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Some insights into galaxy bias with a toy model

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Cosmology/Lens seminar
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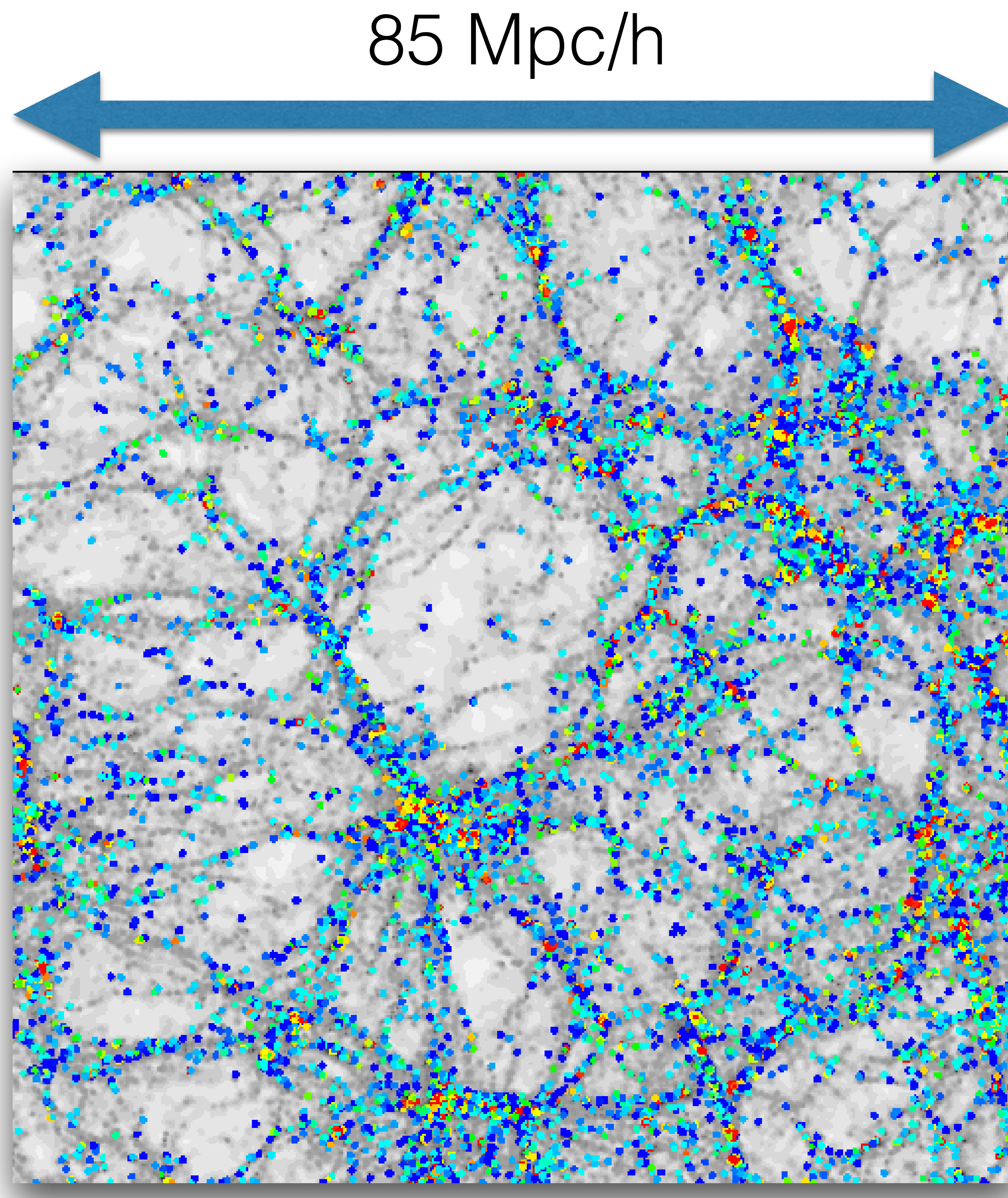


- simulation slice from z=0; LCDM
- gray: dark matter
- dots: B-V colors of galaxies

Credit: J. Colberg and A. Diaferio; GIF simulations (1998) modes of density fluctuations (random fields):

$$\tilde{\delta}(\boldsymbol{k}) = \int \mathrm{d}^3x\, \delta(\boldsymbol{x})\, \mathrm{e}^{-\mathrm{i}\boldsymbol{x}\cdot\boldsymbol{k}} \; .$$

complete second-order statistics of fluctuations:

$$\begin{aligned}
\langle \tilde{\delta}_\mathrm{m}(\boldsymbol{k}) \tilde{\delta}_\mathrm{m}(\boldsymbol{k}') \rangle &= (2\pi)^3 \delta_\mathrm{D}(\boldsymbol{k} + \boldsymbol{k}') P_\mathrm{m}(k) \; ; \\
\langle \tilde{\delta}_\mathrm{m}(\boldsymbol{k}) \tilde{\delta}_\mathrm{g}(\boldsymbol{k}') \rangle &= (2\pi)^3 \delta_\mathrm{D}(\boldsymbol{k} + \boldsymbol{k}') P_\mathrm{gm}(k) \; ; \\
\langle \tilde{\delta}_\mathrm{g}(\boldsymbol{k}) \tilde{\delta}_\mathrm{g}(\boldsymbol{k}') \rangle &= (2\pi)^3 \delta_\mathrm{D}(\boldsymbol{k} + \boldsymbol{k}') \left(P_\mathrm{g}(k) + \bar{n}_\mathrm{g}^{-1} \right) \; ,
\end{aligned}$$

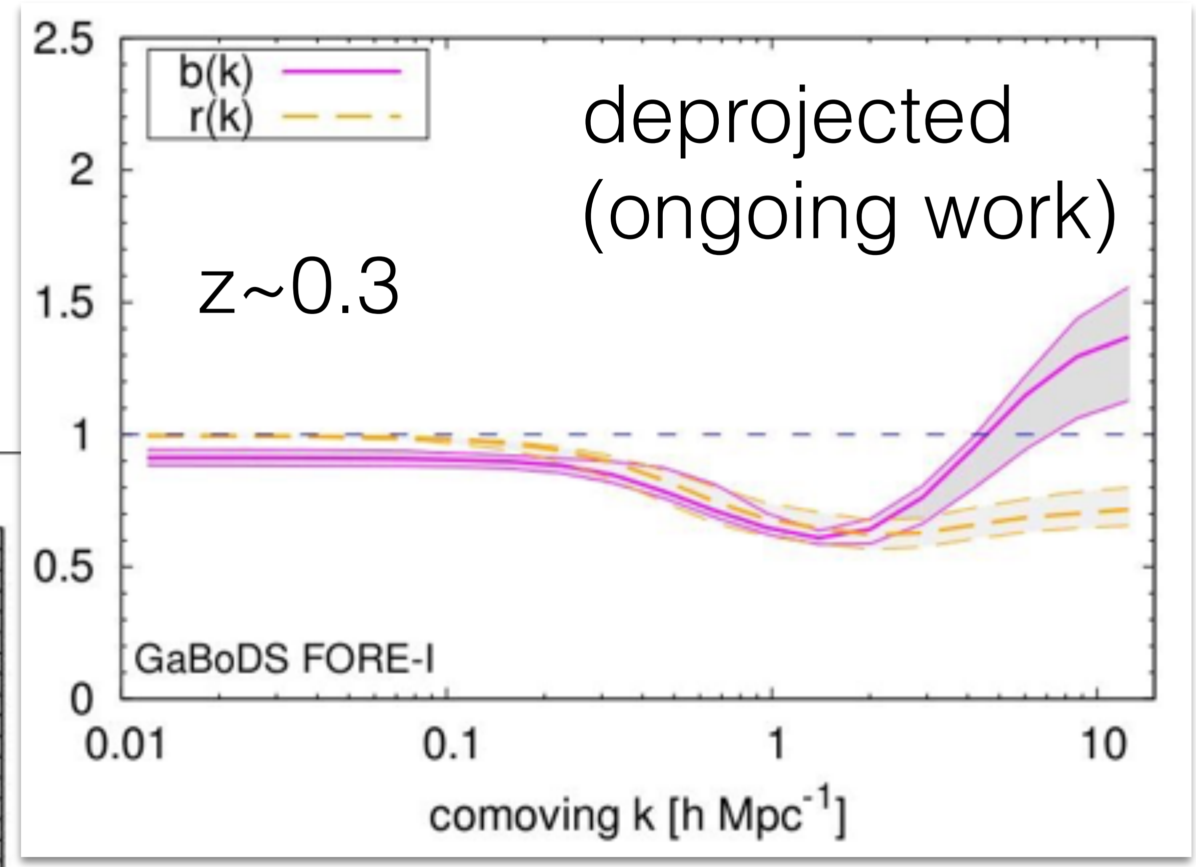
Biasing functions (linear stochastic bias):

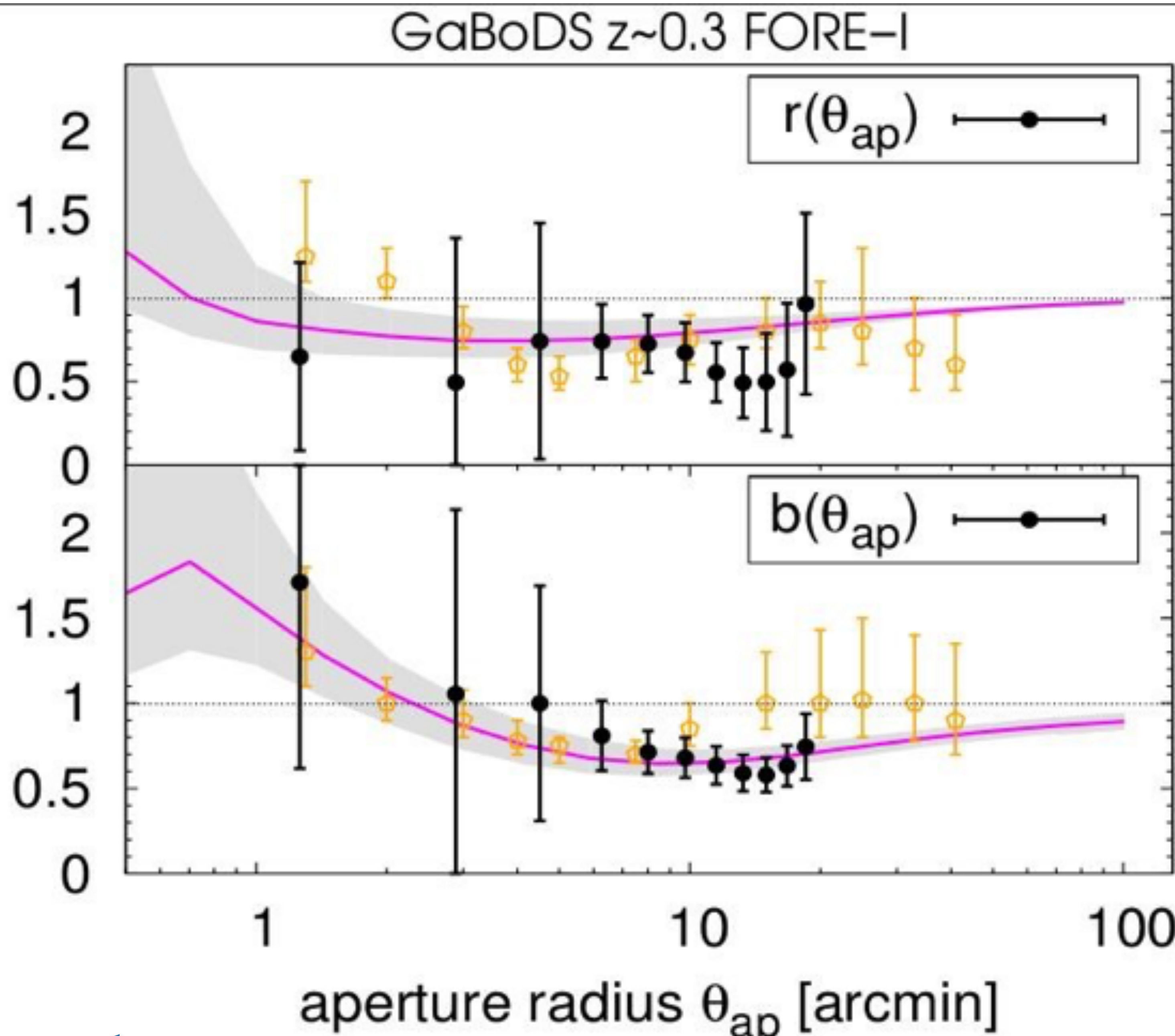
bias factor

$$b(k) = \sqrt{\frac{P_\mathrm{g}(k)}{P_\mathrm{m}(k)}} \; ; \; r(k) = \frac{P_\mathrm{gm}(k)}{\sqrt{P_\mathrm{g}(k)\, P_\mathrm{m}(k)}} \; .$$

correlation factor

Bias measured with lensing (on the sky inside apertures)



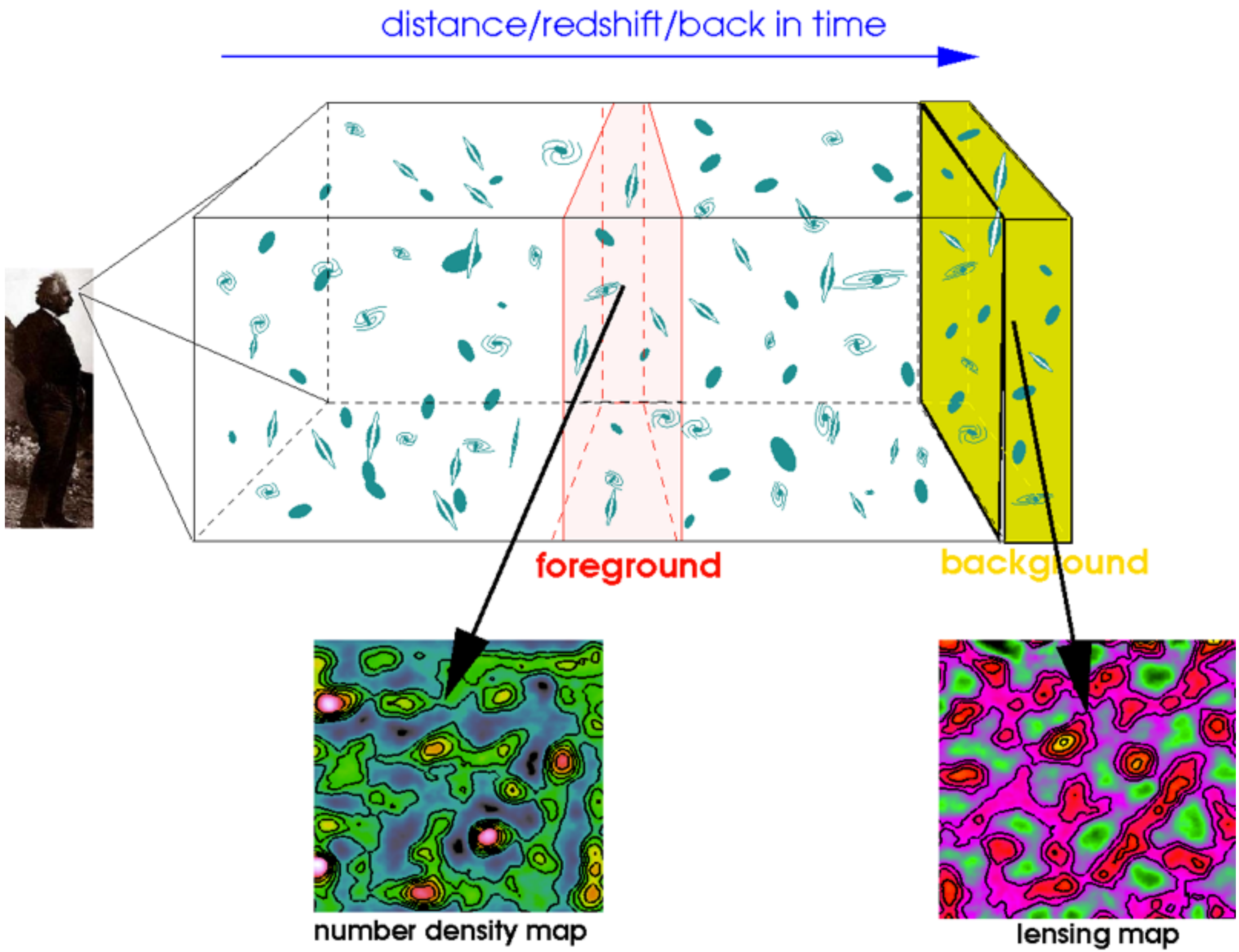


wave number k

Hoekstra et al., 2002, ApJ
RCS/VIRMOS-DESCART

Simon et al., 2007, A&A
GaBoDS

more recent:
Jullo et al., 2012, ApJ
Buddendiek et al., 2016, A&A

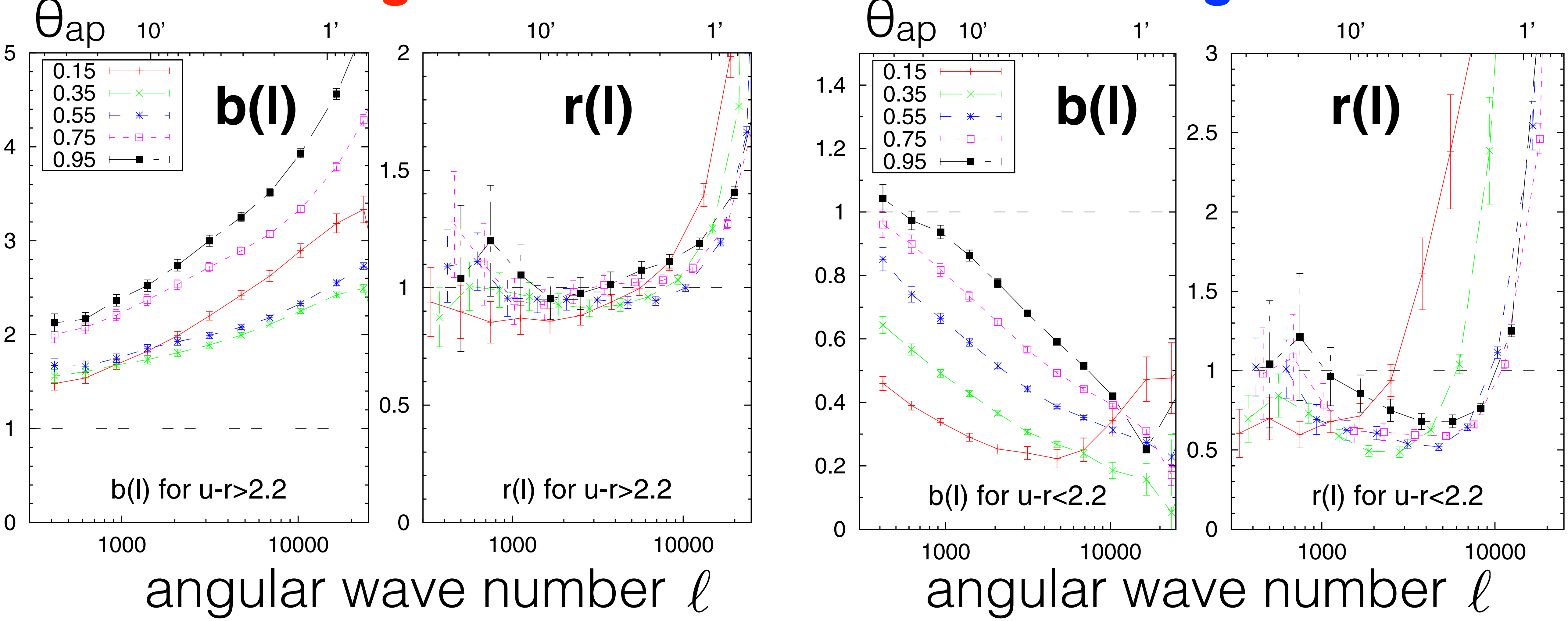


details: e.g. Simon et al., 2007, A&A, 861

In simulation: Millennium Simulation + SAMs

(by colour and redshift; flux limit r < 25 mag)

red galaxies **blue galaxies**

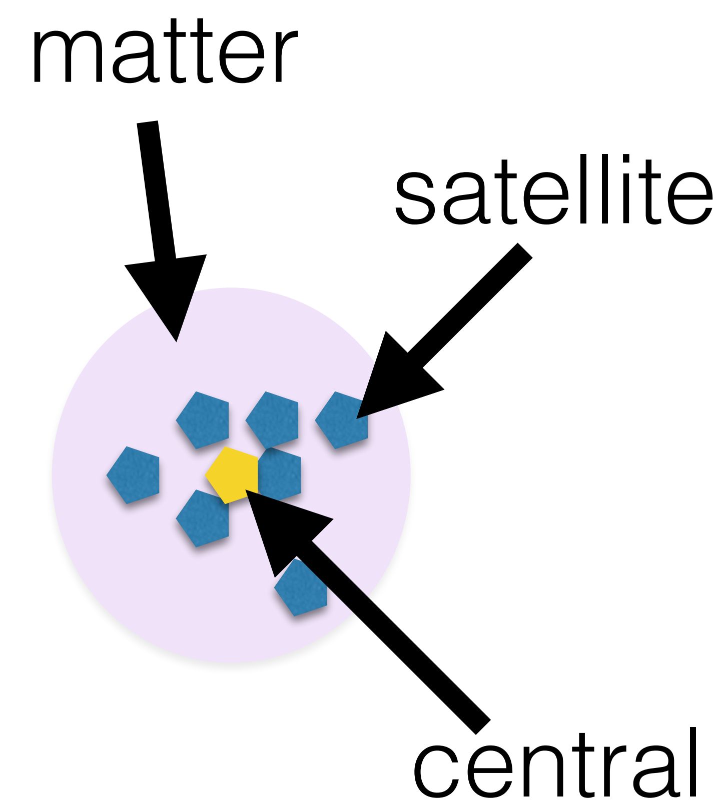


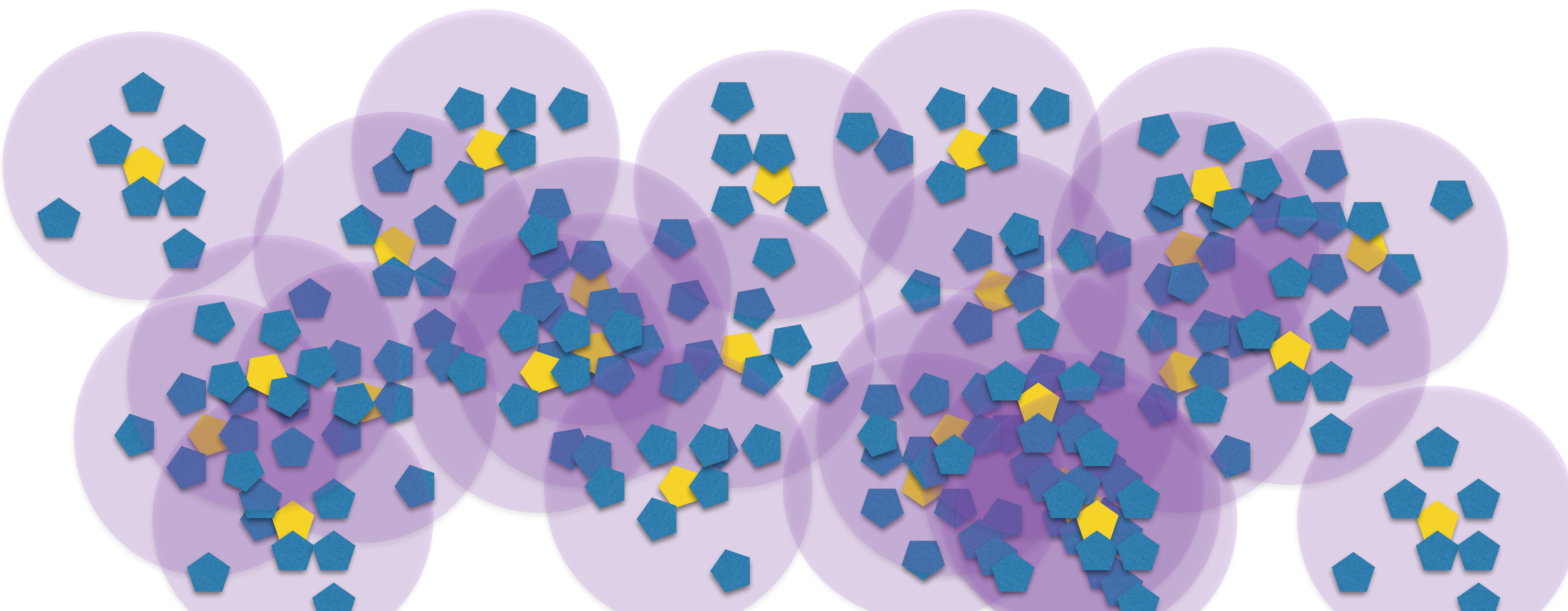
from Simon, P., 2013, A&A, 516

SAMs: Guo et al. (2011); dark matter: Springel et al. (2005)

Halo-model inspired toy model that features:

- unclustered matter halos;
- single-mass halos with same density profile (mass m_0);
- optionally mix of satellite and central galaxies or just satellite galaxies;





Ingredients of model:

$\langle N \rangle$	mean galaxy number in halo;
$\langle N(N-1) \rangle$	mean number of galaxy pairs in halo;
$u_{\rm m}(x)$	radial matter density profile;
$u_{\rm g}(x)$	radial galaxy density profile;
$n_{\rm h}$	halo number density;

$$\tilde{u}(k) := \frac{\int_0^\infty \mathrm{d}x \, x \, k^{-1} \, u(x) \, \sin{(kx)}}{\int_0^\infty \mathrm{d}x \, x^2 \, u(x)}$$

Fourier transform of radial profile (normalised!)

Implementation by Seljak, 2000, MNRAS, 218:

$$P_{\mathrm{g}}(k) = \frac{n_{\mathrm{h}}}{\bar{n}_{\mathrm{g}}^2}\, \tilde{u}_{\mathrm{g}}^{2p}(k)\, \langle N(N-1)\rangle = \frac{\langle N(N-1)\rangle}{n_{\mathrm{h}}\, \langle N\rangle^2}\, \tilde{u}_{\mathrm{g}}^{2p}(k)$$

$$P_{\mathrm{m}}(k) = \frac{m_0^2\, n_{\mathrm{h}}}{\bar{\rho}_{\mathrm{m}}^2}\, \tilde{u}_{\mathrm{m}}^2(k) = \frac{\tilde{u}_{\mathrm{m}}^2(k)}{n_{\mathrm{h}}}$$

$$P_{\mathrm{gm}}(k) = \frac{n_{\mathrm{h}}\, m_0}{\bar{\rho}_{\mathrm{m}}\, \bar{n}_{\mathrm{g}}}\, \tilde{u}_{\mathrm{m}}(k)\, \tilde{u}_{\mathrm{g}}^q(k)\, \langle N\rangle = \frac{\tilde{u}_{\mathrm{m}}(k)\, \tilde{u}_{\mathrm{g}}^q(k)}{n_{\mathrm{h}}}$$

uses $\bar{\rho}_{\mathrm{m}} = n_{\mathrm{h}}\, m_0\,,\ \bar{n}_{\mathrm{g}} = n_{\mathrm{h}}\, \langle N\rangle\,,\ n(m) = n_{\mathrm{h}}\, \delta_{\mathrm{D}}(m - m_0)\,,$ and

$$p = \begin{cases} 1 & \langle N(N-1)\rangle > 1 \\ 1/2 & \text{otherwise} \end{cases} \qquad q = \begin{cases} 1 & \langle N\rangle > 1 \\ 0 & \text{otherwise} \end{cases}$$

controls impact of central galaxies

Correlation factor for scale *k*:

$$r(k) = \frac{P_{\rm gm}(k)}{\sqrt{P_{\rm g}(k)\, P_{\rm m}(k)}} = \frac{\tilde{u}_{\rm g}^{q-p}(k)\, \langle N \rangle}{\sqrt{\langle N(N-1) \rangle}} = \tilde{u}_{\rm g}^{q-p}(k) \left(1 + \frac{\Delta\sigma_{\rm N}^2}{\langle N \rangle^2}\right)^{-1/2}$$

where we have introduced the excess variance

$$\Delta\sigma_{\rm N}^2 := \sigma_{\rm N}^2 - \sigma_{\rm N}^2|{\rm Poisson}$$

$$= \langle N^2 \rangle - \langle N \rangle^2 - \langle N \rangle = \langle N(N-1) \rangle - \langle N \rangle^2$$

$\Delta\sigma_{\rm N}^2 = 0$ variance of N as in Poisson statistic;

$\Delta\sigma_{\rm N}^2 < 0$ sub-Poisson variance;

$\Delta\sigma_{\rm N}^2 > 0$ super-Poisson variance

$$r(k) = \tilde{u}_{\rm g}^{q-p}(k) \left(1 + \frac{\Delta\sigma_{\rm N}^2}{\langle N \rangle^2}\right)^{-1/2}$$

We learn about two ways to produce r(k) > 1:

1. dominating central galaxies; for q-p = -1/2, r(k) can become arbitrarily large for k >> 1:

$$r(k) \propto \tilde{u}_{\rm g}^{-1/2}(k)$$

2. no/negligible central galaxies and sub-Poisson variance; hence p = q = 1 and $\Delta\sigma_N^2 < 0$;

Note: impact of 2. becomes small if $\Delta\sigma_N^2 \ll \langle N \rangle^2$

Bias factor for scale *k*:

$$b(k) = \sqrt{\frac{P_{\rm g}(k)}{P_{\rm m}(k)}} = \frac{\tilde{u}_{\rm g}^{p}(k)\,\sqrt{\langle N(N-1)\rangle}}{\tilde{u}_{\rm m}(k)\,\langle N\rangle} = \frac{\tilde{u}_{\rm g}^{q}(k)}{\tilde{u}_{\rm m}(k)}\,\frac{1}{r(k)}$$

1. even without central galaxies and identical radial profiles we do not necessarily have b(k) = r(k) = 1; yet we find b(k)r(k) = 1;

2. only a Poisson variance in addition to 1. ensures b(k) = r(k) = 1; truly unbiased galaxies require a Poisson-like variance of N;

3. for satellite-dominated halos, b(k) reflects the difference between matter and galaxy radial profile; hence q = 1 and r(k) = const;

- Central galaxies or a sub-Poisson variance of N can make r(k) > 1 because

  1. $P_g(k)$ is defined in excess of Poisson shot noise of discrete galaxies. If galaxy sampling is actually sub-Poisson, this over-corrects the shot noise power (Seljak 2000; Guzik & Seljak 2001).

  2. Putting one galaxy *always* at the halo centre is not a Poisson sampling of the halo density profile; so is non-Poisson $\Delta\sigma_N^2$.

- Is r(x) > 1 also possible for the real-space biasing functions (shot noise only at zero lag)?

$$b(x) = \sqrt{\frac{\xi_{\rm g}(x)}{\xi_{\rm m}(x)}}\ ;\ r(x) = \frac{\xi_{\rm mg}(x)}{\sqrt{\xi_{\rm g}(x)\,\xi_{\rm m}(x)}}\ .$$

-> transform three power spectra of toy model to real-space correlation functions via (back Fourier transform)

$$\xi(x) = [P](x) := \frac{1}{2\pi^2 x} \int_0^\infty \mathrm{d}k\, k\, P(k)\, \sin(k\, x)\ ,$$

and you get real-space counterparts of b(k) and r(k):

$$b(x) \quad = \quad \frac{[\tilde{u}_\mathrm{m} \cdot \tilde{u}_\mathrm{g}^q](x)}{|[\tilde{u}_\mathrm{m}^2](x)|} \frac{1}{r(x)}\ ;$$

$$r(x) \quad = \quad \frac{[\tilde{u}_\mathrm{m} \cdot \tilde{u}_\mathrm{g}^q](x)}{\sqrt{[\tilde{u}_\mathrm{g}^{2p}](x)\, [\tilde{u}_\mathrm{m}^2](x)}} \left(1 + \frac{\Delta\sigma_\mathrm{N}^2}{\langle N \rangle^2}\right)^{-1/2}$$

Interestingly, r(x) depends also on matter density profile.

Assume: identical density profiles $u_{\rm m}(x) = u_{\rm g}(x)$!

- without central galaxies — p = q = 1 — we have

$$b(x)\, r(x) = 1 \;;\; r(x) = \left(1 + \frac{\Delta\sigma_{\rm N}^2}{\langle N\rangle^2}\right)^{-1/2}$$

- with centrals dominating and Poisson variance — p = 1/2, q = 0, and $\Delta\sigma_{\rm N}^2$ = 0 — we have

$$b(x)\, r(x) = \frac{[\tilde{u}_{\rm m}](x)}{|[\tilde{u}_{\rm m}^2](x)|} \;;\; r(x) = \sqrt{\frac{[\tilde{u}_{\rm m}](x)}{[\tilde{u}_{\rm m}^2](x)}} \geq 1$$

Summary and conclusions

- biasing functions of linear stochastic bias fully capture the differences in distributions of galaxies vs. matter for two-point statistics; can be measured with lensing;

- their definition assumes Poisson shot-noise for galaxies; can give seemingly curious values $r > 1$ if sampling is actually sub-Poisson;

- demonstrated with toy model that $r(k) > 1$ or $r(x) > 1$ can arise through central galaxies or a sub-Poisson variance of galaxy numbers inside matter halos;

- even if galaxies perfectly trace matter, we still have *not* $b = r = 1$ for all scales if sampling variance is not Poisson;