Particle Astrophysics and Cosmology (SS 08)
Homework no. 9 (June 25, 2008)

Tutorials: Wednesday, 17:15 to 18:45, AVZ, room 116 (first floor)

1 Indirect WIMP Detection in Neutrino Telescopes

In class we saw that one way to look for WIMPs is to search for muon neutrinos produced from WIMP annihilation in the center of the Sun or Earth. Here one actually detects ultra–relativistic muons, produced in charged–current ν_μ interactions in or below the detector.

(a) Show that the muon production cross section on matter, $\sigma(\nu_\mu + A \to \mu^- + X)$, scales linearly with the neutrino energy, in the relevant energy range 1 GeV $\leq E_\nu \leq$ 1 TeV. *Hint:* Draw the corresponding Feynman diagram, and remember the simple rules about bosonic propagators and the mass dimension of cross sections.

(b) The energy loss of muons in matter can be described by

$$\frac{dE}{dx} = -(a + bE)\,. \tag{1}$$

Here the constant term is due to the ionization and excitation of atoms, and the second term is due to radiative processes. a and b only depend weakly on the traversed material, if the length is expressed as "column length", measured in g/cm^2, such that a physical length $l = x/\rho$, ρ being the mass density (in g/cm^3) of the material: $a \simeq 2$ MeV / (g/cm^2), $b \simeq 10^{-5}$/(g/cm^2). Determine the range of a muon with incident energy E by solving eq.(1). For what range of energies does one find an approximately linear dependence of the range on the incident energy?

(c) From the considerations in (a) and (b) we find that, for most relevant energies, the signal is proportional to E_ν^2. In most WIMP models, the neutrinos are not produced directly in WIMP annihilation; rather, the WIMPs annihilate into heavy particles X, which then decay into neutrinos. The signal is then proportional to the second moment of the neutrino spectrum, given by

$$\langle z^2 \rangle \equiv \frac{1}{m_\chi^2 \sigma(\chi\chi \to X\bar{X})} \int_{E_{\min}}^{E_{\max}} \frac{d\sigma(\chi\chi \to \nu)}{dE_\nu} E_\nu^2 dE_\nu\,. \tag{2}$$

Calculate $\langle z^2 \rangle$ for $X = W^+$. Assume that the W bosons are produced unpolarized, so that the decay $W^+ \to \mu^+\nu_\mu$ is isotropic in the W^+ rest frame. *Hint:* Calculate the energy distribution of the neutrinos first in the W^+ rest frame, and then boost into the $\chi\chi$ rest frame. Remember that the WIMPs annihilate nearly at rest, i.e. Mandelstam$-s = 4m_\chi^2$.

(d) Is $\langle z^2 \rangle$ bigger or smaller than in (c) if X undergoes three–body decay into a neutrino and two other particles, e.g. $b \to \bar{\nu}_\mu \mu^- c$?

2 WIMP Annihilation in the Halo of a Galaxy

In this exercise, we will compute the "boost factor", originating from the "clumpiness" of the Dark Matter (DM) halos, which can modify the WIMP annihilation rate, thus playing an important role in indirect DM detection. These clumps or minihalos are thought to be embedded in the halo. Here we are interested in length scales much smaller than the galaxy as a hole, so that the smooth component of the WIMP distribution can be taken to be constant.

In general, the boost factor of the WIMP annihilation signal in the halo is defined as

$$B = \frac{\langle n_\chi^2 \rangle}{\langle n_\chi \rangle^2}\,, \tag{3}$$

where $\langle \dots \rangle$ denotes averaging over a sufficiently large volume.

(a) Consider $N_{\rm tot}(\gg 1)$ DM particles in a large box with volume V. If the particles are uniformly distributed, then $n_\chi = N_{\rm tot}/V$. What is the boost factor in this case?

(b) Now allow some fraction f of the $N_{\rm tot}$ particles to be bound in clumps, i.e. $fN_{\rm tot}$ particles are in the minihalos. Each minihalo has a density profile such that

$$\rho = \rho_0 \left(\frac{r}{r_s}\right)^{-1} \left[1 + \left(\frac{r}{r_s}\right)^2\right]^{-1.5} \tag{4}$$

Calculate the total number of WIMPs ($N_{\rm WIMP}$) in such a clump, and re–write eq.(4) by trading ρ_0 for $N_{\rm WIMP}$. Express f in terms of $N_{\rm WIMP}, N_{\rm halo}, N_{\rm tot}$, where $N_{\rm halo}$ is the number of clumps in the box. *Hint:* You can assume that the minihalo is much smaller than the total volume we are considering; hence the upper limit of integration over a minihalo can be sent to ∞.

(c) Now compute the boost factor as a function of f. Consider two cases: *(i)* $N_{\rm WIMP}$ =const., *(ii)* $N_{\rm halo}$ =const. *Hint:* First compute B for a single minihalo. Each minihalo contributes equally to the numerator in eq.(3). Remember that the density of WIMPs in the smooth (constant) component diminishes with increasing f.