Particle Astrophysics and Cosmology (SS 08)

Homework no. 12 (July 16, 2008)

Tutorials: Wednesday, 17:15 to 18:45, AVZ, room 116 (first floor)

1 Maximum temperature during reheating

Let us consider a model universe with two components: inflaton field energy ρ_ϕ and radiation energy density ρ_R. The decay rate of the inflaton field energy density is $\Gamma_\phi = \alpha_\phi m_\phi$ and the light degrees of freedom (i.e. radiation) are in local thermodynamic equilibrium.

With the above assumptions, the Boltzmann equations describing the redshift and interchange in the energy density among the different components are:

$$\begin{aligned} \dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi \rho_\phi &= 0 \ , \\ \dot{\rho}_R + 4H\rho_R - \Gamma_\phi \rho_\phi &= 0 \ , \end{aligned} \tag{1}$$

where the dots denote time derivatives.

1. Since it is convenient to work with dimensionless quantities that can absorb the effect of expansion in the universe we introduce the quantities: $\Phi \equiv \rho_\phi m_\phi^{-1}(aR_c)^3$ and $\Psi \equiv \rho_R (aR_c)^4$, where R_c is some (arbitrary) reference length scale. (Recall that a is dimensionless.) It is also convenient to use the scale factor, rather than time, for the independent variable, so we define a variable $x \equiv aR_c m_\phi$. Show that with this choice the system of equations (1) can be written as

$$\begin{aligned} \Phi' &= -c_1 \tfrac{x}{\sqrt{\Phi x + \Psi}} \Phi \ , \\ \Psi' &= c_1 \tfrac{x^2}{\sqrt{\Phi x + \Psi}} \Phi \ , \end{aligned} \tag{2}$$

   where the prime denotes d/dx and c_1 is given by

$$c_1 = \sqrt{3} \frac{M_P}{m_\phi} \alpha_\phi \ , \tag{3}$$

   M_P being the reduced Planck mass.

2. Find an expression $\Phi(x_I) = \Phi_I$ for the initial conditions at $x = x_I$ and $\Psi(x_I) = 0$ in terms of the expansion rate at x_I and the initial Hubble parameter H_I.

3. Consider now the early-time solution for Ψ where $H \gg \Gamma_\Phi$ is valid, i.e. before a significant fraction of the inflaton energy density is converted to radiation. Solve the system of equations (2) at early times for $\Phi \simeq \Phi_I$ and $\Psi \simeq 0$ by obtaining

$$\Psi \simeq \frac{2}{5} c_1 \left(x^{5/2} - x_I^{5/2} \right) \Phi_I^{1/2} \ . \tag{4}$$

4. Now express Ψ in terms of the temperature T in order to derive the early-time solution for T:

$$\frac{T}{m_\phi} \simeq \left(\frac{12}{\pi^2 q_{\rm tot}}\right)^{1/4} c_1^{1/4} \left(\frac{\Phi_I}{x_I^3}\right)^{1/8} \left[\left(\frac{x}{x_I}\right)^{-3/2} - \left(\frac{x}{x_I}\right)^{-4}\right]^{1/4}, \tag{5}$$

where $q_{\rm tot}$ denotes the number of relativistic degrees of freedom contributing to ρ_R.

5. Find the value x/x_I at which the temperature T becomes maximal.

6. Express α_ϕ in the derived equation for T_{max} in terms of the reheating temperature T_{RH} in order to obtain

$$\frac{T_{MAX}}{T_{RH}} \simeq 1.15 \left(\frac{9}{2\pi^3 q_{\rm tot}}\right)^{1/4} \left(\frac{H_I M_P}{T_{RH}^2}\right)^{1/4} . \tag{6}$$

7. As an example, consider now the simplest model of chaotic inflation $H_I^2 \sim M_P m_\phi$ with $m_\phi \simeq 10^{13}$ GeV. What value in terms of $q_{\rm tot}$ do you find for the ratio of maximal to reheating temperature for $T_{RH} = 10^9$ GeV?