Physics of the interstellar medium

Prof. U. Klein
P.D. J. Kerp
timeline

• Today: Introduction (JK)
• 28. April 2009: line radiation, fundamentals, radiation transfer (JK)
• 5. May 2009: continuum radiation processes (UK)
• 12. May 2009: heating and cooling (JK)
• 19. May 2009: dust (JK)
• … (UK)

• Written exam: 21. July 2009 16:00 Hörsaal 0.03

• Second date: 08. September 2009 10:00 Hörsaal 0.03
Further reading

• James Lequeux „The interstellar medium“ Springer Verlag 2005

• Ewine van Dishoeck „Master lecture Leiden University 2006“ http://www.strw.leidenuniv.nl/~dave/ISM
The interstellar medium: line radiation processes, physical conditions of the gas

J. Kerp
Outline

• The interstellar medium in general
• History of the discovery
• Composition
• Phases of the ISM
• Transitions
• Basic physical quantities
The interstellar medium in general
The interstellar medium in general

- The radiation of a galaxy is dominated by the stellar light
- The mass of a galaxy is dominated by the stellar mass distribution
- Studies of the stellar populations of galaxies are essential to disentangle the formation history of a galaxy
- The radiation of the ISM is faint and the bulk of photons are emitted in the radio and infrared wavelength regime
- The ISM hosts only a few percent of the total mass of a galaxy
- The composition of the ISM is continuously modified by the stellar evolution
- The distribution and chemical composition of the ISM is continuously modified by accretion of matter by the galaxy
- **Density and temperature variations within the ISM trigger the star formation rate of a galaxy**
Galaxy: M82

Visible light/HAST Starformation leads to transfer of matter outside the stellar distribution
Galaxy: M82

Blue: X-ray/Chandra
Red: Infrared/Spitzer
Visible light/HST
Dark clouds
Dark clouds: Horsehead Nebula
Dark clouds: Horsehead Nebula


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Dark clouds: Horsehead Nebula


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Dark clouds: Horsehead Nebula


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The ISM in general

Coronal Gas

- radiative cooling
- Supernova blast

H II

- radiative recombination
- photo-ionization

H I

- H₂ formation on dust
- photo-dissociation

Diffuse H₂

- cloud collapse

Dense H₂

- star formation

Stars

- Cool Winds, Plan. Nebulae
- Fast Winds

SN blast

SN ejecta

van Dishoeck ISM lecture

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The discovery of the interstellar medium
Stellar distribution

- First attempts to determine the distribution of the stars were made by Thomas Wright (1750) and Immanuel Kant (1755). Systematic star counts were performed by Wilhelm Herschel (1738-1822). He produced the first map of the stellar density distribution. According to his star counts, the Sun is thought to be in the very center of the Milky Way Galaxy.
Stellar distribution
Stellar distribution

- We assume,
  - that the stars fill entirely homogeneously the space.
  - All stars have the same luminosity.
  - The apparent magnitude of a star $m$ is a function of $r^2$.
- Accordingly, stars with an apparent magnitude in excess of $m$ follows the relation
  \[ \log r = 0.2 \, m + \text{const}. \]
- The number of stars in space increases proportional to $r^3$
- Accordingly we find
  \[ \log N(m) = 0.6 \, m + \text{const} \]
Stellar distribution

Abb. 5.2.1. Sternzahlen $N(m)$, d. h. Anzahl der Sterne heller als $m$ pro Quadratgrad nach Zählungen von F. H. Scales (1928) am galaktischen Äquator ($b = 0^\circ$) und am galaktischen Pol ($b = 90^\circ$) (ausgezogene Kurven). Berechnete Kurven (gestrichelt): $\log N(m) = 0.6 m + \text{const}$ für konstante Sterndichte, ohne galaktische Absorption. (Die Konstante wurde für $m = 4$ mag den Beobachtungen angepaßt)

"Neuer Kosmos" Herm. Reclam, Springer Verlag
## Stellar distribution

<table>
<thead>
<tr>
<th>Apparent magnitude [mag]</th>
<th>Plane [N/square degrees]</th>
<th>Polar region [N/square degrees]</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,0</td>
<td>0,25</td>
<td>0,06</td>
</tr>
<tr>
<td>11,0</td>
<td>50</td>
<td>10</td>
</tr>
<tr>
<td>16,0</td>
<td>6000</td>
<td>350</td>
</tr>
<tr>
<td>21,0</td>
<td>200,000</td>
<td>3000</td>
</tr>
</tbody>
</table>
Stellar distribution

• Considering the vertical distribution of the stars, we can approximate this by a hydrostatic density distribution.
• According to this approach, we can characterise the distribution by a single quantity, the scale height $\beta$. $z$ denotes the vertical distance from the galactic plane and $\rho_0$ is the mid-plane stellar density.

$$\rho(z) = \rho_0 \cdot e^{-\left(\frac{z}{\beta}\right)}$$
Z-distribution of the stellar populations

Scale-heights for different types (Objektgruppe) of stars

<table>
<thead>
<tr>
<th>Objekt-Gruppe</th>
<th>$\beta$ pc</th>
<th>Objekt-Gruppe</th>
<th>$\beta$ pc</th>
<th>Objektgruppe</th>
<th>$\beta$ pc</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>50</td>
<td>G V</td>
<td>350</td>
<td>Delta-Cephei-Sterne</td>
<td>45</td>
</tr>
<tr>
<td>B</td>
<td>60</td>
<td>K V</td>
<td>350</td>
<td>Offene Sternhaufen</td>
<td>80</td>
</tr>
<tr>
<td>A</td>
<td>120</td>
<td>M V</td>
<td>350</td>
<td>Kugelsternhaufen</td>
<td>4000</td>
</tr>
<tr>
<td>F</td>
<td>190</td>
<td>G III</td>
<td>400</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>K III</td>
<td>270</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

“Astronomie II” Gondolatsch, Groschopf, Zimmermann, Klett Studienbücher
Discovery of the ISM

- ~ W. Herschel compiles the first catalog of "nebulae"
- Until ~1900 absorption lines have been discovered, but it is unclear whether they are of stellar (circum-stellar) or interstellar origin
- 1919 Barnard compiled the first catalog of "Dark Clouds"
Discovery of the ISM

• 1933 Plasket & Pearce found a correlation between the CaII absorption line strength and the stellar distance.
Discovery of the ISM

• ~1937 the first interstellar molecules CH, CH$^+$ and CN were discovered
• 1945 van der Hulst predicted the detect ability of the HI 21-cm line
• 1949 discovery of interstellar magnetic field by polarization measurements
• 1950`s maps of the Milky Way in HI (10% of the stellar mass is in HI)
Discovery of the ISM

• 1945 van der Hulst predicted that the hyperfine transition of the radiation emerging from the hydrogen ground level will be detectable within the Milky Way

• 1951 Ewen & Purcell and Oort & Muller detected independently the HI 21-cm line of neutral hydrogen.

• This was the starting point to explore the Milky Way Galaxy in HI 21-cm emission.
  – The HI disk is much larger than the stellar disk
  – The total gas mass of the HI disk is about 10% of the stellar mass
  – The average volume density of atoms is 1 cm⁻³
Properties of the HI 21-cm line

Excited State:
Proton and electron spins are parallel

Ground State:
Proton and electron spins are anti-parallel

Photon emitted
Detection of the 21-cm line

\[ \theta \propto \frac{\lambda}{D} \]

35' with a 25m dish
9' with a 100m dish
\( \leq 2 \text{cm surface accuracy} \)
Distribution of the line intensity

http://www.astro.rug.nl
Properties of the HI 21-cm line

• Natural line width is $10^{-16}$ km s$^{-1}$, accordingly the line is ideal suited to trace turbulence and Doppler motions.
• The thermal line width is $\sigma \sim 0.09 \sqrt{T}$, which corresponds to 2 km s$^{-1}$ at $T = 100$ K
• Doppler shift

$$v' = \left(1 - \frac{v}{c}\right) \cdot v$$

$$\Delta v = \frac{v}{c} \cdot v$$
HI in the Milky Way

HI 21-cm rotation curve of the Milky Way galaxy. Shown is the velocity measured relative to the local standard of rest versus the galactic longitude (Burton 2001)
HI in the Milky Way

• Assuming that the gas encircles the center of the Milky Way on concentric orbits, it is feasible to determine its distance (tangential point method).

• The method is restricted to the inner galaxy. However, it probes a large fraction of the baryonic mass.
HI in the Milky Way
HI in the Milky Way

- The tangent point method is only applicable for \( R < R_{\text{Sonne}} \).
- The outer galaxy can be explored by the HI 21-cm line only via continuing the rotation curve. The derived dynamical distances are only estimates with partly high uncertainties.
HI in the Milky Way

HI in the Milky Way

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Discovery of the ISM

http://www.astron.nl
Discovery of the ISM

- 1960 discovery of the soft X-ray background

http://www.mpe.mpg.de
Discovery of the ISM

• 1963 the first interstellar maser had been discovered (OH)

• 1968 \( \text{NH}_3 \), the "thermometer" in the Universe was observed for the first time

• 1970 \( ^{12}\text{CO}(1 \rightarrow 0) \), the second most abundant molecule in the Universe was discovered
Discovery of the ISM

- 1970`s infrared astronomy opens the window to the most abundant molecule H$_2$
$\text{H}_2$: necessity for tracers
Discovery of the ISM

• ~1990 Sub-millimeter astronomy opened the window to molecular clouds and star forming regions
• 1990 COBE studied the distribution of the dominant cooling line of the ISM CII.
• 1995 allowed detailed spectroscopic studies of the dust, the vibrationally excited H$_2$ emission line and the infrared dark clouds
Discovery of the ISM

**COBE FIRAS 158 µm C⁺ Line Intensity**

**COBE FIRAS 205 µm N⁺ Line Intensity**
Discovery of the ISM

- Infrared Dark Clouds
Discovery of the ISM
Discovery of the ISM

• 1998-2006 SWAS studied the distribution of H$_2$O, O$_2$, CI up to 500 GHz
• 2000-today FUSE observation of H$_2$ in absorption against background continuum sources, observation of the vertical structure of highly ionized gas like OVI and NV
• 2003-today Spitzer studies the ISM with high angular resolution
Basic properties of the ISM
Basic properties of the ISM

- Confined to the Galactic Plane (much flatter than a compact disk!)
- The interstellar medium consists mainly of hydrogen and helium.
  - All elements heavier than hydrogen are denoted as “metals”
- Temperature range $4 \text{ K} < T < 10^6 \text{K}$
  - The temperature is used as a measure for the physical conditions of the interstellar gas. The “phases” of the interstellar medium are characterized by the average gas temperature.
- Densities $10^{-4} \text{ cm}^{-3} < n < 10^7 \text{ cm}^{-3}$
- Far from thermal equilibrium!

Not easy to calculate!
## Abundances of elements

<table>
<thead>
<tr>
<th>Element</th>
<th>Abundance</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>1.00</td>
</tr>
<tr>
<td>He</td>
<td>0.075</td>
</tr>
<tr>
<td>C</td>
<td>2.5 \times 10^{-4}</td>
</tr>
<tr>
<td>N</td>
<td>6.3 \times 10^{-5}</td>
</tr>
<tr>
<td>O</td>
<td>4.5 \times 10^{-4}</td>
</tr>
<tr>
<td>Na</td>
<td>2.1 \times 10^{-6}</td>
</tr>
<tr>
<td>Mg</td>
<td>4.2 \times 10^{-5}</td>
</tr>
<tr>
<td>Al</td>
<td>3.1 \times 10^{-6}</td>
</tr>
<tr>
<td>Si</td>
<td>4.3 \times 10^{-5}</td>
</tr>
<tr>
<td>S</td>
<td>1.7 \times 10^{-5}</td>
</tr>
<tr>
<td>Ca</td>
<td>2.2 \times 10^{-6}</td>
</tr>
<tr>
<td>Fe</td>
<td>4.3 \times 10^{-5}</td>
</tr>
</tbody>
</table>
Abundances of the ISM

• "Metalls"
  – He about 10%
  – C, N and O
  – Si, Ca and Fe bound in dust grains
• Grains
  – About 1% of the ISM mass
• Photons
• Magnetic fields
• Cosmic rays
Phases of the interstellar matter
Classification of the ISM

- Chemical composition of the ISM comparable to the elements abundance of the Solar System
- The state of Hydrogen determines the state of the ISM
  - Molecular region $\leftrightarrow H_2$
  - Neutral region $\leftrightarrow HI$
  - Ionized region $\leftrightarrow H^+$
Molecular regions

• Molecular clouds are defined by the presence of molecular hydrogen. We differentiate between:
  • Diffuse molecular clouds
    – $T \approx 40 \ldots 80$ K
    – $n \approx 100$ cm$^{-3}$
  • Dark clouds
    – $T \approx 10 \ldots 50$ K
    – $n \approx 10^4 \ldots 10^6$ cm$^{-3}$
  • Dust is an important constituent of molecular clouds.
Molecular regions (sub-mm range)
Molecular regions (sub-mm range)
Molecular regions (UV-range)
Neutral gas regions (radio-range)

• Dusty cirrus clouds
  – \( T \approx 80 \) K
  – \( n \approx 1 \) cm\(^{-3}\)

• Warm neutral gas
  – \( T \approx 6000 \) K
  – \( n \approx 0.05 \ldots 0.2 \) cm\(^{-3}\)
Neutral gas regions (infrared range)
Neutral gas regions (infrared range)
Ionized gas regions

- HII regions are envelopes of early-type stars
  - $T \approx 10^4 \text{ K}$
  - $n \approx 0.1 \ldots 10^4 \text{ cm}^{-3}$
Ionized gas regions

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Ionized gas regions

van Dishoeck ISM lecture 1
Ionized gas regions

- Coronal Gas
  - $T \approx 10^6$ K
  - $n \approx 0.005$ cm$^{-3}$
Ionized gas regions

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Phase transitions: Orion nebula
Phase transitions

Coronal Gas

H II

Radiative cooling
Supernova blast

Radiative recombination
Photo-ionization

H I

H_2 formation on dust
Photo-dissociation

Diffuse H_2

SN blast

Cool Winds, Plan. Nebulae

Dense H_2

Cloud collapse

Stars

SN ejecta

Fast Winds

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Phase transitions: M82

Blue: X-ray/Chandra
Red: Infrared/Spitzer
Visible light/HST
**Phase transitions**

![Diagram showing the flow of matter through different stages, from Extragalactic Matter to Interstellar medium, then to Stars, and further to Stellar Remnants and Black holes.](diagram)

- **Extragalactic Matter**
  - Infall \(\leq 1 \, M_\odot \, \text{yr}^{-1}\)
  - Galactic Wind?

- **Interstellar medium**
  - ~\(5 \times 10^9 \, M_\odot\)
  - Star formation \(3\ldots10 \, M_\odot \, \text{yr}^{-1}\)
  - Stellar ejecta \(\sim 1 \, M_\odot \, \text{yr}^{-1}\)
  - Stellar Winds \(\sim 0.3 \, M_\odot \, \text{yr}^{-1}\)
  - Planetary nebulae \(0.3\ldots1 \, M_\odot \, \text{yr}^{-1}\)
  - Novae \(0.003 \, M_\odot \, \text{yr}^{-1}\)
  - Supernovae \(0.03 \, M_\odot \, \text{yr}^{-1}\)

- **Stars**
  - A few \(M_\odot \, \text{yr}^{-1}\)

- **Stellar Remnants**
  - (white dwarfs, neutron stars)

- **Black holes**
  - (stellar, also supermassive?)

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Physical quantities
Atoms and Ions

• The dominant part of baryonic matter in the universe is in neutral or partly ionized state (Warm Hot Intergalactic Medium, 80% of all baryons)

• Line emission and absorption of atoms and ions are the dominant sources of photon emission and absorption

• Basic knowledge of atomic physics is necessary to understand the processes in the universe
Fraunhofer lines of the Sun, from 390 nm to 690 nm

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Fraunhofer lines

- Fraunhofer lines can be observed in laboratory experiments. A continuous light source is observed through a gas. Using a spectrometer, absorption lines are observed, which are unique for individual chemical elements.
- If the continuous light source is switched off, the absorption lines change into emission lines in the spectrum.
- Fraunhofer lines originate via resonance absorption. The gas attenuates the continuous light emission only at those wavelengths (energies) which corresponds exactly to the spacing of the energy levels.
Linien radiation

Top: continuous spectrum
Middle: emission spectrum of the gas
Bottom: absorption spectrum of the gas

http://de.wikipedia.org
Line radiation processes

Transition energy, $E_{12} = E_2 - E_1 = h\nu_{12}$

EINSTEIN A & B COEFFICIENTS

http://www.homepages.ucl.ac.uk/~ucapphj/EinsteinAandB.jpg
Line radiation: hydrogen
Line radiation: hydrogen

 Diagram showing transitions for hydrogen with different series: Lyman, Balmer, and Paschen.
Line radiation: hydrogen

Energy levels in Hydrogen:

\[ E_n = -\frac{2\pi^2 \mu e^4}{\hbar^2} \cdot \frac{1}{n^2} \propto -\frac{1}{n^2} \]

\( n \in \mathbb{N}; \) Balmer formula

(1eV = 1 electron volt = 1.6x10^{-19}J)
Line radiation: line width

- The natural line width is determined by the time-energy uncertainty due to the Heisenberg uncertainty principle

\[
\frac{\hbar}{2} \leq \Delta E \cdot \Delta t
\]

- This relates the lifetime of an excited state with the energy uncertainty of the transition. The natural line width can be described by a Lorentzian profile

\[
L(x, \gamma) = \frac{\gamma}{\pi \cdot (x^2 + \gamma^2)}
\]

- \( \gamma \) determines the FWHM and the amplitude \( \approx 1/\gamma \pi \)
Linien radiation: line broadening

Radiation from a background source is absorbed by the gas cloud G. The absorbent photon energy is emitted isotropically. The total amount of absorbed energy is released after the absorption processes but without any preferred direction.
Line radiation: line broadening

Maxwell-Boltzmann distribution

Inelastic scattering

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Line radiation: line broadening

- An absorption line is produced when a atom absorbs an amount of energy $\Delta E = \frac{h \cdot c}{\lambda}$ at a wavelength $\lambda$.
- The electron which is bound to the electronic state $E_m$ is excited after the absorption to the higher energy state $E_k$.

\[ E_k - E_m = \frac{h \cdot c}{\lambda} \]
Equivalent width

Definition of the full-width half maximum equivalent width $\Delta \lambda$. $I_K$ denotes the intensity of the continuum at the wavelength of the absorption line. $I_0$ is the intensity minimum and $I_\lambda$ is the residual intensity at the wavelength $\lambda$. $A_\lambda$ is the corresponding equivalent width, which has the same area as the absorption line.
The advantage of using the full-width half maximum definition is, that we apply the assumption, that the absorbing/emitting gas atoms are following a Maxwell-Boltzmann distribution.

According to this assumption we find

\[
f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-x_0)^2}{2\sigma^2}}
\]

\[
FWMH = \sqrt{8 \ln 2} \cdot \sigma \approx 2.35482 \cdot \sigma
\]
Maxwell-Boltzmann FWHM

\[ P(\nu) d\nu = \sqrt{\frac{m}{2\pi kT}} e^{-\left(\frac{m\nu^2}{2kT}\right)} d\nu \]

\[ P(f) df = \sqrt{\frac{mc^2}{2\pi kTf_0^2}} e^{-\left(\frac{mc(f-f_0)^2}{2kTf_0^2}\right)} df \]

\[ \sigma_f = \sqrt{\frac{kT}{mc^2} f_0} \]

\[ \Delta f_{FWHM} = \sqrt{\frac{8kT \ln 2}{mc^2}} f_0 \]
Line radiation: line broadening

- We apply the non-relativistic Doppler formula
  \[ \Delta \lambda = \frac{\vartheta}{\lambda} \]

- For a gas with the molecular mass \( M^* \) we can calculate the average velocity dispersion of the gas atoms
  \[ \Delta \lambda_D = \frac{\lambda}{c} \sqrt{\frac{2 \cdot R \cdot T}{M^*}} \]

- Applying the Doppler shift formula, we can calculate the average line width due to thermal motion.

- A typical value for the interstellar medium is
  \[ \Delta \lambda_D = 5.491 \times 10^{-7} \text{m} \sqrt{\frac{2 \cdot 8,314 \times 10^3 \text{J/K} \cdot \text{kmol}^{-1} \cdot 6 \times 10^3 \text{K}}{47.9 \text{kg kmol}^{-1}}} \]

  \[ \Delta \lambda_D = 0.0026 \text{nm} \]

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Line shape

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Abb. 4.28 Profil der Wasserstoff-Linie H_α (Wellenlänge \( \lambda = 656,3 \) nm) aus dem Utrechter photometrischen Atlas des Sonnen- spektrums (Beispiel einer „starken“ Linie)

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Line radiation: line broadening

- As shown in the viewgraph before, we have two processes which lead to the broadening of an absorption/emission line.
  - Thermal broadening (temperature $T$)
  - Pressure broadening (temperature $T$ and volume density $n$)
- Thermal and pressure broadening are different in their physical behaviour.
  - Thermal broadening implies a folding of the Lorentzian profile (natural line width) with the Maxwell-Boltzmann velocity distribution.
  - Pressure broadening leads to a shift of the atomic energy levels due to the interaction process. In general, the interaction with ambient gas atoms is on shorter time scale than the spontaneous emission. Here, the volume density $n$ is of prime importance, $T$ is of second interest.

\[
\Delta I \propto e^{-\frac{(\lambda-\lambda_0)^2}{2\Delta \lambda_D}} \\
\Delta I \propto \frac{1}{(\lambda-\lambda_0)^2 + \Delta \lambda_D^2}
\]
Evaluation the strength of an absorption line

- A photon of the wavelength $\lambda$ will be absorbed by a ion which has the ionization degree $i$ will move the excited electron from orbit $m$ ($E_{i,m}$) to orbit $k$ ($E_{i,k}$).
- The „strength“ of the absorption line is proportional to the path length through the absorbing medium $H$.
- The strength of the absorption line is also proportional to the number of ions in the same energetic state $E_{i,m}$.
- Finally, the strength of absorption is a function of the probability that the absorption events occurs quantum mechanically. This probability is describe by the oscillator strength $f_{m,k}$

$$A_\lambda \approx f_{m,k} \cdot N_{i,m} \cdot H$$
Evaluation the strength of an absorption line

1. Determination of $A_\lambda$.
2. The gas atom, its ionization state $i$, energy levels $E_{i,m}$ and $E_{i,k}$ and $f_{m,k}$ are known, accordingly $A_\lambda$ yields directly $N_{i,m} \cdot H$
3. Using the **Boltzmann equation** one can calculate the population of the ions at a certain energy level $(E_{i,m})$

$$\frac{N_{i,m}}{N_{i,1}} = e^{-\left(\frac{E_{i,m} - E_{i,1}}{kT}\right)}$$
Evaluation the strength of an absorption line

- We know from quantum mechanics, that most of the energy levels show up with a degeneracy in the orbital and magnetic quantum number. These „hidden“ energy levels are well known as fine-structure levels. According to this degeneracy, we have to apply some weightening to the number of energy levels.

\[
\frac{N_{i,m}}{N_{i,1}} = \frac{g_{i,m}}{g_{i,1}} \cdot e^{-\left(\frac{E_{i,m} - E_{i,1}}{kT}\right)}
\]
Fig. 5. Rotational levels of NH$_3$ (left) and OH (right) (adapted from Watson 1982)
Evaluation the strength of an absorption line

- Finally, we need the number of elements which are within the excited state \( E_{i,m} \).
- The population of elements in the state is a function of the temperature. It follows the Boltzmann equation but deals with the ions.
- The equation is the Saha equation

\[
\frac{N_{1,1} \cdot N_e}{N_{0,1}} = 2 \cdot \frac{g_{1,1}}{g_{0,1}} \left( \frac{\sqrt{2\pi m_e kT}}{\hbar} \right)^3 \cdot e^{-\left( \frac{E_{1,1} - E_{0,1}}{kT} \right)}
\]
Evaluation the strength of an absorption line

• Using the Saha and the Boltzmann equation, we can calculate
  Temperature
  Elektron density
  Element abundance
  Elektron pressure
• Accordingly, we can determine the physical state of a gas!