Particle Astrophysics and Cosmology (SS 08)
Homework no. 1 (April 16, 2008)

Tutorials: Wednesday, 17:15 to 18:45, AVZ, room 116 (first floor)

1 Natural Units

Quite often in particle physics and cosmology so–called natural units are used, which you get by setting the speed of light c, the quantum of action or spin $\hbar$ and the Boltzmann konstant k_B to unity. The only unit left can be chosen to be that of energy (GeV). Which units do length, time, mass and temperature have in this system?

1.1 The Planck Mass

A way to define the Planck mass is to express Newton's gravitational constant G_N in natural units.

- What is the relation between M_{Pl} and G_N?
  (Hint: Use the ansatz $M_{Pl} = G_N^\alpha \times \hbar^\beta \times c^\gamma$.)
- Determine the value of the Planck mass in natural units.

1.2 Temperature and Energy

Given the Boltzmann constant k_B, what energy does a room temperature of 300 K correspond to?

1.3 Critical Density

- Which unit does the critical density $\rho_{crit,0} = 3H_0^2/(8\pi G_N)$ have when you use the system of natural units?
- How many protons per unit volume correspond to the critical density? Calculate the value of the critical density in natural units!

1.4 Hubble's Constant

Calculate the Hubble constant H_0 in natural units.

$G_N = 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2} = 1.069 \times 10^{-20} m^5 s^{-4} GeV^{-1}$
$\hbar c = 197\text{MeV fm}$, $\hbar = 6.6 \times 10^{-22}$ MeV s, $k_B = 8.6175 \times 10^{-5}\text{eV K}^{-1}$

2 Relativistic Doppler Shift

In class, we introduced the redshift defined by

$$z = \frac{\lambda_{obs} - \lambda_{emit}}{\lambda_{emit}}, \tag{1}$$

where λ_{emit} and λ_{obs} are the wavelengths of light at the points of emission (e.g. a distant galaxy) and observation (us). This is basically the Doppler effect applied to light waves. If an object is receding in radial direction at a speed v, then the Doppler shift is given as

$$1 + z = \frac{\lambda_{obs}}{\lambda_{emit}} = \frac{1 + v/c}{\sqrt{1 - v^2/c^2}} \tag{2}$$

Derive Eq.(2).
(Hint: One way to do this is writing down a wave in a frame K, and in another frame K' which moves away from the source with velocity v, and using the the fact that a Lorentz tranfsormation between these frames leaves the speed of light unchanged.)