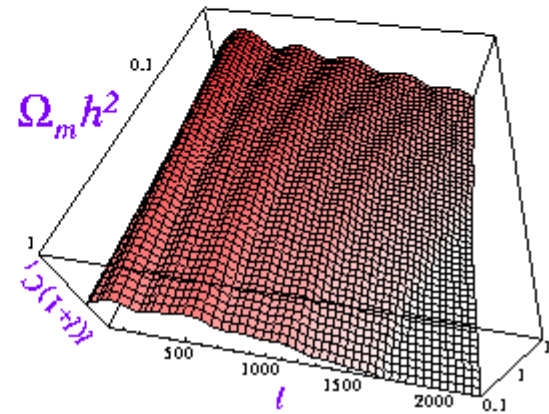


Cosmological Parameters in the CMB

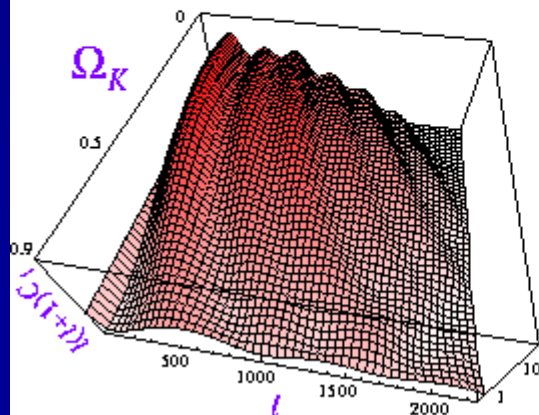
Baryon-Photon Ratio



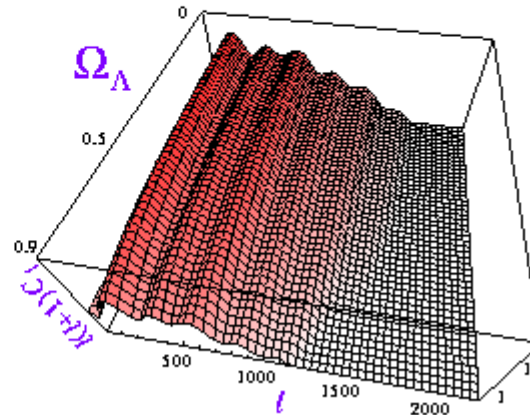
Matter-Radiation Ratio



Curvature



Cosmological Constant

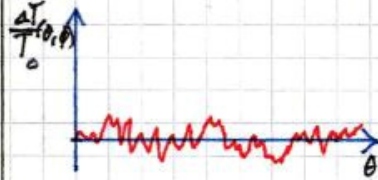


deriving the power spectrum

Analogy

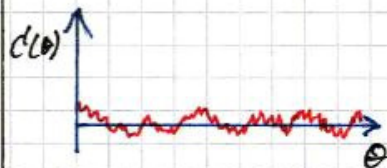
temperature fluctuations

$$\frac{\Delta T}{T_0}(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} a_{lm} \cdot Y_{lm}(\theta, \phi)$$



2-point correlation function

$$C(\theta) = \left\langle \frac{\Delta T}{T_0}(\theta, \phi) \cdot \frac{\Delta T}{T_0}(\theta', \phi') \right\rangle$$



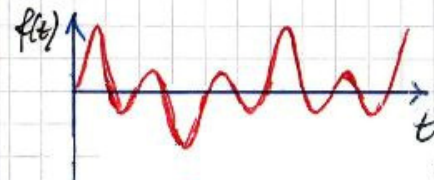
coefficients of spherical harm.

$$a_{lm} = \iint_{4\pi} \frac{\Delta T}{T_0}(\theta, \phi) \cdot Y_{lm}^*(\theta, \phi) d\Omega$$

Complex!

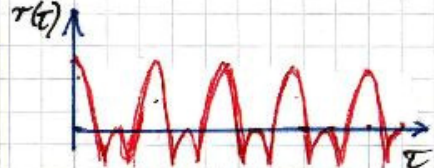
time-dependent signal

$$f(t) = \sum_{n=-\infty}^{+\infty} g_n \cdot e^{-i2\pi \cdot n \cdot \nu \cdot t}$$



autocorrelation function

$$r(\tau) = \frac{1}{2T} \int_{-T}^{+T} f(t) \cdot f(t-\tau) dt$$



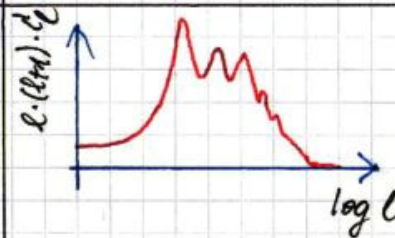
Fourier coefficients

$$g_n = \int_{-20}^{+20} f(t) \cdot e^{i2\pi \cdot n \cdot \nu \cdot t} dt$$

Complex!

power spectrum

$$C_l = \frac{1}{2l+1} \sum_{m=-l}^{+l} a_{lm} \cdot a_{lm}^* = \langle |a_{lm}|^2 \rangle$$



power spectrum

$$P(\nu) = g_n \cdot g_n^* = |g_n|^2$$



"autocorrelation theorem"

$$C(\theta) = \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1) \cdot P_l(\cos \theta) \cdot C_l$$

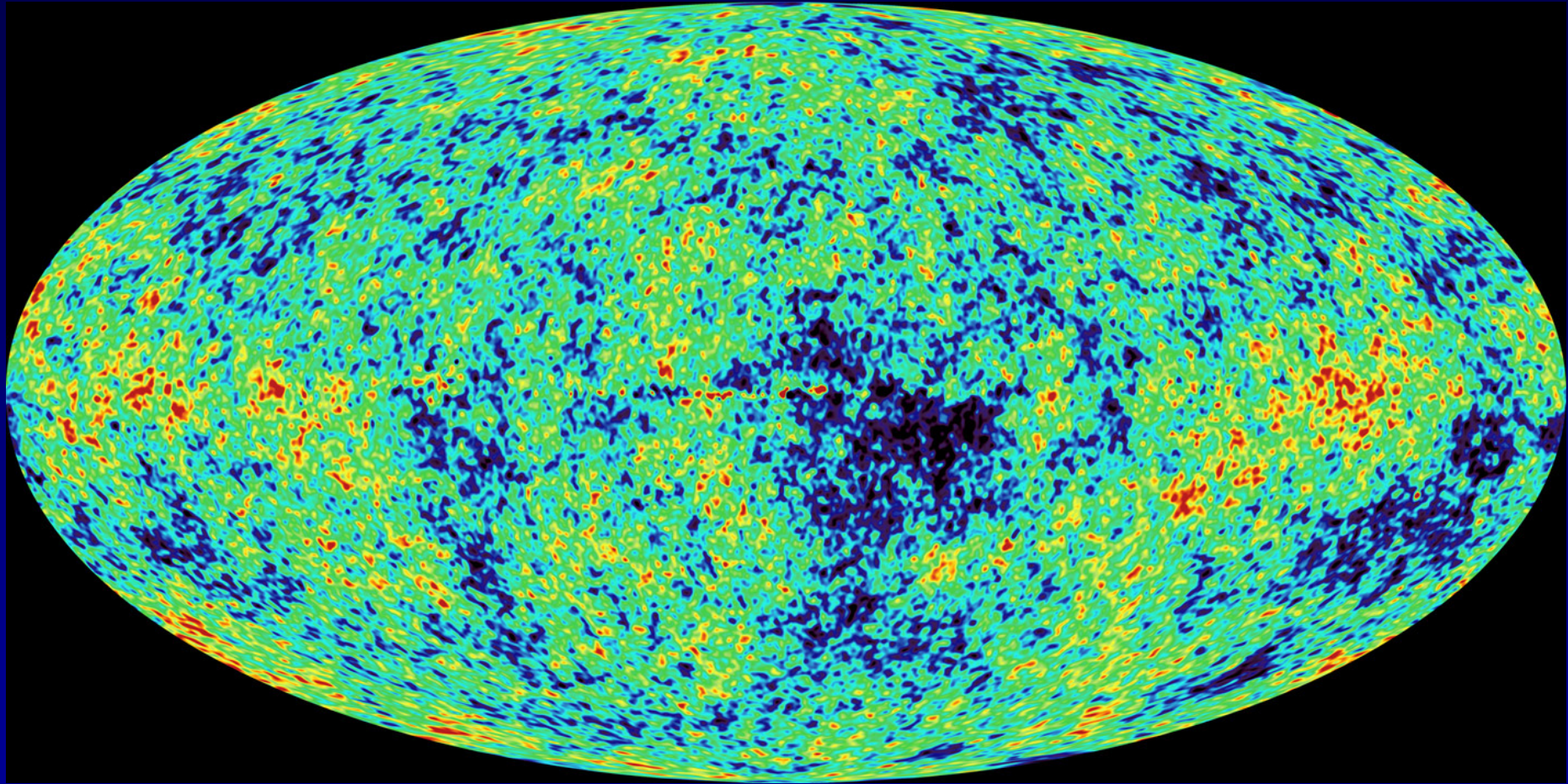
autocorrelation theorem

$$r(\tau) = \int_0^{\infty} P(\nu) \cdot e^{-i2\pi \nu \tau} d\nu$$

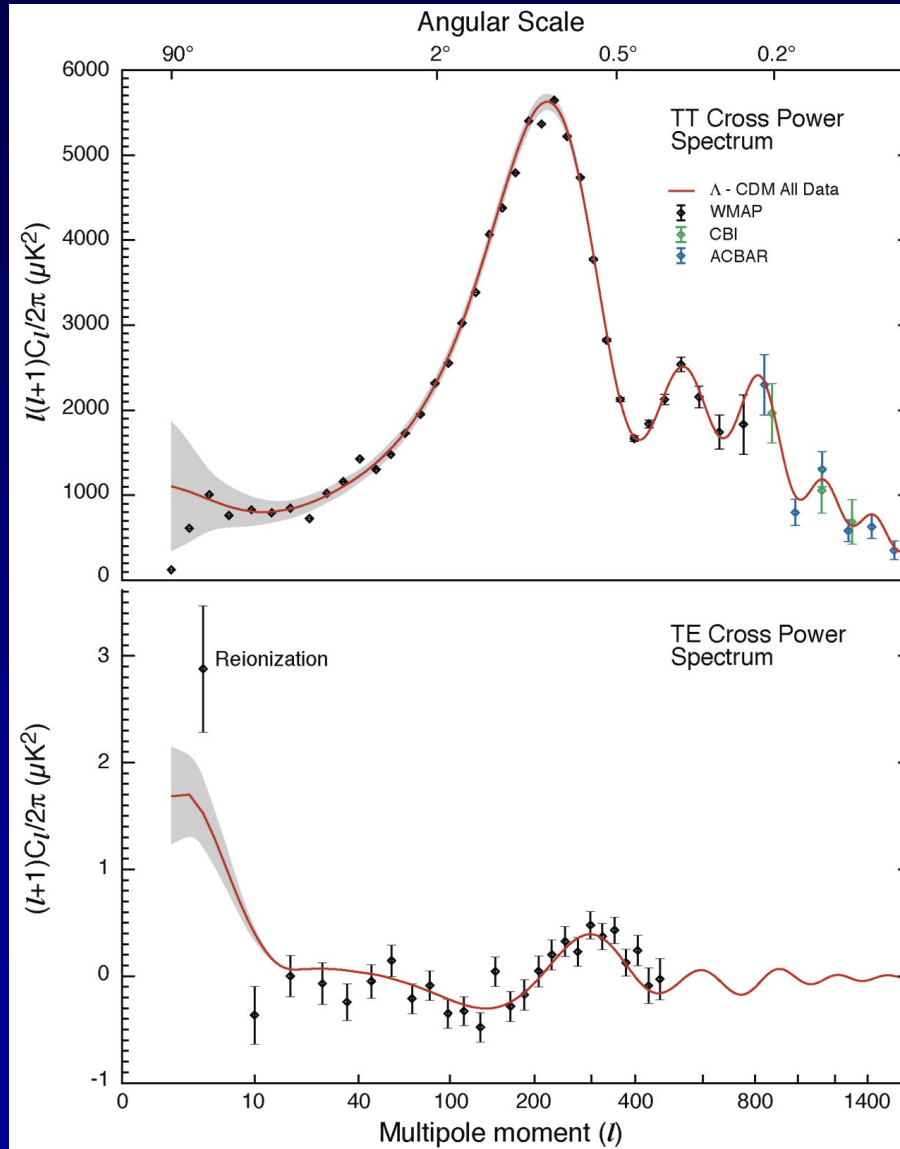
↑ because $Y_{lm} = \sqrt{\frac{2l+1}{4\pi}}$

ANS

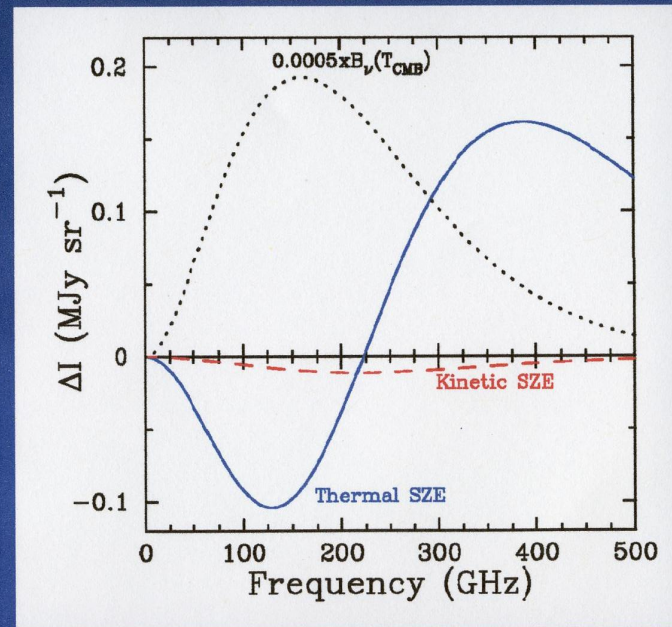
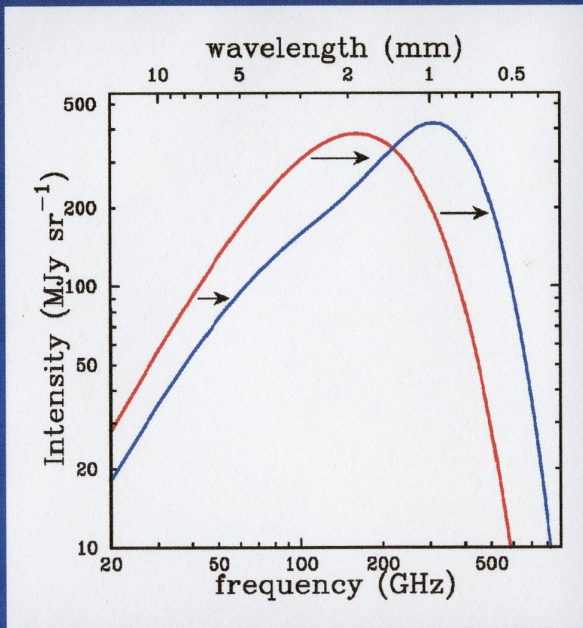
WMAP CMB anisotropy



WMAP power spectrum



Sunyaev-Zeldovich effect



Carlstrom, Holder & Reese, ARAA, 2002

