Stellar-mass black holes in star clusters II

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Contents

- Astrophysical black holes
- Runaway mass segregation (a qualitative discussion)
- Stellar mass black holes in star clusters

Gravitational waves from star clusters
Gravitational waves & detection rates from N-body computations
Effect on cluster's structure & evolution

What happens to these BHs ?

- Compact remnants (NS/BH) can receive birth / *"natal" velocity kick* due to asymmetry in supernova ejecta which carries net momentum.

- Amount of kick for BH uncertain (in theory & observation).

- Can be observationally inferred from "back-tracing" orbital motion of Galactic BH X-ray binaries [e.g. ,Willems et al., 2005, ApJ, 625, 324, Repetto et al.] --- indicate *very low to high natal kicks*.

- Computations of core-collapse supernova also support a wide range of natal kicks (Janka et al.).

- "Electron Capture" mechanism necessarily produces remnants with small kick velocities.

What happens to these BHs ?

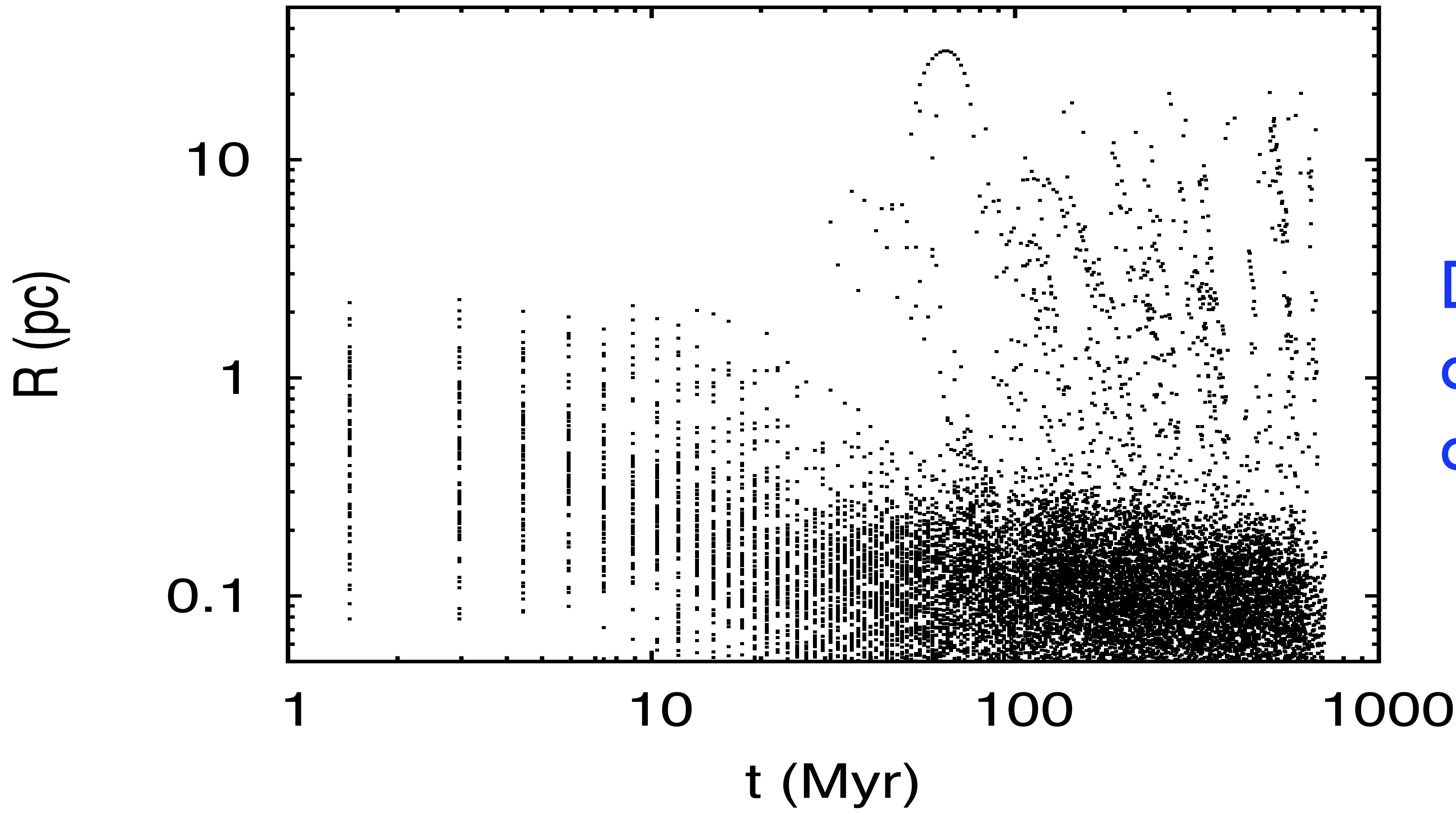
- If retained in significant number (>10%), the BHs never attain complete equipartition.
- Continual / runaway sinking towards cluster center.
- "Mass stratification" or "Spitzer" instability (otherwise dynamical friction)— see Lyman Spitzer's book
- The Spitzer mass-stratification stability criterion:

$$\chi < \chi_{max} = 0.16$$

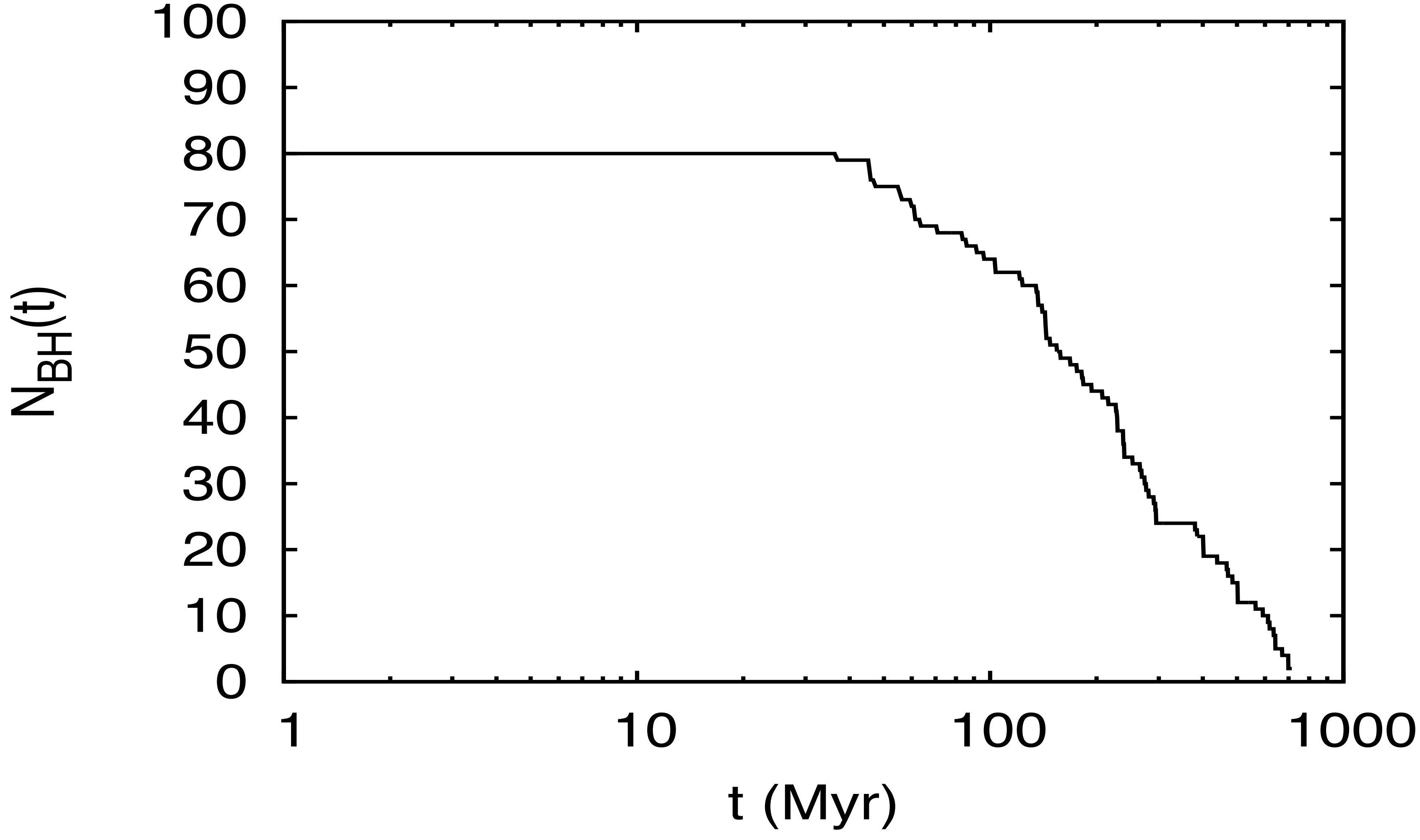
$$\chi = \frac{M_2}{M_1}\left(\frac{m_2}{m_1}\right)^{\frac{3}{2}}$$

$m_1 =$ mass of background component
$m_2 =$ mass of segregated component
$M_X =$ total mass of componet X

- Highly dense, dynamically isolated sub-cluster purely of BHs forms in cluster center.



Direct N-body computation of $N = 4.5 \times 10^4$ cluster, $r_h(0) = 1$ pc, $N_{BH} \approx 100$(full retention).



Two phases: (a) initial segregation: $N_{BH} \approx$ const (b) formation of BH-core: N_{BH} depletes due to super-elastic dynamical encounters.

BH-core (or "Dark core") phase have potential for a wide variety of physical phenomena

BH-core phenomena

- BH-BH inspiral via GW emission.
  dynamical formation of tight BH-BH binary, inspiral within or outside cluster

- Heating and expansion of cluster core: delay of core collapse.
  due to K.E. energy deposition of ejected BHs in core

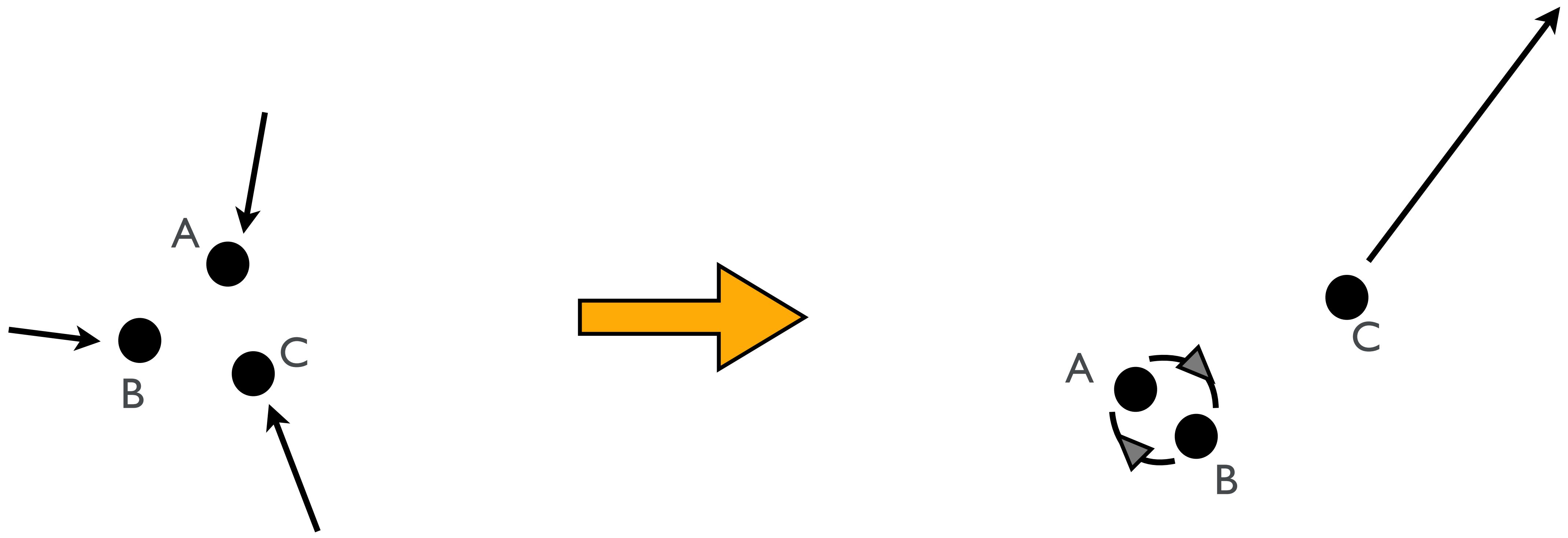
- Formation of BH X-ray binaries.
  due to dynamical encounters of BHs with normal stars

- Formation of "dark star clusters".
  due to rapid removal of stars by galactic tidal field close to galactic center

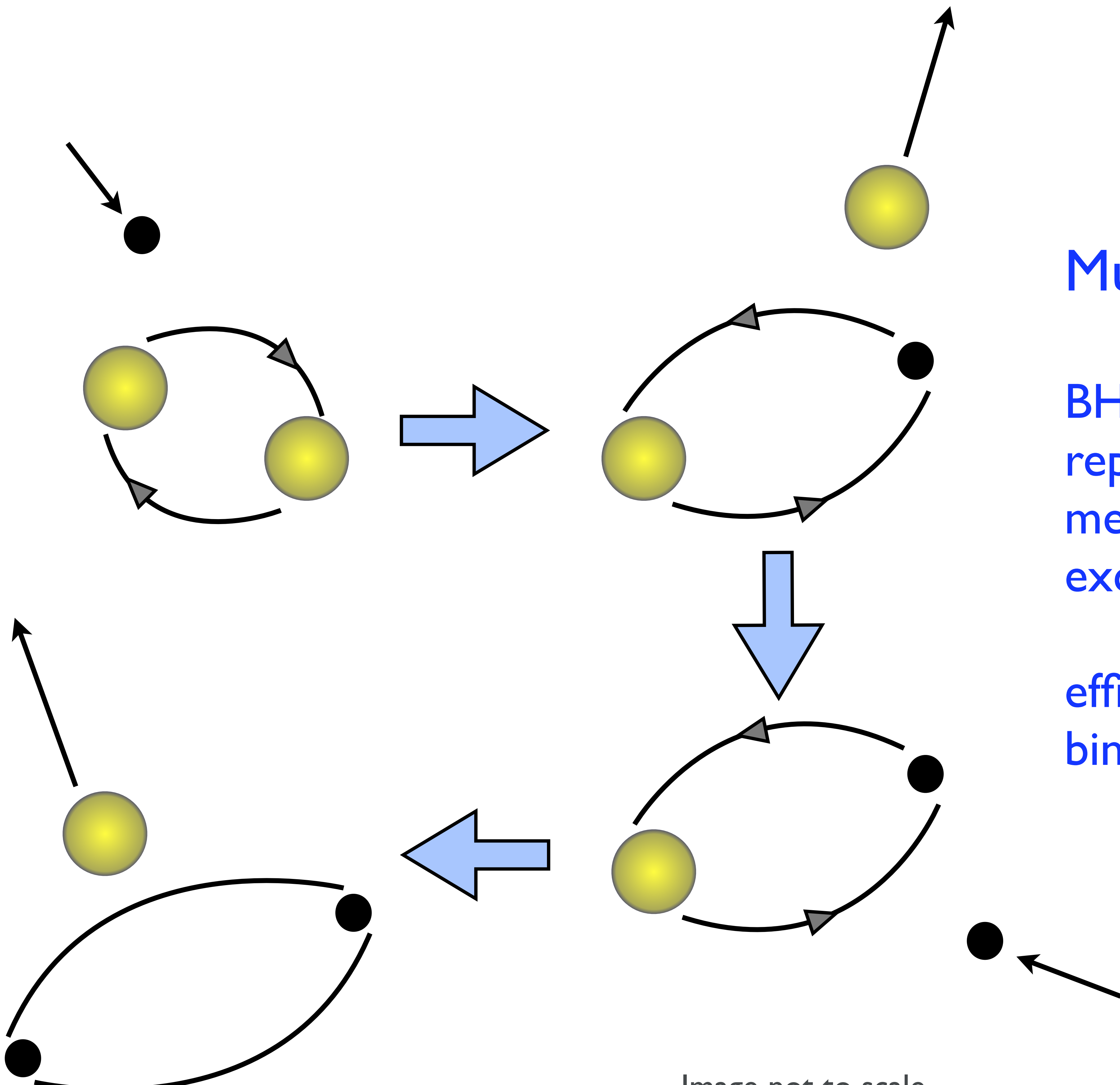
Dynamical formation of BH-BH binaries

3-body binary formation in dense BH-core:

in close encounter among 3 BHs, two of them get bound while third escape with the excess K.E.



BH-BH binaries from primordial binaries



Multiple exchange:

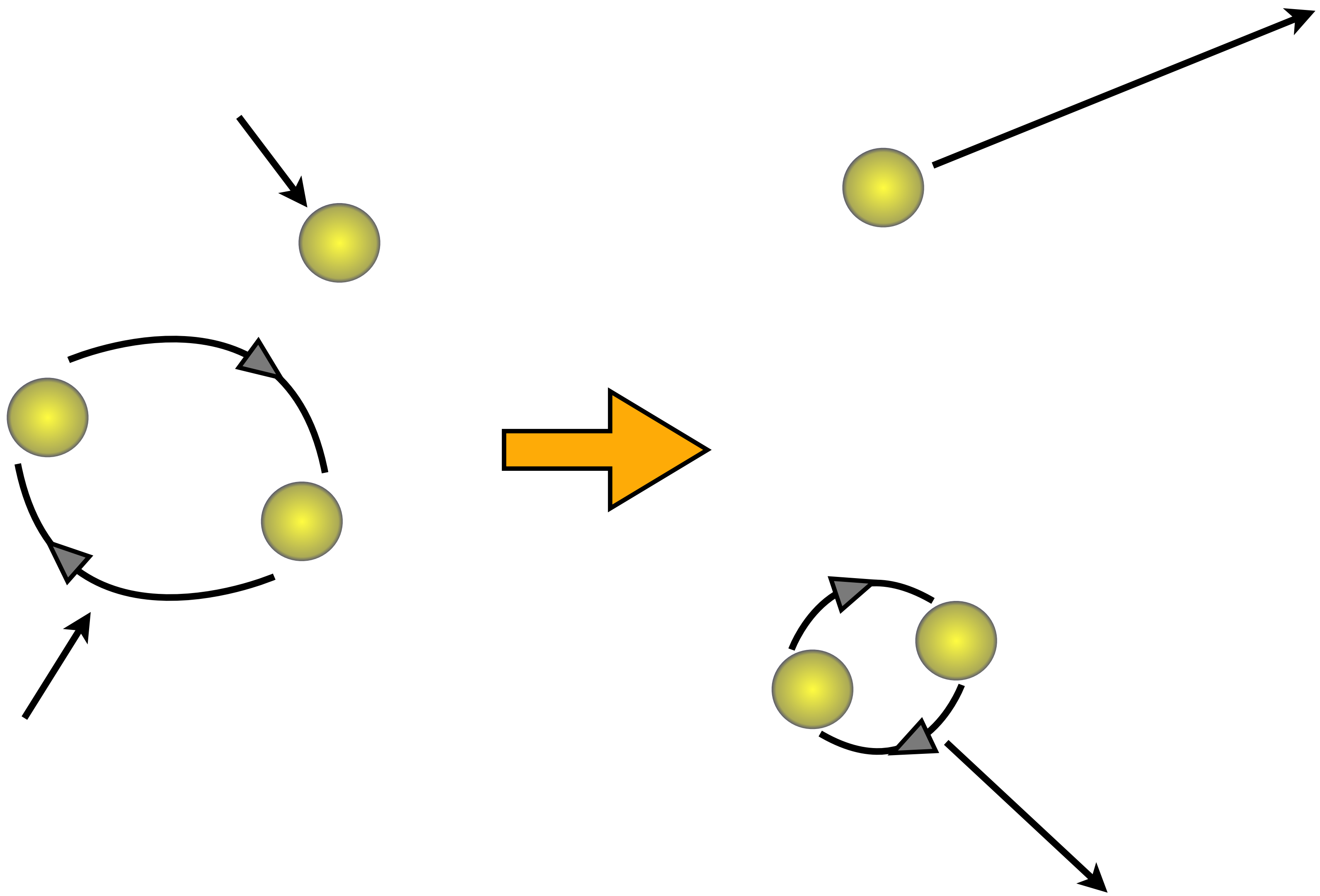
BHs being more massive replace stellar binary members in successive exchange encounters;

efficient with primordial binaries

Image not to scale

Encounter/collisional hardening

Dynamically formed BH-BH binaries are "hard":
total binding energy greater than mean stellar K.E.



Heggie's Law: "hard binary hardens" --- encounter/collisional hardening
(hardening => increase of binding energy, i.e., shrinking of semi-major-axis)

Both intruder star & binary get recoiled with larger total K.E.

N.B. : soft binary softens, hence easily dissociated

Consequence of "negative specific heat" of a single binary

Image not to scale

Encounter/collisional hardening

Statistical effect over many encounters: theoretically predicted & verified through numerical experiments. [Heggie, D.C, 1975, MNRAS, 173, 729]

Encounters with hard binaries "super-elastic":
hard binaries supply K.E. to encountering stellar environment as they shrink --- *energy source*.

"Binary burning": has profound consequences on cluster's dynamical evolution; halt's core collapse.

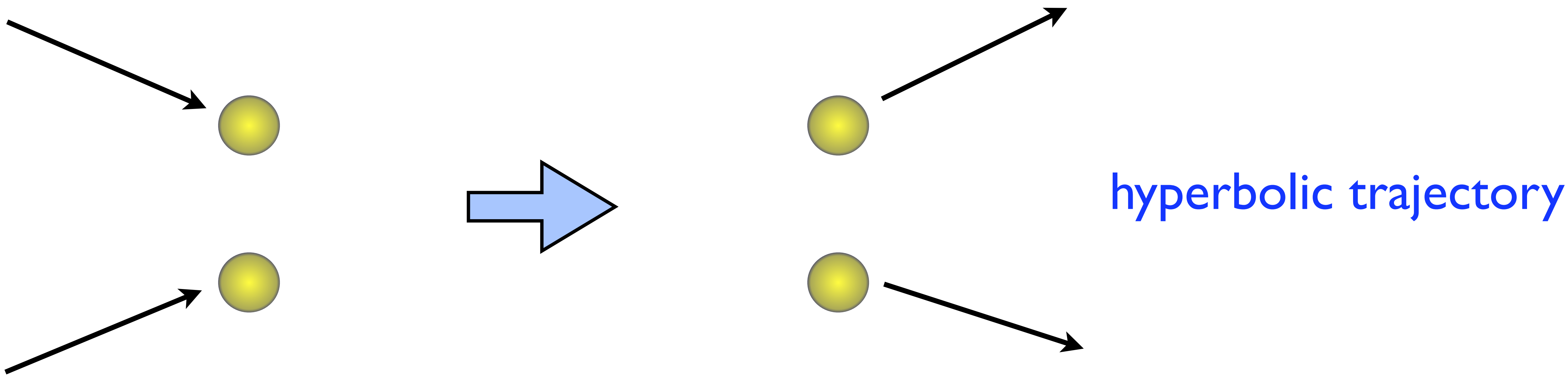
Hardening rate $\dot{a} \sim a^2$ (roughly), rate decreases as binary shrinks: too close binaries ($P \lesssim 10^3$ days typically for globular clusters) behave essentially as single stars.

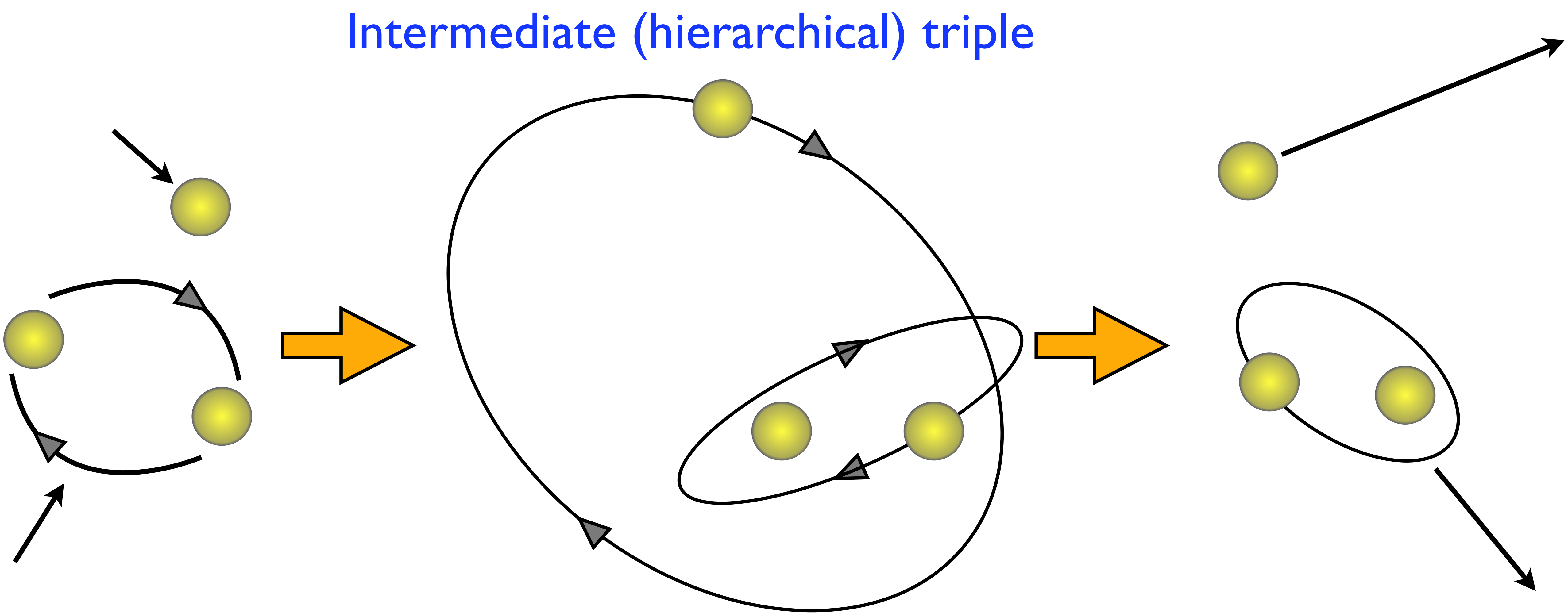
Rate of collisional hardening for distant ($d >> a$) encounters :

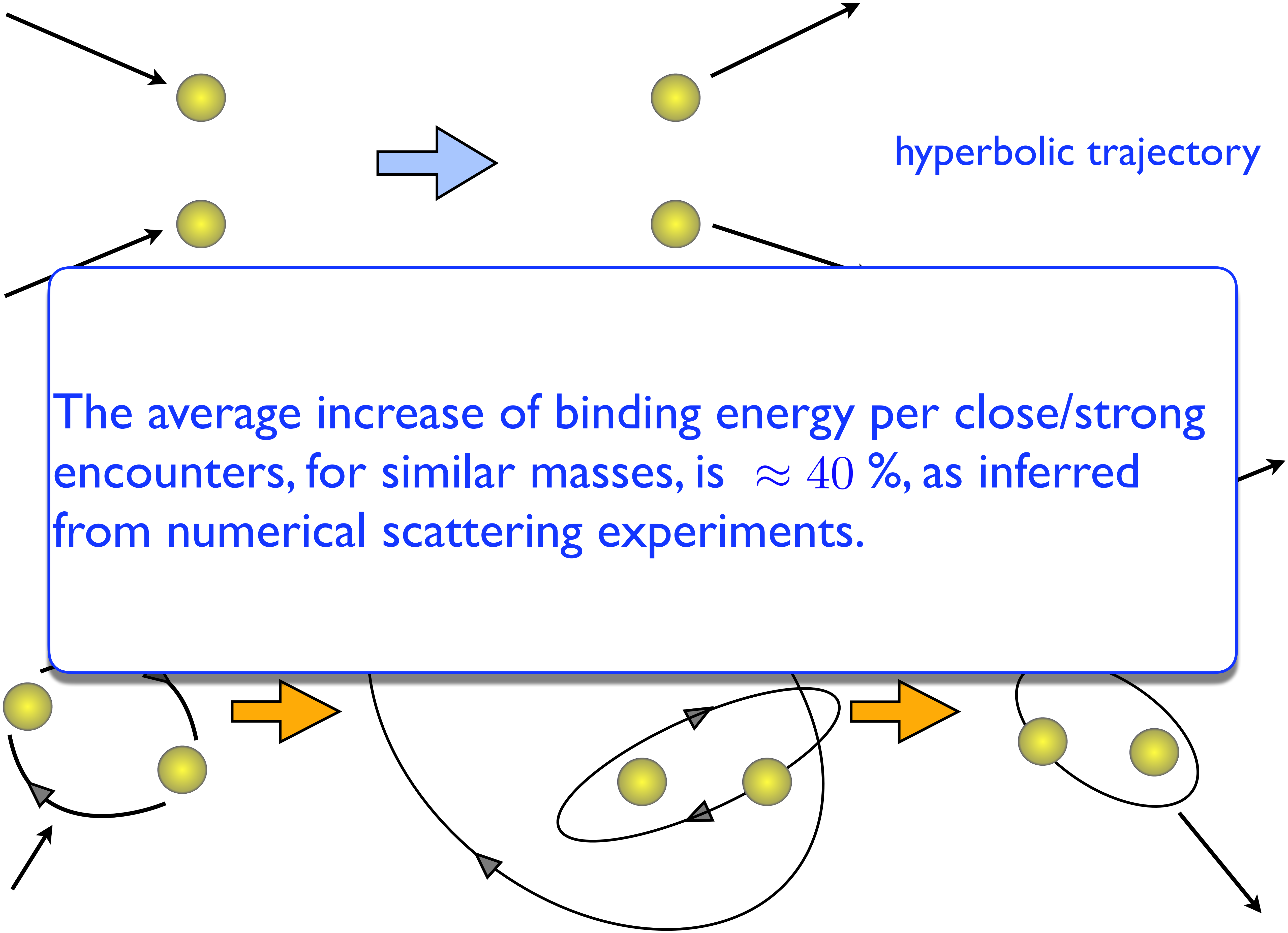
$$\dot{a}_{\rm coll} \approx -2.36 \times 10^{-7} \left(\frac{\langle m \rangle^3}{m_1 m_2} \right) \left(\frac{\rho}{v} \right) a^2 \;\; R_\odot \;\, {\rm Gyr}^{-1}$$

Shull (1979), Heggie & Hut (2003), Banerjee & Ghosh (2006)

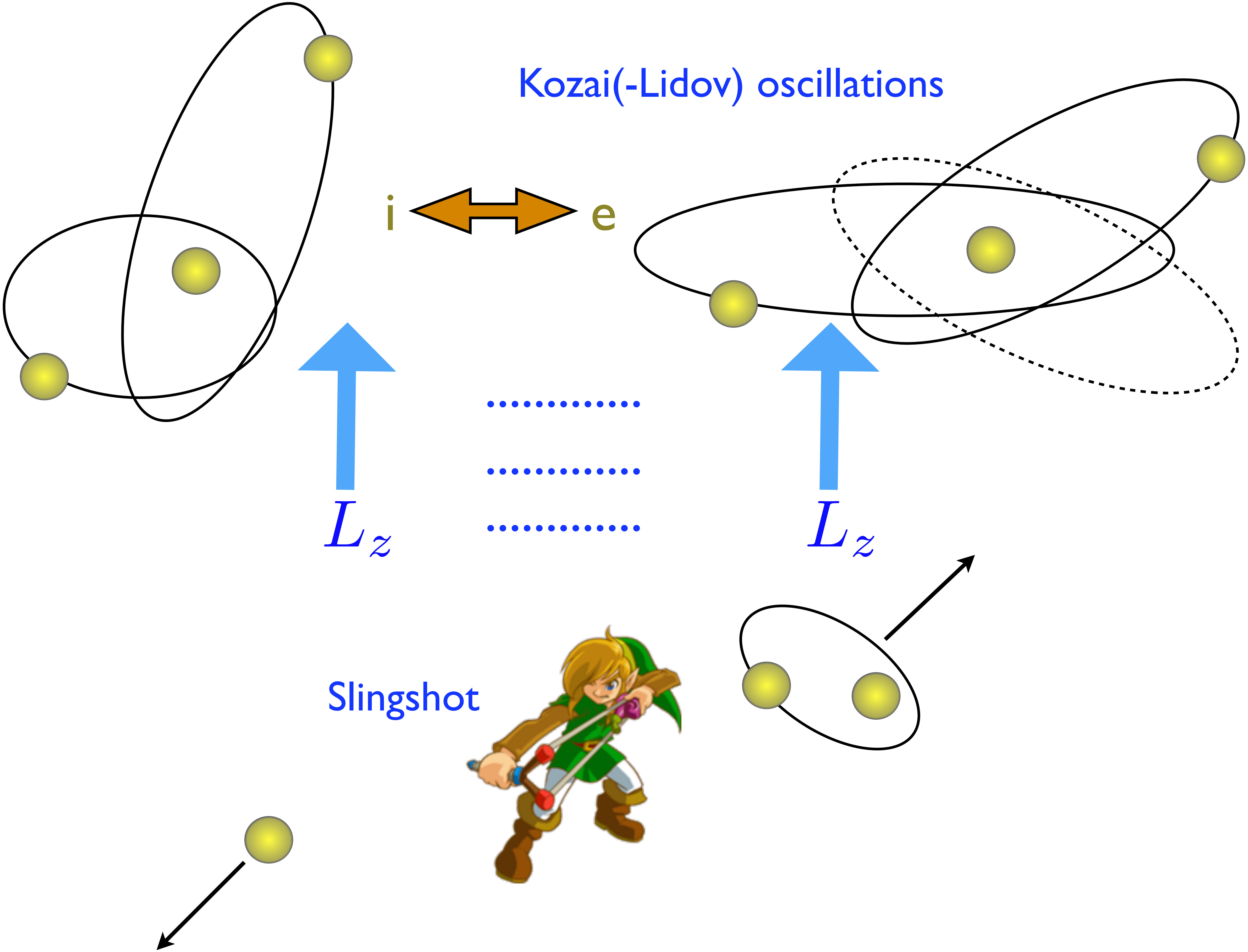
Much stronger hardening is possible through close ($d \sim a$) encounters!


hyperbolic trajectory


Intermediate (hierarchical) triple



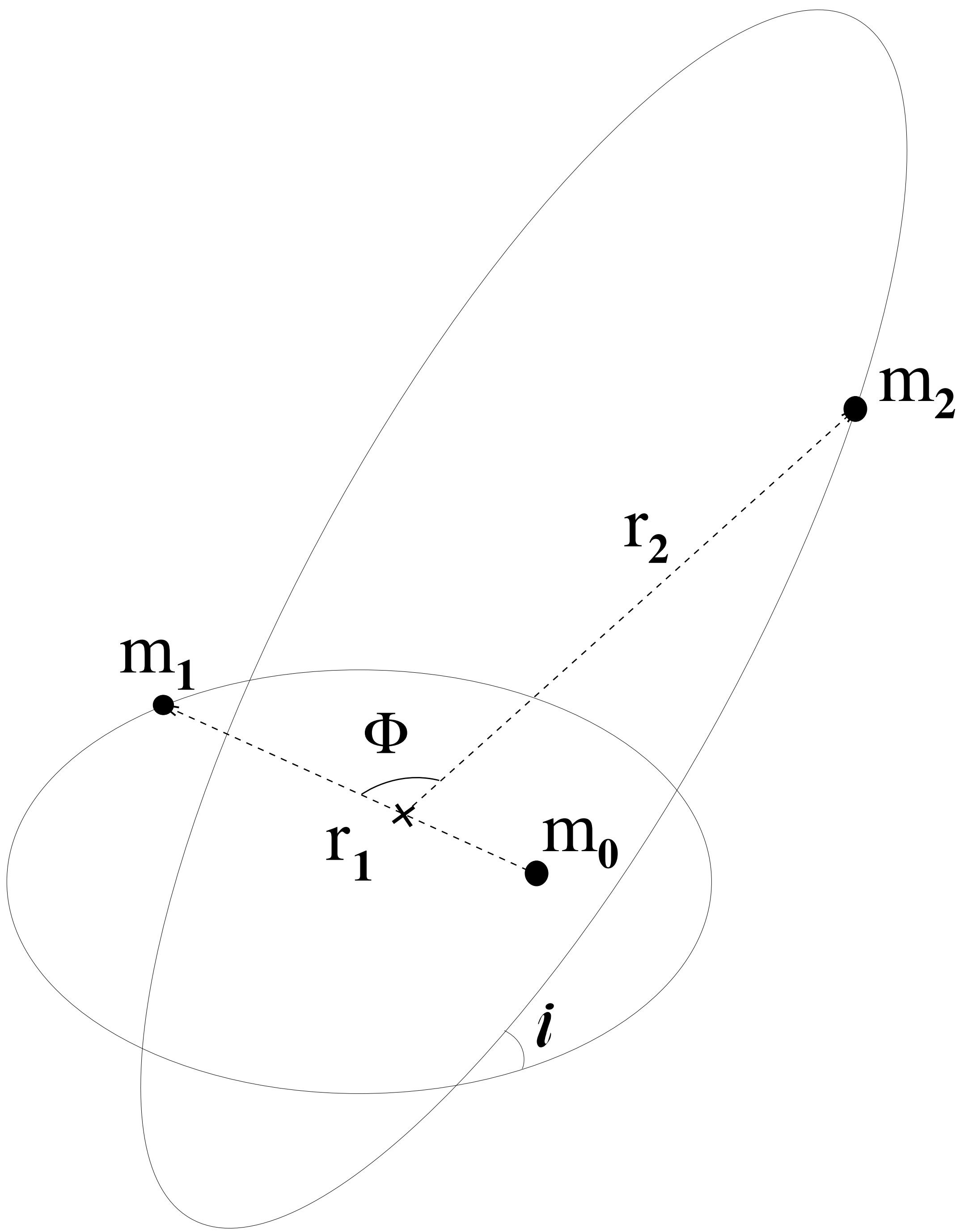
The average increase of binding energy per close/strong encounters, for similar masses, is ≈ 40 %, as inferred from numerical scattering experiments.



Kozai(-Lidov) oscillations
i
e
L_z
L_z
Slingshot

$$P_e \approx 2\pi \sqrt{\frac{a_1^3}{G(m_0+m_1)}} \left(\frac{m_0+m_1}{m_2}\right) \left(\frac{a_2}{a_1}\right)^3 (1-e_2^2)^{3/2}$$

[1 => inner orbit, 2 => outer orbit]



$$e_{\max} \approx \sqrt{1-(5/3)\cos^2 i_0} \quad (e_{1_0}=0)$$

E.g.,

$a_1 = 1\ \mathrm{AU}, a_2 = 100\ \mathrm{AU}, e_2 = 0$

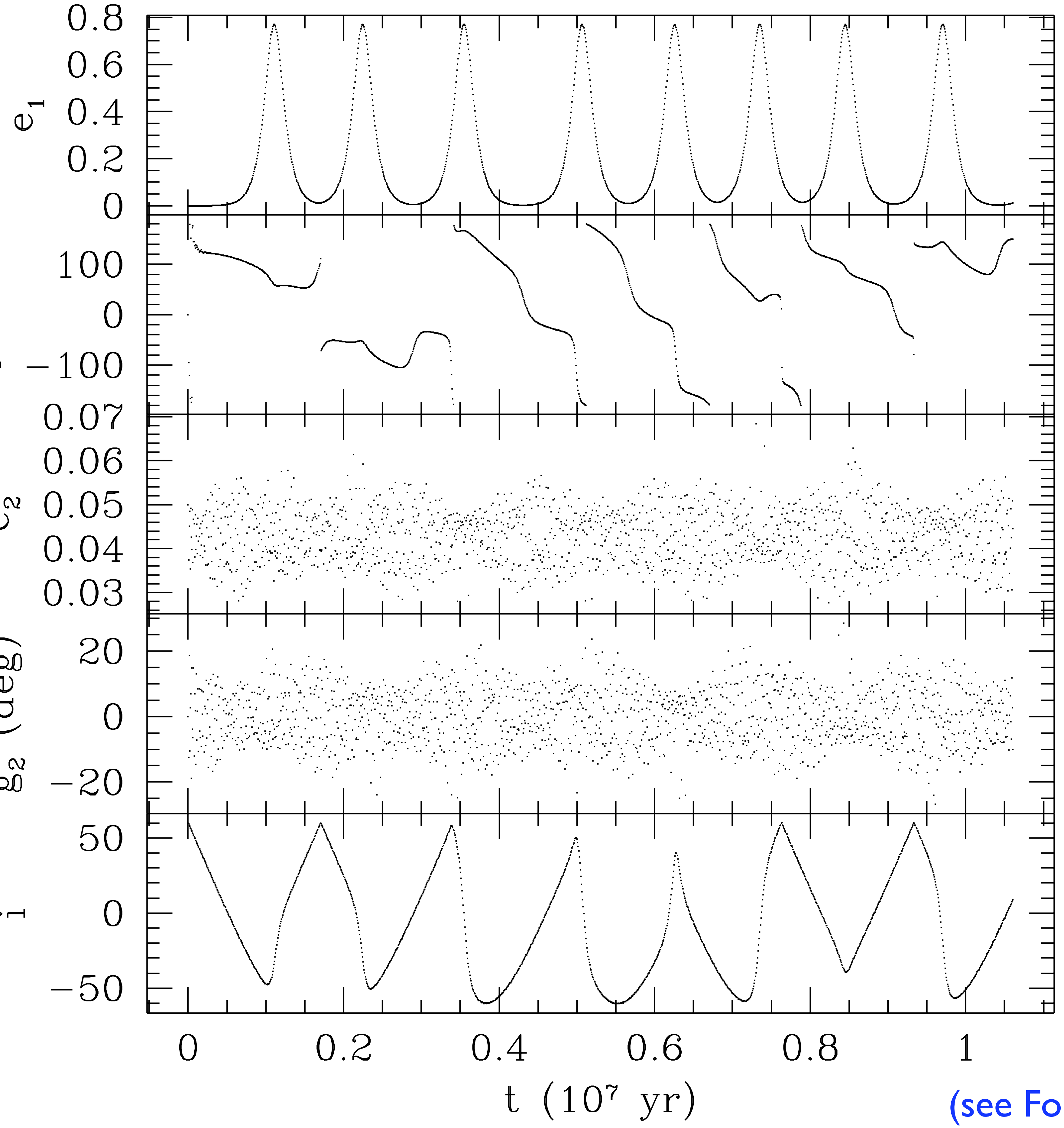
$m_0 = m_1 = m_2 = 10 M_\odot$

$\Rightarrow P_e \approx 0.4\ \mathrm{Myr}$

(longer than dynamical encounter time in a dense cluster)

Kozai oscillation formulae

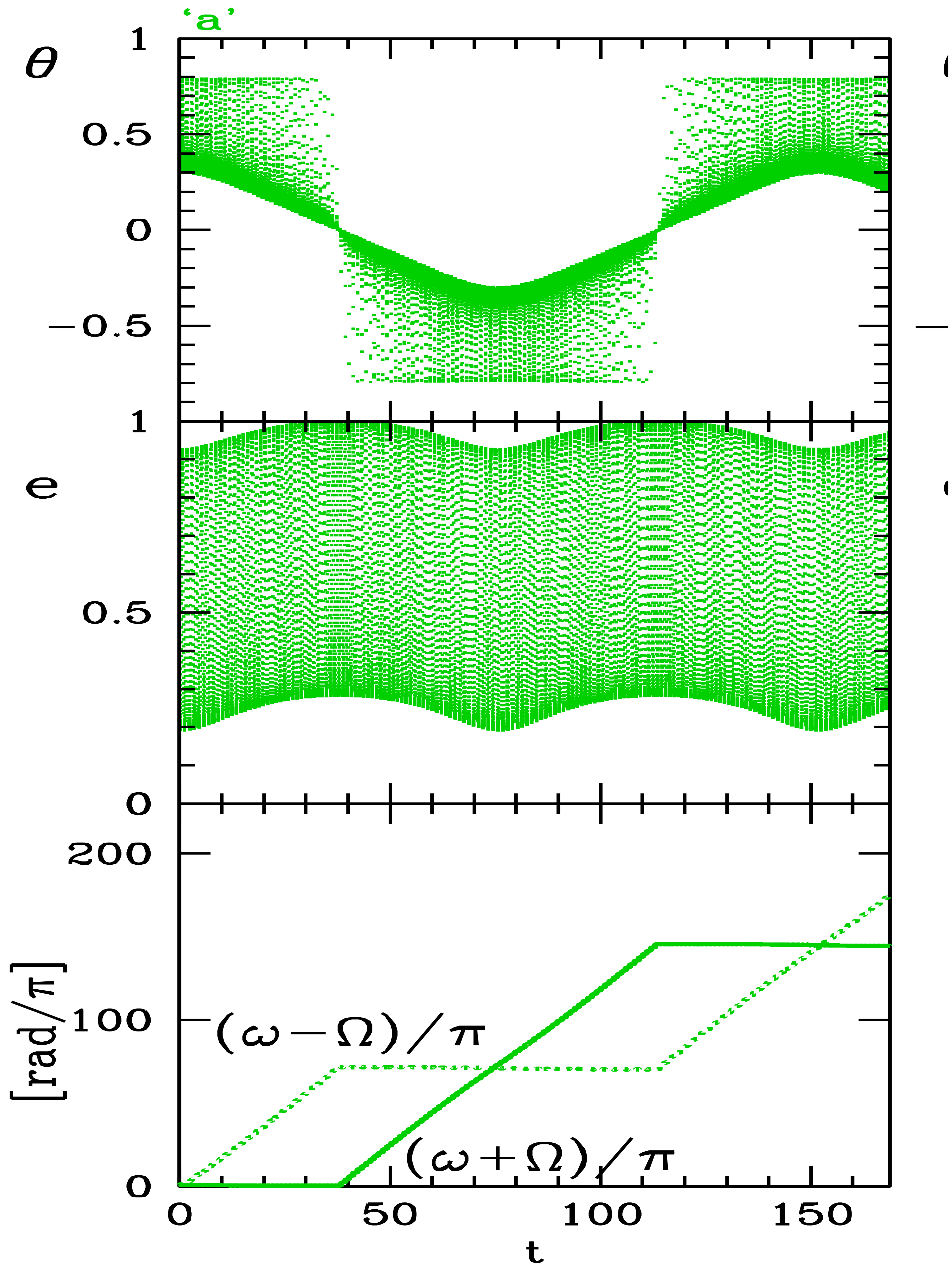
(see Ford et al. 2000, ApJ, 535, 385)



Standard
Kozai(-Lidov) Mechanism
(SKM)

$(e_{1_0} = 0)$

(see Ford et al. 2000, ApJ, 535, 385)



Eccentric
Kozai(-Lidov) Mechanism
(EKM)

$(e_{1_0} > 0)$

Lithwick & Naoz 2011, ApJ, 742, 94

Kozai osc. of BH triple / inner BH binary

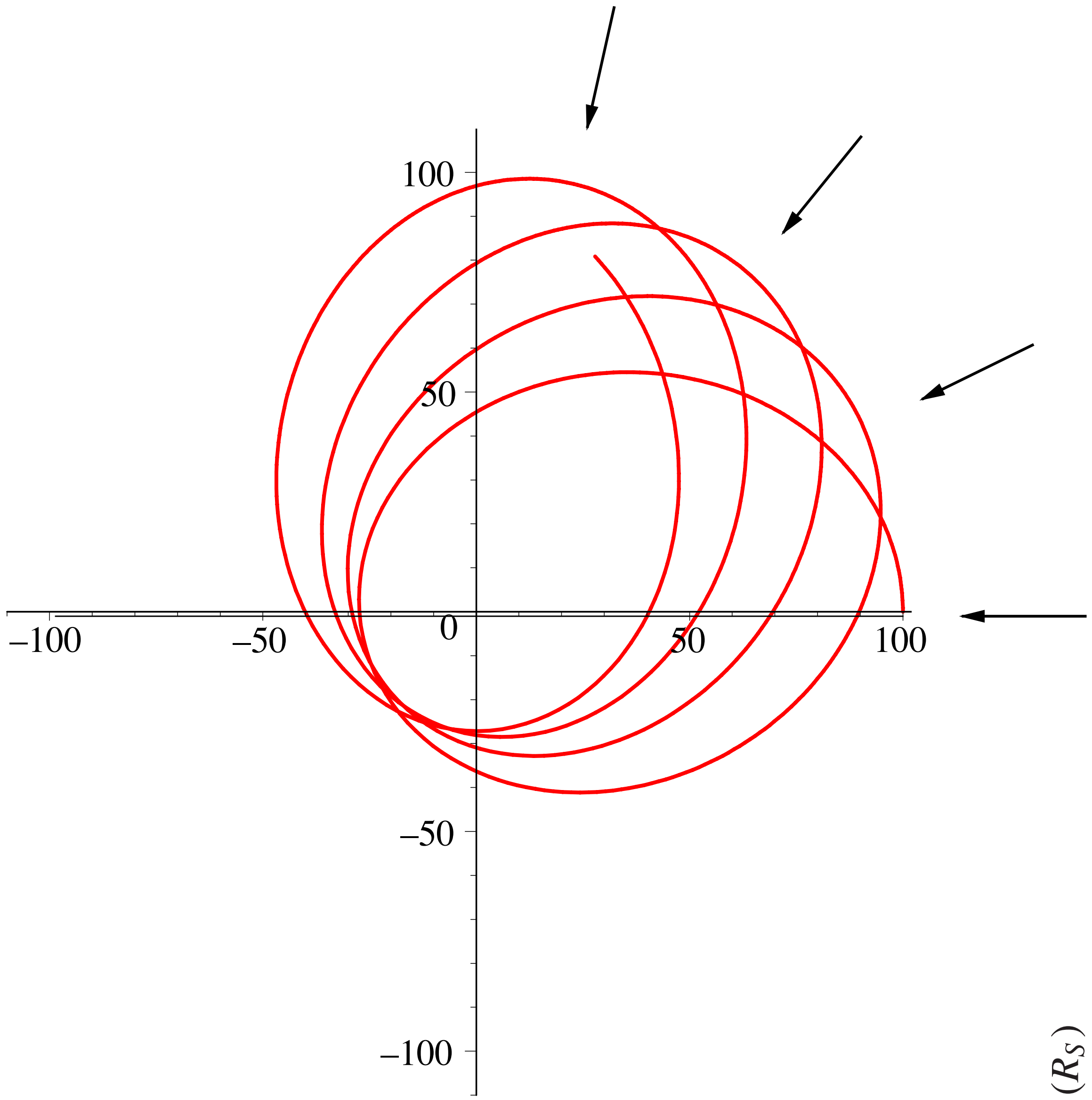
High inner eccentricity => gravitational-wave radiation

Recall Peters' formula of orbit-shrinkage via GW

$$\left\langle \frac{da}{dt} \right\rangle = -\frac{64G^3}{5c^5} m_1 m_2 (m_1 + m_2) a^{-3} (1 - e^2)^{-\frac{7}{2}}$$

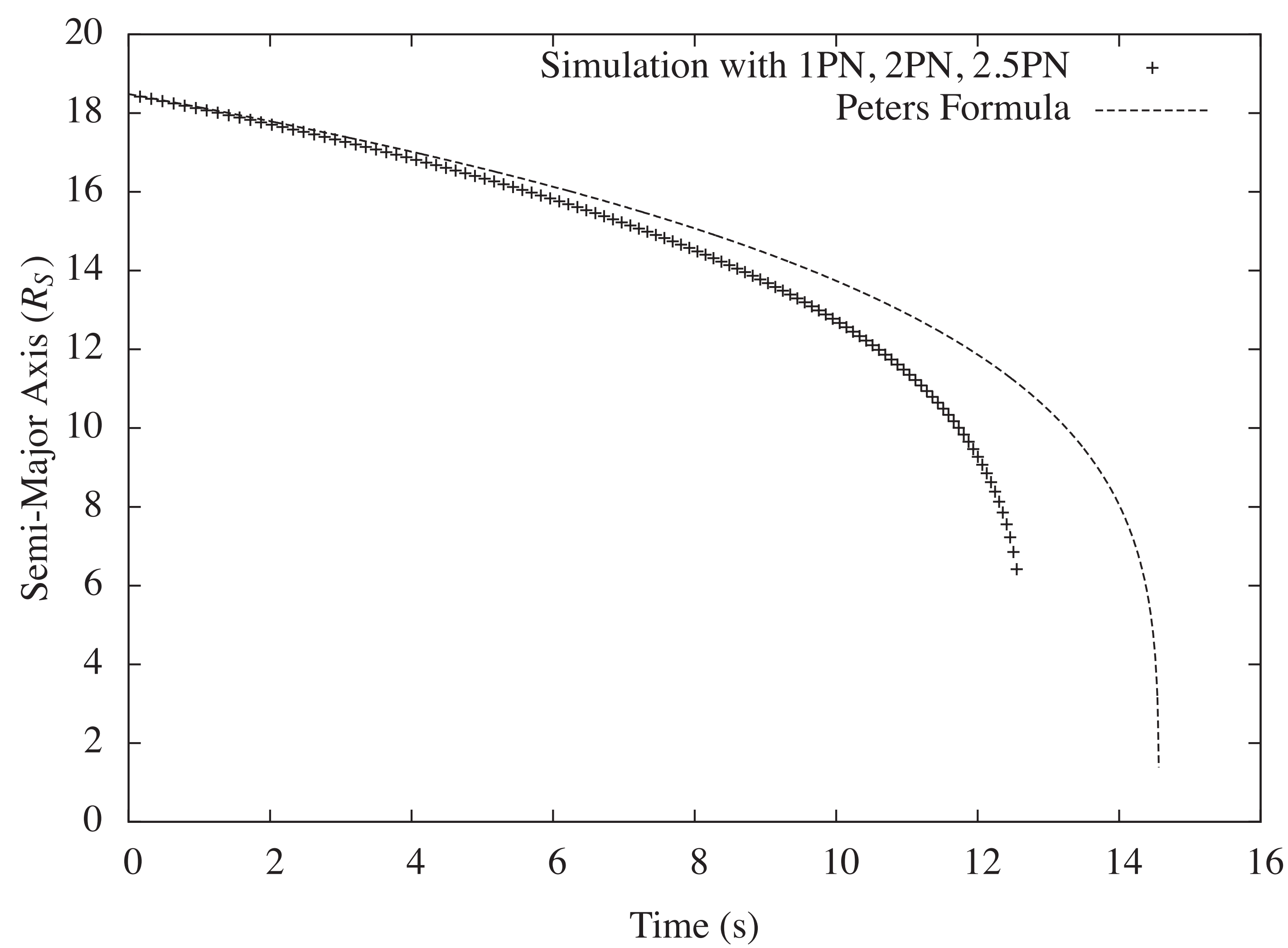
$$\left\langle \frac{de}{dt} \right\rangle = -\frac{304G^3}{15c^5} m_1 m_2 (m_1 + m_2) a^{-4} e (1 - e^2)^{-\frac{5}{2}}$$

Inner orbit in a BH-BH-BH becomes relativistic via Kozai mechanism

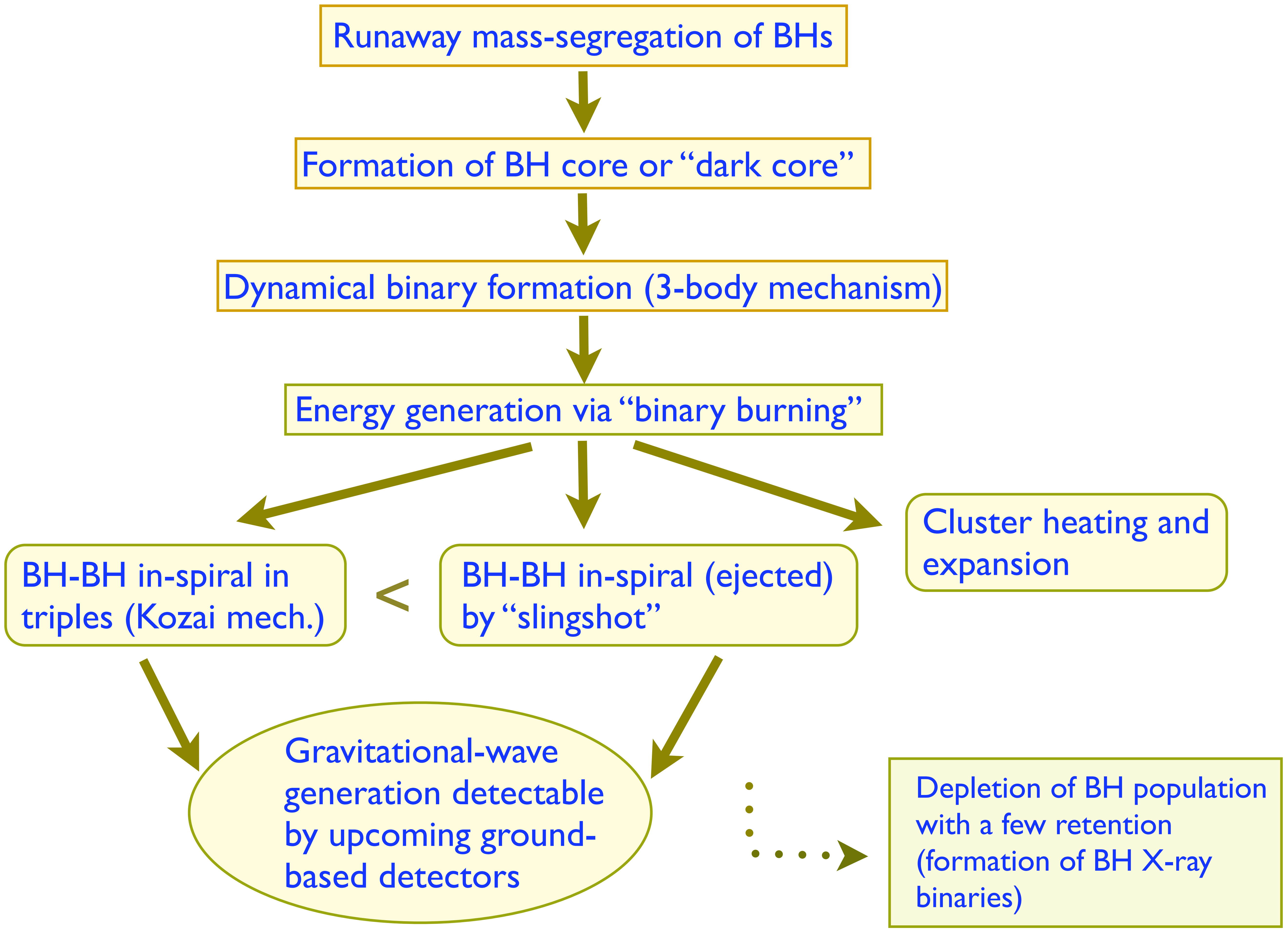


GR precession partially "detunes" Kozai cycle and delays onset of GW in-spiral for inner binary

In-spiral still happens as seen in numerical studies



In-spiral process itself becomes quicker

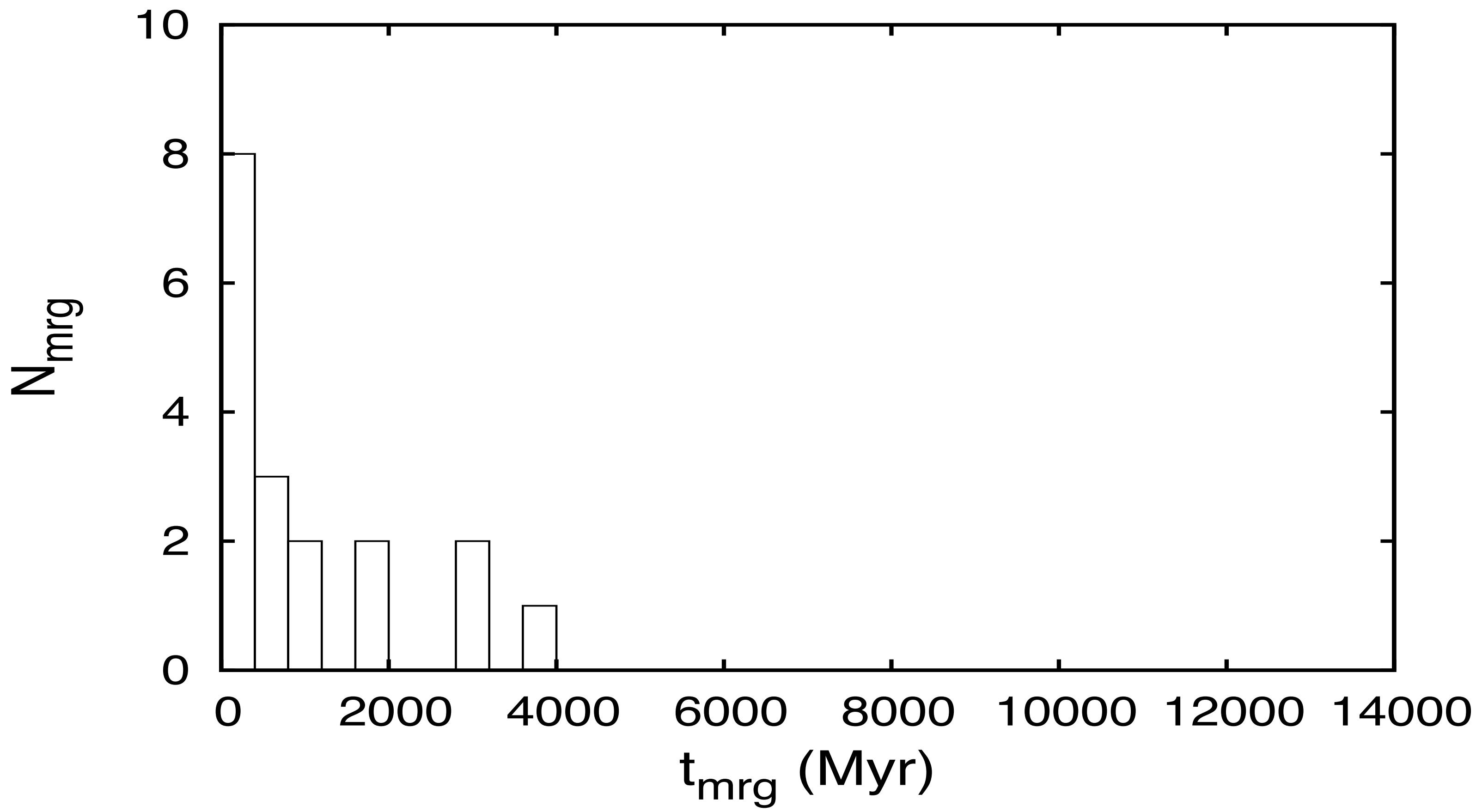


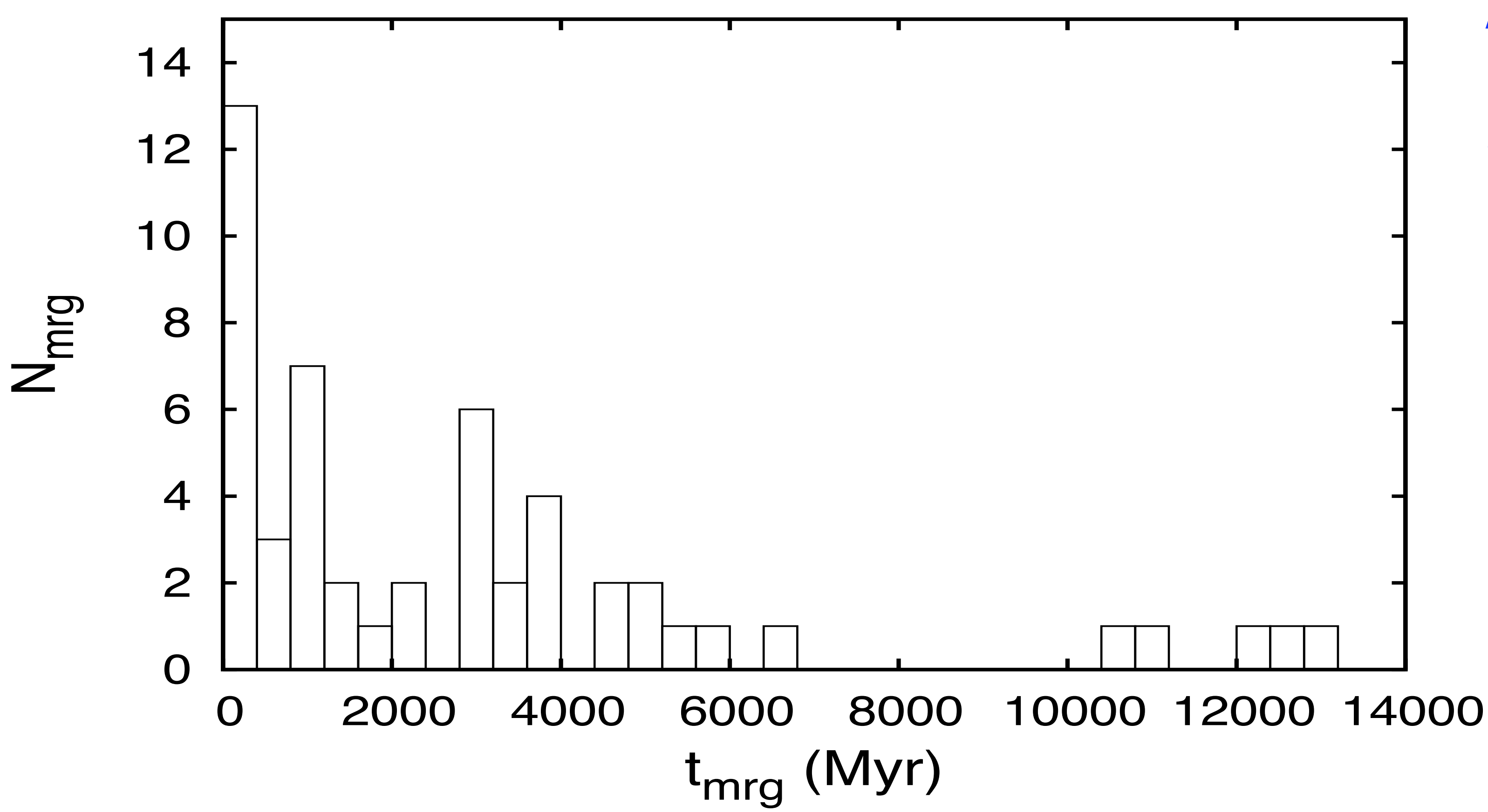
Runaway mass-segregation of BHs
Formation of BH core or "dark core"
Dynamical binary formation (3-body mechanism)
Energy generation via "binary burning"
BH-BH in-spiral in triples (Kozai mech.)
<
BH-BH in-spiral (ejected) by "slingshot"
Cluster heating and expansion
Gravitational-wave generation detectable by upcoming ground-based detectors
Depletion of BH population with a few retention (formation of BH X-ray binaries)

BH-BH mergers in computed clusters (Banerjee et al. 2010, MNRAS, 402, 371)

Model name	N	N_{sim}	$r_h(0)$ or R_s (pc)	$N_{BH}(0)$	N_{mrg}	t_{mrg} (Myr)	N_{esc}	$\mathcal{R}_{\mathrm{AdLIGO}}$
				Isolated clusters				
C5K12	5000	10	1.0	12	0	— —	— —	— —
C10K20	10000	10	1.0	20	0	— —	— —	— —
C25K50	25000	10	1.0	50	0	— —	3 1 1	— —
C50K80	45000	1	1.0	80	1	698.3	3 1 0	28(±14)
C50K80.1	45000	1	0.5	80	2	217.1, 236.6	3 2 1	35(±15)
C50K40.1	45000	1	0.5	40	0	— —	1 1 1	7(±7)
C50K200	50000	1	1.0	200	2	100.8, 467.8	0 0 0	14(±10)
C65K110	65000	1	1.0	110	1	314.6	4 2 1	35(±15)
C65K110.1	65000	1	0.5	110	0	— —	4 3 1	28(±14)
C65K55.1	65000	1	0.5	55	1	160.5	1 0 0	14(±10)
C100K80	100000	1	1.0	80	2	219.4, 603.2	5 2 1	42(±15)
C100K200	100000	1	1.0	200	0	— —	5 4 4	28(±14)
				Reflective boundary				
R3K180	3000	1	0.4	180	1	1723.9	5 3 1	35(±15)
R4K180A	4000	1	0.4	180	1	3008.8	2 2 1	21(±12)
R4K180B	4000	1	0.4	180	2	100.2, 1966.5	2 1 0	28(±14)
R3K100	3000	1	0.4	100	2	3052.8, 3645.9	1 1 0	18(±10)
R4K100A	4000	1	0.4	100	2	104.4, 814.2	3 3 1	28(±14)
R4K100B	4000	1	0.4	100	1	1135.3	3 3 3	28(±14)

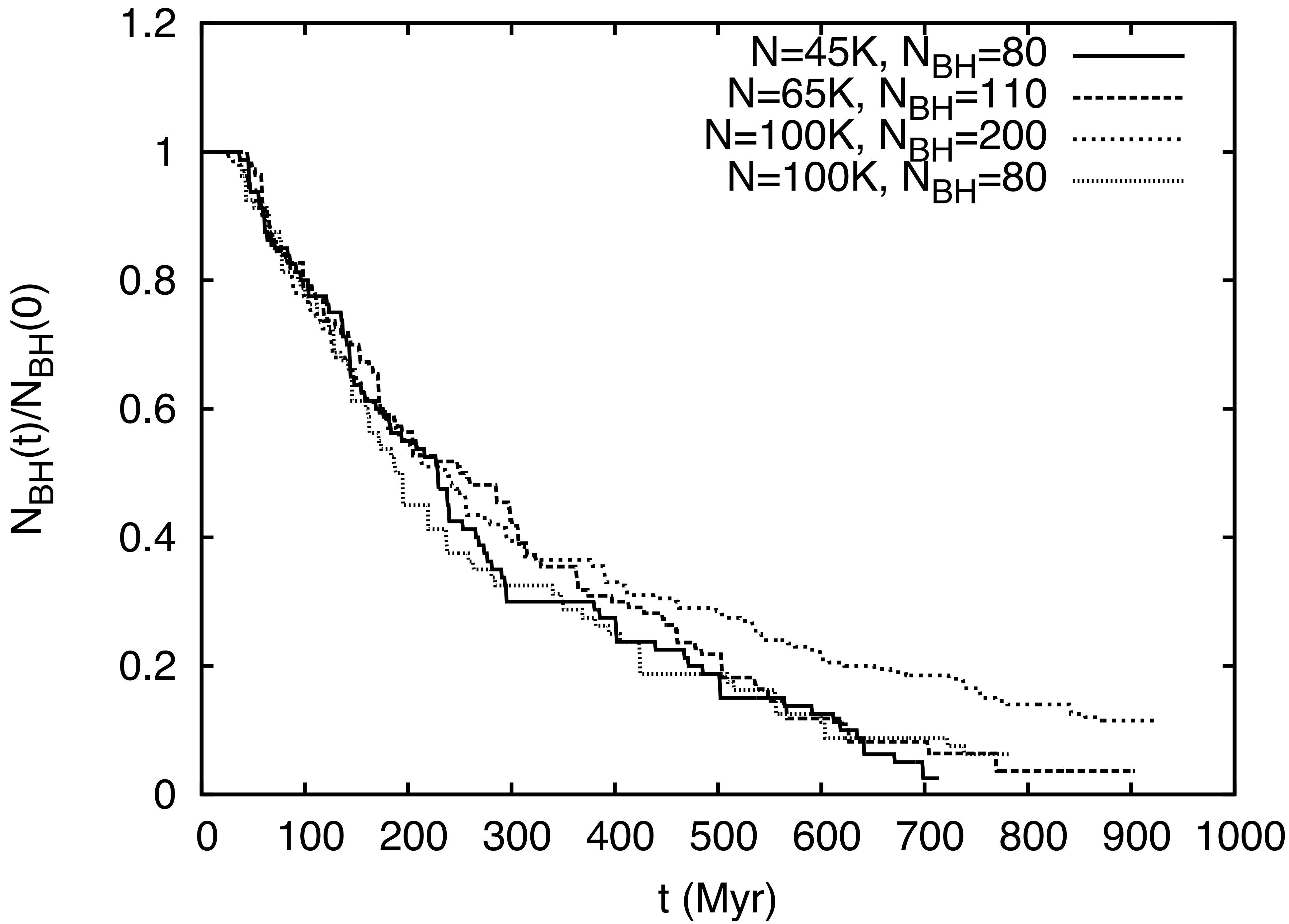
Merger-time distribution





Most mergers happen within first few Gyr.

BH-cluster depletion



Depletion timescale (~ 1 Gyr) of BH sub-cluster nearly independent of cluster mass & BH retention fraction

Which clusters are best candidates?

Inferences from N-body computations:

(a) *Concentrated star clusters with* $N \gtrsim 5.0 \times 10^4$ *and significant BH-retention produce dynamical BH-BH binaries that merge within Hubble time.*

(b) *Most mergers occur within first few Gyr cluster evolution (for both in-cluster & escaped BH-binaries).*

=> chances of detecting BH-BH GW source would increase with redshift (thanks to Matt Benacquista for pointing this out!)

Also, mergers would preferentially happen among the most massive stellar-mass BHs (i.e., with highest "chirp mass" for stellar BHs; see Rodriguez et al. 2015)

BH-BH merger detection rate

- Total detection rate of BH-BH mergers from IMCs

$$\mathcal{R}_{\mathrm{GW}} = \frac{4}{3}\pi D^3 \rho_{cl} \mathcal{R}_{mrg}$$

$D =$ max. distance for detection of compact-binary inspiral. For $10M_\odot$ BH-pair, $D \approx 1500$ Mpc (AdLIGO), ≈ 100 Mpc (LIGO). $\rho_{cl} \approx 1.4$ Mpc^{-3} (density of young populous clusters, Portegies Zwart & McMillan (2000)).

- Considering isolated clusters with full BH retention and power-law IMC mass function with index = −2 (ICMF in spiral/starburst galaxies),

$$\mathcal{R}_{\mathrm{AdLIGO}} \approx 31(\pm 7)\ \mathrm{yr}^{-1}$$

- *Dynamical BH-BH binaries may constitute dominant contribution to stellar mass BH-BH merger events in the Universe.*

See Banerjee, S., Baumgardt, H. & Kroupa, P., 2010, MNRAS, 402, 371 for more.

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D = max. distance for detection of compact binary inspiral. For $10 M_\odot$

BH ... c^{-3}

(de ...)).

Theoretical estimates of dynamically-induced BH-BH in-spiral detection rate by LIGO2 ranges from a few - 100 per year.

C

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Effect on cluster’s structure & evolution

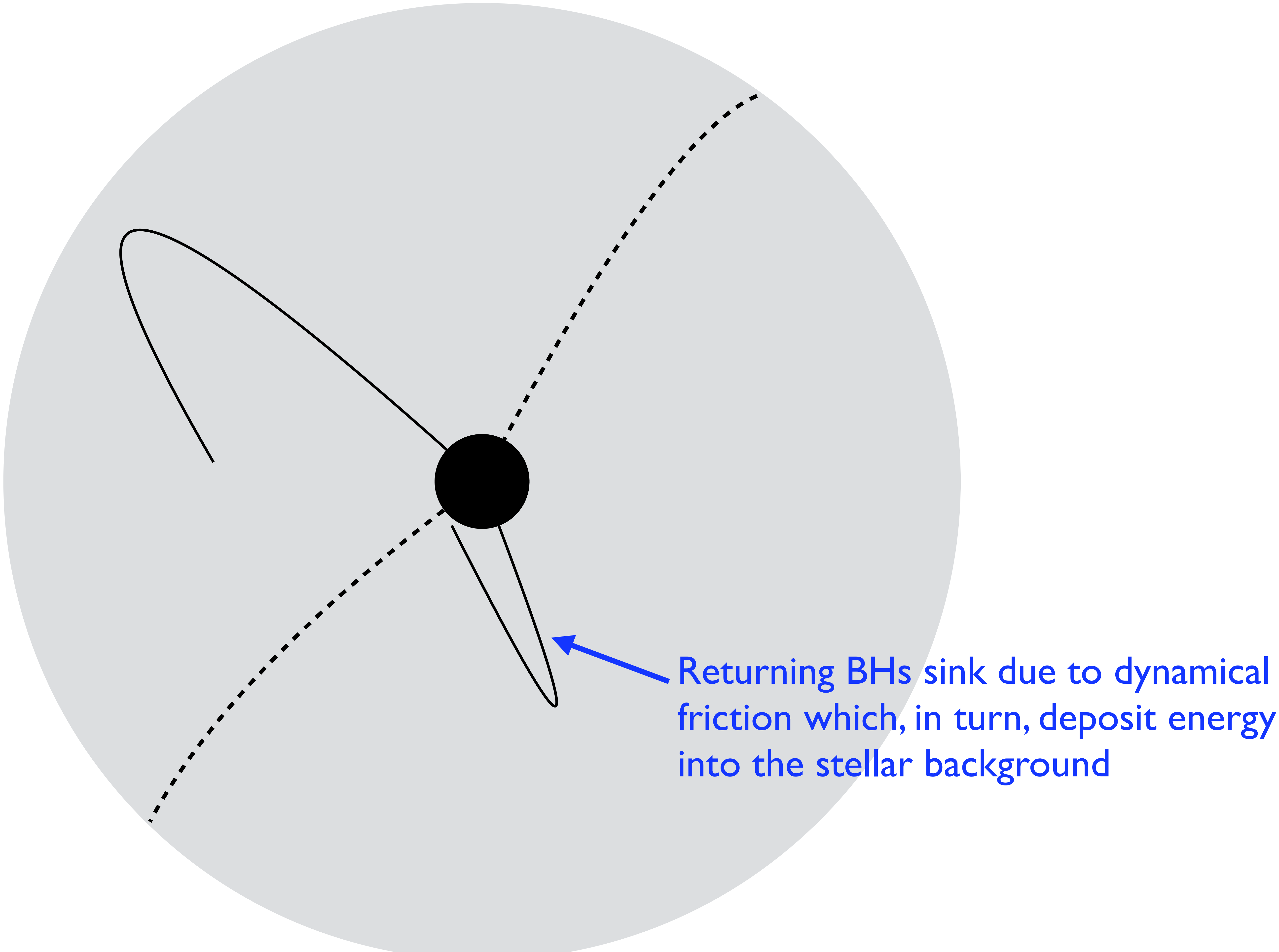
Heating of cluster core

- Close & super-elastic BH-binary---single-BH encounters in BH core eject BH-binaries & single BHs from core to cluster halo if not escaped from cluster.

- BHs return towards cluster core due to *"Dynamical friction": retardation of a massive object moving through a dense background made up of significantly lower mass particles*. Dynamical friction continually shrinks C.M. orbits of single/binary-BHs.

- Loss of orbital energy of BHs deposited in stellar background. Energy deposition most efficient in cluster core due to highest stellar density.

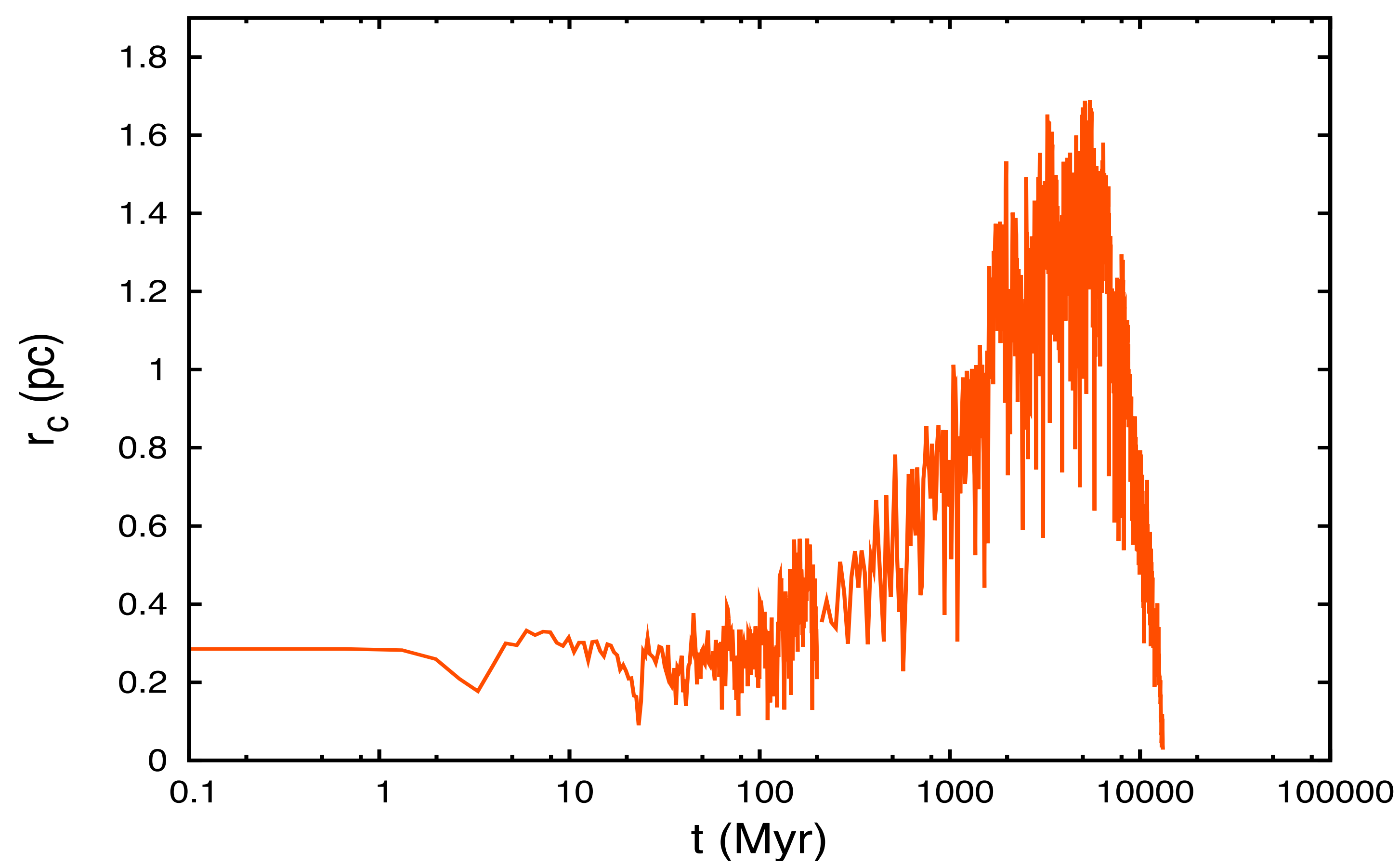
- Results in significant core expansion, delays core collapse.

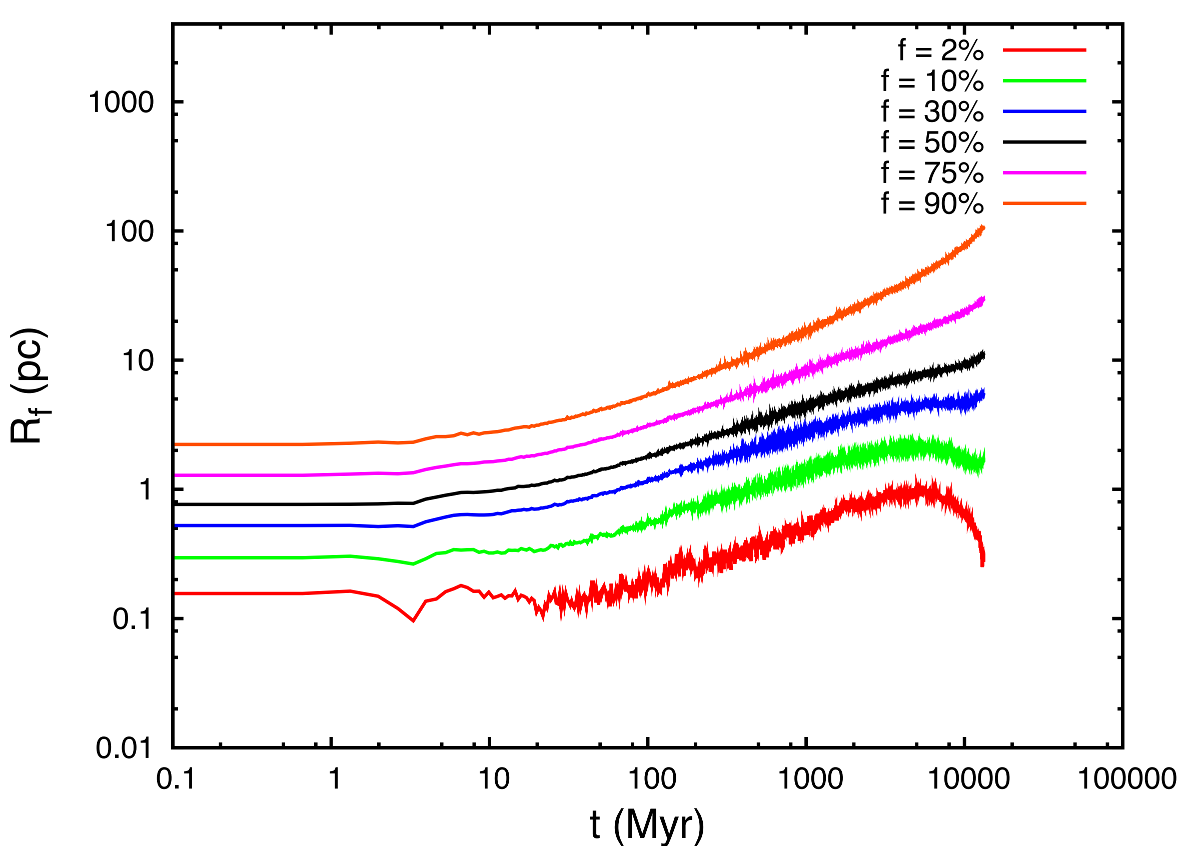

Returning BHs sink due to dynamical friction which, in turn, deposit energy into the stellar background

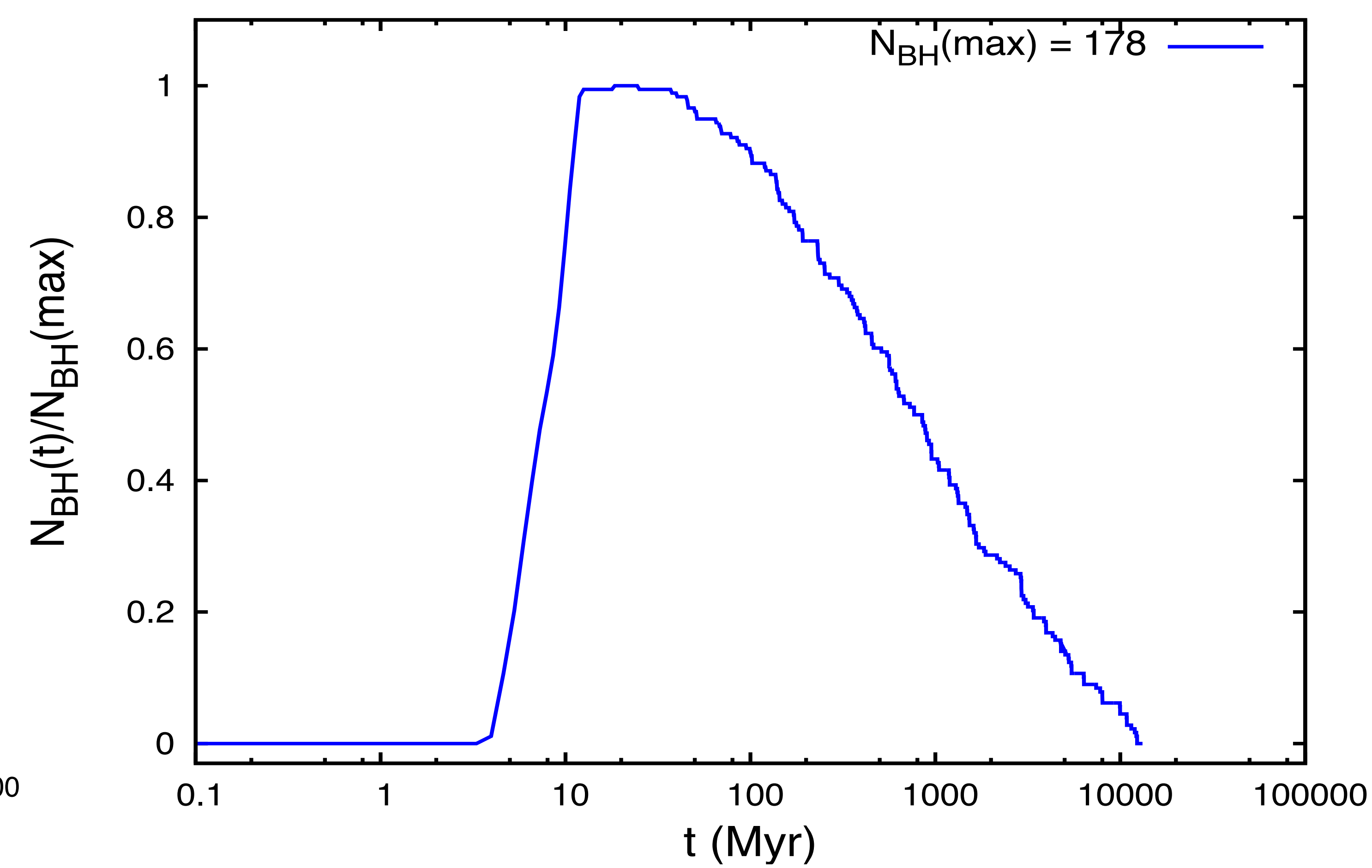
Heating of cluster core

$N \approx 8 \times 10^4$ Plummer cluster, Kroupa IMF, $Z = 0.01 Z_{\odot}$, no primordial binaries. 100% BH retention.

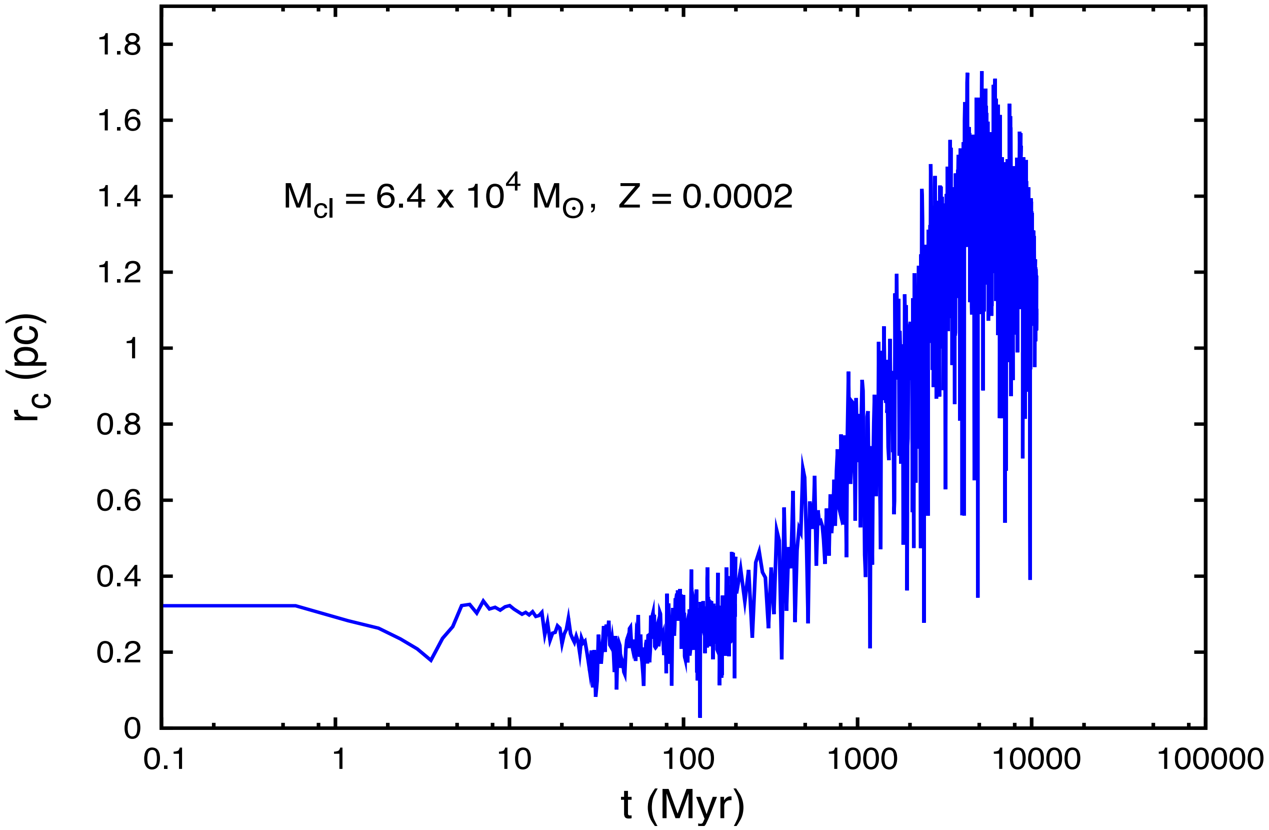
Direct N-body computation.

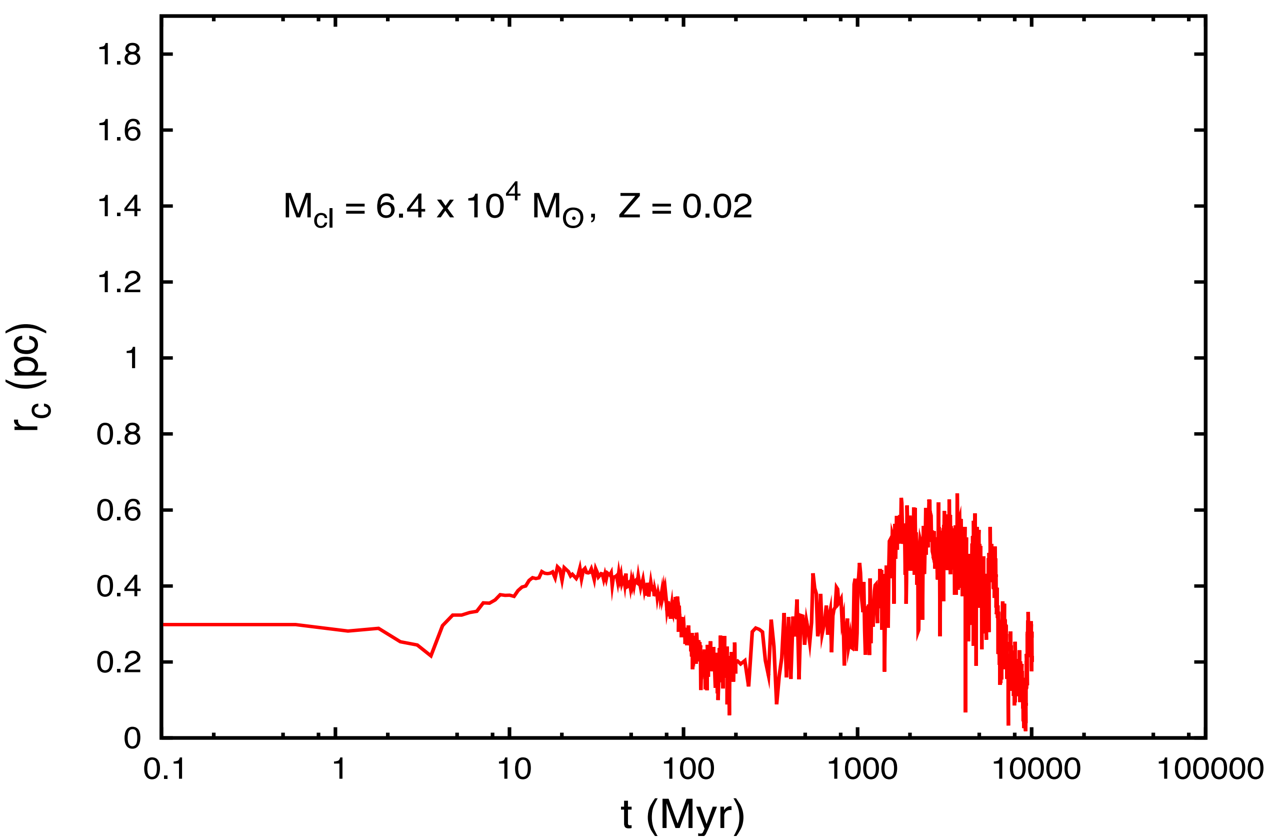






Heating of cluster core: effect of metallicity

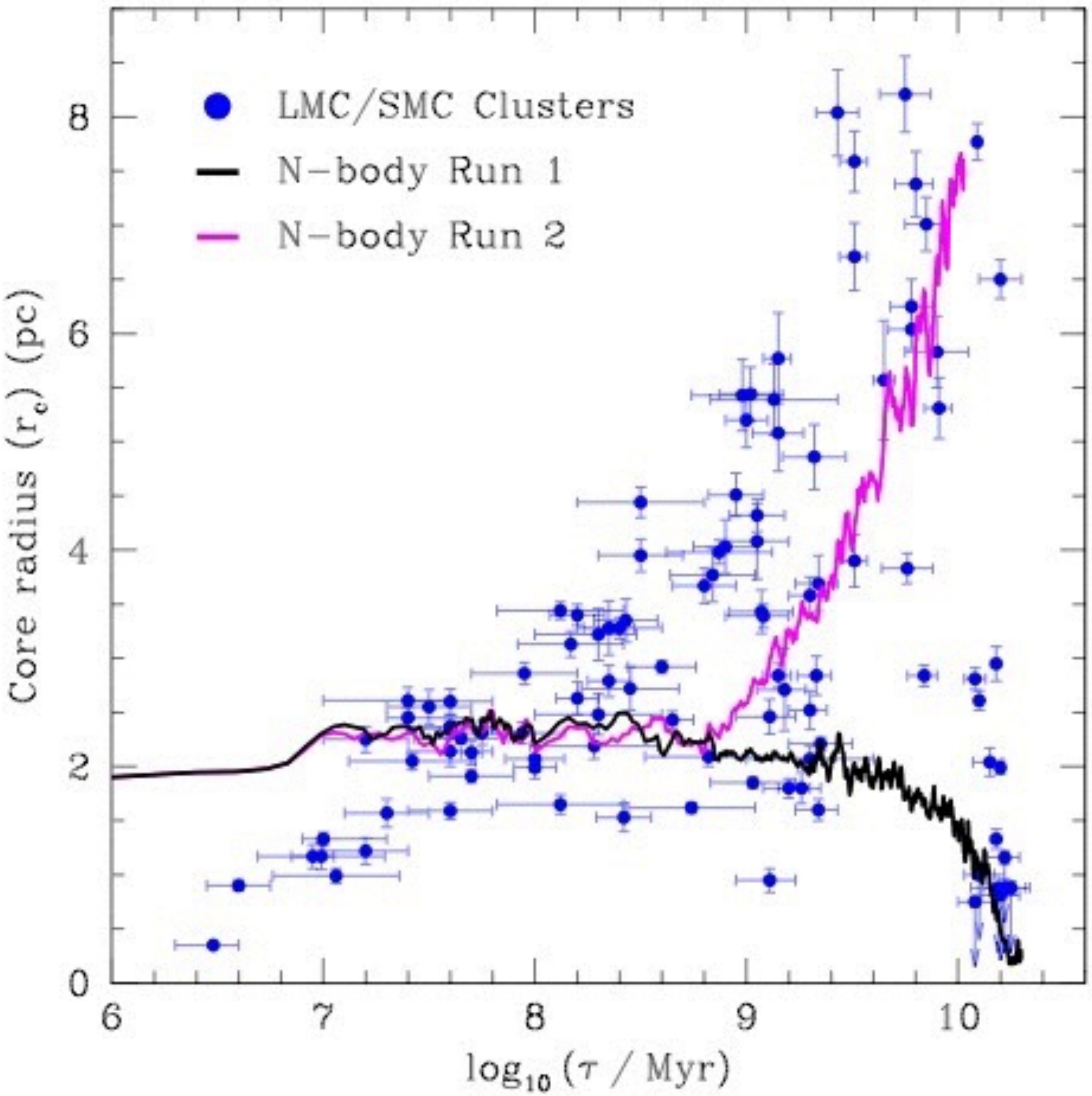




Lower Z yields more massive BHs, hence core expansion stronger.

Also, lower Z tends to produce more BH-BH mergers:

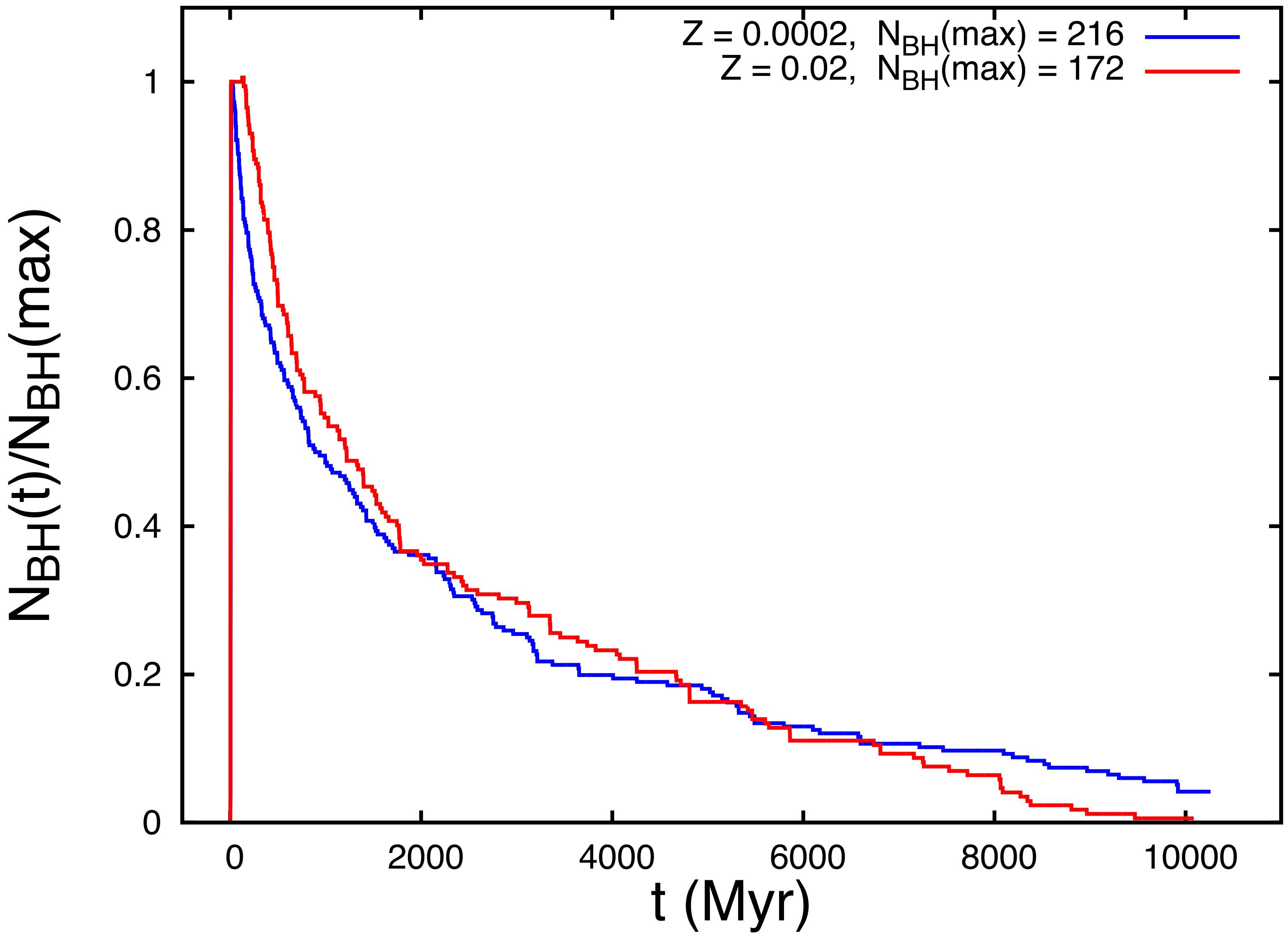
low Z computation: 3 mergers within Hubble time, high Z computation: 1 merger.



Cluster core expansion in similar N-body calculations by Mackey et al., 2008, MNRAS, 386, 65 consistent with observed age-core radius relation of LMC/SMC clusters!

Evidence of high BH retention following supernovae collapse (low BH natal kick)?

Can BH X-ray binaries form in star clusters?



BH-normal star interaction essential for X-ray binary formation.

BH-cluster dynamically isolated (Spitzer unstable), BHs mostly interact among themselves.

When nearly depleted, Spitzer instability ceases (see Spitzer's book): BHs encounter frequently with stars.

<u>A few</u> BHs can easily retain and interact with stellar members

<u>Multiple</u> BH X-ray binary formation in principle possible at later stage of dynamical evolution.

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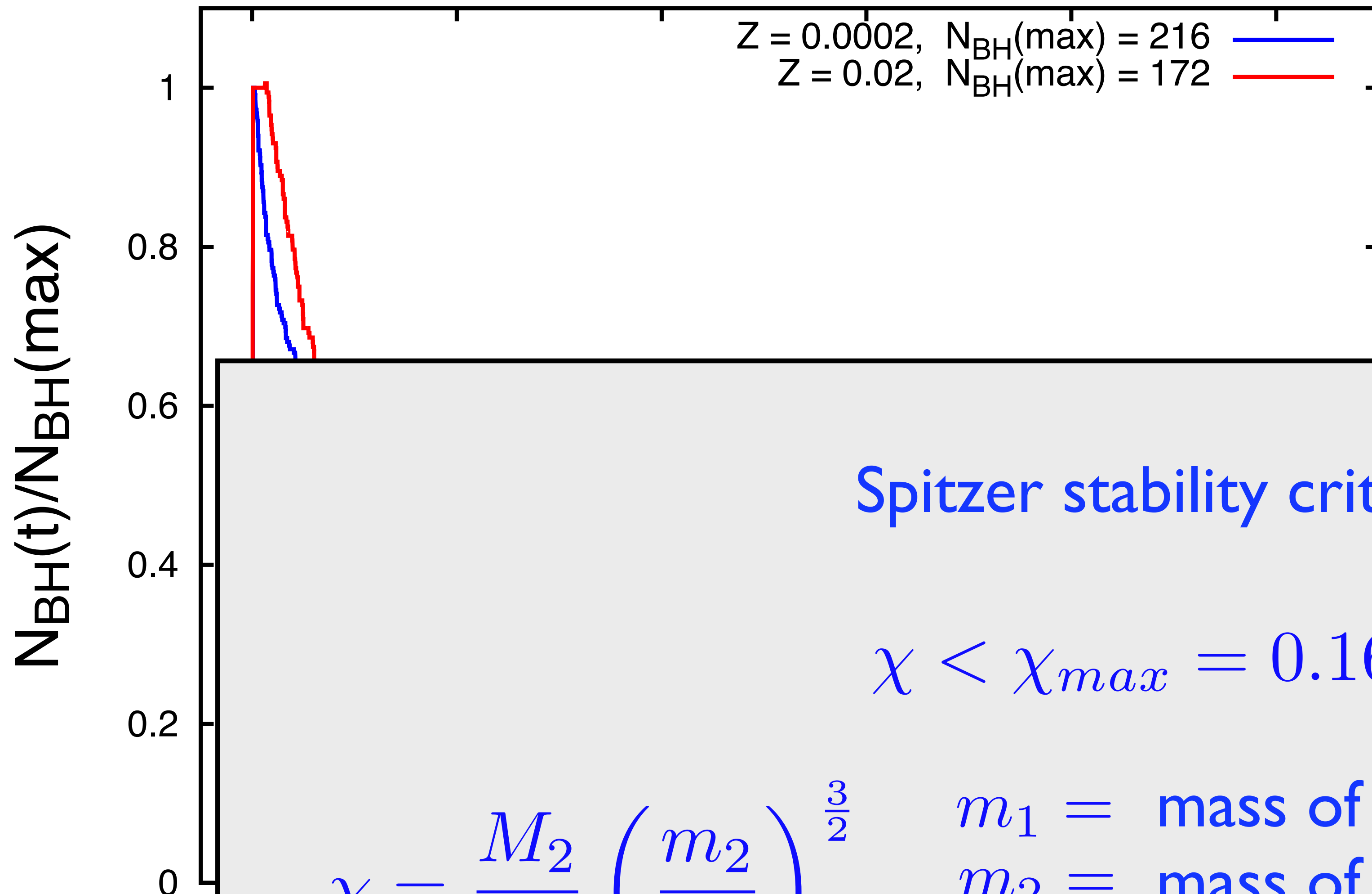

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Spitzer
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Spitzer stability criterion

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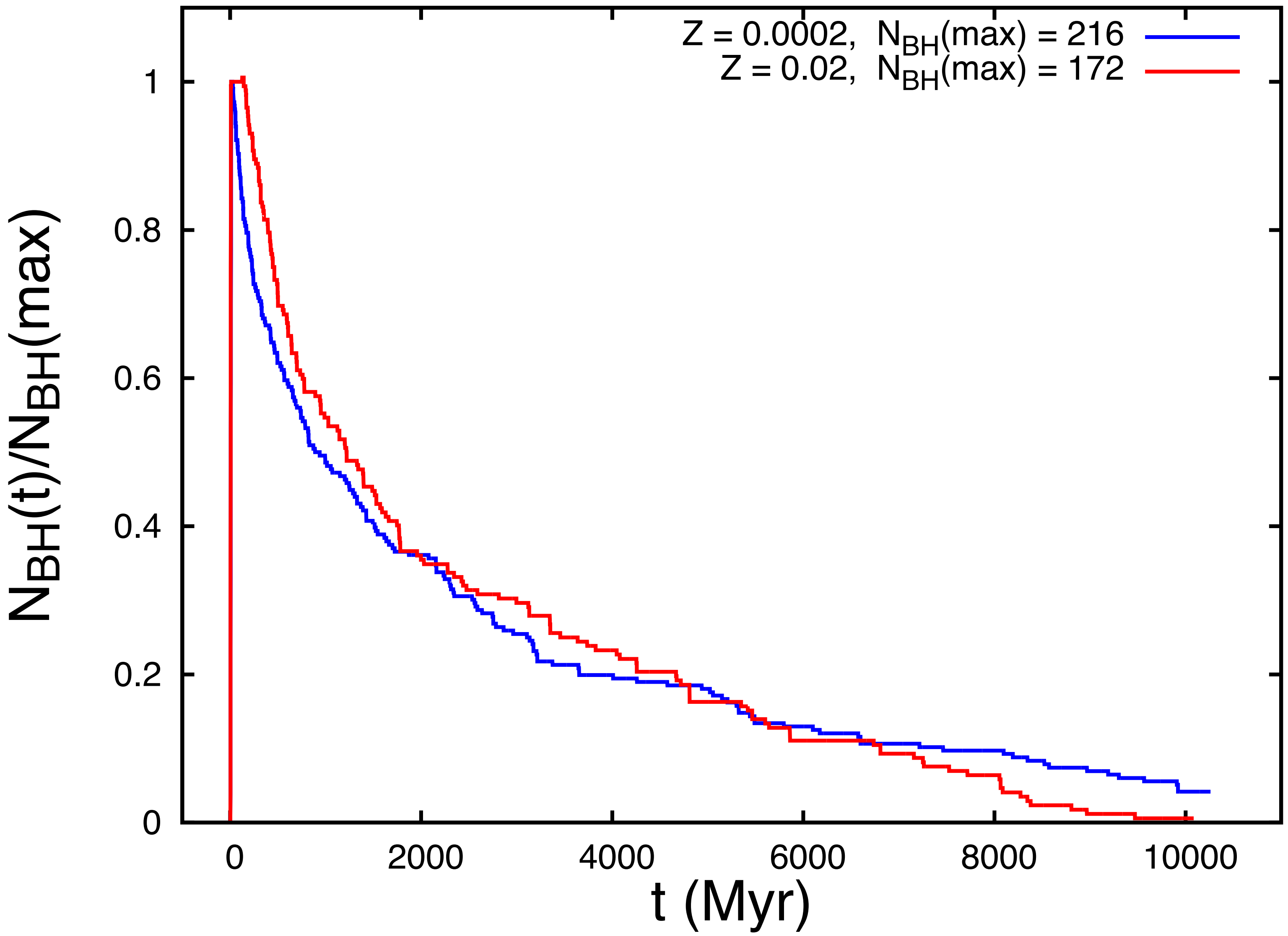

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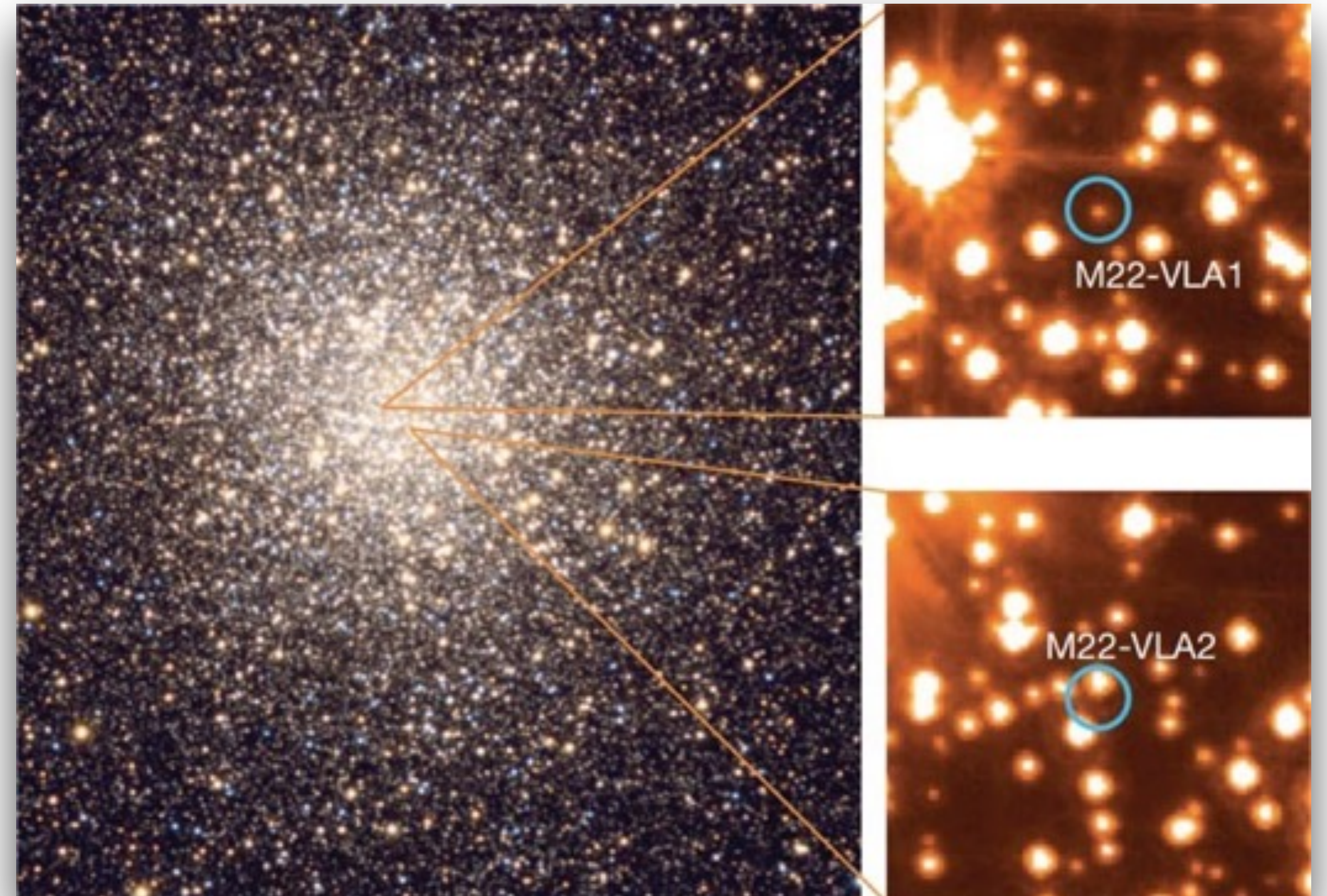
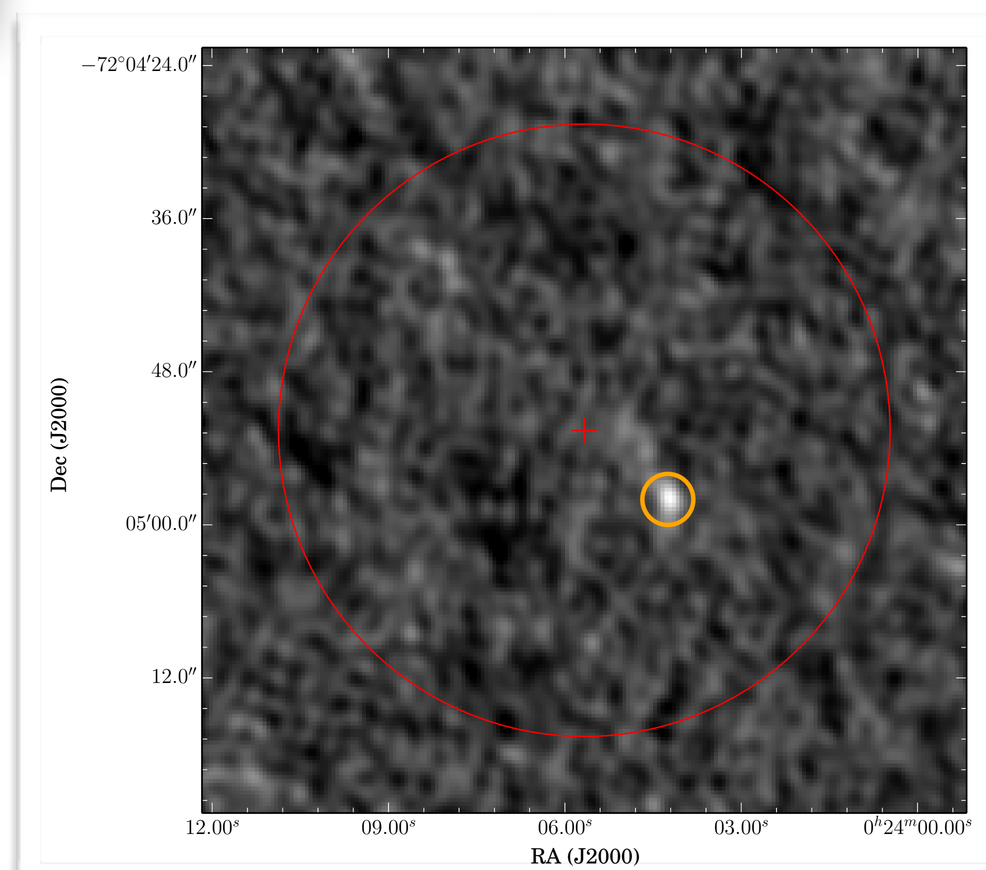


M22 BH candidates (Strader et al. 2012)

Stellar-mass BH candidates identified in globular clusters



ACTA 5.5-Ghz image of 47 Tuc core (Miller-Jones et al. 2015)