Universität Bonn Summer Term 2009

Physics of the ISM

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Exercises VII

Inclass Problems

1 Masses of molecular cloud

The mass of a spherical molecular cloud assumed in virial equilibrium is given by

$$M_{vir} = k_1' \cdot \frac{\sigma_v^2 \, R}{G} = k_1 \cdot \left(\frac{\Delta v}{\mathrm{kms}^{-1}} \right)^2 \left(\frac{R}{\mathrm{pc}} \right) \; , \tag{1}$$

where

$$k_1' = \frac{5 - 2n}{3 - n} \; , \tag{2}$$

accounts for a radial density distribution

$$\rho(r) \propto r^{-n} \; , \;\; n < 3 \tag{3}$$

(a) Derive the above expression (**??**), noting that the observed line width Δv differs from the (one-dimensional) velocity dispersion σ_{obs} by

$$\Delta v = \sqrt{8 \cdot \ln 2} \cdot \sigma_{obs} \; , \tag{4}$$

and that the full (three-dimensional) velocity dispersion is obviously related to the one-dimensional one by

$$\sigma_{obs}^2 = \frac{1}{3} \cdot \sigma_v^2 \; . \tag{5}$$

(b) The CO luminosity in the radio-astronomical jargon is

$$L_{CO} = D^2 \cdot \int_{cloud} \int_{-\infty}^{+\infty} T_b \; dv \; d\Omega \quad \mathrm{K \; km \; s^{-1} \; pc^2}, \tag{6}$$

Show that the ratio X_{CO} of the H_2 column density to the observed velocity-integrated CO intensity

$$W_{CO} = \int_{cloud} T_b \; dv \quad \mathrm{K \; km \; s^{-1}} \tag{7}$$

is equivalent to the ratio of virial mass to CO luminosity (as defined above) of a cloud, i.e.

$$X_{CO} = \frac{N_{\mathrm{H}_2}}{W_{CO}} = \frac{M_{vir}}{L_{CO}} \quad \mathrm{cm^{-2} \; (K \; km \; s^{-1})^{-1}} \tag{8}$$

if the cloud is virialised and if the brightness temperature is constant across the cloud.

<u>Homework</u>

2 Line frequencies

The $^{12}C^{16}O$ molecule has a rotational constant $B_e = 57.6360$ GHz and a stretching constant of $D_e = 0.185$ MHz. The energy levels with rotational quantum number J are given by

$$E(J) = h \cdot \left\{ B_e J(J+1) - D_e [J(J+1)]^2 \right\} \ , \tag{9}$$

Calculate the energies in K for the rotational quantum numbers $J = 1, 2, 3, 4$, and the frequencies (in GHz) for the transitions J = 1 → 0, J = 2 → 1, J = 3 → 1, and J = 4 → 3. These values are precisely known and given in the table below. Compare your results with these "literature values". Are there differences and, if so, think about a reason.

J	$E(J)$ [K]	$\nu(J \to J-1)$ [GHz]
1	5.532	115.271203
2	16.706	230.538001
3	33.191	345.796000
4	55.317	461.040768