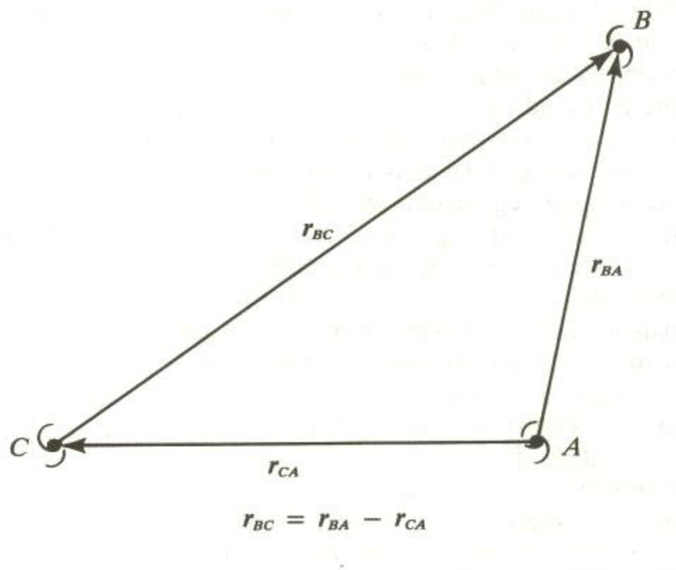
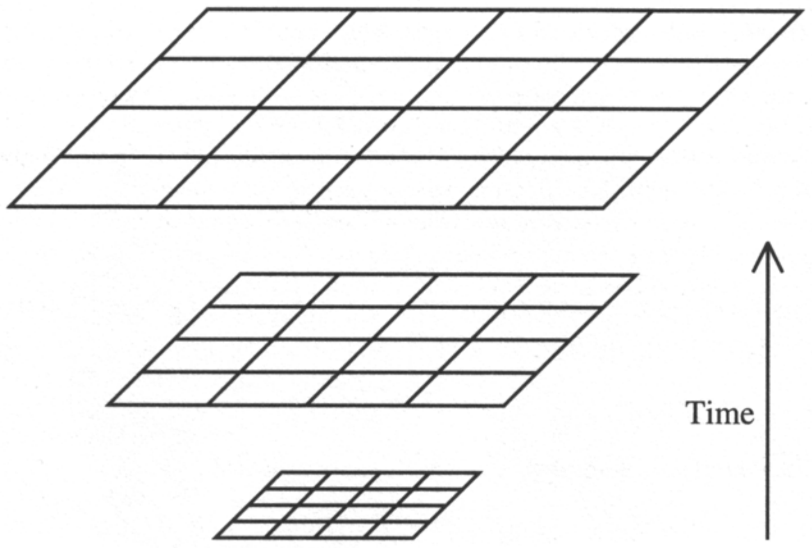
Chapter 2

Standard Cosmology

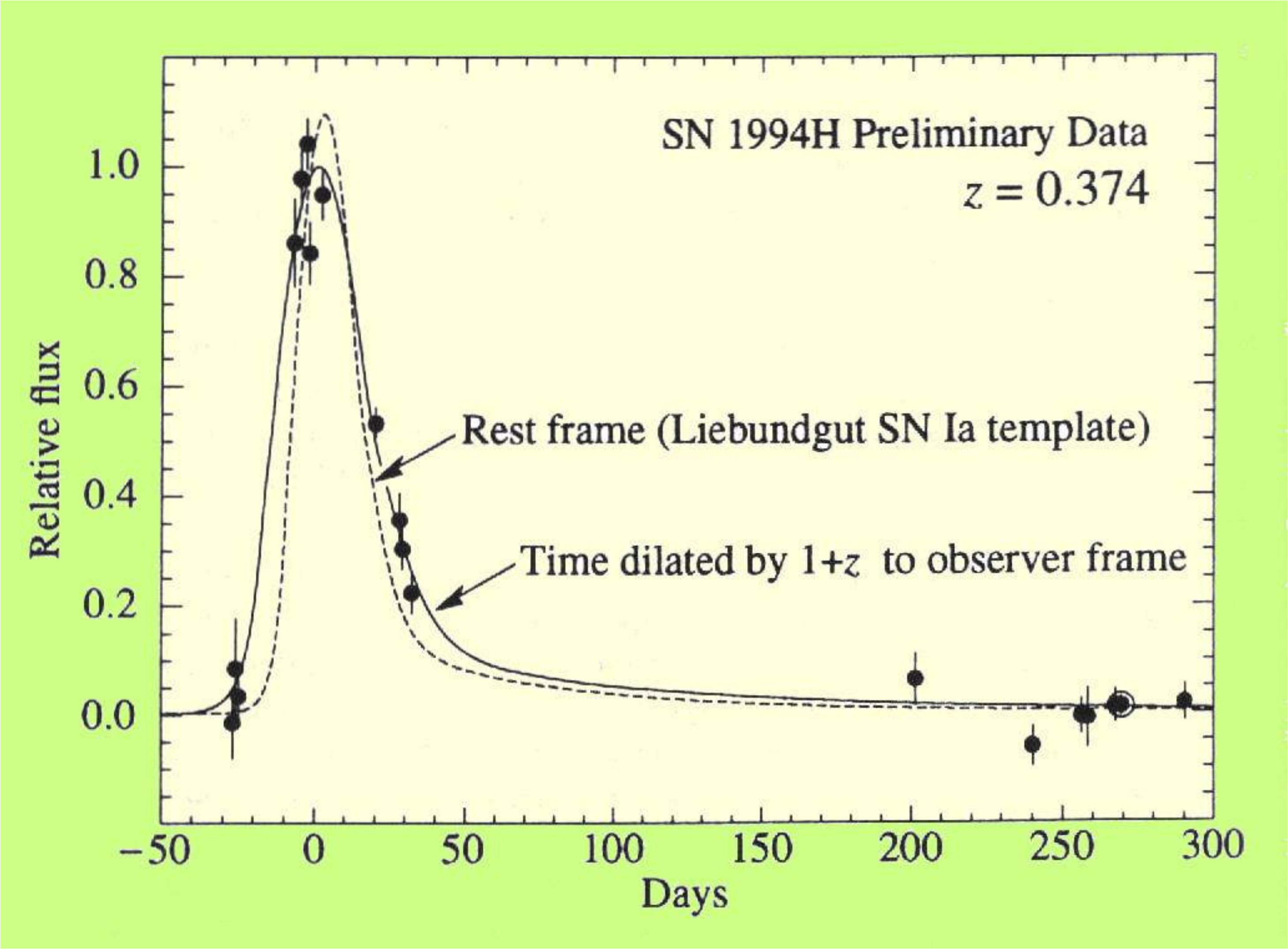
Hubble expansion



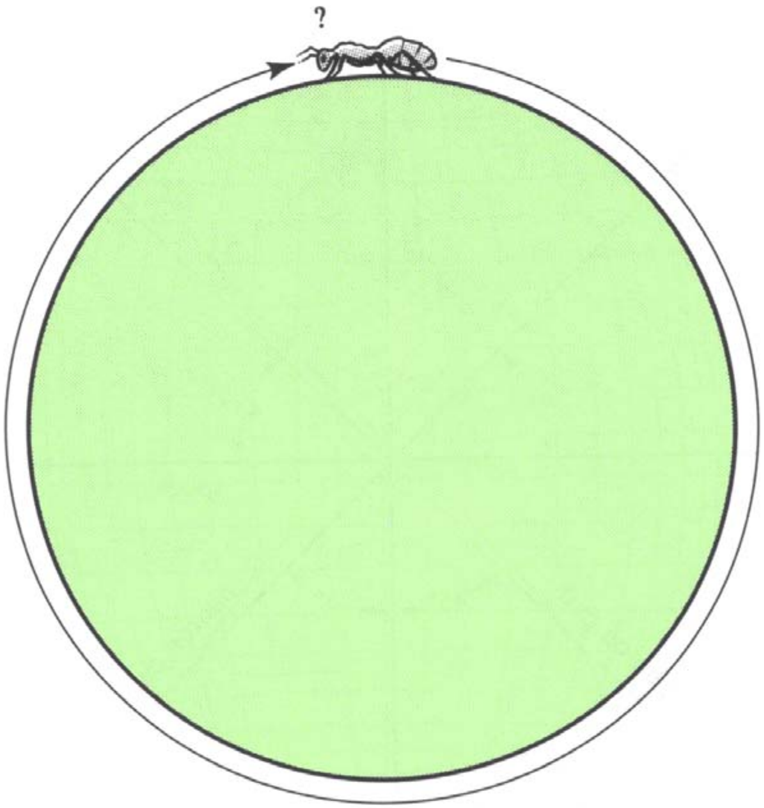


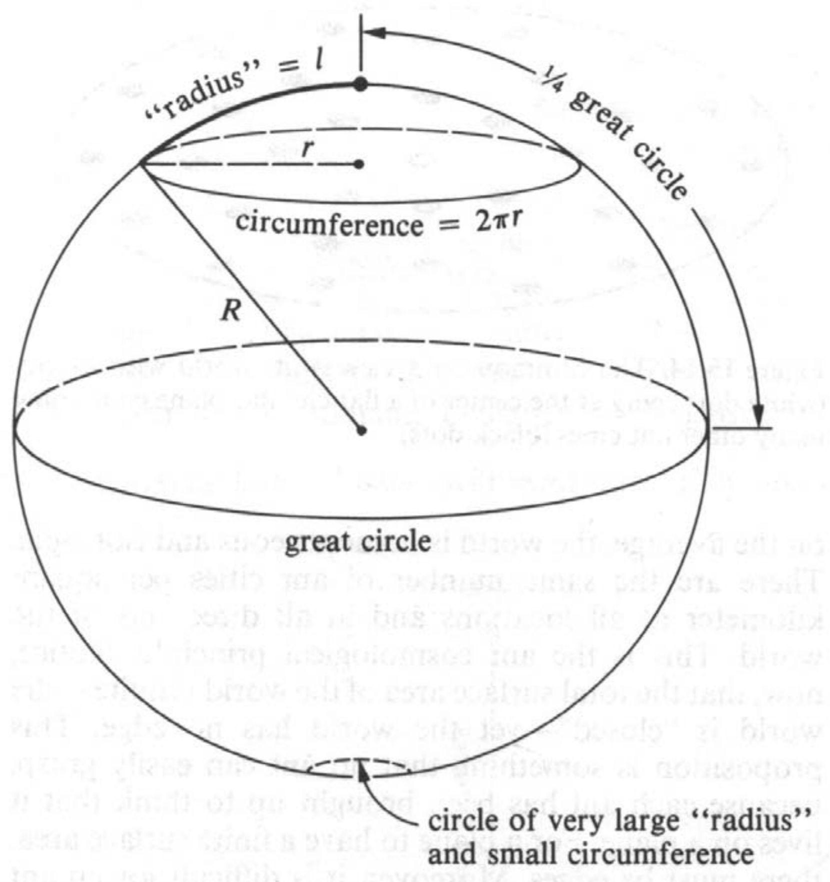
$$r(t) = a(t) \cdot r(t_0) = a(t) \cdot r_0$$

Time dilation

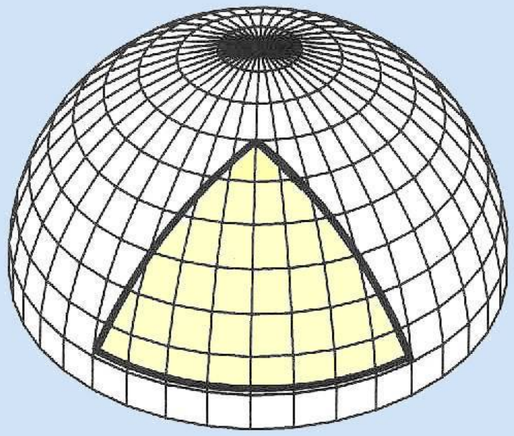


Innocent beings



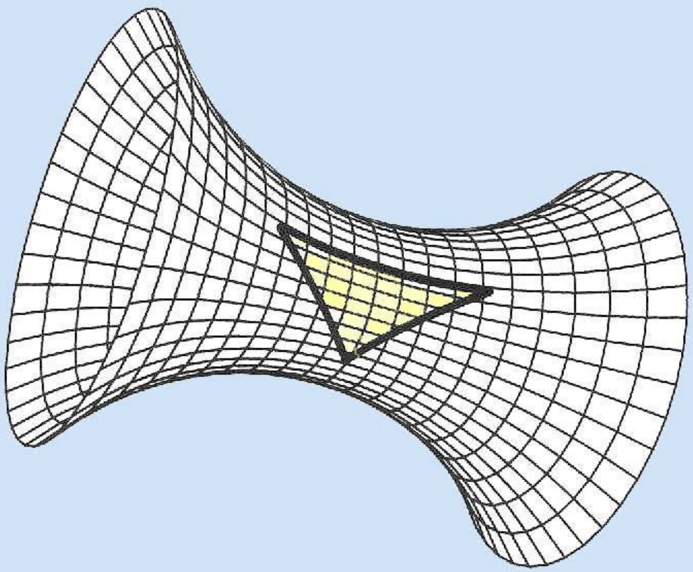


Curved space



positively curved space

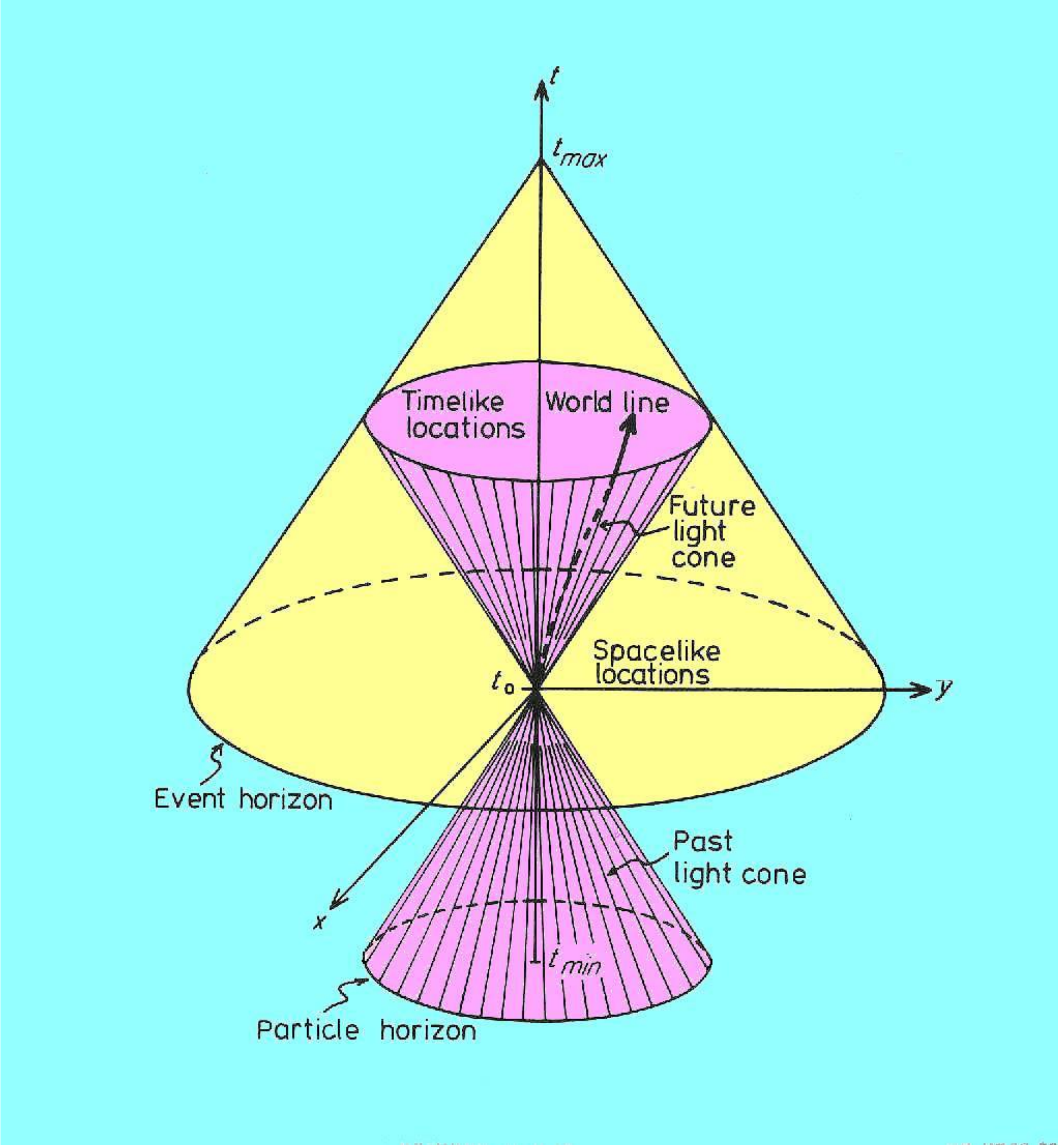
$$\sum_{triangles} > 180^0$$



negatively curved space

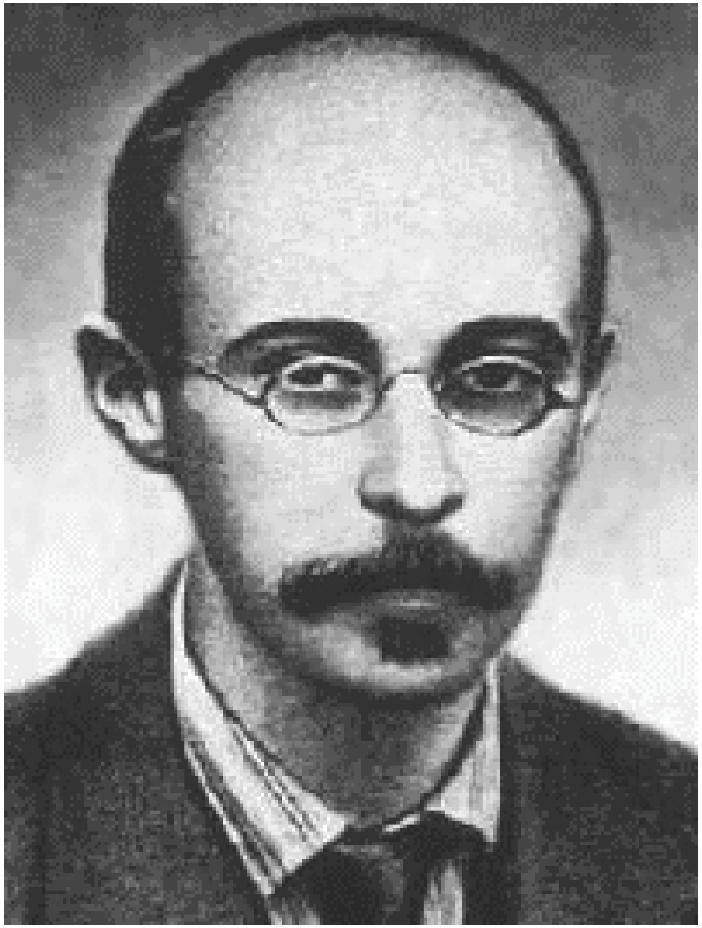
$$\sum_{triangles} < 180^0$$

Spacetime



t
t_{max}
Timelike locations
World line
Future light cone
Spacelike locations
t_0
y
Event horizon
Past light cone
x
t_{min}
Particle horizon

Some history



Aleksander Friedman(n)

1888 - 1925

1915 Einstein GRT

1917 Einstein: cosmological constant

1922 Friedman's first paper (GRT + RWM)

1924 Friedman's 2nd paper

→ expanding universes with open and closed geometries

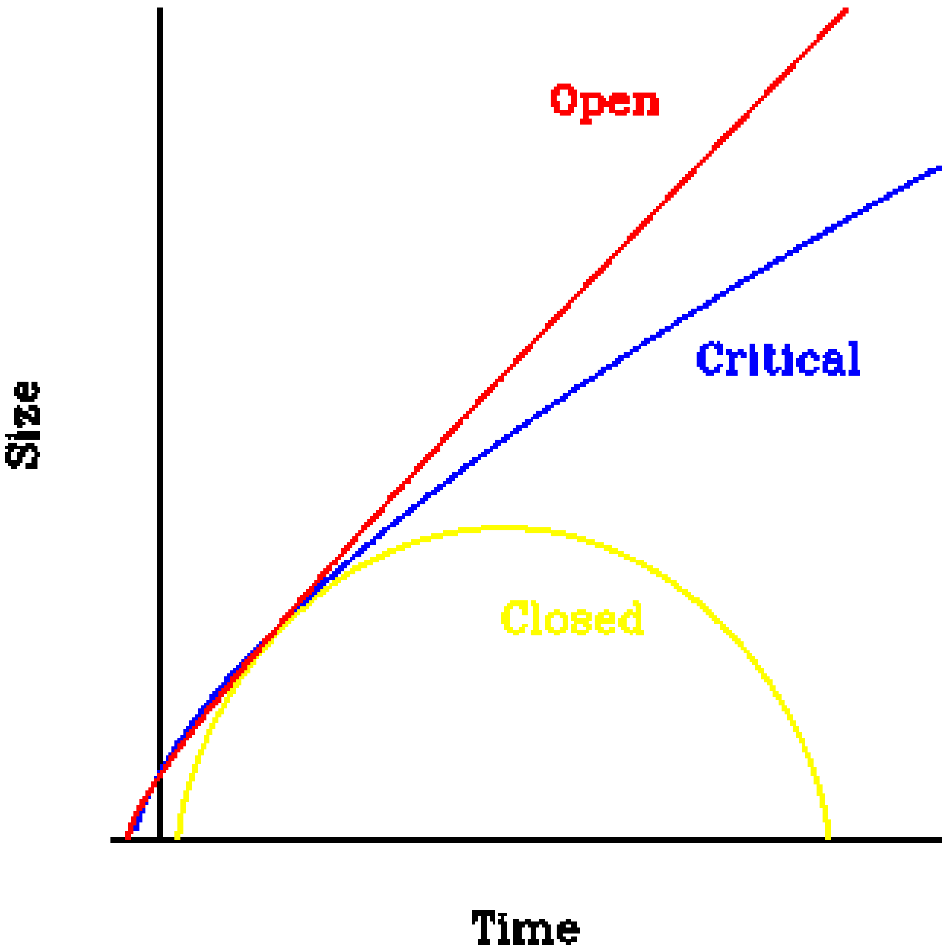
1925 Friedman dies

1927 Lemaitre "rediscovers" Friedman's findings

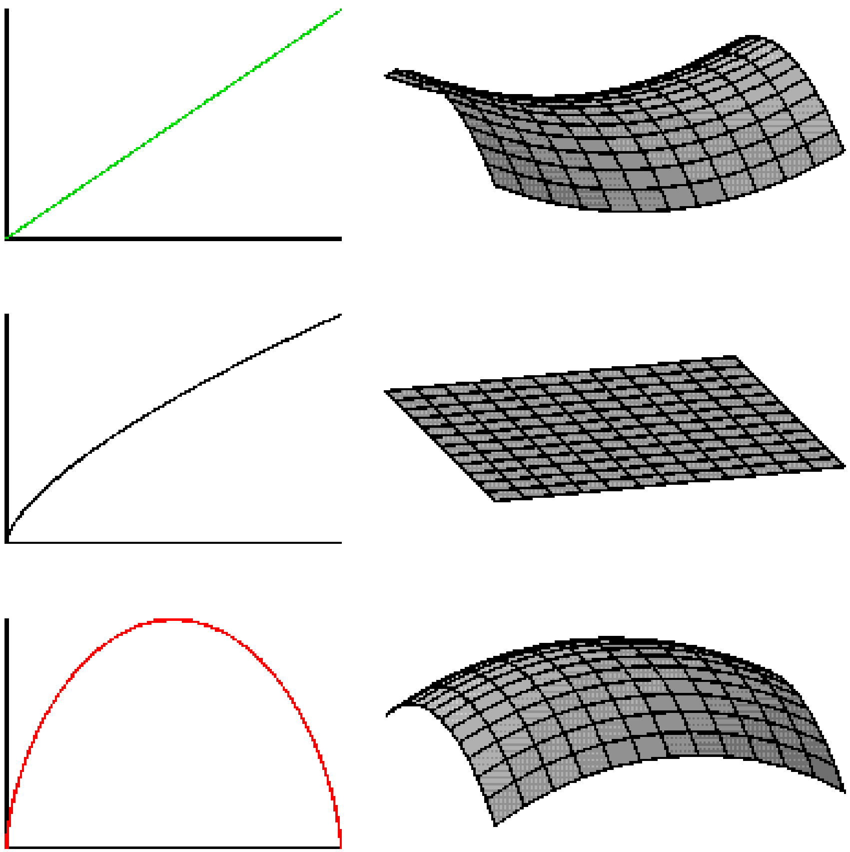
1929 Hubble: receeding galaxies

➔ Einstein withdraws cosm. constant ("my biggest blunder")

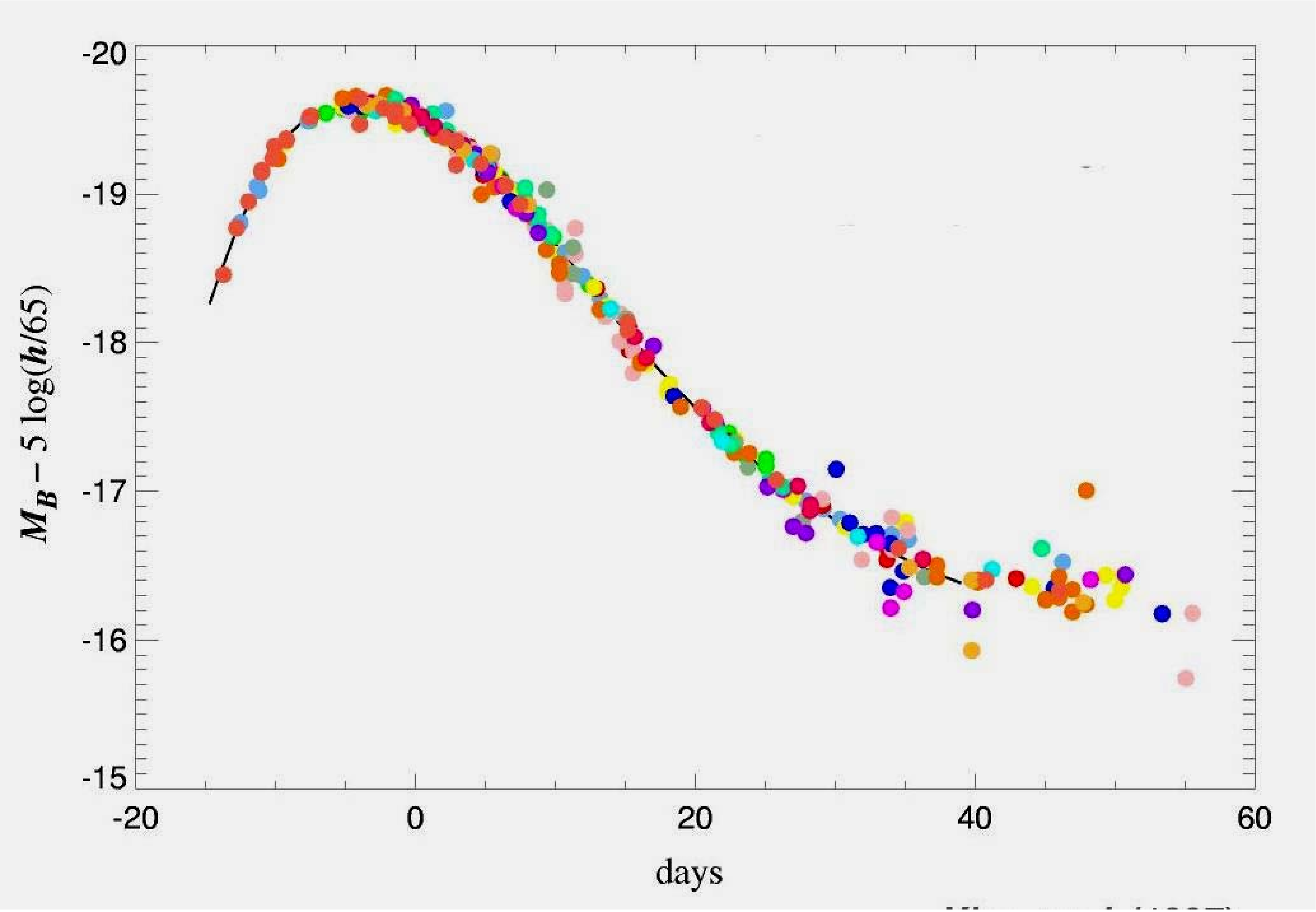
Evolution of the universe in “dust models” Λ = 0



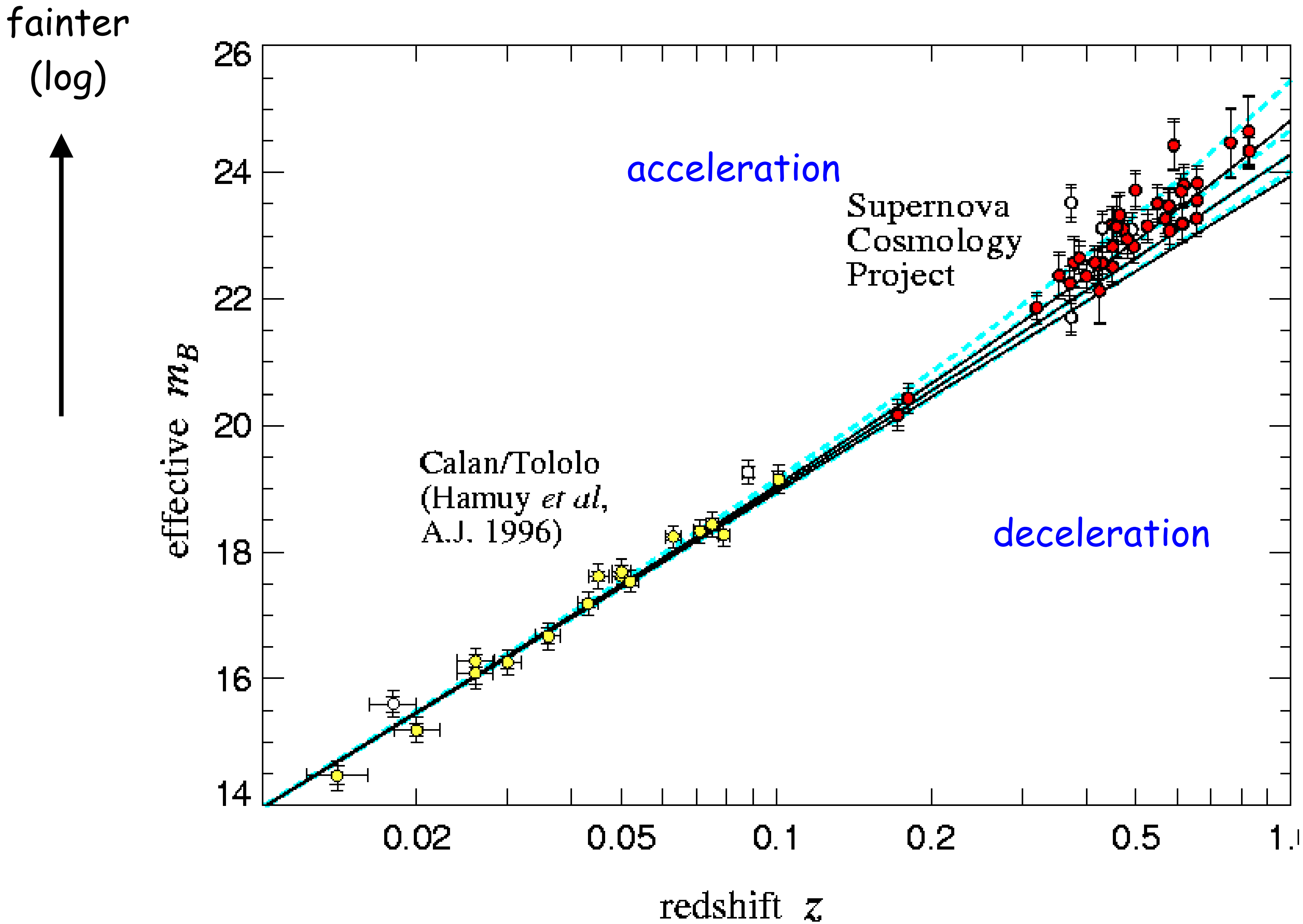
2D analogy



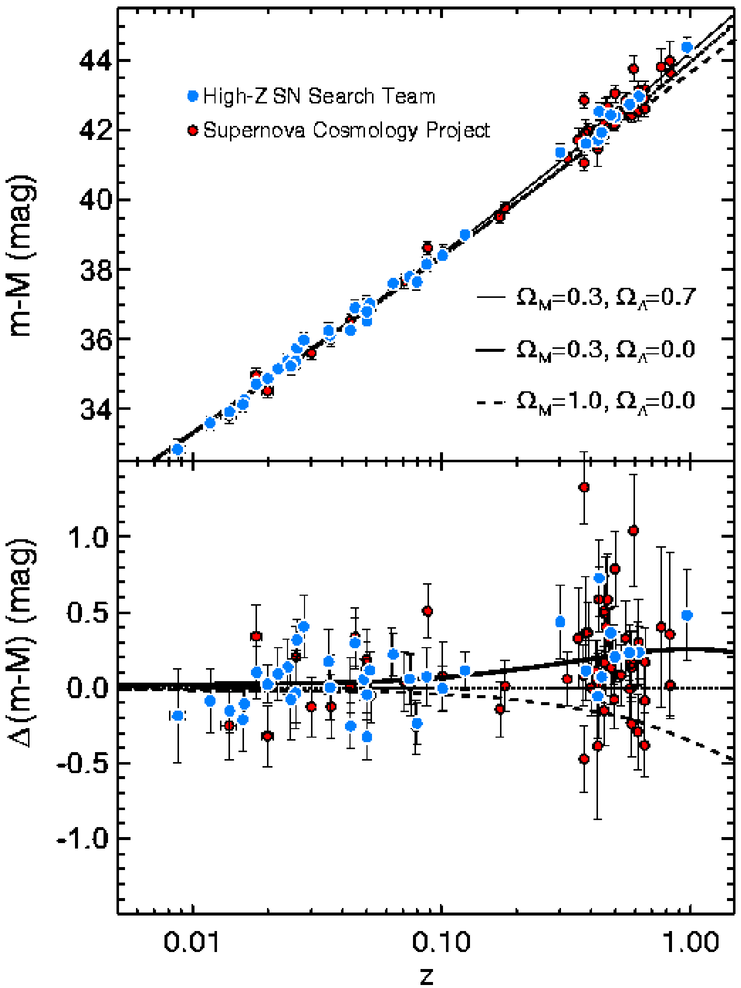
Supernovae as candles (identical light curves)



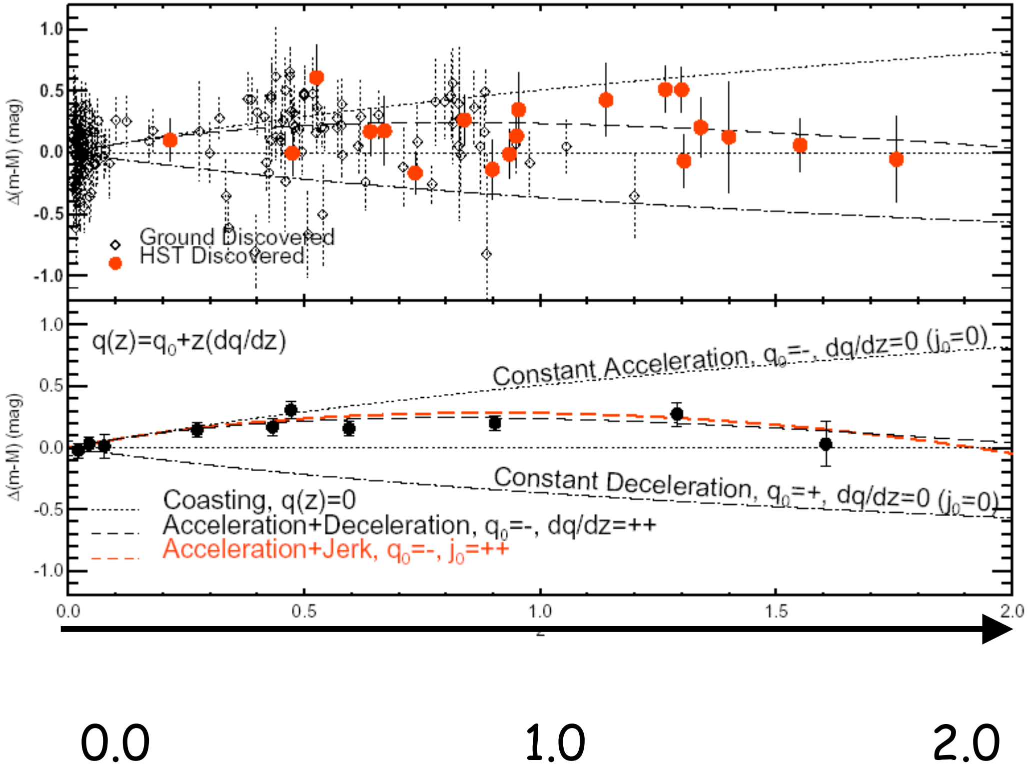
Supernovae (~2001)



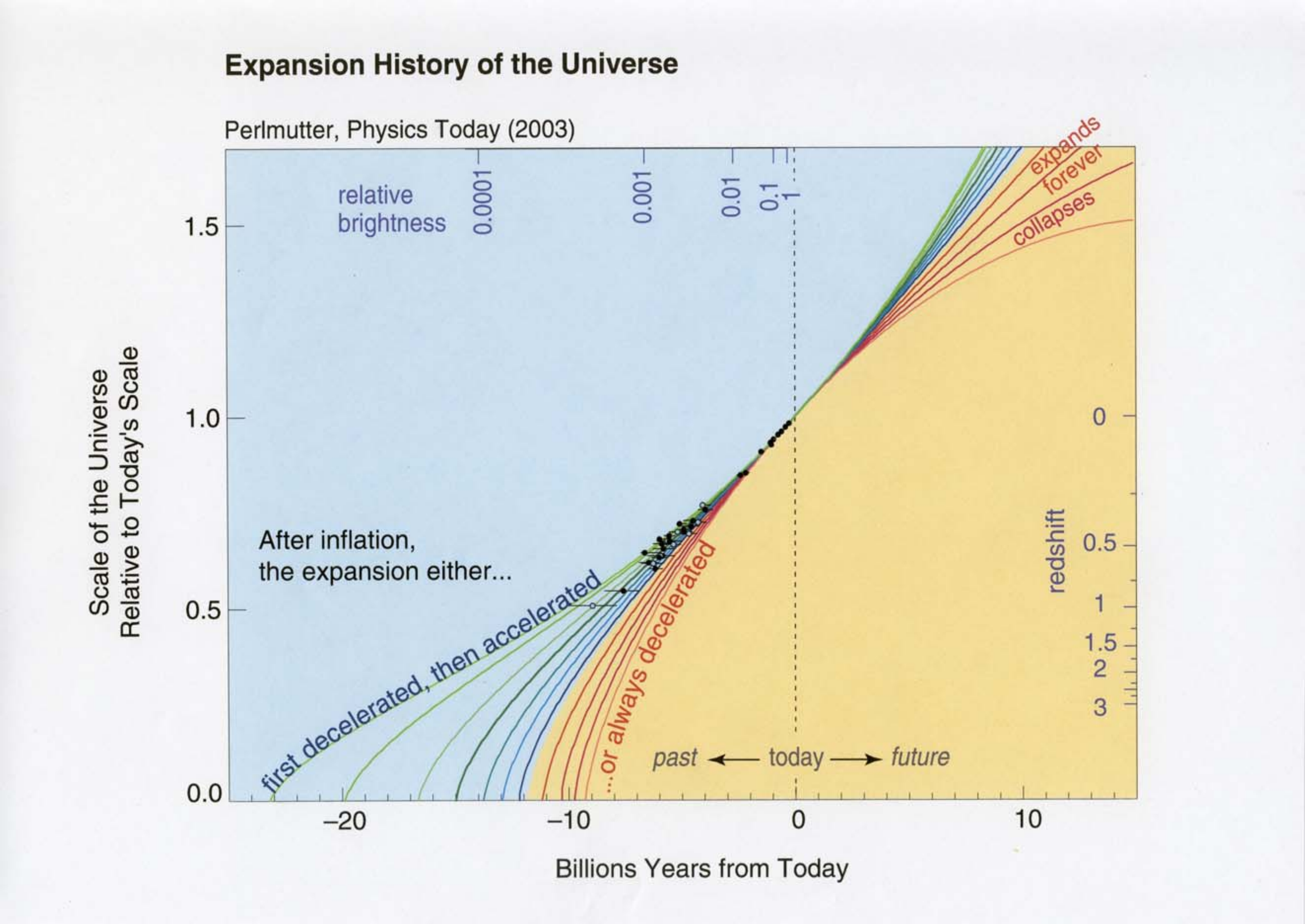
SN Ia red shifts



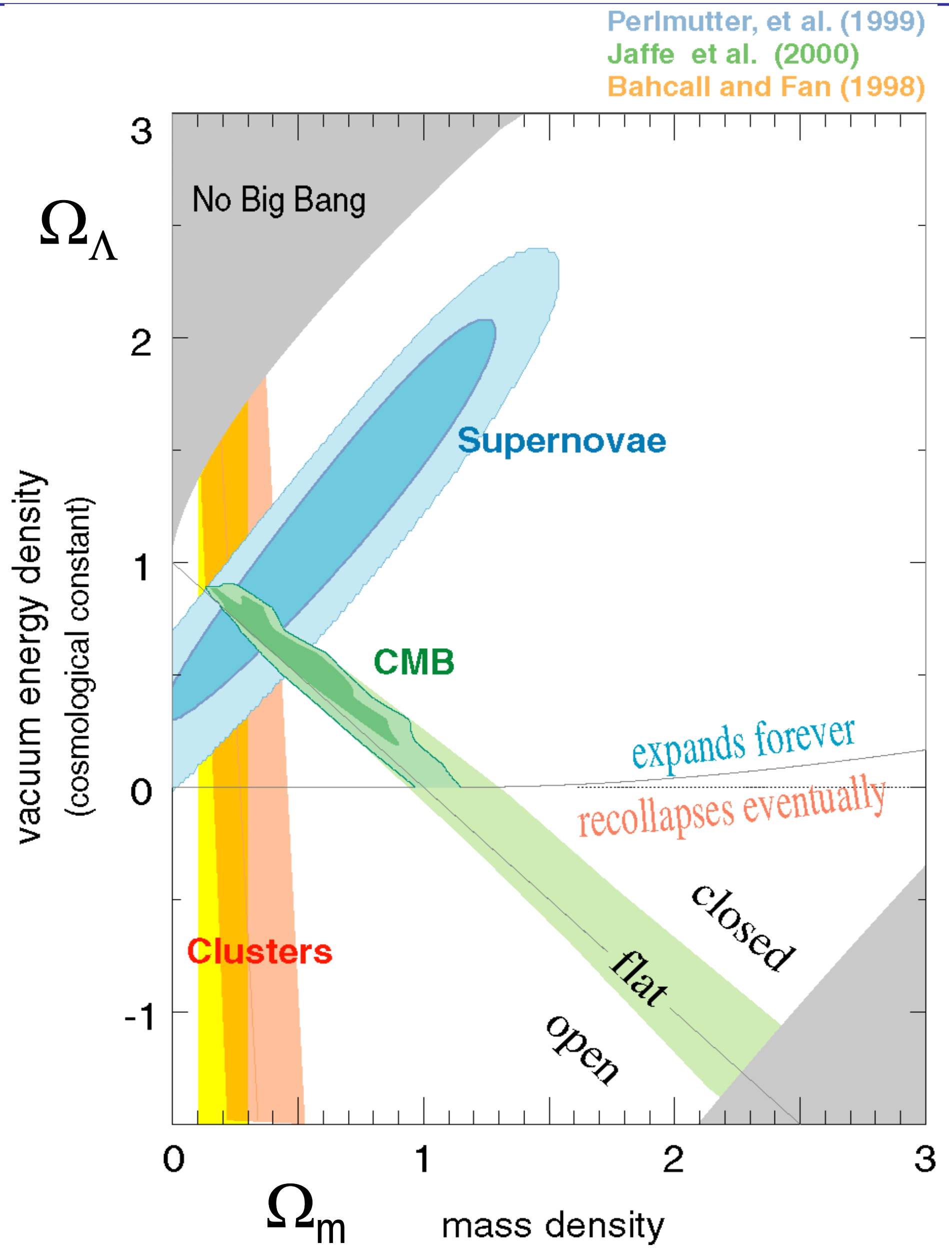
HST: Feb 2004



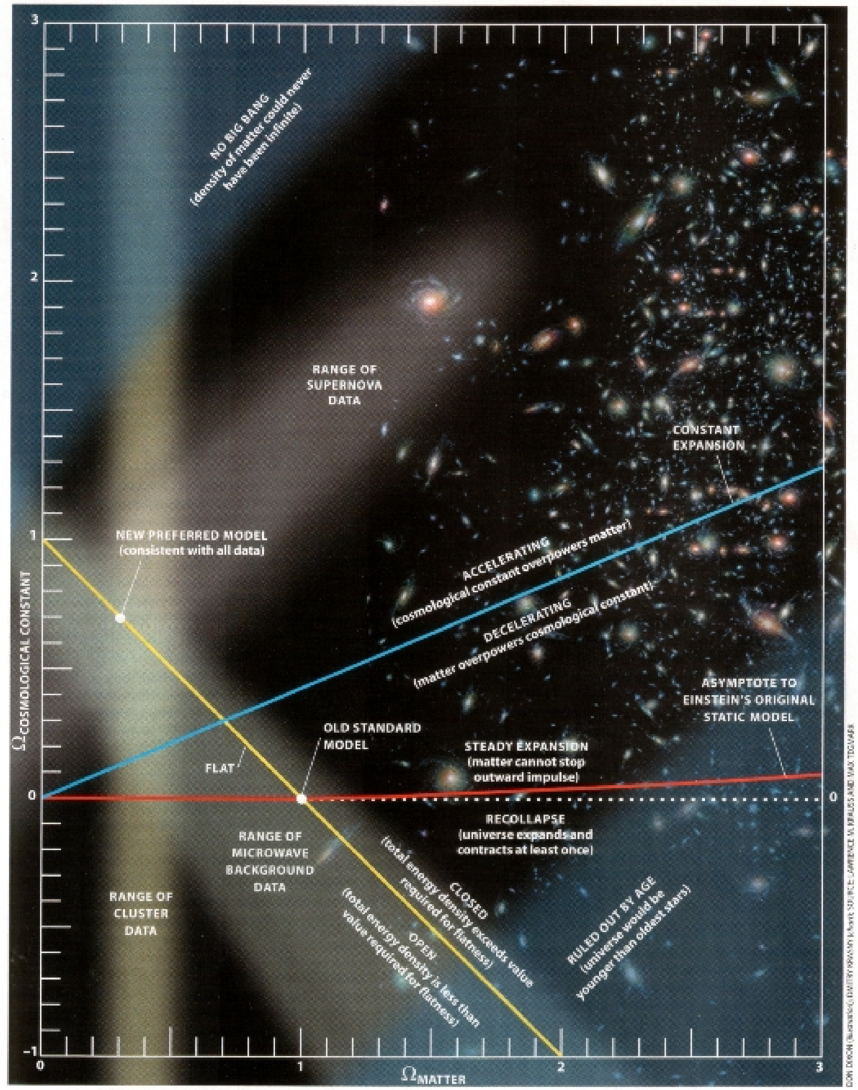
Past Deceleration, Present Acceleration



The present state of knowledge …



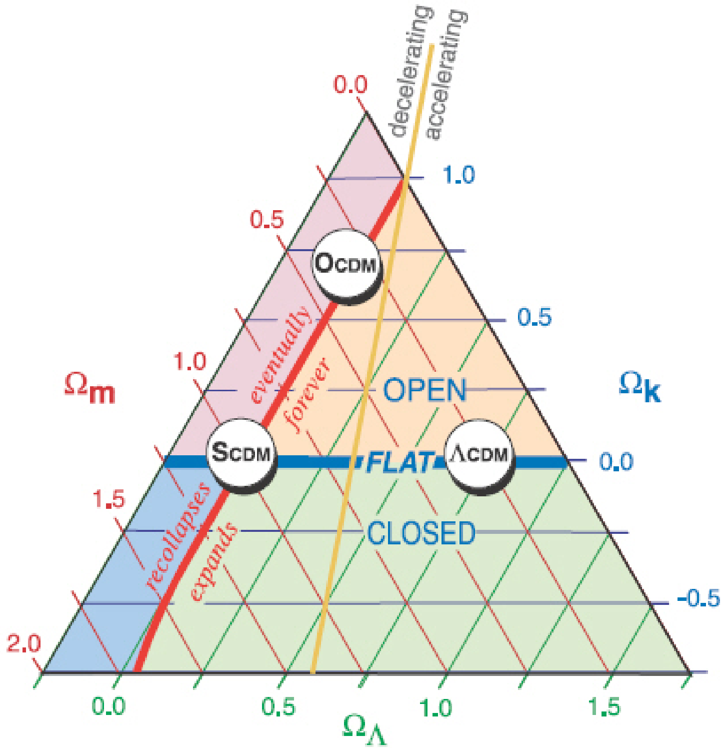
HST SN-Ia and Cosmic Sum Rule



MAP OF MODELS shows how the unfolding of the universe depends on two key cosmological quantities: the average density of matter (*horizontal axis*) and the density of energy in the cosmological constant (*vertical axis*). Their values, given here in standard cosmological units, have three distinct effects. First, their sum (which represents the total cosmic energy content) determines the geometry of space-time (*yellow line*). Second, their difference (which represents the relative strength of expansion and gravity) determines how the expansion rate changes over time (*blue line*). These two effects have been probed by recent observations (*shaded regions*). The third, a balance of the two densities, determines the fate of the universe (*red line*). The three effects have many permutations—unlike the view of cosmology in which the cosmological constant is assumed to be zero and there are only two possible outcomes.

Cosmological Antigravity SCIENTIFIC AMERICAN January 1999 39

The Cosmic Triangle



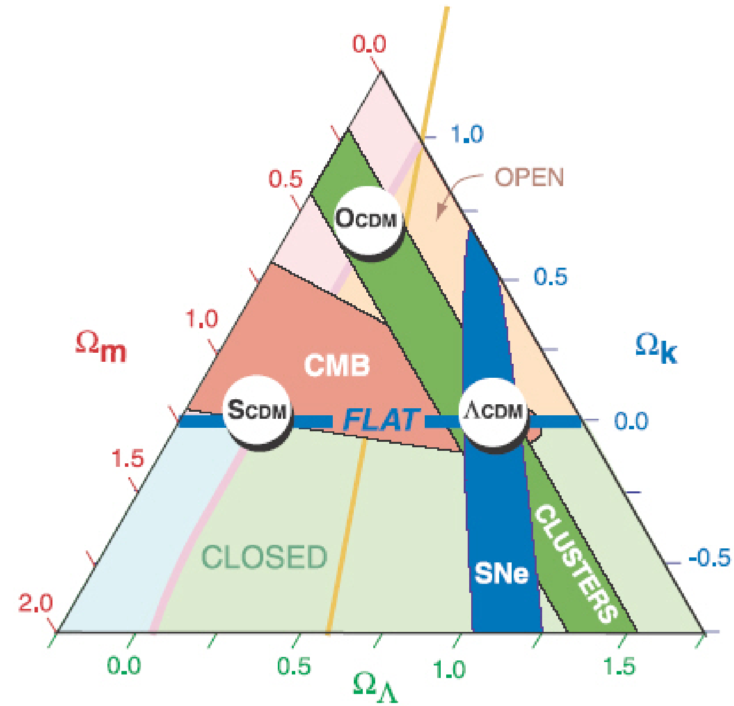
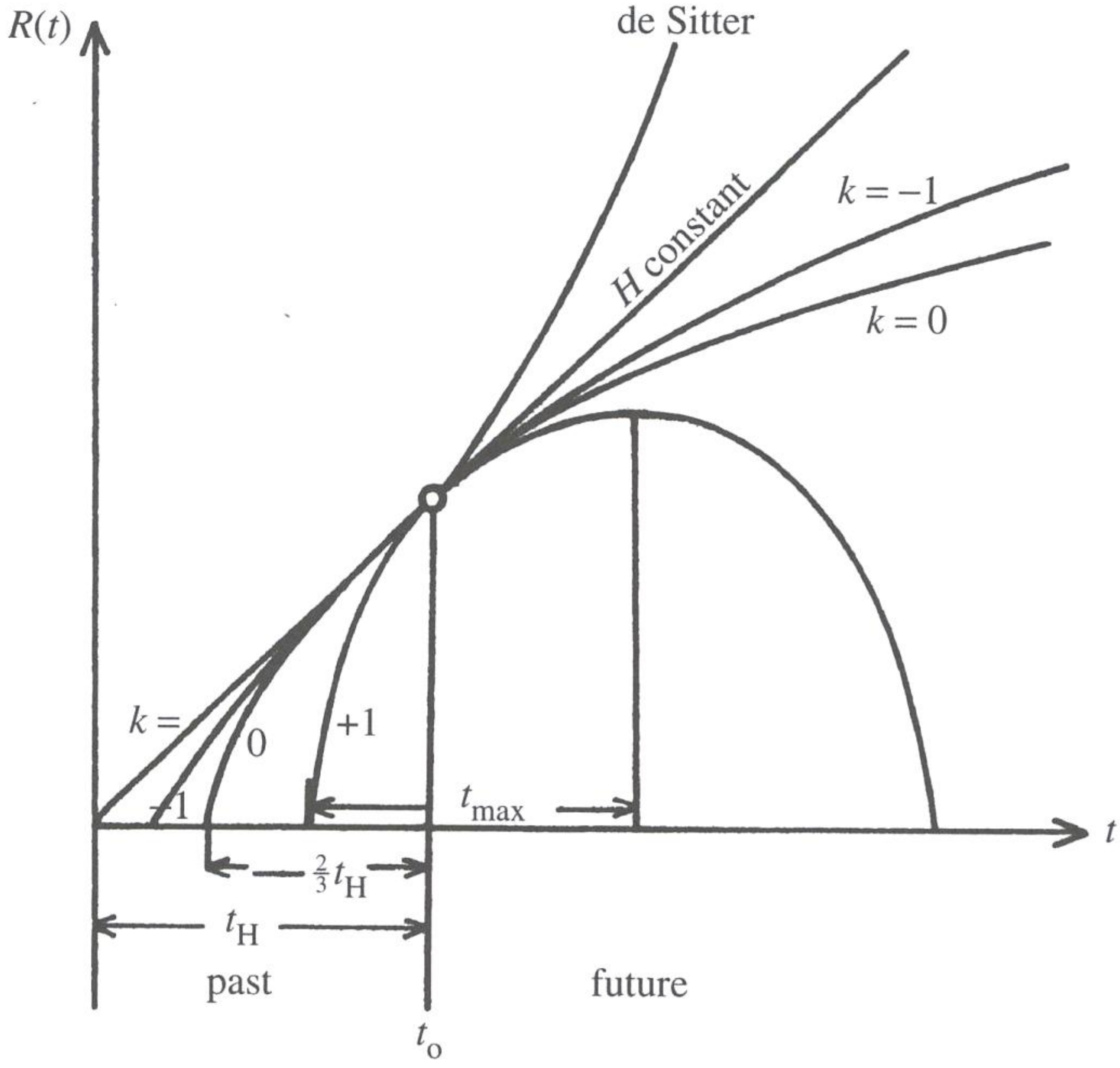
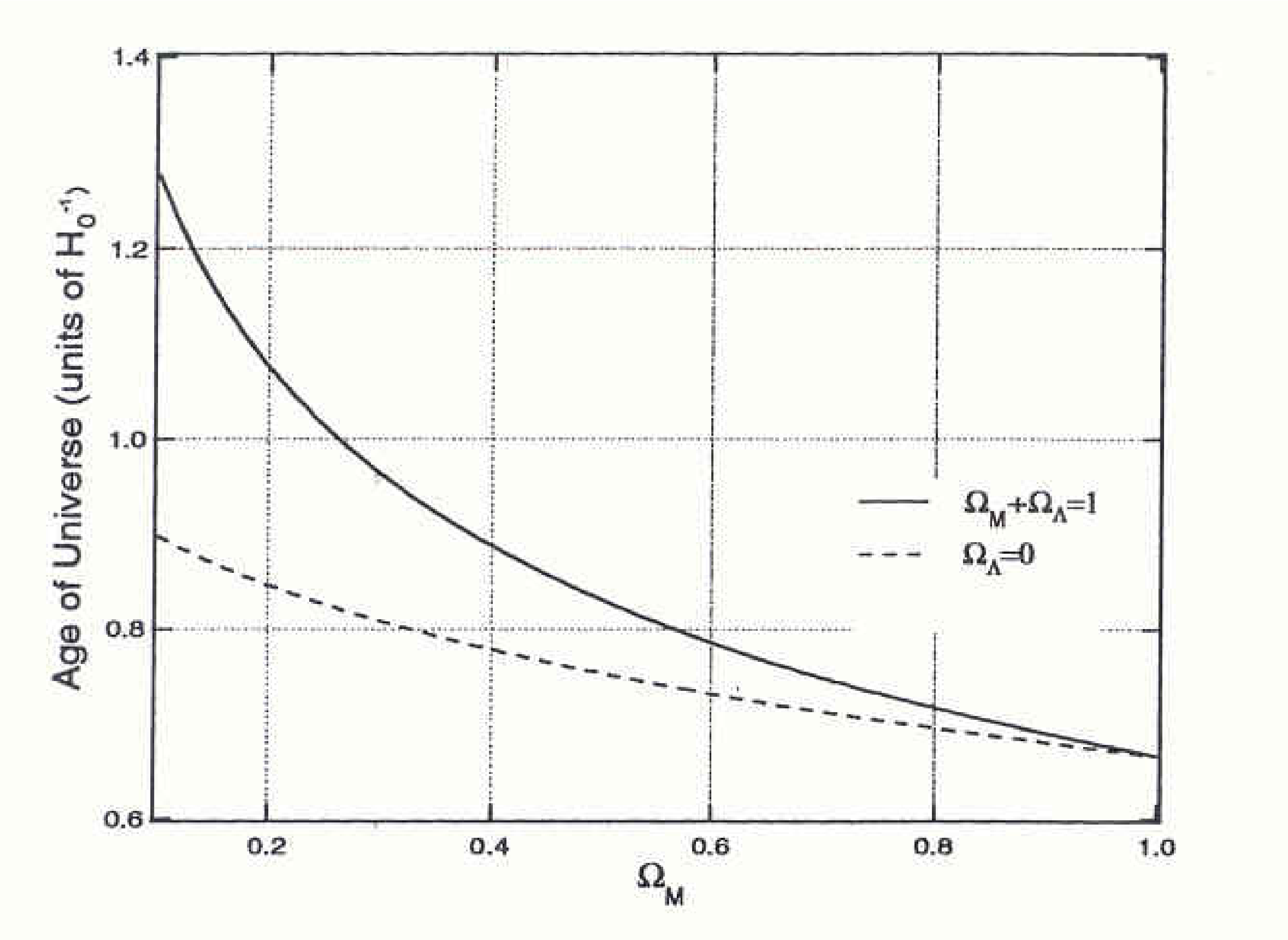


Fig. 1 (left). The Cosmic Triangle. This triangle represents the three key cosmological parameters (Ω_m, Ω_Λ, and Ω_k), where each point in the triangle satisfies the sum rule $\Omega_m + \Omega_\Lambda + \Omega_k = 1$. The horizontal line (marked "FLAT") corresponds to a flat universe ($\Omega_m + \Omega_\Lambda = 1$), separating an open universe from a closed one. The red line, nearly along the $\Lambda = 0$ line, separates a universe that will expand forever (approximately $\Omega_\Lambda > 0$) from one that will eventually recollapse (approximately $\Omega_\Lambda < 0$). And the yellow, nearly vertical line separates a universe with an expansion rate that is currently decelerating from one that is accelerating. The locations of three key models are highlighted: SCDM, dominated by matter ($\Omega_m = 1$) and no curvature or cosmological constant; flat (ΛCDM), with $\Omega_m = 1/3$, $\Omega_\Lambda = 2/3$, and $\Omega_k = 0$; and OCDM, with $\Omega_m = 1/3$, $\Omega_\Lambda = 0$, and $\Omega_k = 2/3$. (The variant tilted TCDM model is identical in its position to SCDM.) **Fig. 2 (right). The Cosmic Triangle Observed.** This triangle represents current observational constraints. The tightest constraints from measurements at low redshift (clusters, including the mass-to-light method, baryon fraction, and cluster abundance evolution), intermediate redshift (SNe), and high redshift (CMB) are shown by the three color bands (each representing 1σ uncertainties). Other tests that we discuss are consistent with but less constraining than the constraints illustrated here. The cluster constraints indicate a low-density universe, the SNe constraints indicate an accelerating universe, and the CMB measurements indicate a flat universe. The three independent bands intersect at a flat model with $\Omega_m \approx 1/3$ and $\Omega_\Lambda \approx 2/3$; the model contains a cosmological constant or other dark energy.

Age of the universe - I

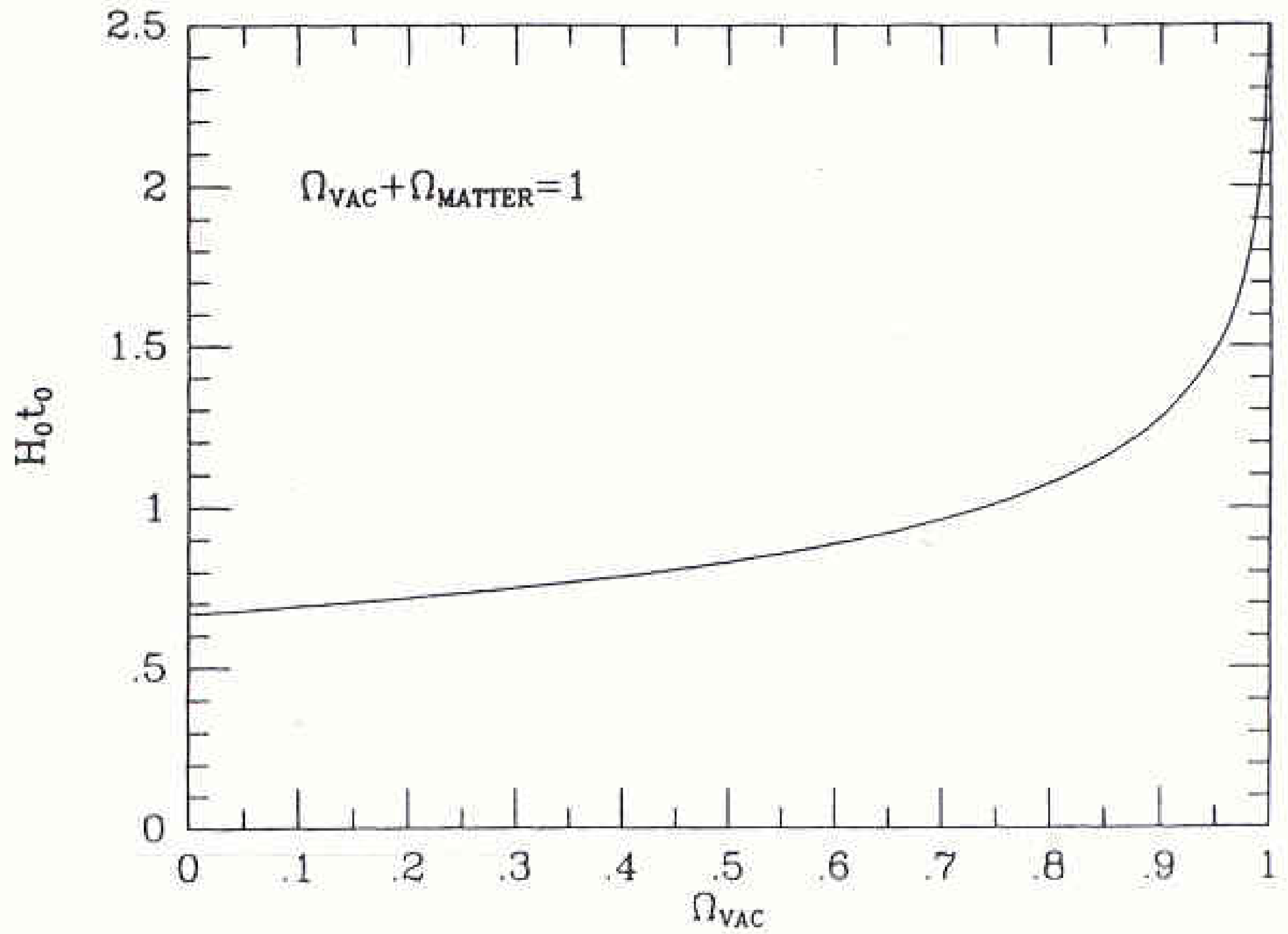


Age of the universe - II



Age of the universe - III

age becomes larger with increasing Ω_Λ



Age of the universe - IV

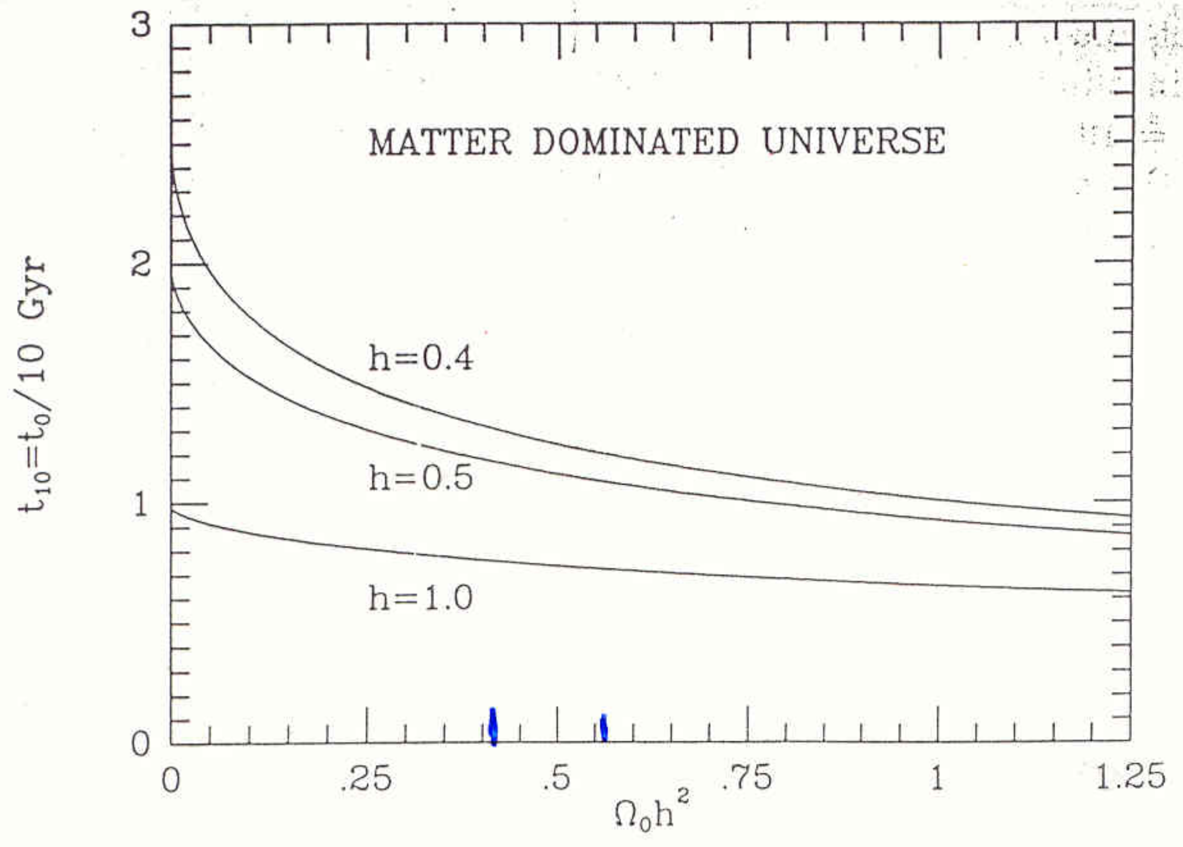


Fig. 3.3: The age of a matter-dominated Universe as a function of $\Omega_0 h^2$.

Solution to the Friedman Eqn

Scale factors $R(t)$ and critical values of the cosmological constant Λ in Friedmann universes

James E. Felten*

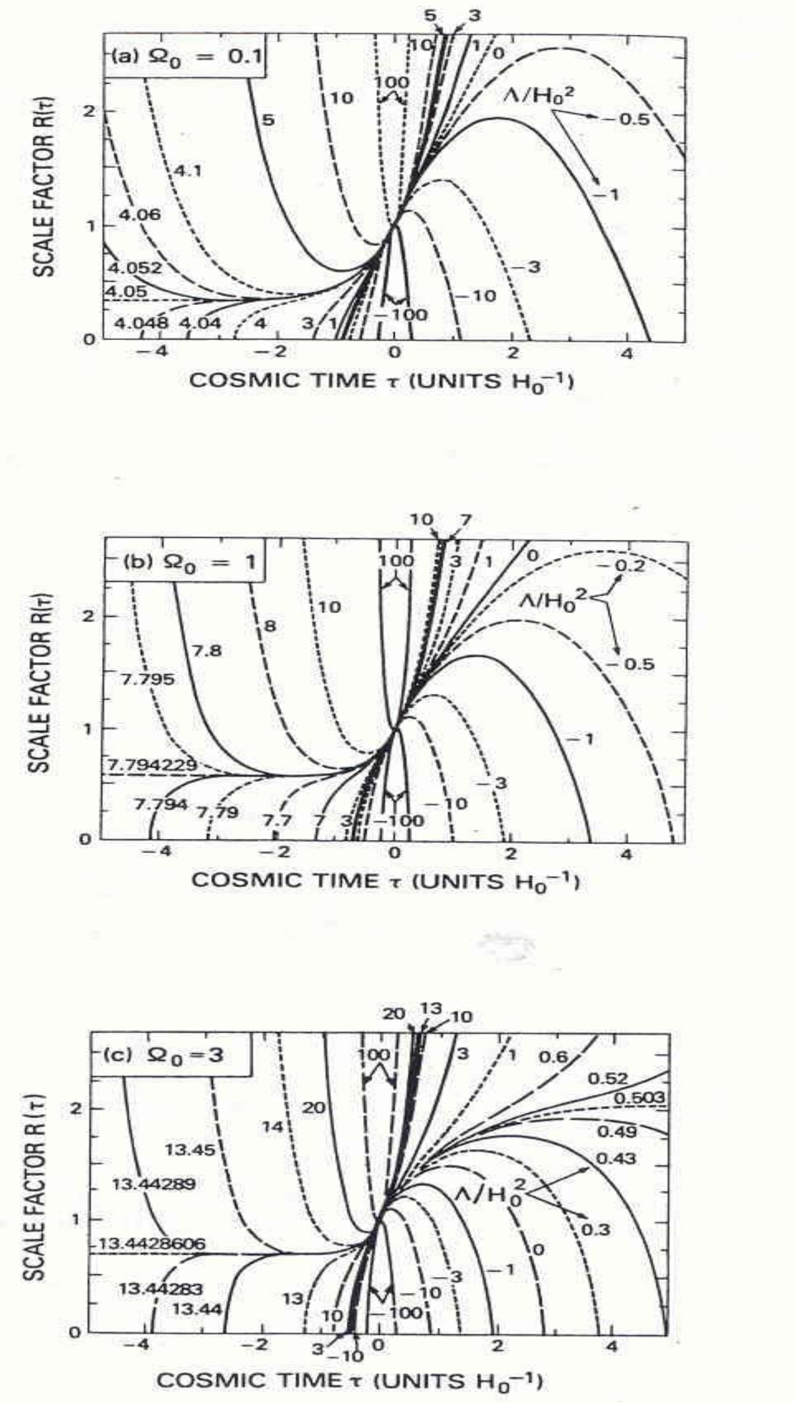
Code 697, NASA Goddard Space Flight Center, Greenbelt, Maryland 20771 and Astronomy Program, University of Maryland, College Park, Maryland 20742

Richard Isaacman

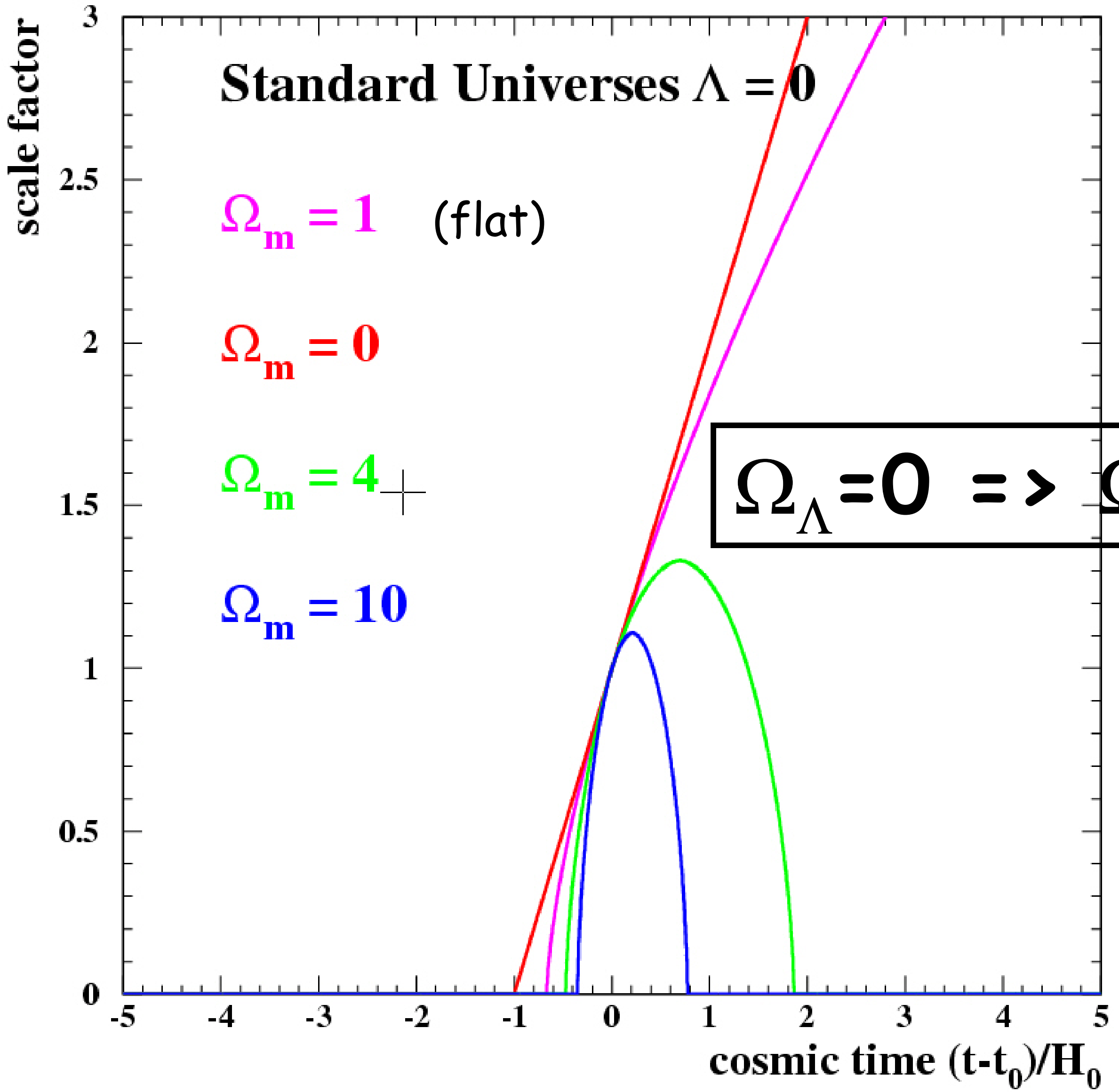
Applied Research Corporation, 8201 Corporate Drive, Landover, Maryland 20785

The authors review the equations, notational choices, and confusing terminology of the Friedmann (zero-pressure) and Lemaître cosmological models, retaining cgs units as far as practical and in particular retaining units cm^{-2} for the present Gaussian curvature K_0 of three-space. They integrate the Friedmann equation numerically, requiring solutions to match the present Hubble parameter H_0 and mass-density ("closure") parameter Ω_0 at present time $t_0=0$, and generate families of curves showing the scale factor $R(\tau)$ (with $R_0=1$) vs τ (time in units H_0^{-1}) for fixed Ω_0 and various values of the cosmological constant Λ (in units H_0^2). These unusual graphs show the continuity of the solutions and the physical significance of Λ. Families for several values of Ω_0 exhibit known but unfamiliar features. The authors also show the family of "standard models" ($\Lambda=0$) and the family satisfying the "inflationary constraint" ($K_0=0$). They obtain new and simple formulas for the critical value $\Lambda_s(H_0,\Omega_0)$, which separates models with a big bang from those without. Their definition of Λ_s at fixed H_0 and Ω_0 differs from usual practice but proves useful. These formulas also give the quasistatic scale factor R_s and redshift z_s for the corresponding Eddington-Lemaître model, and give R_s and z_s approximately for the neighboring "Lemaître coasting models," which have $\Lambda<\Lambda_s$. The conventional wisdom that $\Lambda=\Lambda_c(1+\varepsilon)$ for the coasting models applies to a different characteristic value Λ_c. A quasistatic state in the future, with a second critical value Λ_{s2}, is possible if $\Omega_0>1$. The parameters Ω_0, Λ/H_0^2, Λ_s/H_0^2, and Λ_{s2}/H_0^2 can be used to classify the Friedmann models.

Rev. Mod. Physics 58 (1986) 689-698

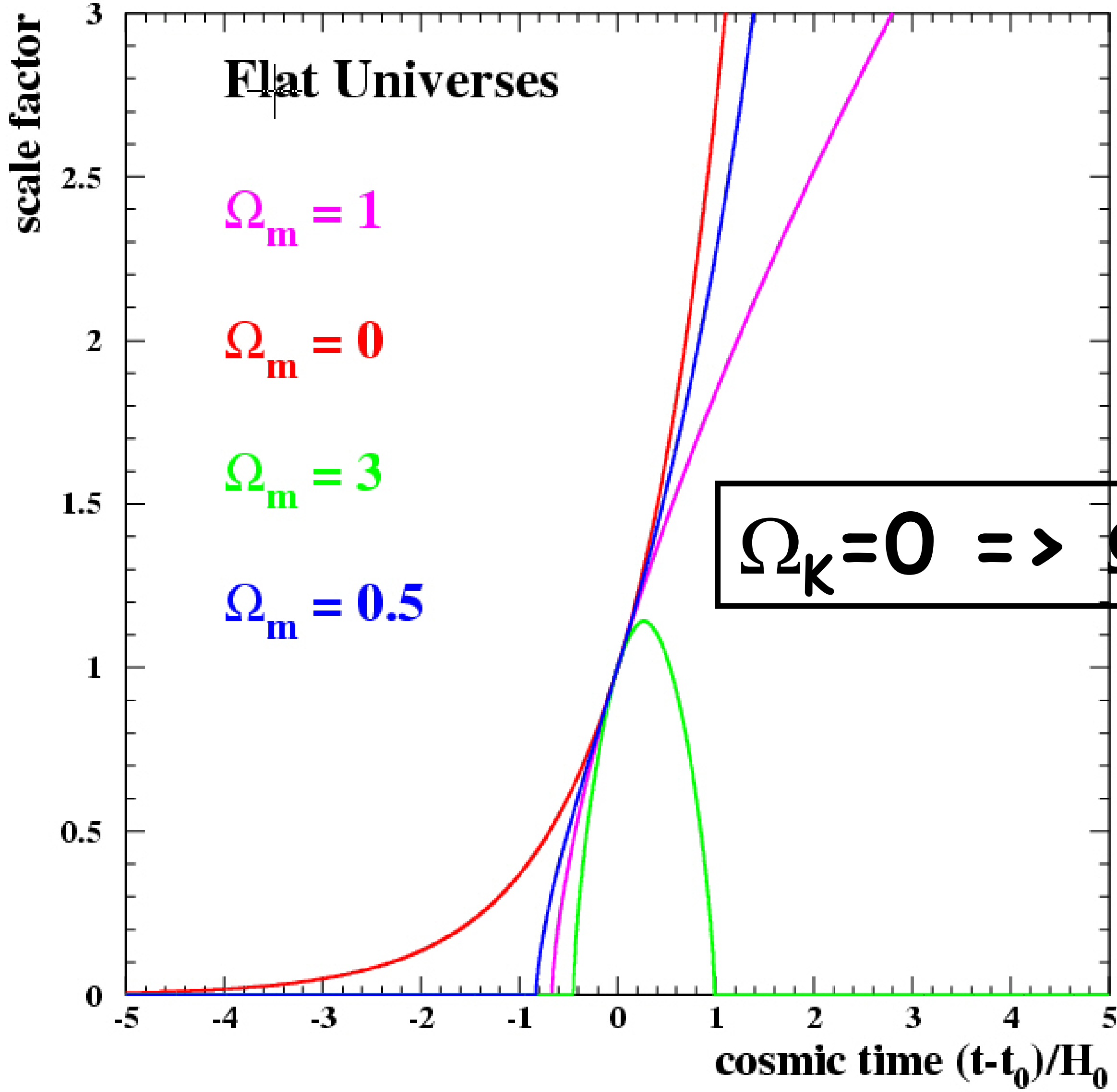


Solution to the Friedman Eqn.- I



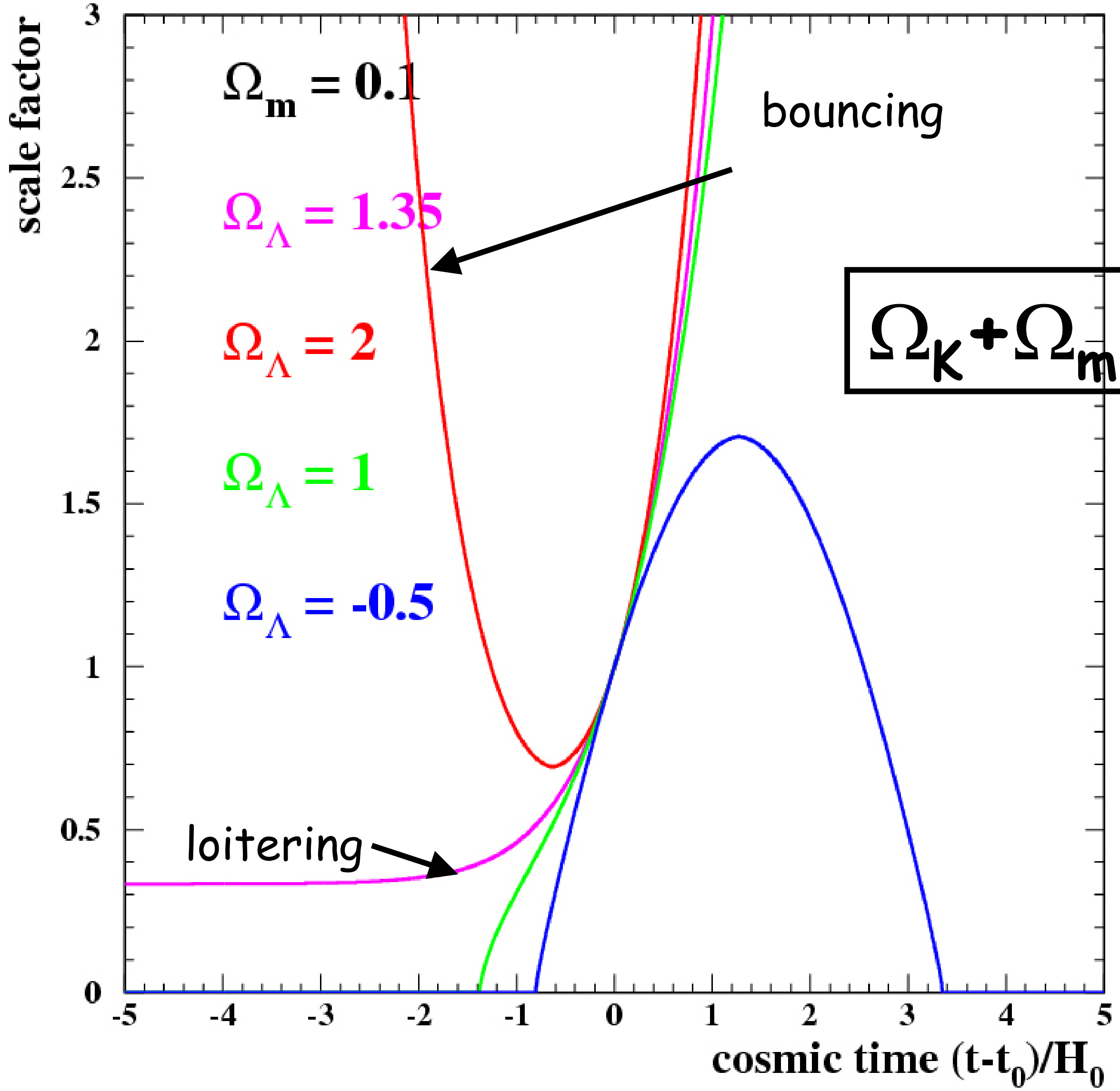
$$\Omega_\Lambda = 0 \Rightarrow \Omega_m + \Omega_K = 1$$

Solution to the Friedman Eqn.- II



$$\Omega_K=0 \Rightarrow \Omega_m+\Omega_\Lambda=1$$

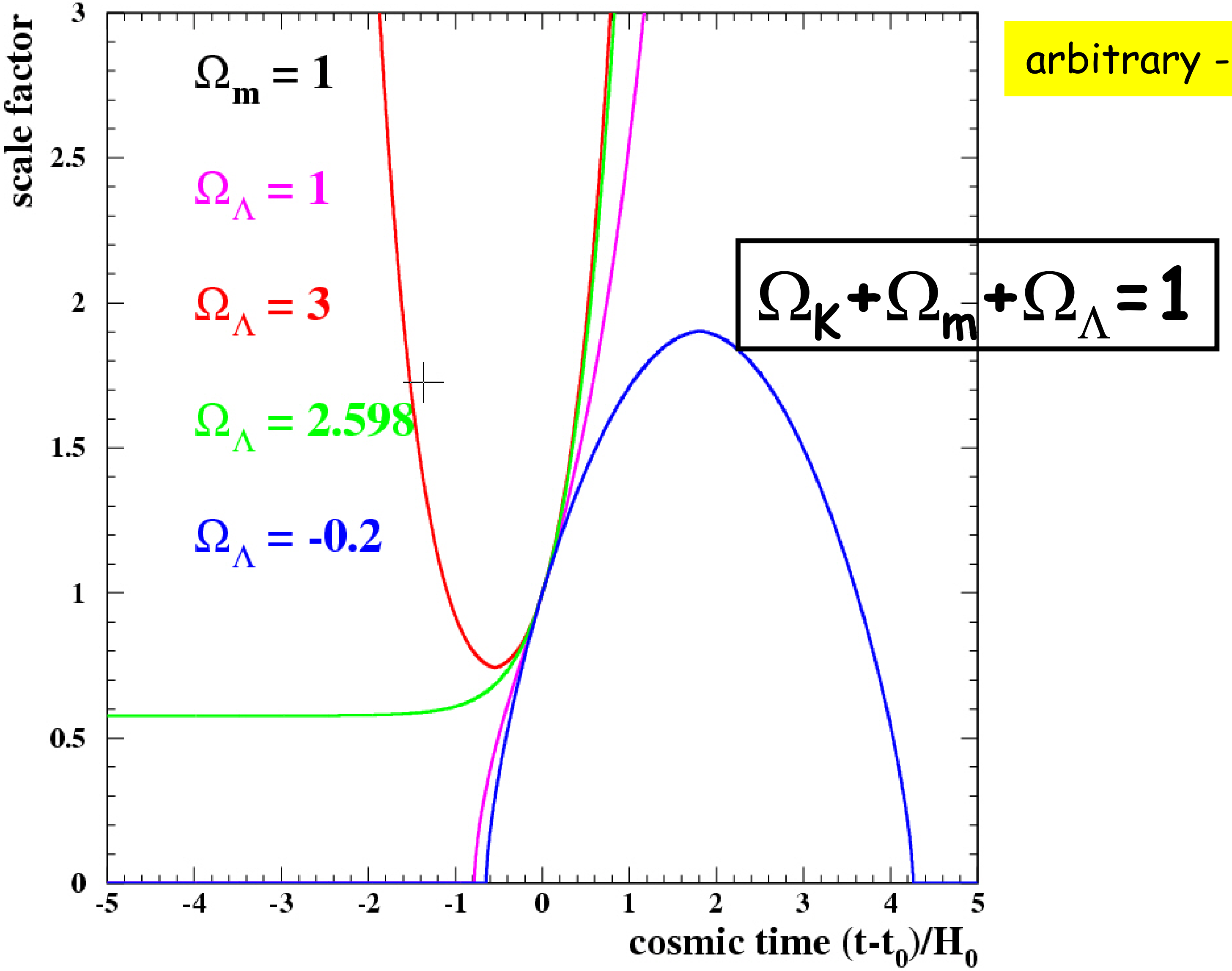
Solution to the Friedman Eqn. - III



arbitrary - I

$$\Omega_K + \Omega_m + \Omega_\Lambda = 1$$

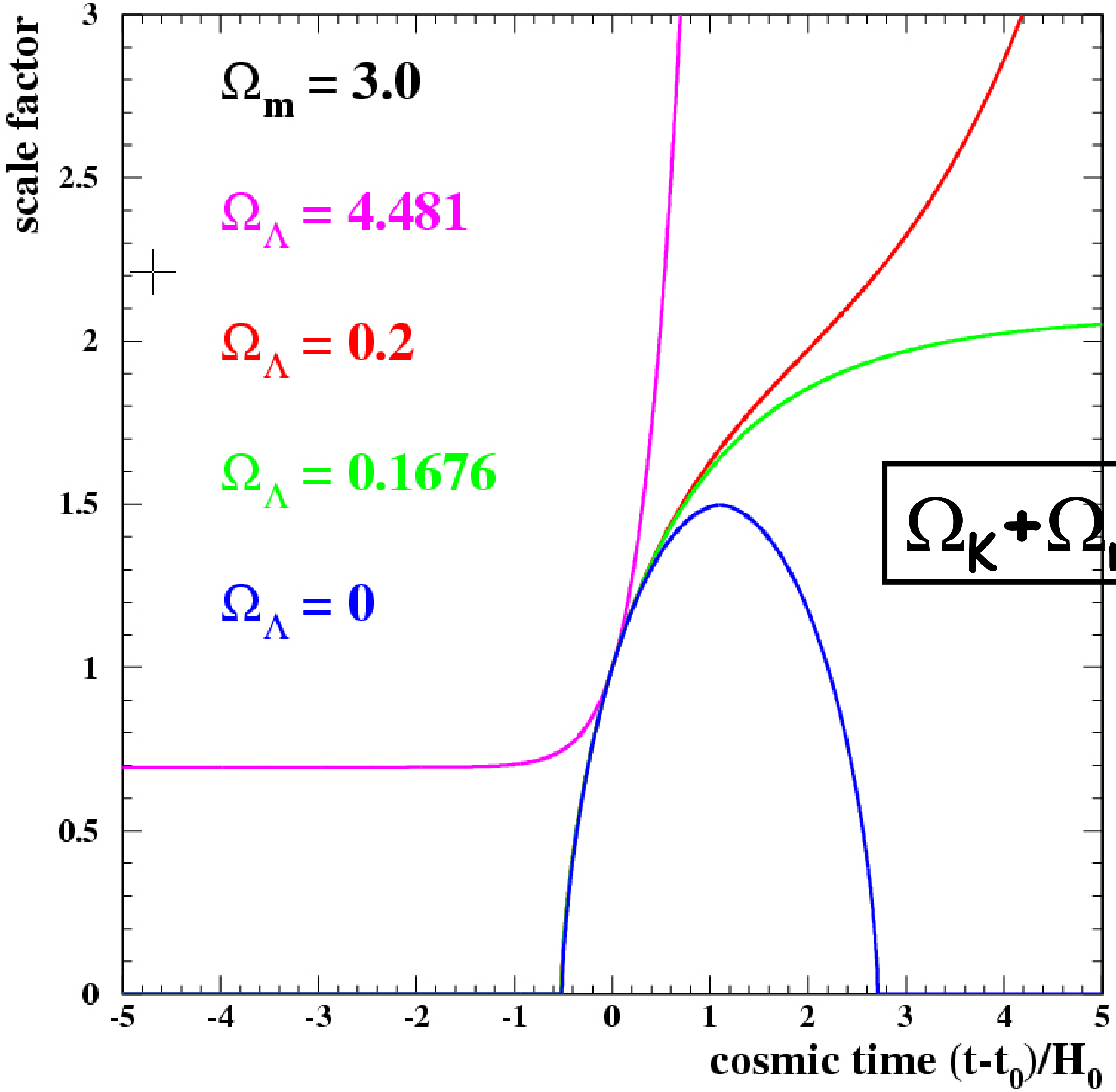
Solution to the Friedman Eqn. - IV



arbitrary - II

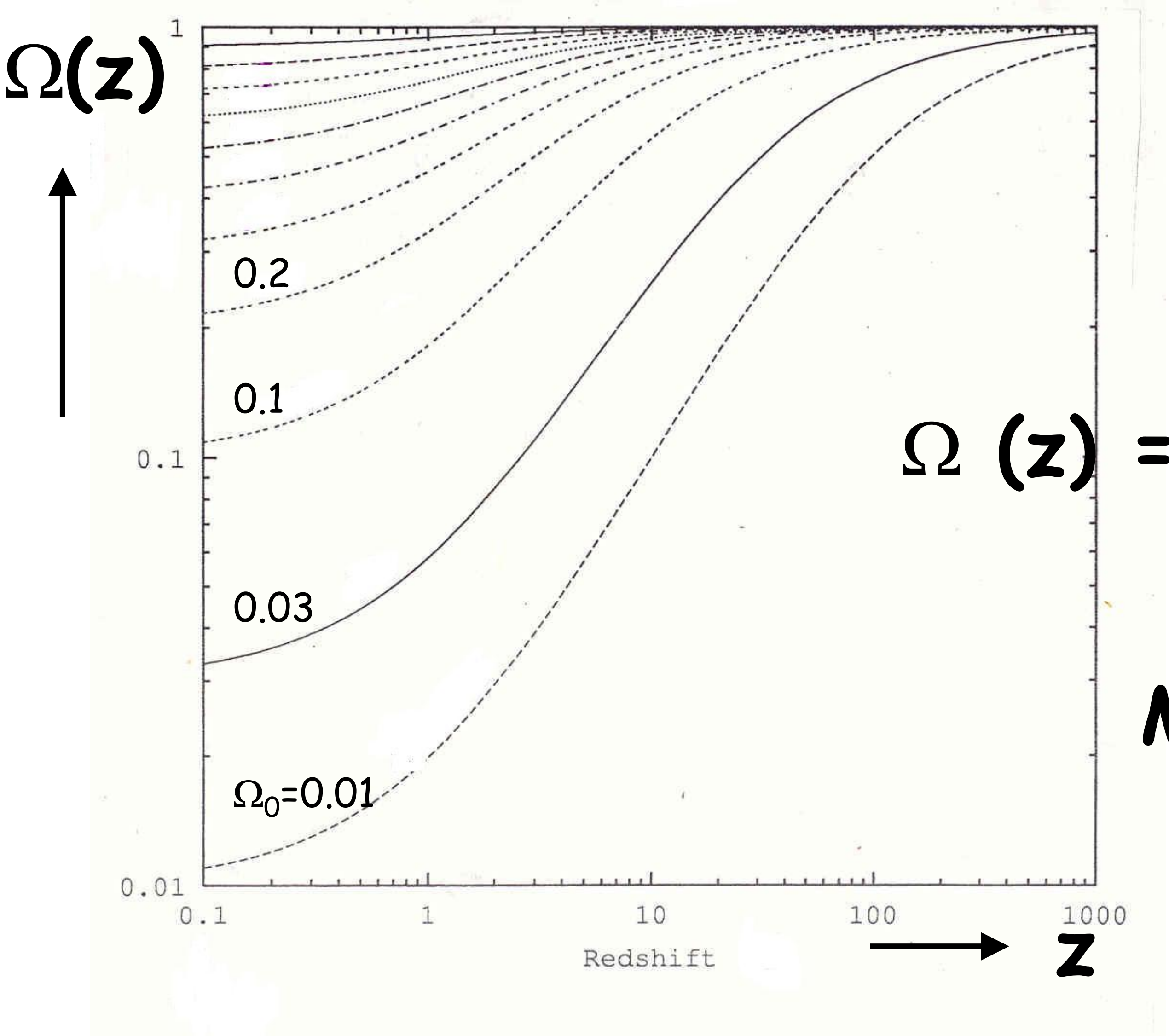
Solution to the Friedman Eqn. - V

arbitrary - III



$$\Omega_K + \Omega_m + \Omega_\Lambda = 1$$

Illustration of Flatness Problem - I



$$\Omega(z) = \frac{\Omega_0 (1+z)}{1 + \Omega_0 z}$$

MD universe

Illustration of Flatness Problem - II

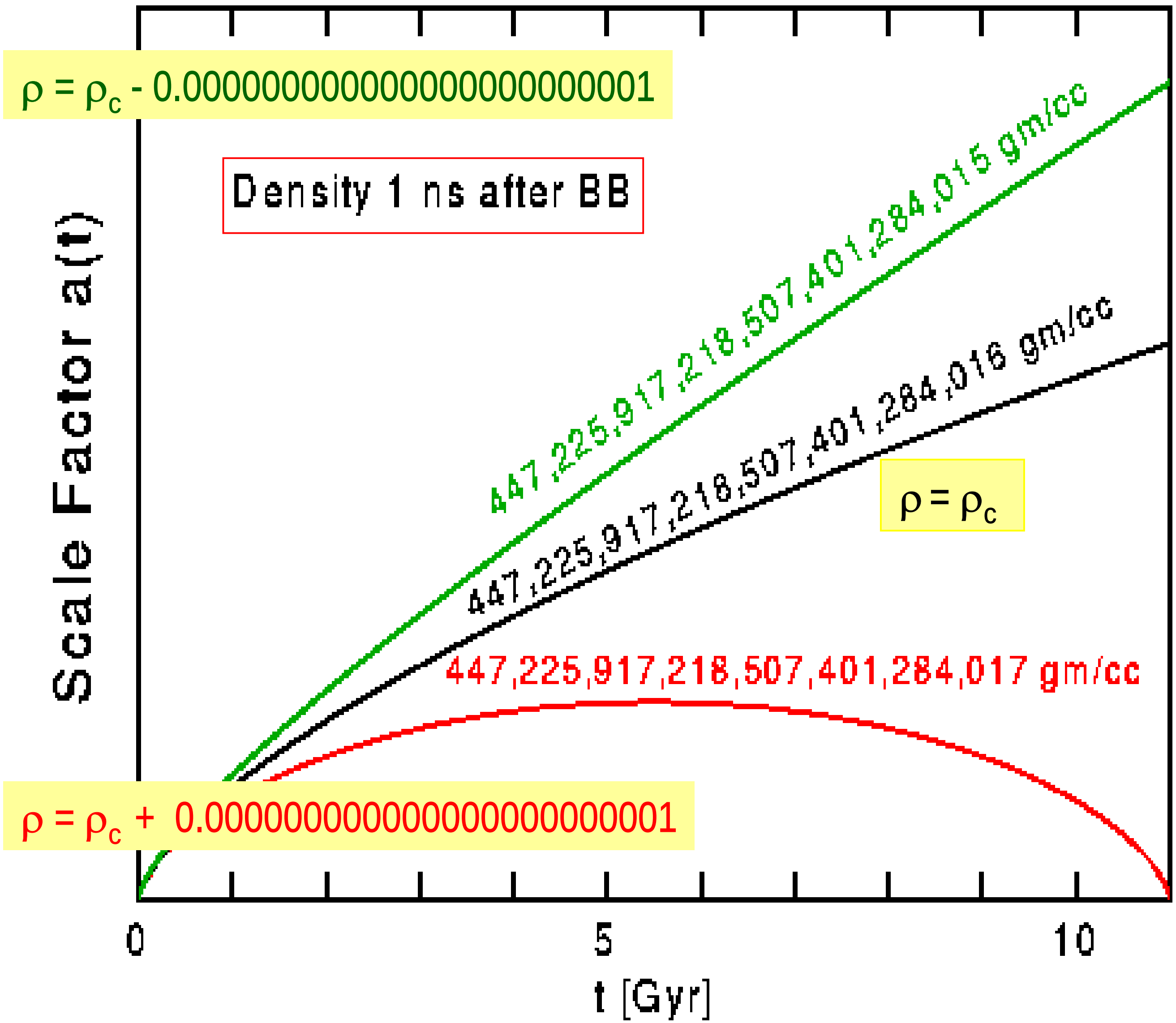


Illustration of Horizon Problem