Physics of the interstellar medium

Prof. U. Klein

P.D. J. Kerp

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timeline

- Today: Introduction (JK)
- 28. April 2009: line radiation, fundamentals, radiation transfer (JK)
- 5. May 2009: continuum radiation processes (UK)
- 12. May 2009: heating and cooling (JK)
- 19. May 2009: dust (JK)
- … (UK)

- Written exam: 21. July 2009 16:00 Hörsaal 0.03

- Second date: 08. September 2009 10:00 Hörsaal 0.03

Further reading

- James Lequeux „The interstellar medium“ Springer Verlag 2005
- Ewine van Dishoeck „Master lecture Leiden University 2006“ http://www.strw.leidenuniv.nl/~dave/ISM

The interstellar medium: line radiation processes, physical conditions of the gas

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Outline

- The interstellar medium in general
- History of the discovery
- Composition
- Phases of the ISM
- Transitions
- Basic physical quantities

The interstellar medium in general

Spiral Galaxy Messier 83
(FORS / VLT)

ESO PR Photo 24b/05 (August 10, 2005) © ESO

The interstellar medium in general

- The radiation of a galaxy is dominated by the stellar light
- The mass of a galaxy is dominated by the stellar mass distribution
- Studies of the stellar populations of galaxies are essential to disentangle the formation history of a galaxy

- The radiation of the ISM is faint and the bulk of photons are emitted in the radio and infrared wavelength regime
- The ISM hosts only a few percent of the total mass of a galaxy
- The composition of the ISM is continuously modified by the stellar evolution
- The distribution and chemical composition of the ISM is continuously modified by accretion of matter by the galaxy
- **Density and temperature variations within the ISM trigger the star formation rate of a galaxy**

Galaxy: M82

www.wikipedia.org

Visible light/HAST
Starformation leads to transfer of matter outside the stellar distribution

Galaxy: M82

www.wikipedia.org

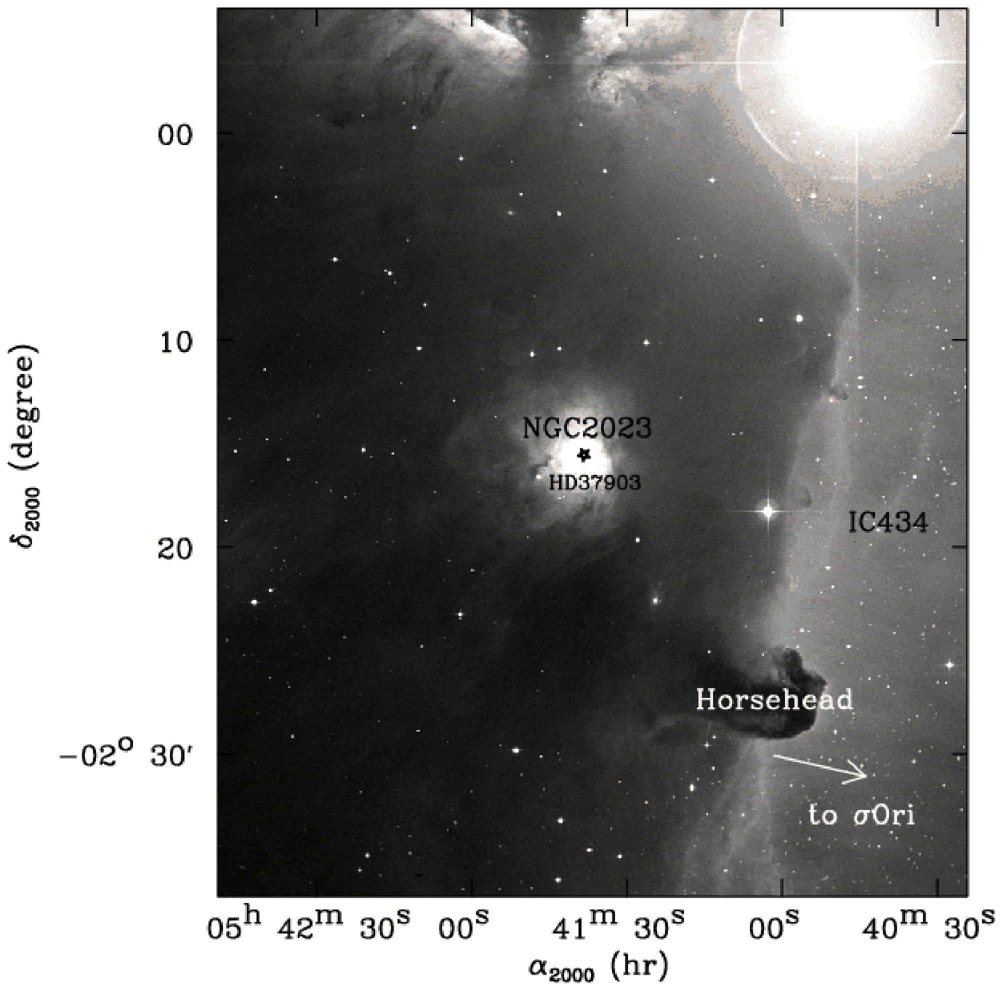
Blue: X-ray/Chandra
Red: Infrared/Spitzer
Visible light/HST

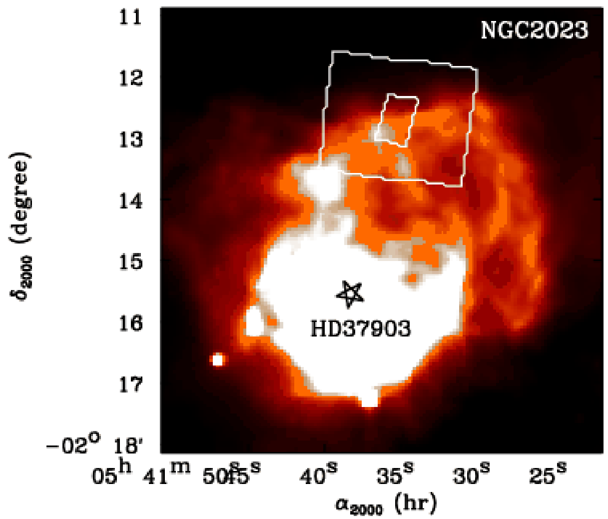
Dark clouds

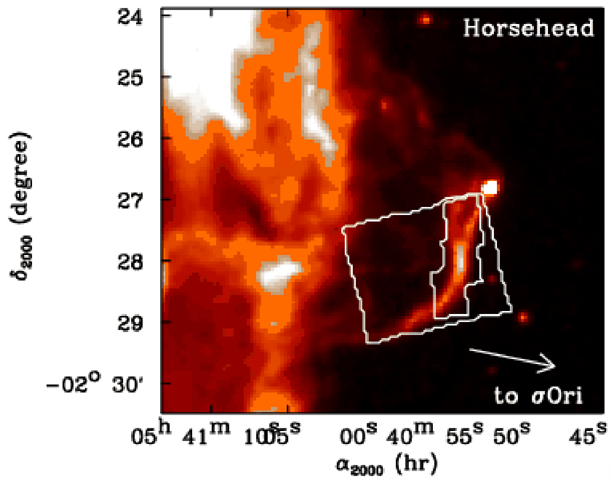

The Dark Cloud FeSt 1–457
2MASS
2 MICRON ALL-SKY SURVEY
Two Micron All Sky Survey
– Southern Facility –
2MASS Atlas Image
Infrared Processing and Analysis Center & University of Massachusetts

Dark clouds: Horsehead Nebula

Dark clouds: Horsehead Nebula







Compiegne et al. 2008, A&A 491, 797

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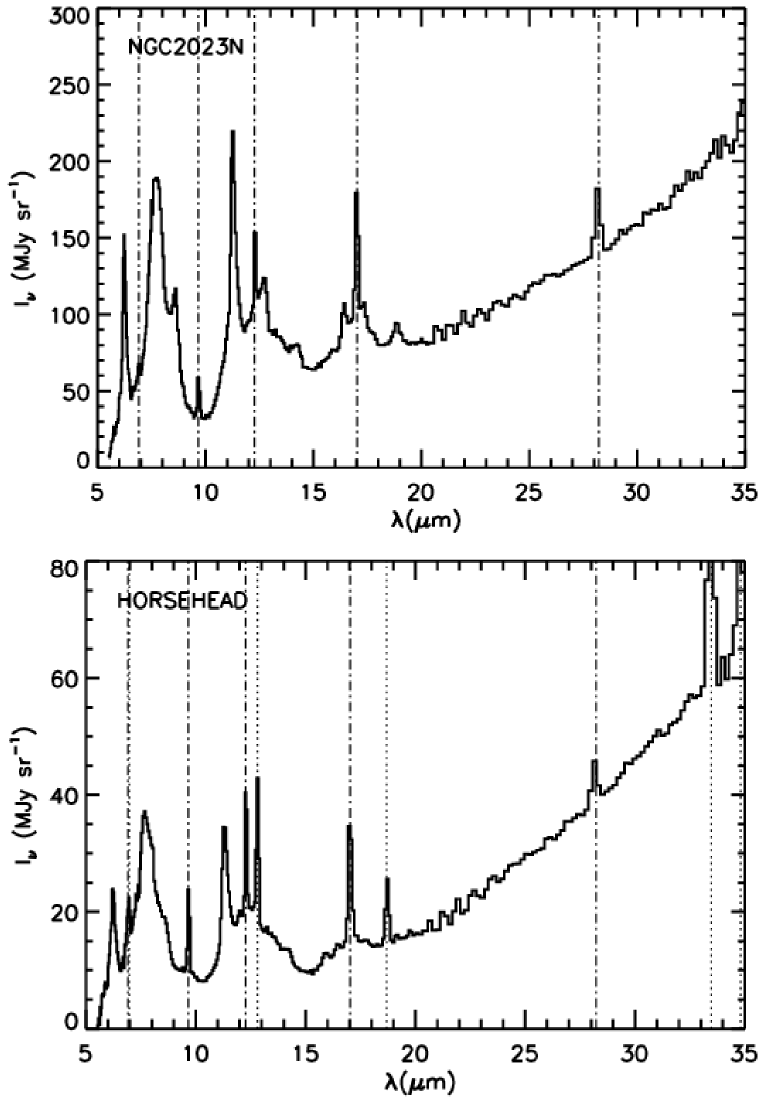
Dark clouds: Horsehead Nebula

Compiegne et al. 2008, A&A 491, 797



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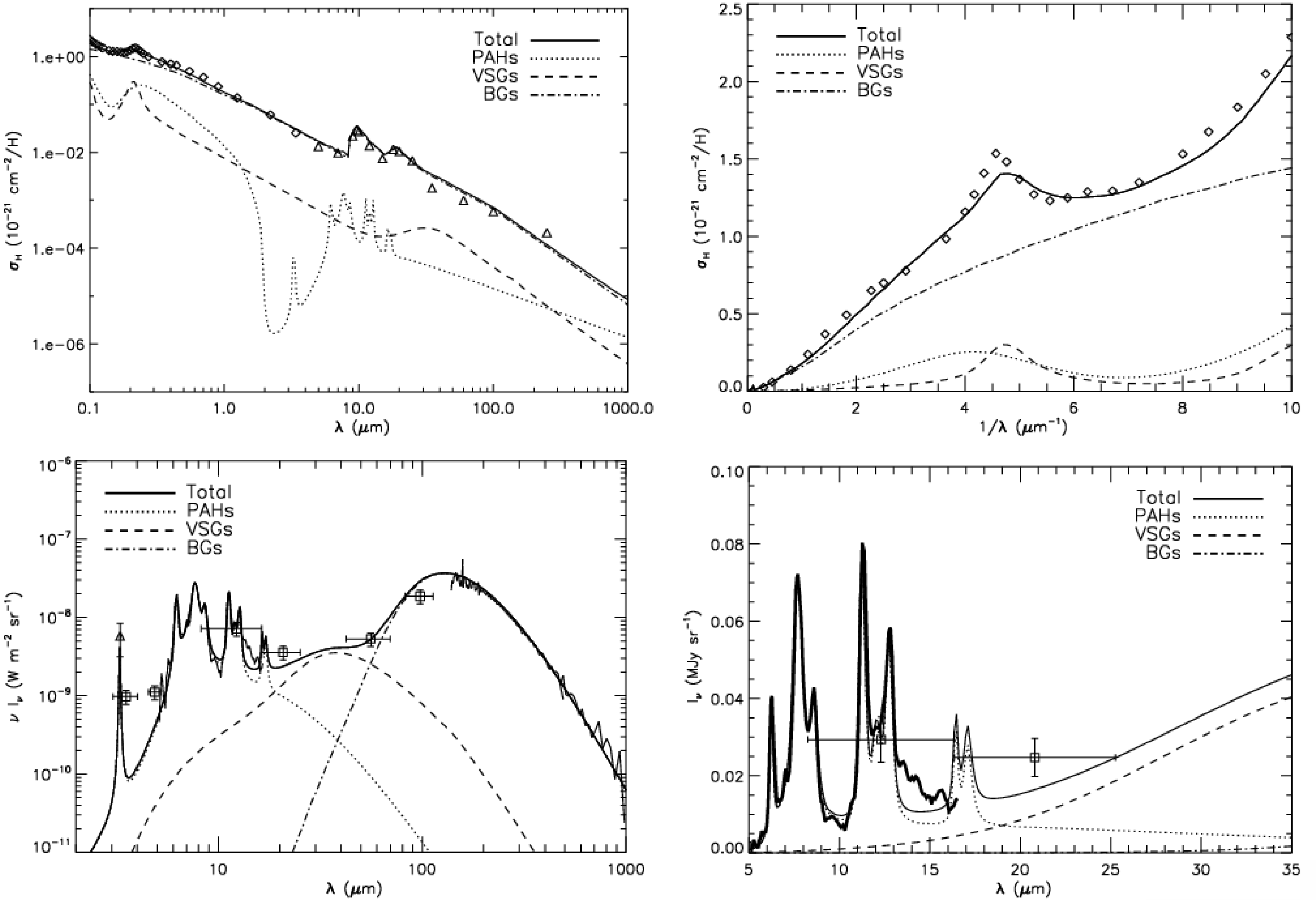
Dark clouds: Horsehead Nebula



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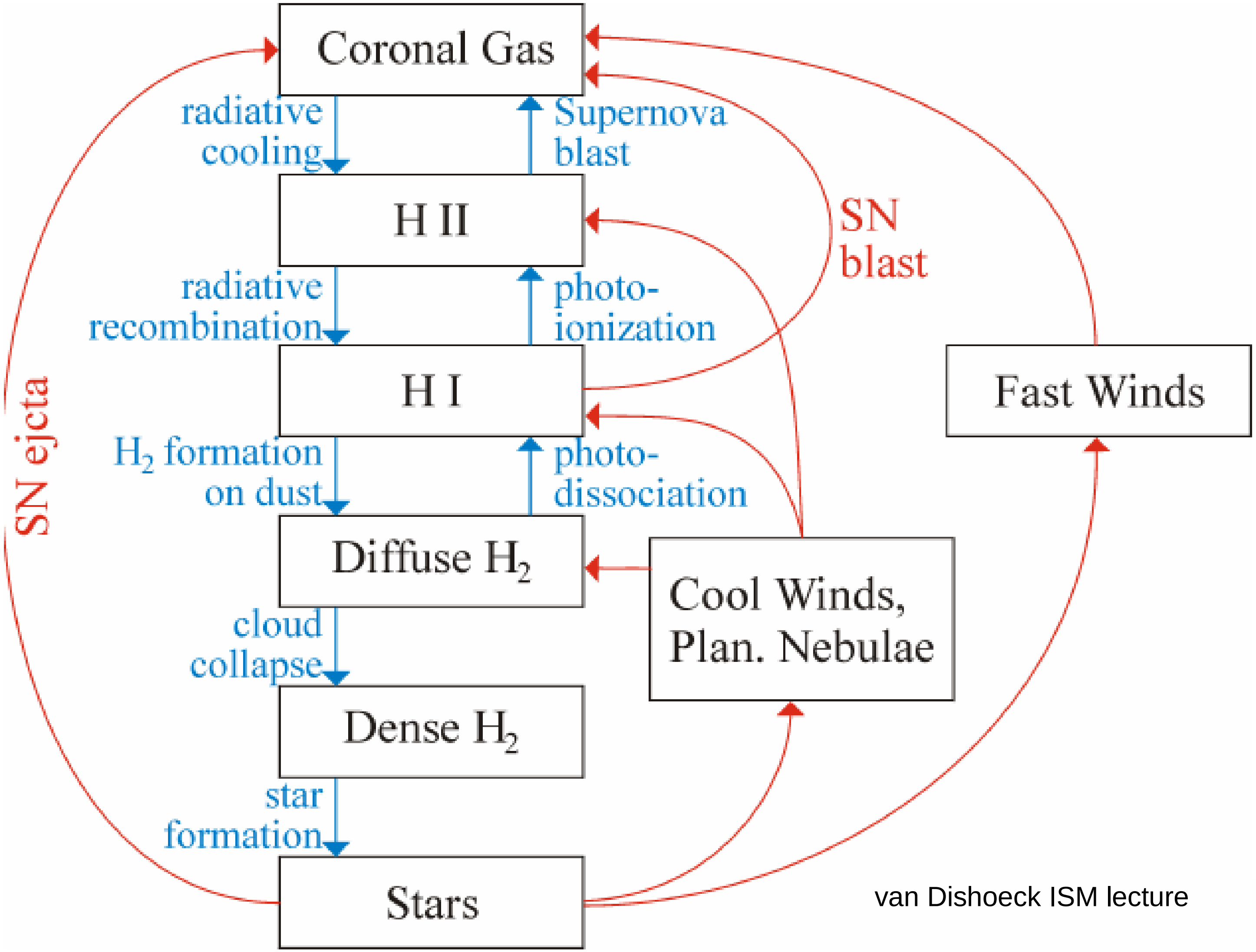
The ISM in general

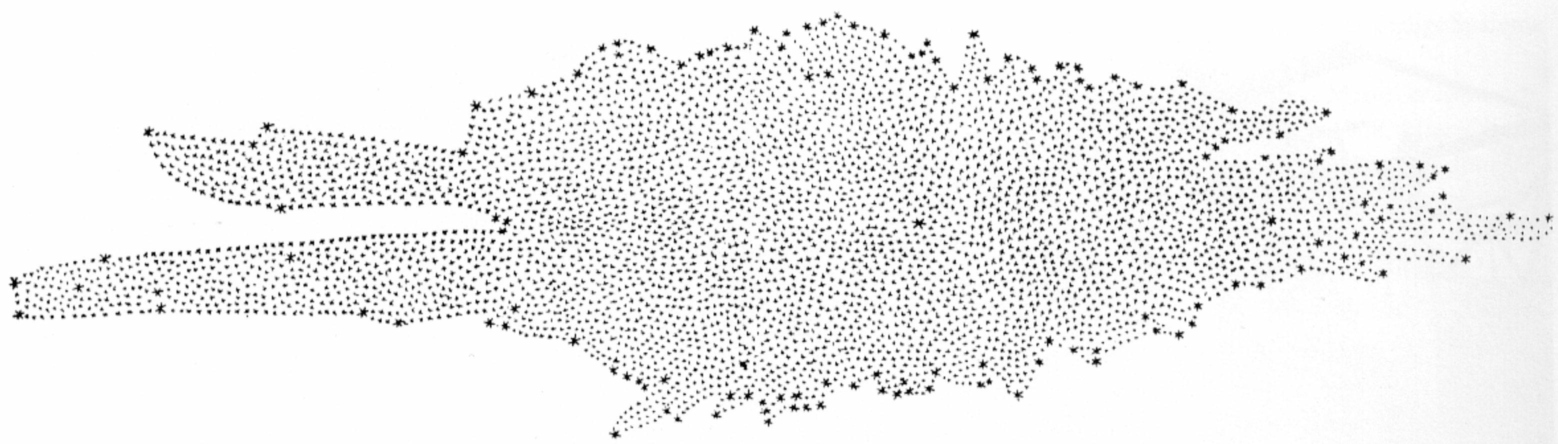


van Dishoeck ISM lecture

The discovery of the interstellar medium

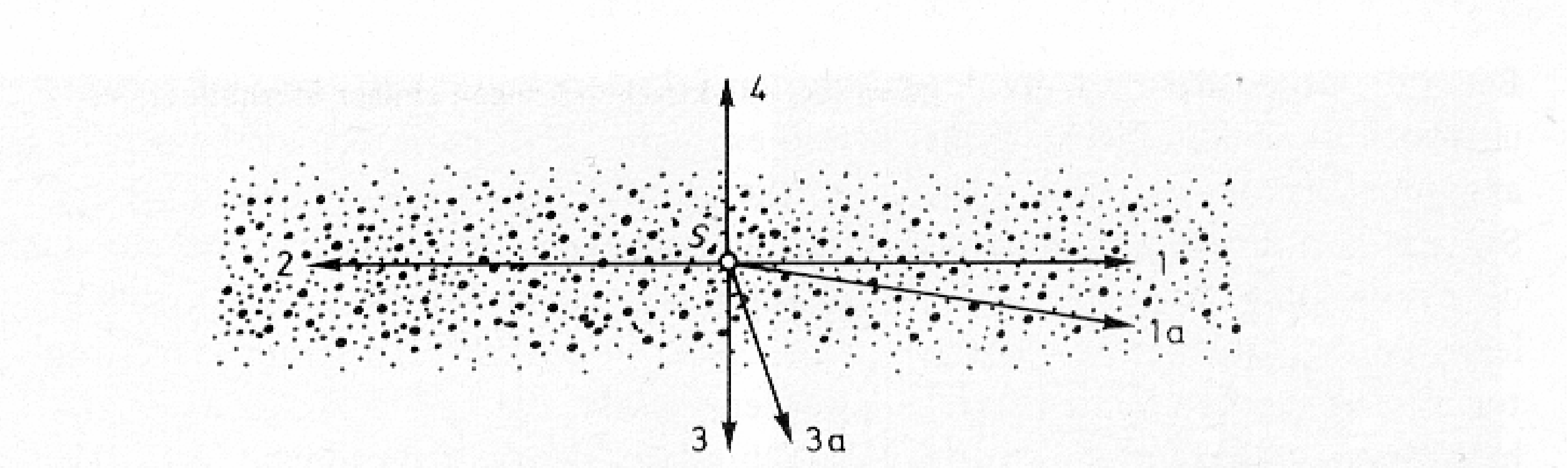
Stellar distribution

- First attempts to determine the distribution of the stars werde made be *Thomas Wright* (1750) and *Immanuel Kant* (1755). Systematic star counts were performed by *Wilhelm Herschel* (1738-1822). He produced the first map of the stellar density distribution. According to his star counts, the Sun is thought to be in the very center of the Milky Way Galaxy.



Herschels Methode der „Sterneichung" – die Darstellung der dreidimensionalen Verteilung der Sterne aufgrund ihrer scheinbaren Helligkeit – führte zu dieser ersten Karte der Galaxis. Tatsächlich war das „Mühlstein"-Modell erstaunlich genau. Doch Herschel verwarf es später, weil er glaubte, daß zu viele dunkle, leere „Tunnel" im Raum (Spalten am Rande des Diagramms) auf die Sonne ausgerichtet seien. Wir wissen heute, daß die „Tunnel" Dunkelwolken sind, die in der Milchstraßenebene liegen.

Stellar distribution



“Astronomie II” Gondolatsch, Groschopf, Zimmermann, Klett Studienbücher

Stellar distribution

- We assume,
  - that the stars fill entirely homogeneously the space.
  - All stars have the same luminosity.
  - The apparent magnitude of a star m is a function of r^{-2}.
- Accordingly, stars with an apparent magnitude in excess of m follows the relation

$$\log r = 0{,}2\, m + \mathrm{const.}$$

- The number of stars in space increases proportional to r^3
- Accordingly we find

$$\log N(m) = 0{,}6\, m + \mathrm{const}$$

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Stellar distribution

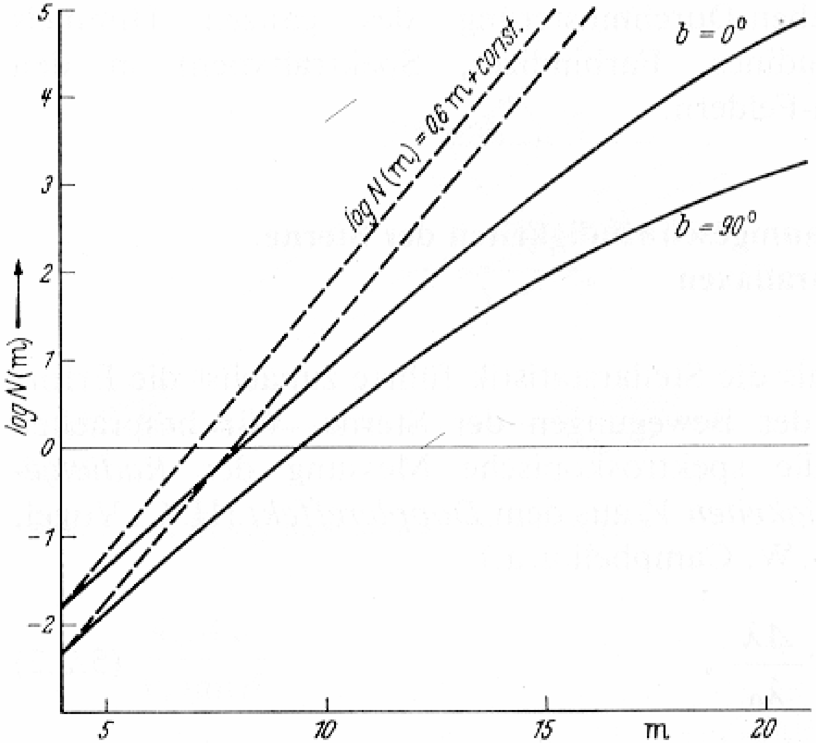


Abb. 5.2.1. Sternzahlen $N(m)$, d.h. Anzahl der Sterne heller als m pro Quadratgrad nach Zählungen von F. H. Seares (1928) am galaktischen Äquator ($b = 0°$) und am galaktischen Pol ($b = 90°$) (*ausgezogene Kurven*). Berechnete Kurven (*gestrichelt*): $\log N(m) = 0.6\,\mathrm{m} + \mathrm{const}$ für konstante Sterndichte, ohne galaktische Absorption. (Die Konstante wurde für $m = 4$ mag den Beobachtungen angepaßt)

Stellar distribution

Apparent magnitude [mag]	Plane [N/square degrees]	Polar region [N/square degees]
6,0	0,25	0,06
11,0	50	10
16,0	6000	350
21,0	200.000	3000

Stellar distribution

- Considering the vertical distribution of the stars, we can approximate this by a hydrostatic density distribution.
- According to this approach, we can characterise the distribution by a single quantity, the scale height β. z denotes the vertical distance from the galactic plane and ρ_0 is the mid-plane stellar density.

$$\rho(z) = \rho_0 \cdot e^{-\left(\frac{z}{\beta}\right)}$$

Z-distribution of the stellar populations

Scale-heights for different types (Objektgruppe) of stars

Objekt-Gruppe	$\frac{\beta}{pc}$	Objekt-Gruppe	$\frac{\beta}{pc}$	Objektgruppe	$\frac{\beta}{pc}$
O	50	G V	350	Delta-Cephei-Sterne	45
B	60	K V	350	Offene Sternhaufen	80
A	120	M V	350	Kugelsternhaufen	4000
F	190	G III	400		
		K III	270		

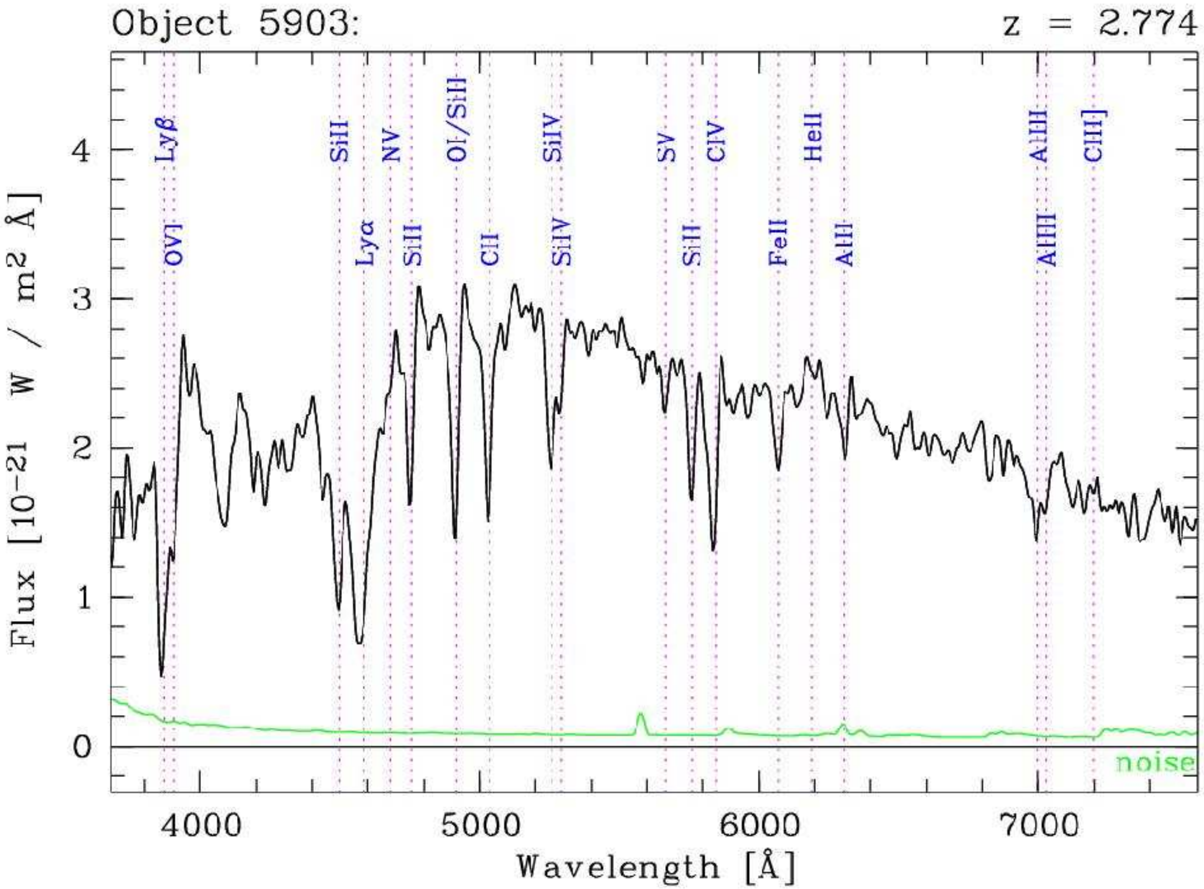
"Astronomie II" Gondolatsch, Groschopf, Zimmermann, Klett Studienbücher

Discovery of the ISM

- ~ W. Herschel compiles the first catalog of "nebulae"
- Until ~1900 absorption lines have been discovered, but it is unclear whether they are of stellar (circum-stellar) or interstellar origin
- 1919 Barnard compiled the first catalog of "Dark Clouds"

Discovery of the ISM

- 1933 Plasket & Pearce found a correlation between the CaII absorption line strength and the stellar distance.



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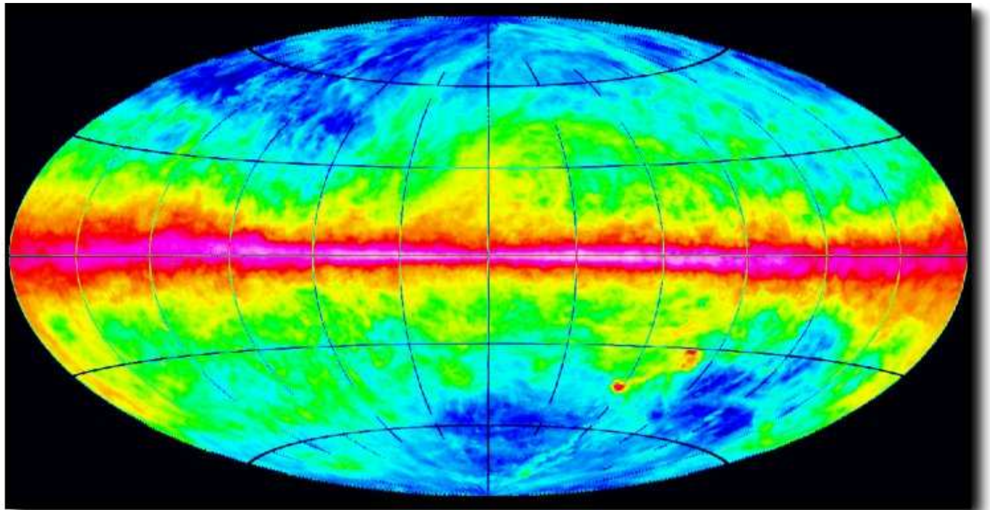
Discovery of the ISM

- ~1937 the first interstellar molecules CH, CH^+ and CN were discovered
- 1945 van der Hulst predicted the detect ability of the HI 21-cm line
- 1949 discovery of interstellar magnetic field by polarization measurements
- 1950`s maps of the Milky Way in HI (10% of the stellar mass is in HI)



LAB Kalberla et al. 2005

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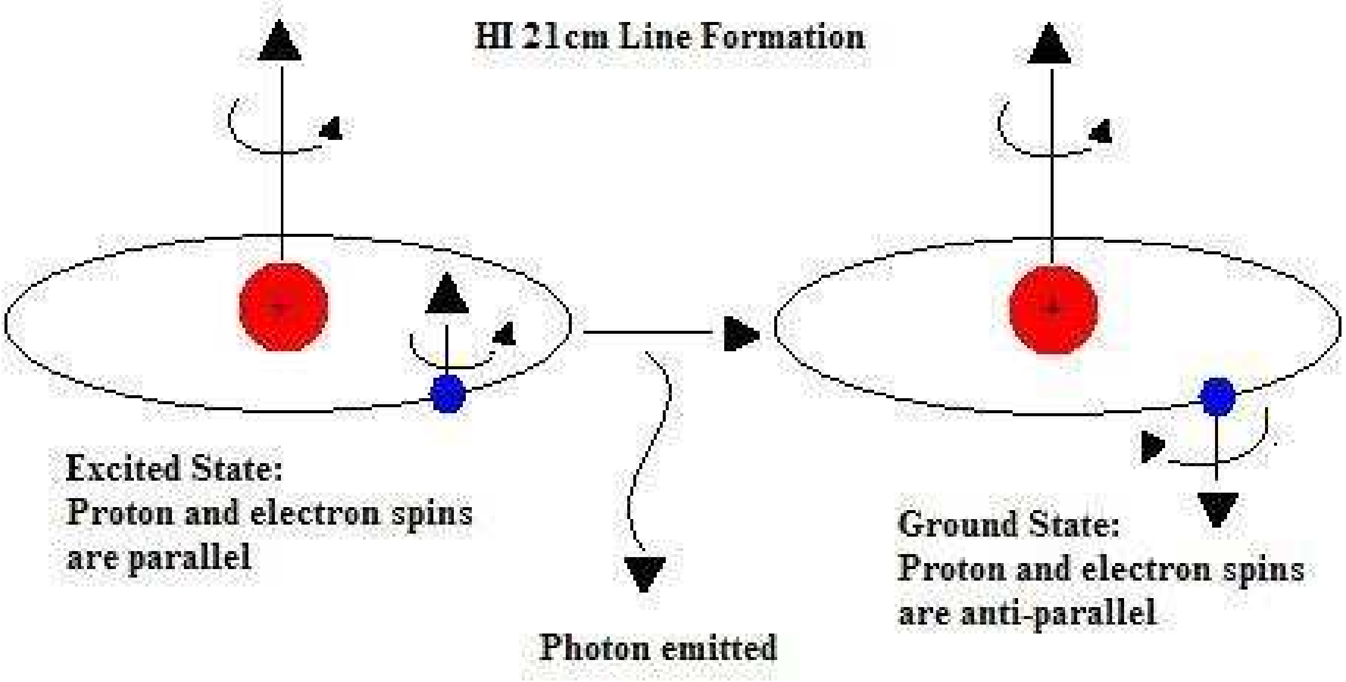
Discovery of the ISM

- 1945 van der Hulst predicted that the hyperfinetransition of the radiation emerging from the hydrogen ground level will be detectable within the Milky Way
- 1951 Ewen & Purcell and Oort & Muller detected idenpendently the HI 21-cm line of neutral hydrogen.
- This was the starting point to explore the Milky Way Galaxy in HI 21-cm emission.
  - The HI disk is much larger than the stellar disk
  - The total gas mass of the HI disk is about 10% of the stellar mass
  - The average volume density of atoms is 1 cm^{-3}

Properties of the HI 21-cm line



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Detection of the 21-cm line

http://www.astro.uni-bonn.de

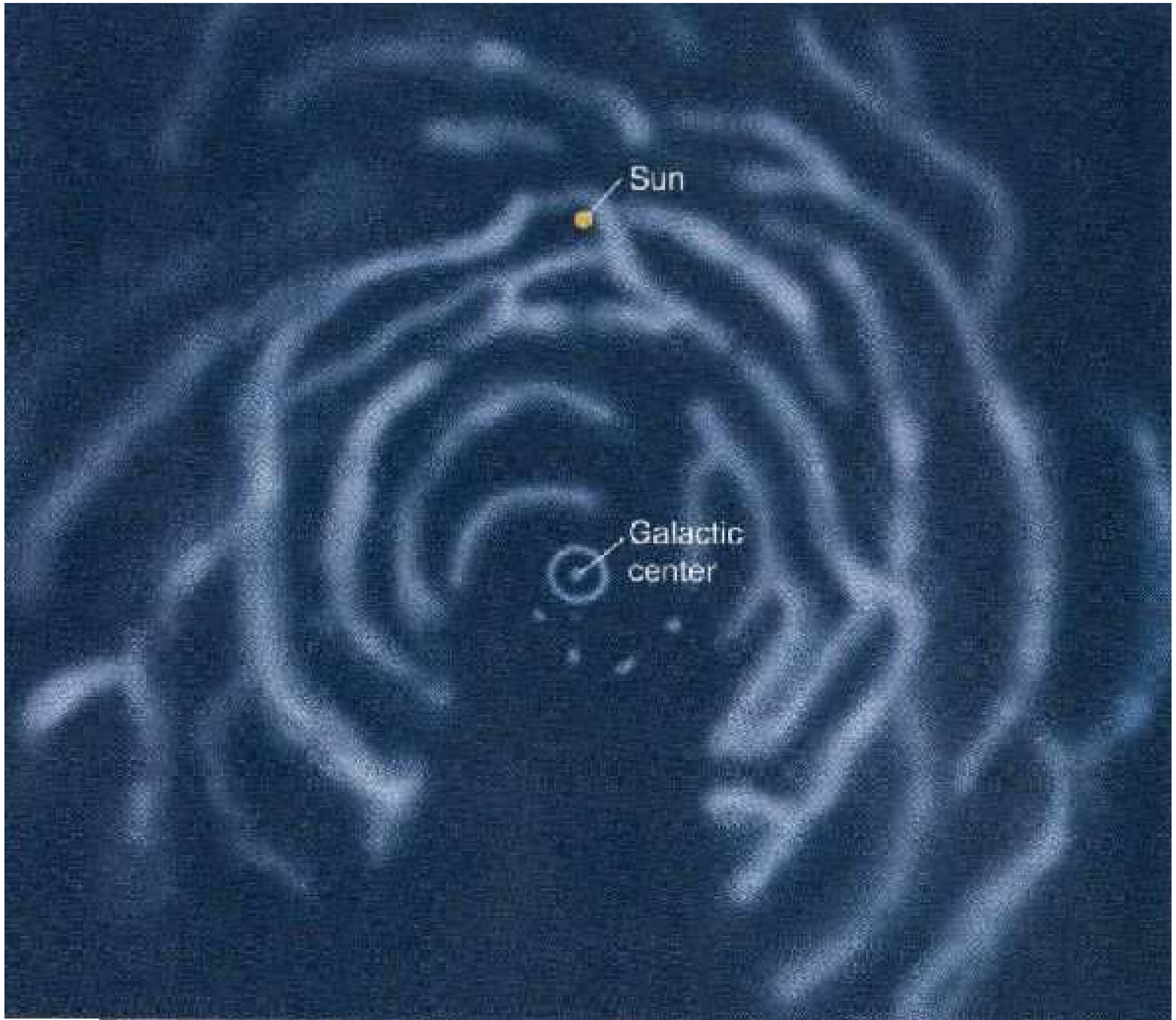
$$\theta \propto \frac{\lambda}{D}$$

$35'$ with a 25m dish

$9'$ with a 100m dish

$\leq$ 2cm surface accuracy

Distribution of the line intensity



http://www.astro.rug.nl

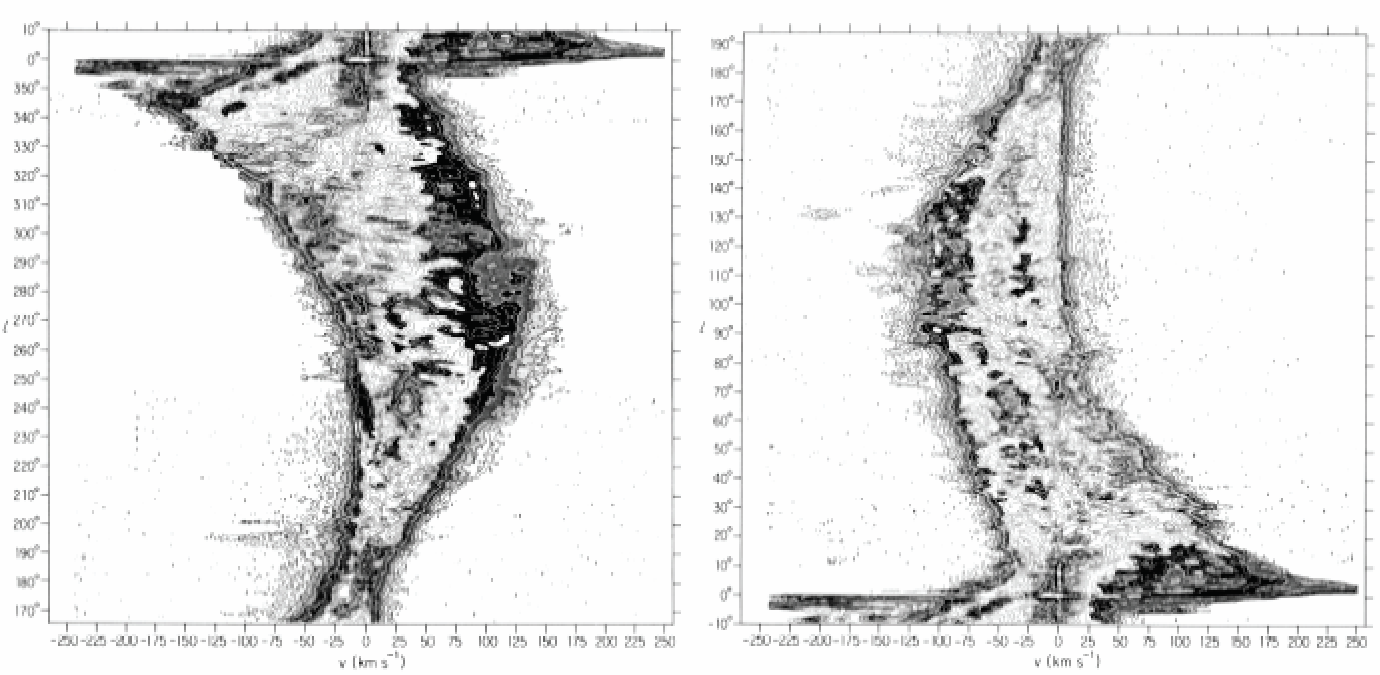
Properties of the HI 21-cm line

- Natural line width is 10^{-16} km s^{-1}, accordingly the line is ideal suited to trace turbulence and Doppler motions.
- The thermal line width is $\sigma \sim 0.09 \sqrt{T}$, which corresponds to 2 km s^{-1} at $T = 100$ K
- Doppler shift

$$\nu' = \left(1 - \frac{\upsilon}{c}\right) \cdot \nu$$

$$\Delta\nu = \frac{\upsilon}{c} \cdot \nu$$

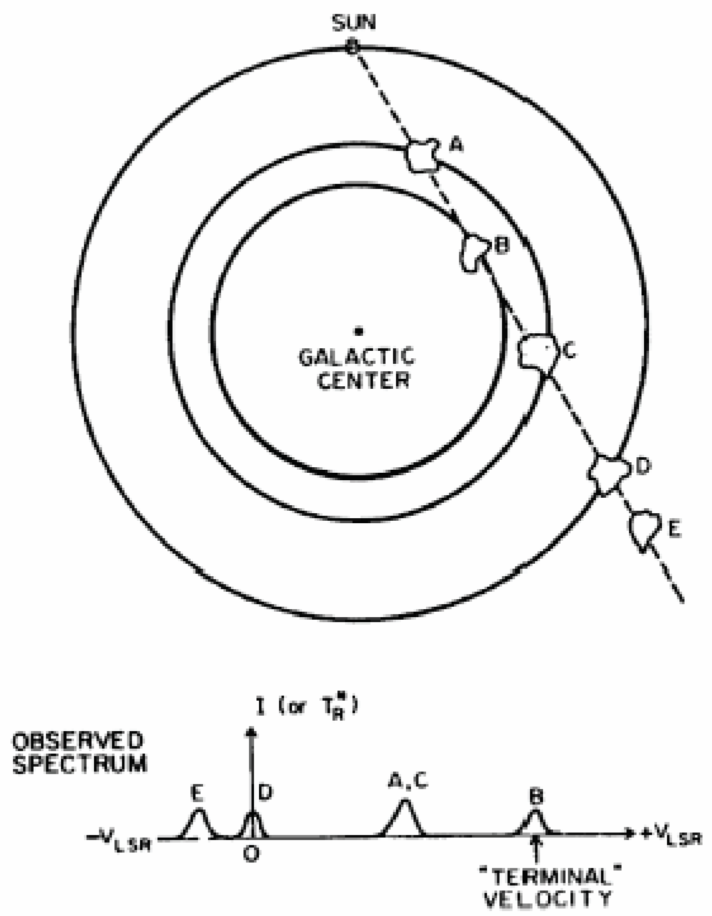
HI in the Milky Way



HI 21-cm rotation curve of the Milky Way galaxy. Shown is the velocity measured relative to the local standard of rest versus the galactic longitude (Burton 2001)

HI in the Milky Way

- Assuming that the gas encircles the center of the Milky Way on concentric orbits, it is feasible to determine is distance (tangential point method).
- The method is restricted to the inner galaxy. However, it probes a large fraction of the baryonic mass.



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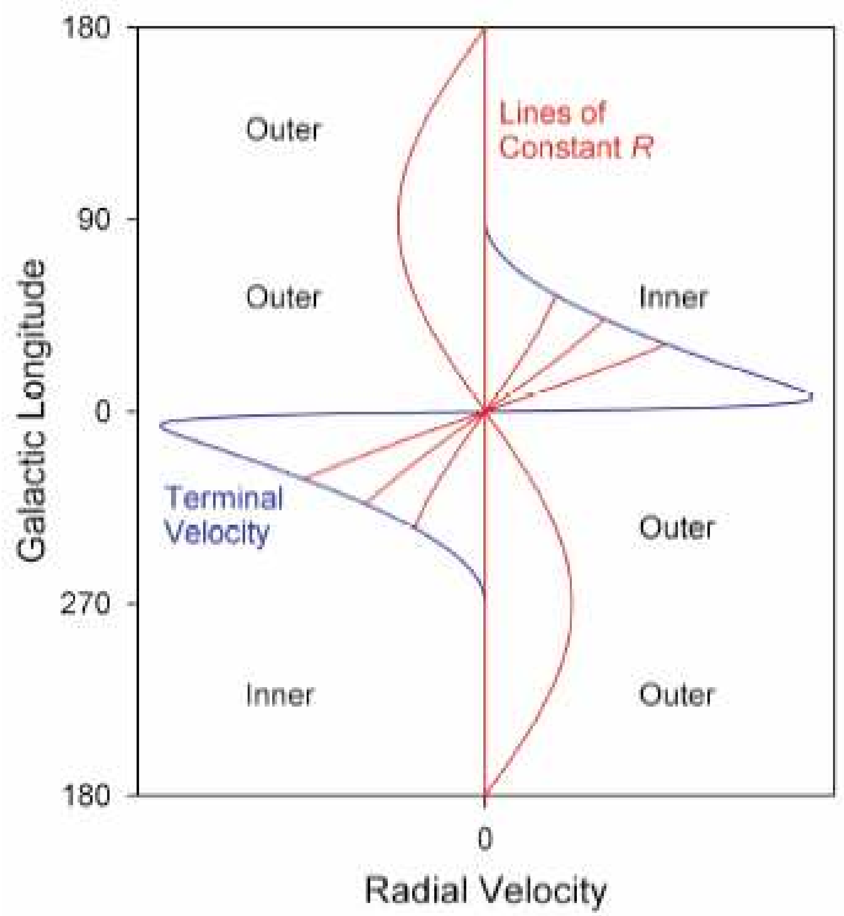
HI in the Milky Way



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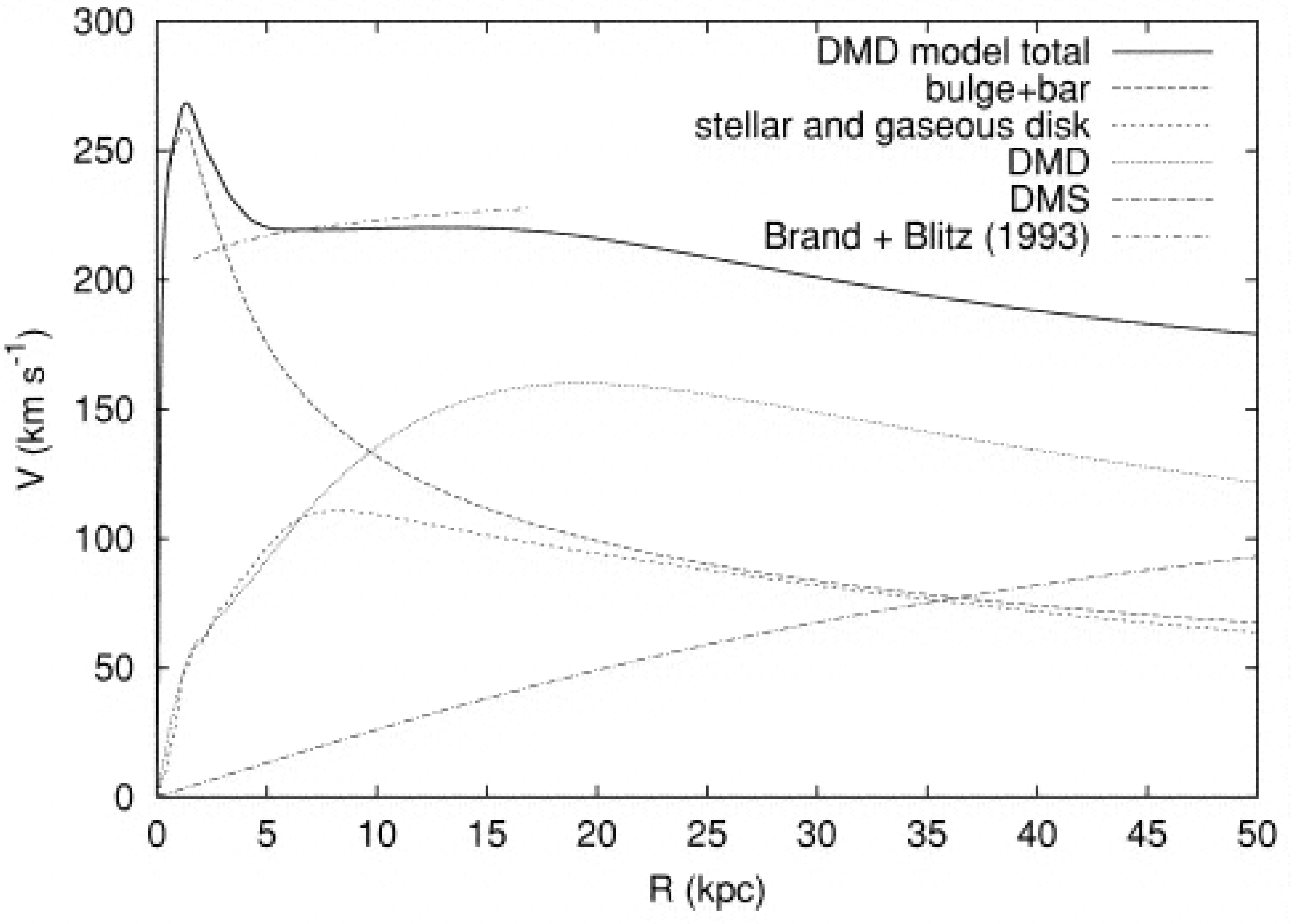
HI in the Milky Way

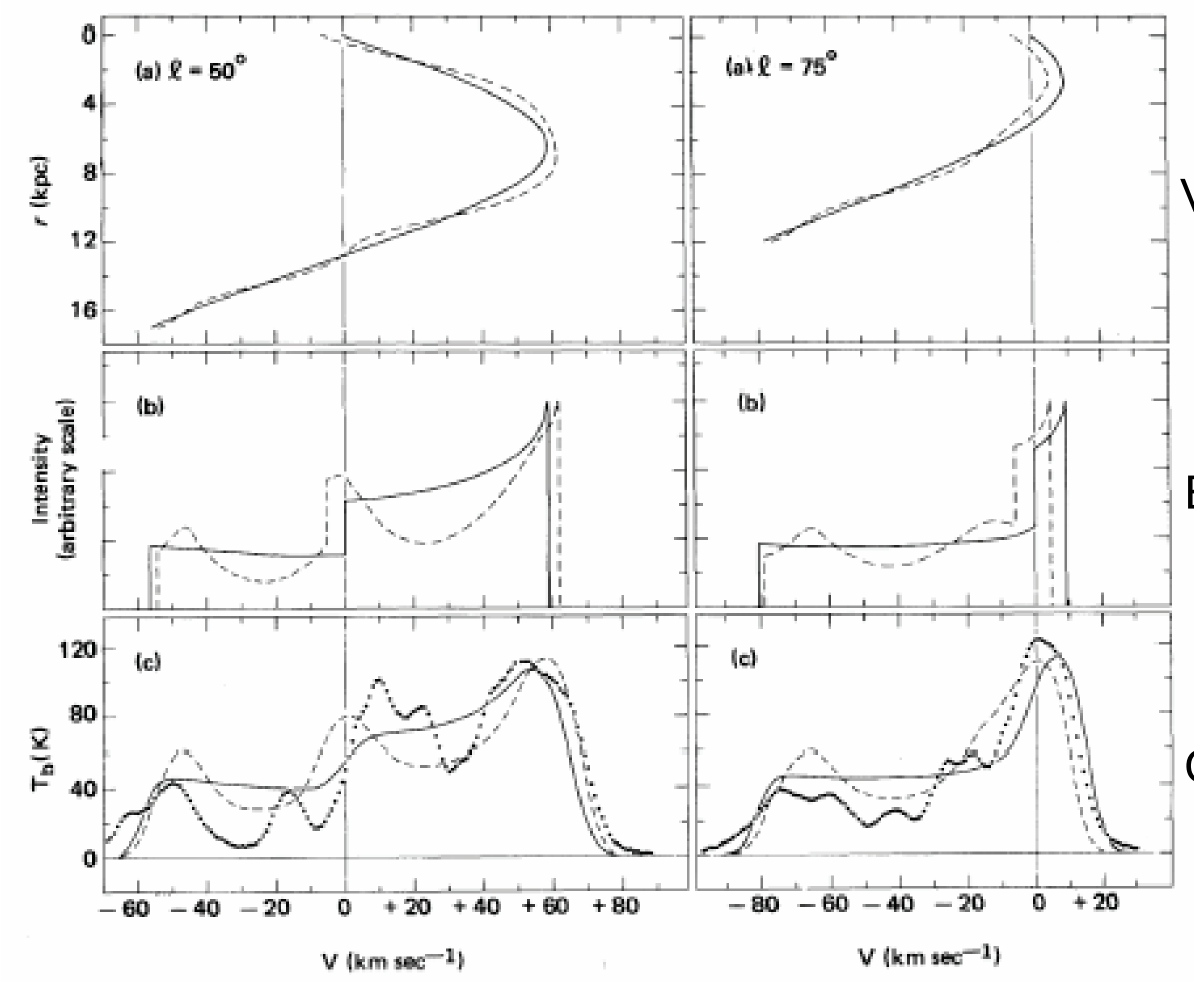
- The tangent point method is only applicable for $R < R_{Sonne}$.
- The outer galaxy can be explored by the HI 21-cm line only via continuing the rotation curve. The derived dynamical distances are only estimates with partly high uncertainties.

HI in the Milky Way



Kalberla, 2003, ApJ 588, 805

HI in the Milky Way



Velocity

Expected line shape

Observed line

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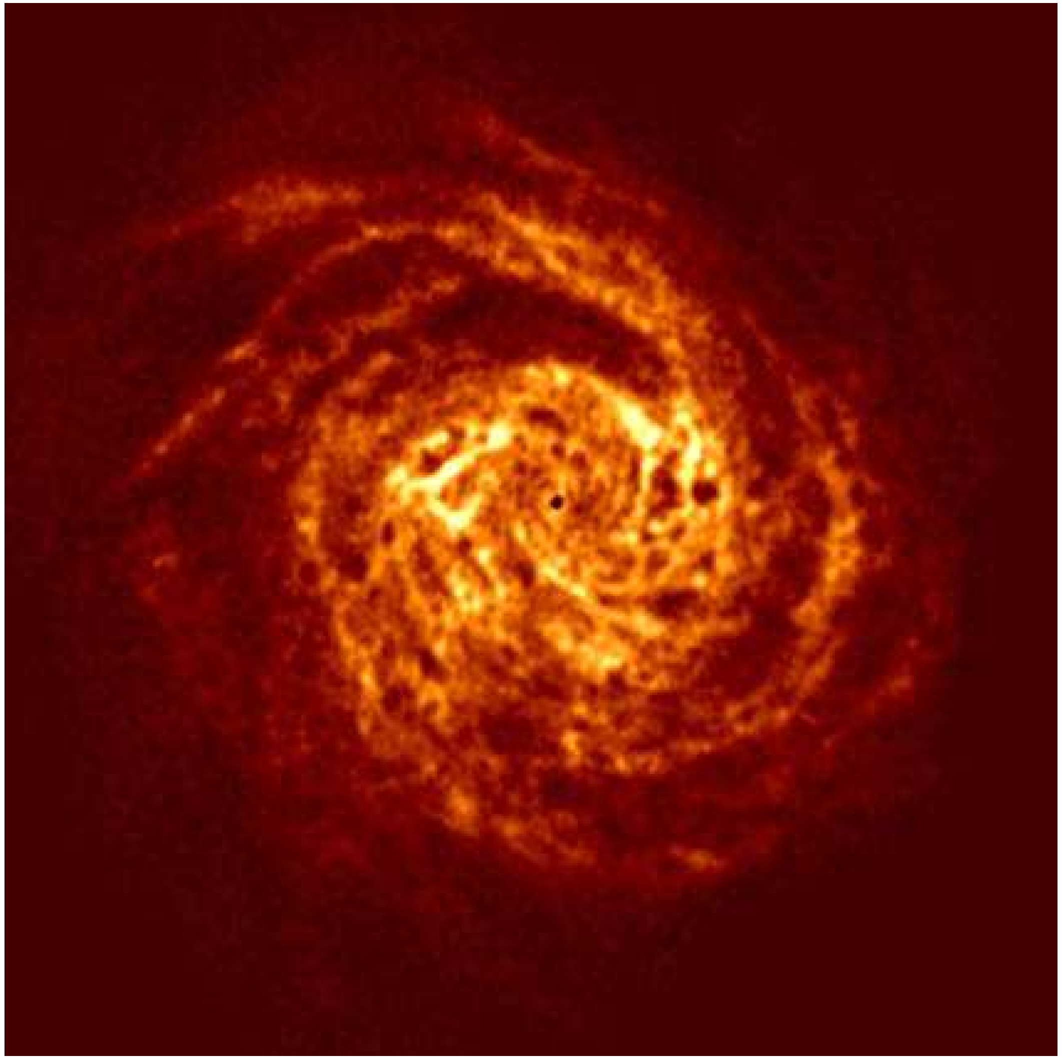
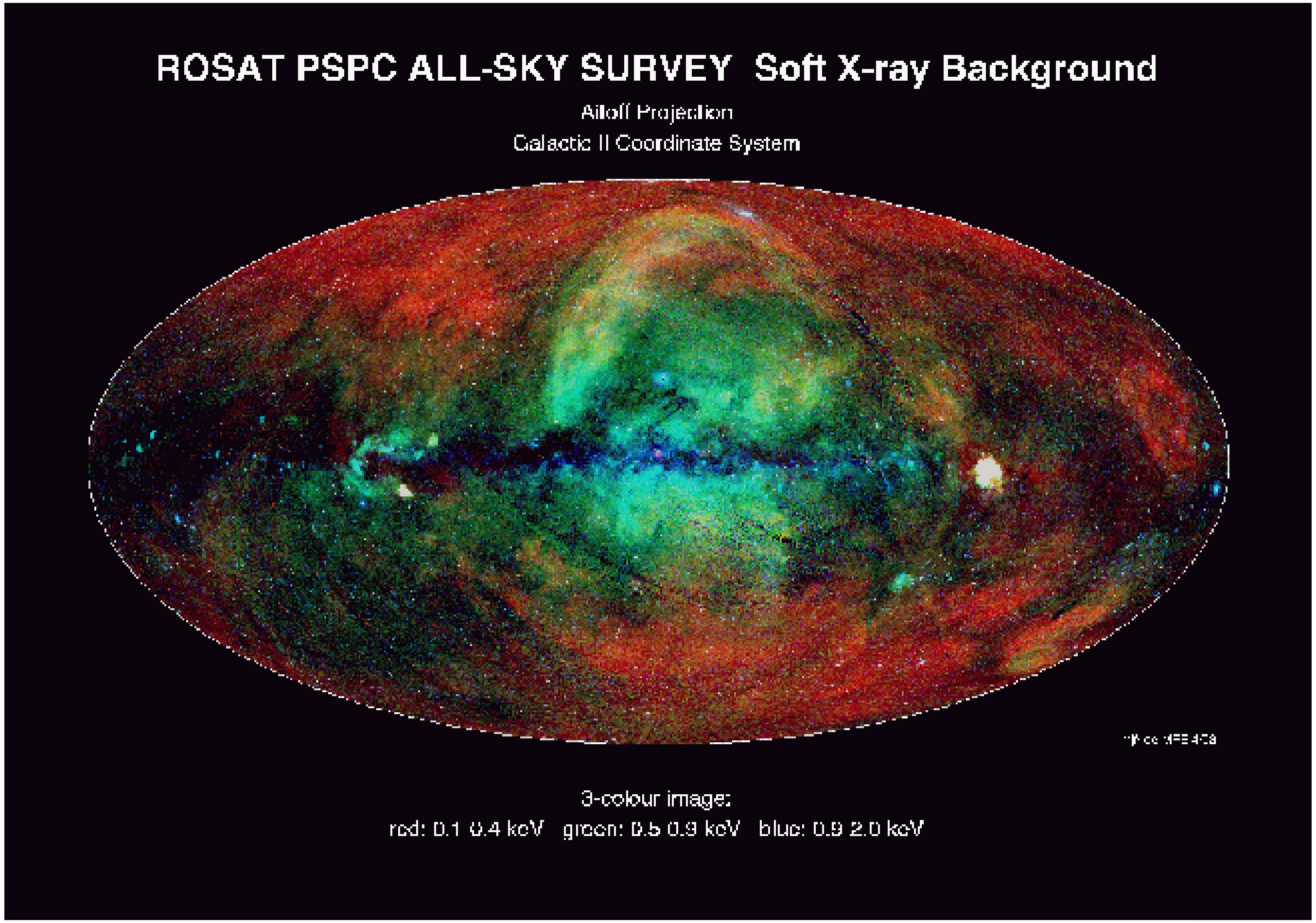
Discovery of the ISM



http://www.astron.nl

Discovery of the ISM

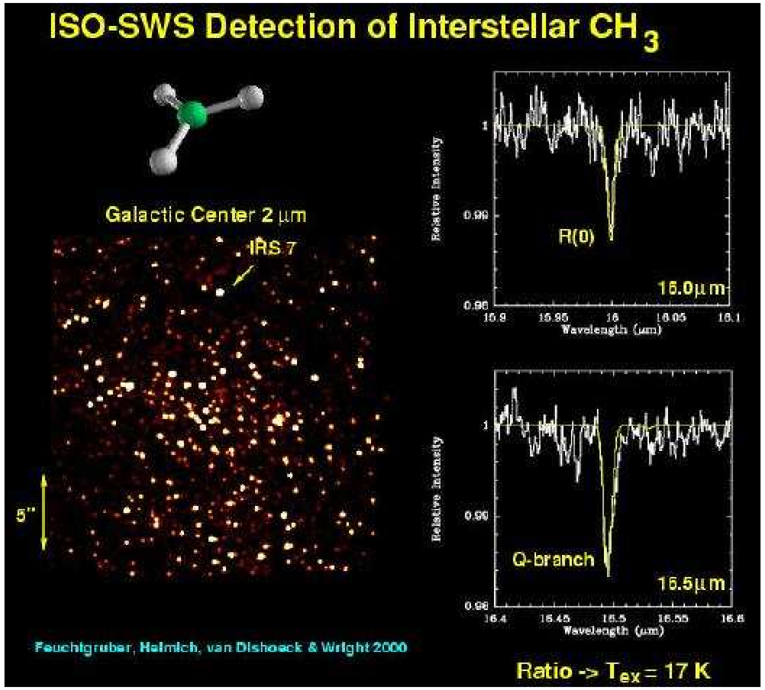
- 1960 discovery of the soft X-ray background



http://www.mpe.mpg.del

Discovery of the ISM

- 1963 the first interstellar maser had be discovered (OH)
- 1968 NH_3, the "thermometer" in the Universe was observed for the first time
- 1970 $^{12}CO(1 \rightarrow 0)$, the second most abundant molecule in the Universe was discovered



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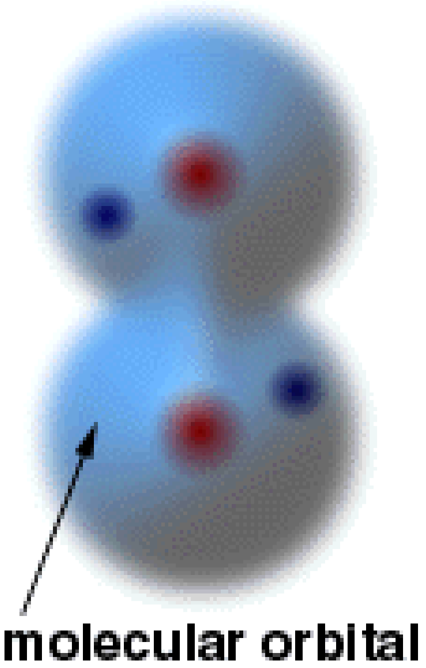
Discovery of the ISM

- 1970`s infrared astronomy opens the window to the most abundant molecule H_2



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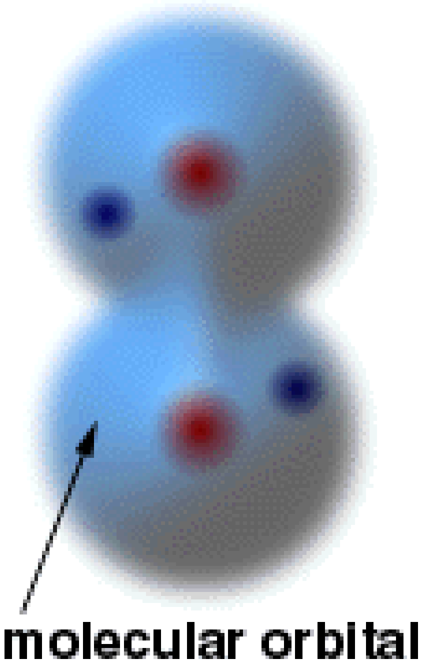
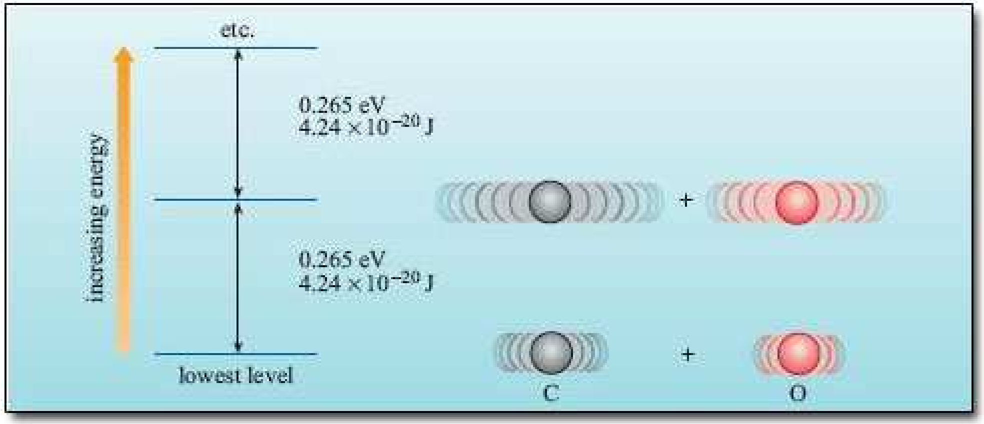
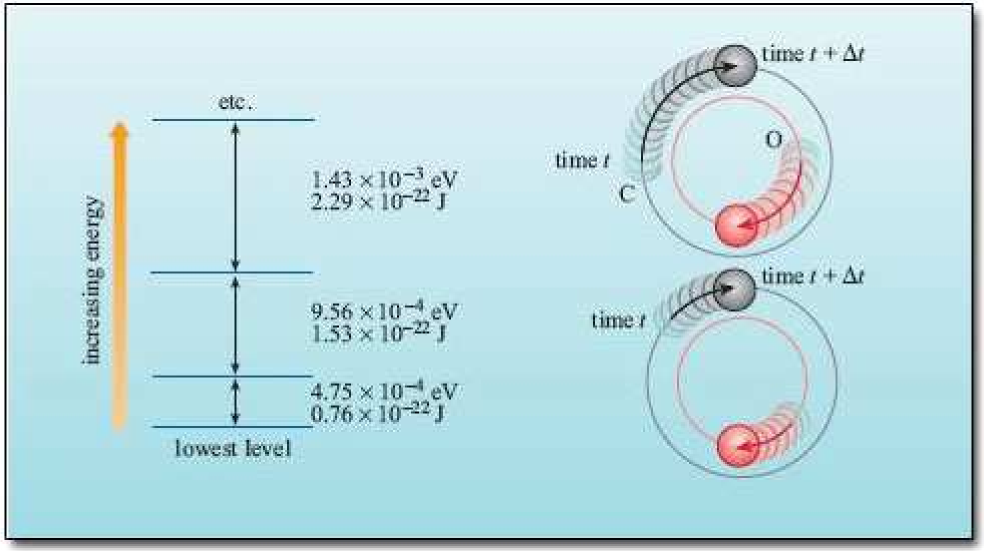
H_2: necessity for tracers

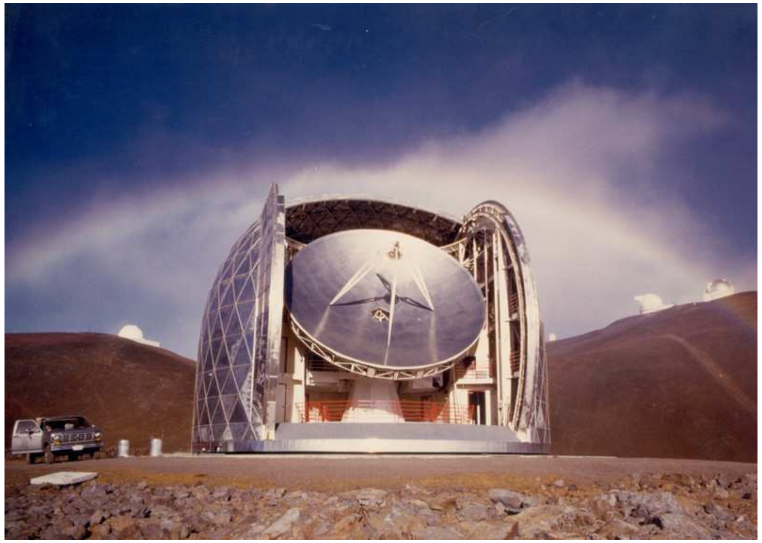




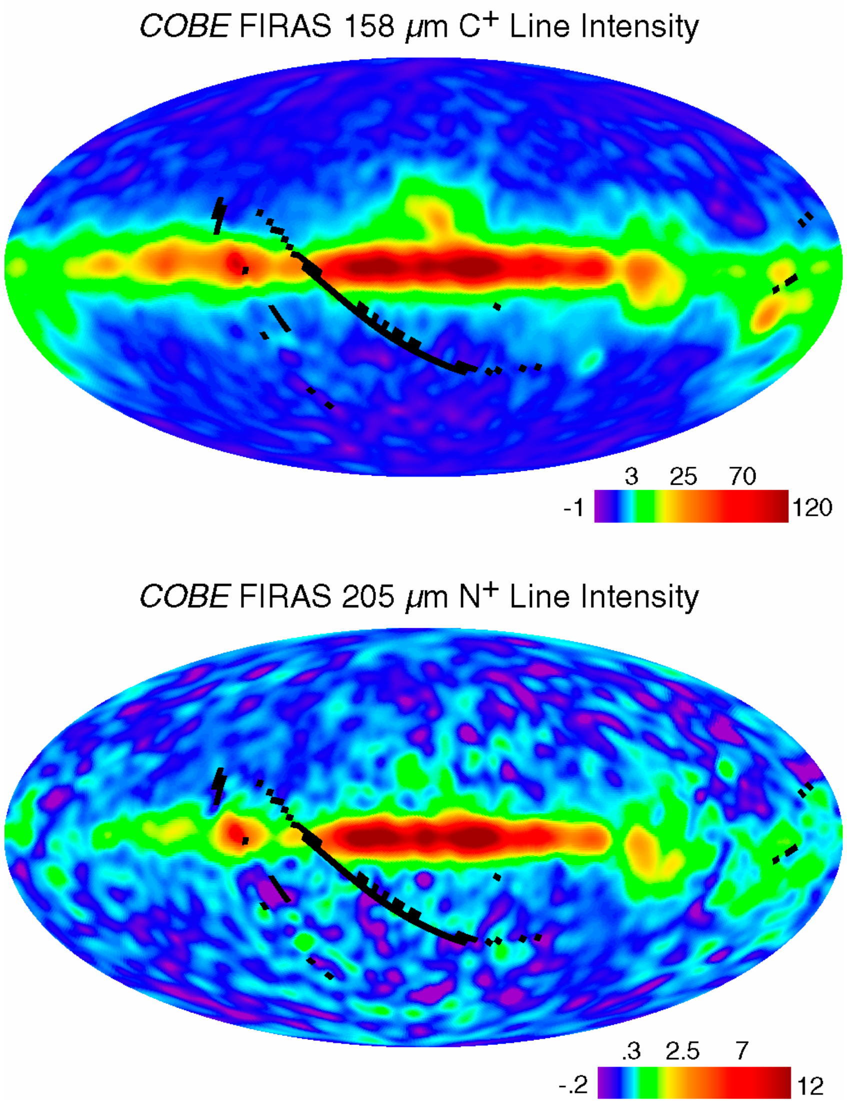


Discovery of the ISM

- ~1990 Sub-millimeter astronomy opened the window to molecular clouds and star forming regions
- 1990 COBE studied the distribution of the dominant cooling line of the ISM CII.
- 1995 allowed detailed spectroscopic studies of the dust, the vibrationally excited H_2 emission line and the <u>infrared dark clouds</u>

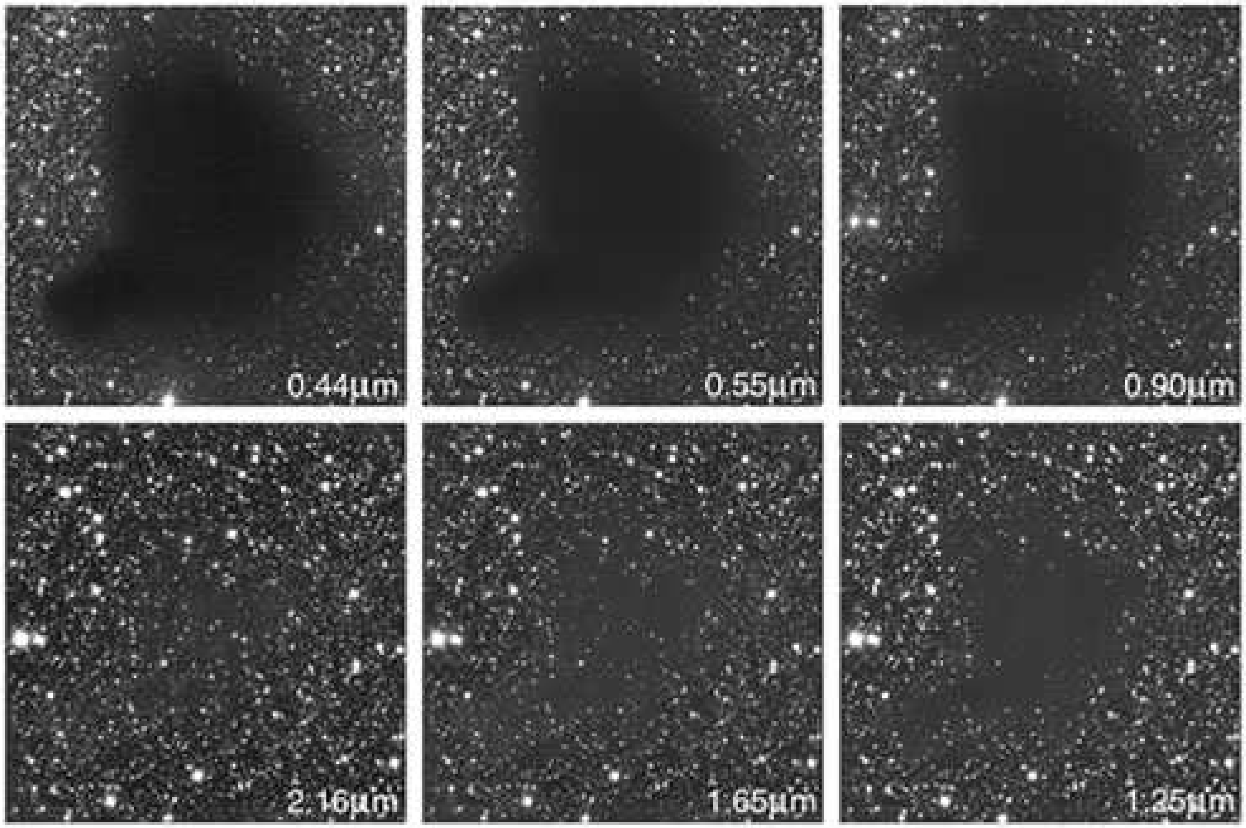


Discovery of the ISM

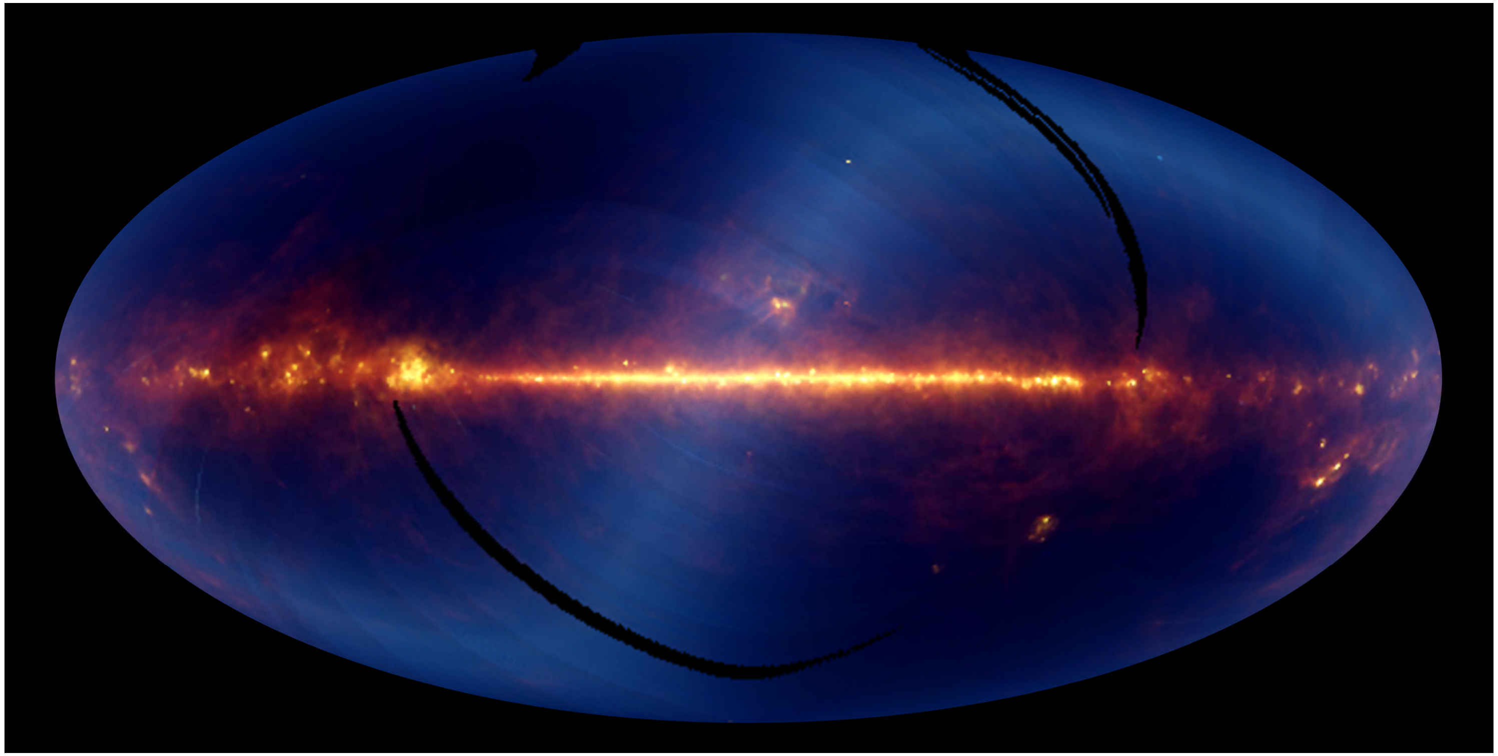


Discovery of the ISM

- Infrared Dark Clouds

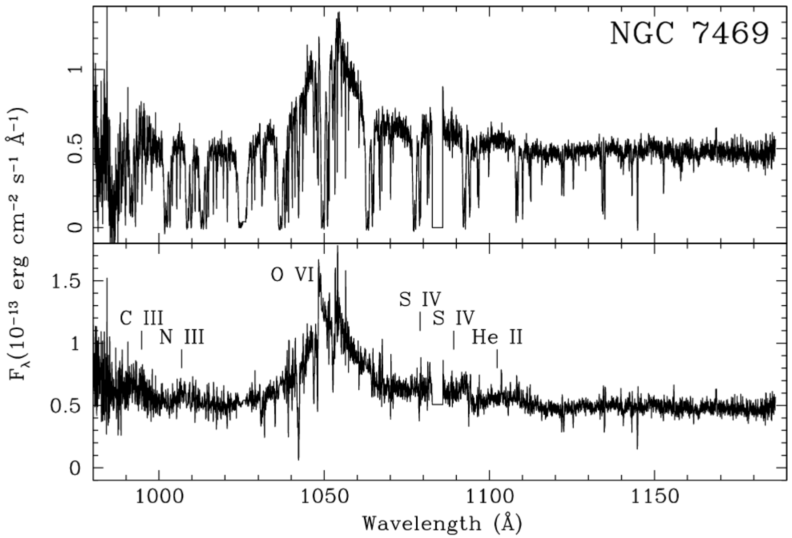


Discovery of the ISM



Discovery of the ISM

- 1998-2006 SWAS studied the distribution of H_2O, O_2, CI up to 500 GHz
- 2000-today FUSE observation of H_2 in absorption against background continuum sources, observation of the vertical structure of highly ionized gas like OVI and NV
- 2003-today Spitzer studies the ISM with high angular resolution



Basic properties of the ISM

Basic properties of the ISM

- Confined to the Galactic Plane (much flatter than a compact disk!)
- The interstellar medium consists mainly of hydrogen and helium.
  - All elements heavier than hydrogen are denoted as “metals”
- Temperature range 4 K < T < 10^6K
  - The temperature is used as a measure for the physical conditions of the interstellar gas. The “phases” of the interstellar medium are characterized by the average gas temperature.
- Densities 10^{-4} cm^{-3} < n < 10^7 cm^{-3}
- Far from thermal equilibrium!

Not easy to calculate!

Abundances of elements

Element	Abundance
H	1.00
He	0.075
C	$2.5 \cdot 10^{-4}$
N	$6.3 \cdot 10^{-5}$
O	$4.5 \cdot 10^{-4}$
Na	$2.1 \cdot 10^{-6}$
Mg	$4.2 \cdot 10^{-5}$
Al	$3.1 \cdot 10^{-6}$
Si	$4.3 \cdot 10^{-5}$
S	$1.7 \cdot 10^{-5}$
Ca	$2.2 \cdot 10^{-6}$
Fe	$4.3 \cdot 10^{-5}$

Abundances of the ISM

- "Metalls"
  - He about 10%
  - C,N and O
  - Si, Ca and Fe bound in dust grains
- Grains
  - About 1% of the ISM mass
- Photons
- Magnetic fields
- Cosmic rays

Phases of the interstellar matter

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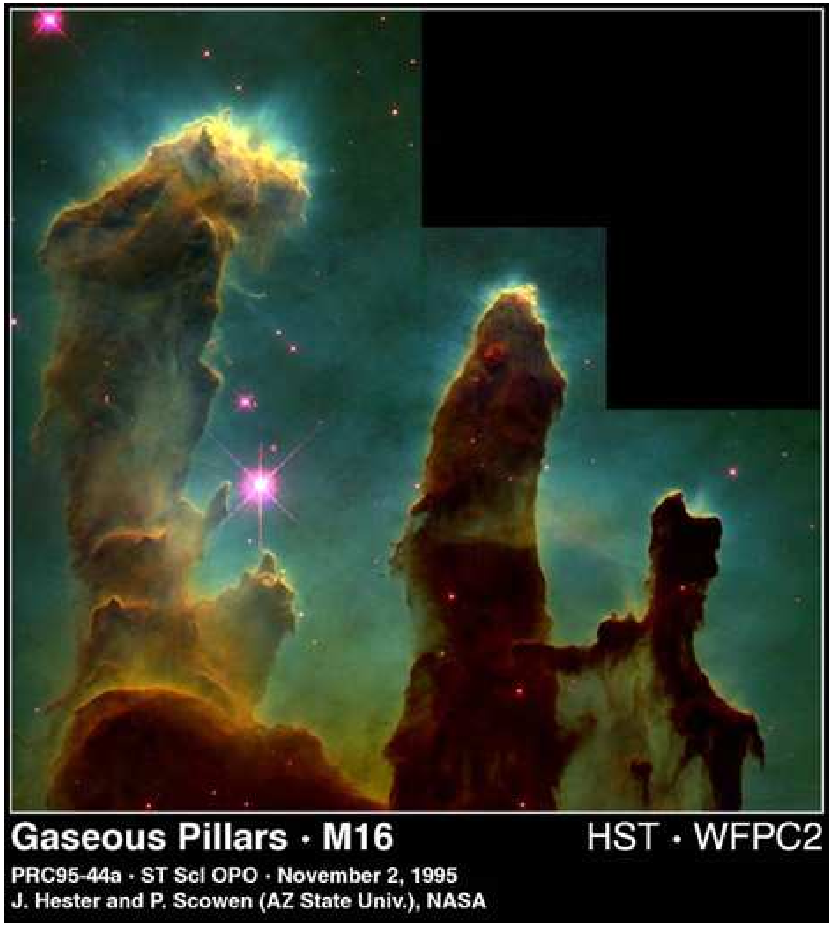
Classification of the ISM

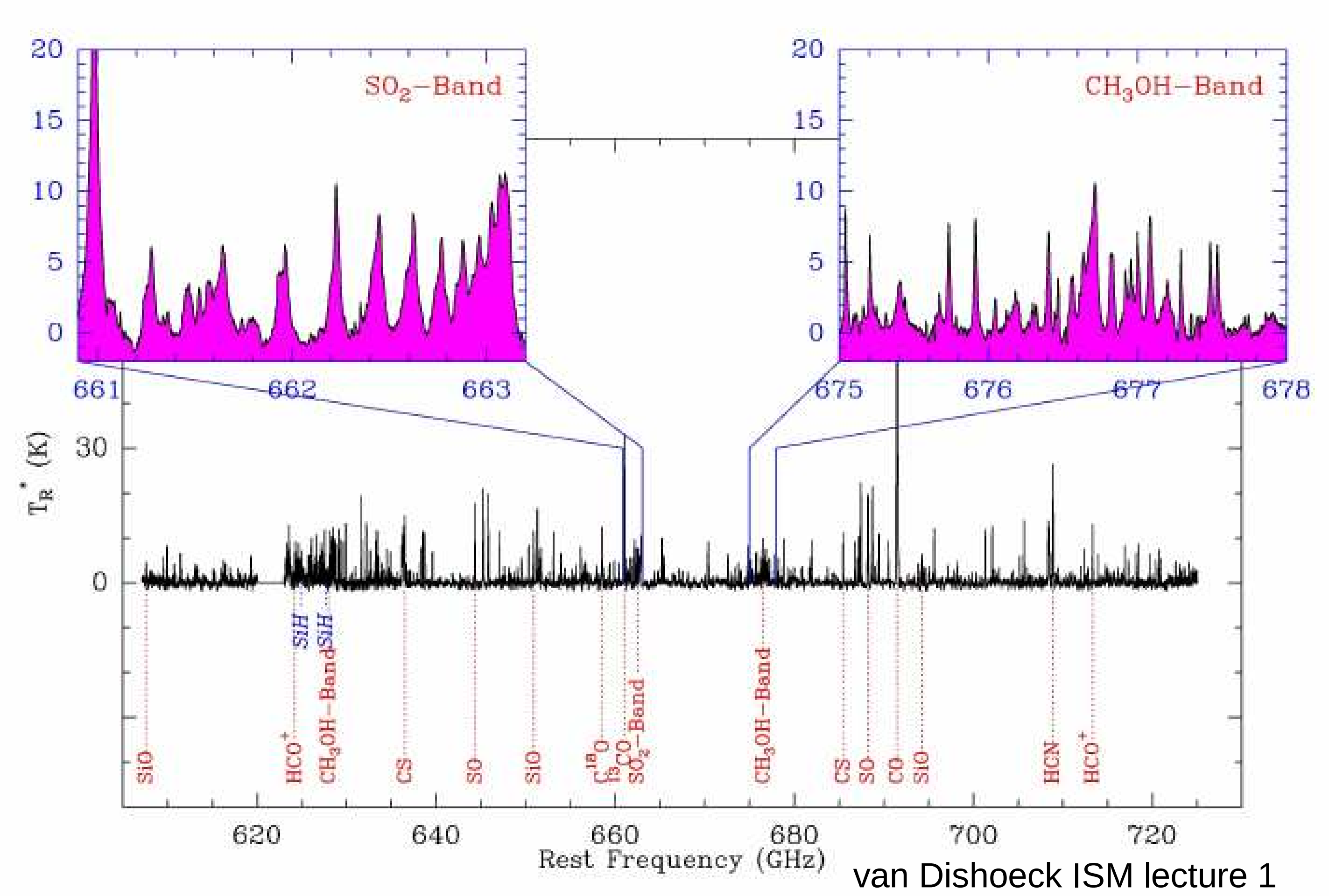
- Chemical composition of the ISM comparable to the elements abundance of the Solar System
- The state of Hydrogen determines the state of the ISM
  - Molecular region $\leftrightarrow H_2$
  - Neutral region $\leftrightarrow$ HI
  - Ionized region $\leftrightarrow H^+$

Molecular regions

- Molecular clouds are defined by the presence of molecular hydrogen. We differentiate between:
- Diffuse molecular clouds
  - $T \approx 40 \ldots 80$ K
  - $n \approx 100$ cm^{-3}
- Dark clouds
  - $T \approx 10 \ldots 50$ K
  - $n \approx 10^4 \ldots 10^6$ cm^{-3}
- Dust is an important constituent of molecular clouds.



Molecular regions (sub-mm range)



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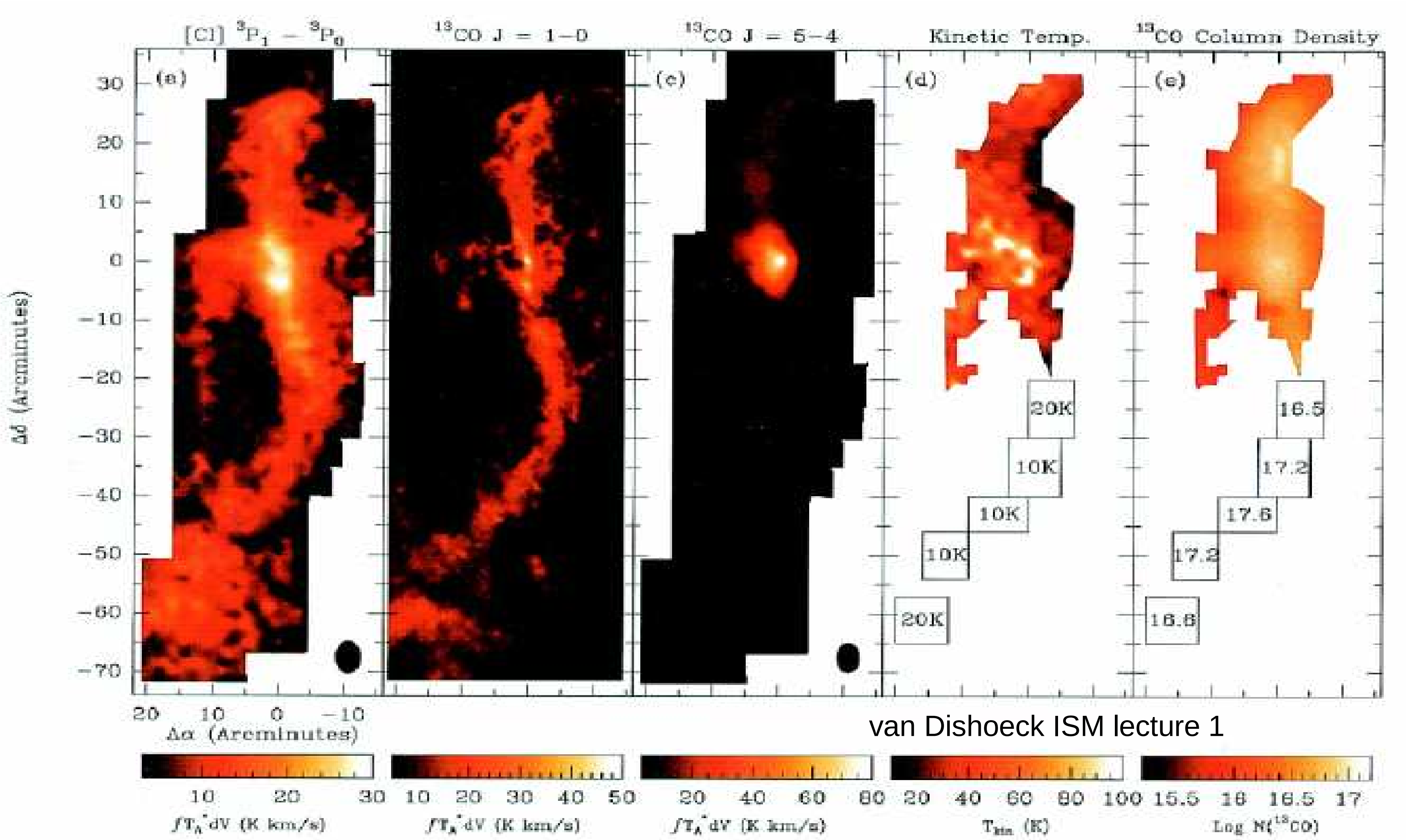
Molecular regions (sub-mm range)



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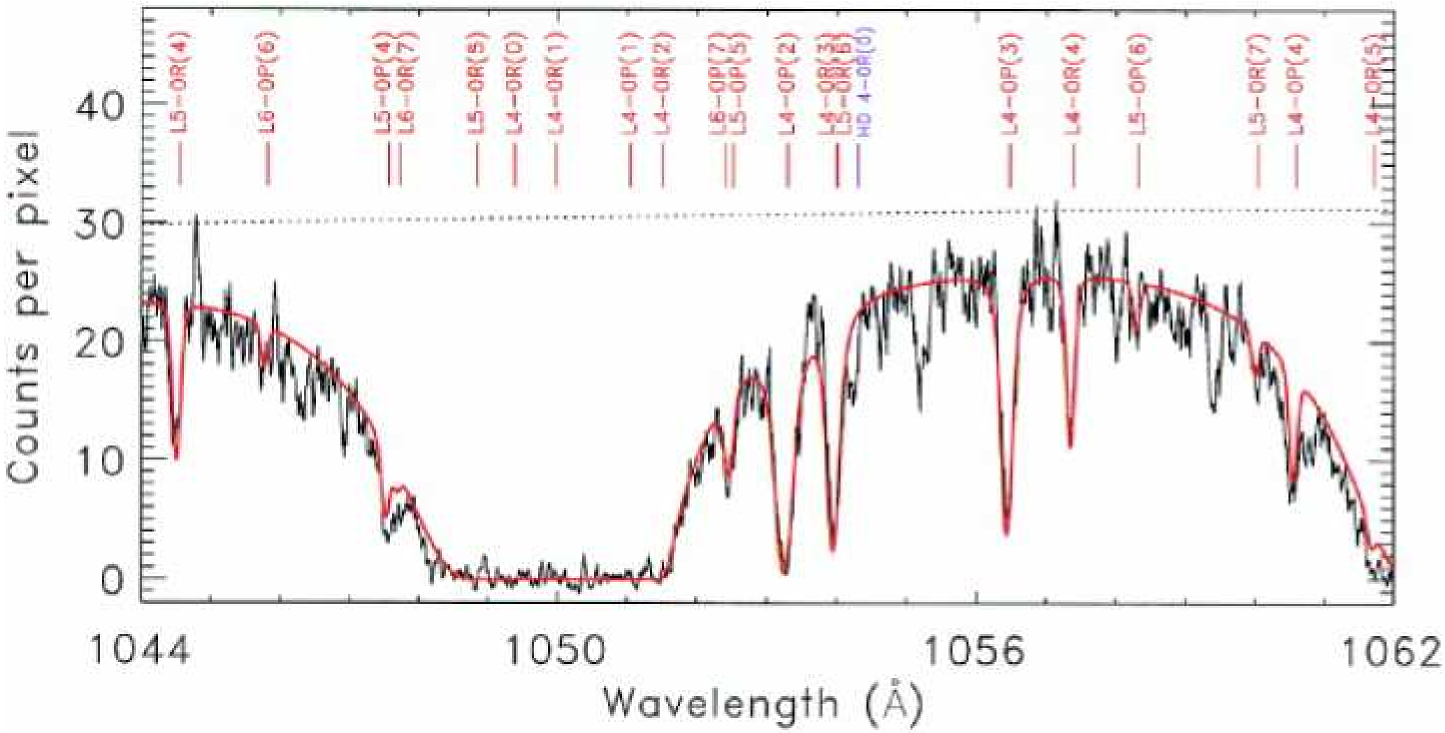
Molecular regions (UV-range)



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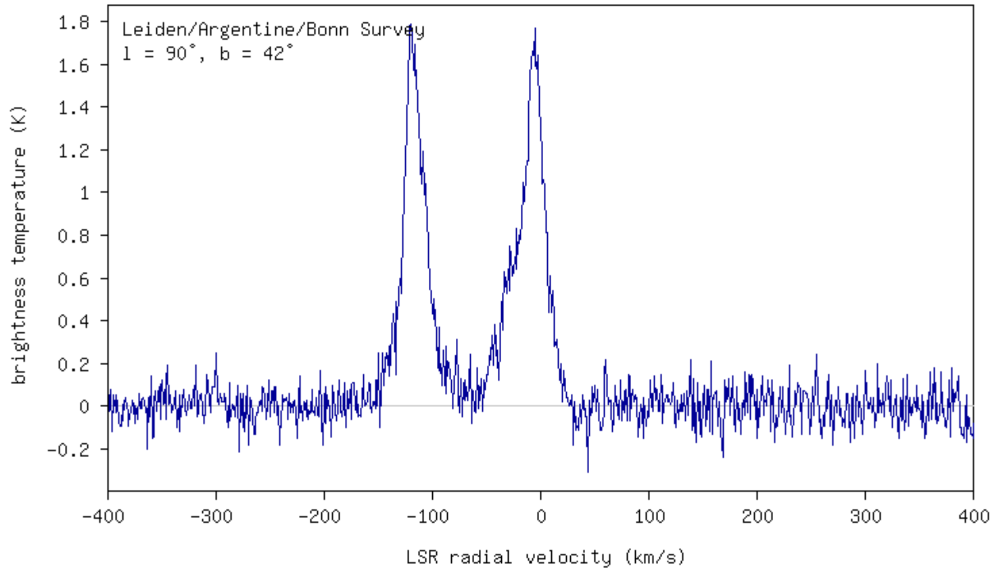
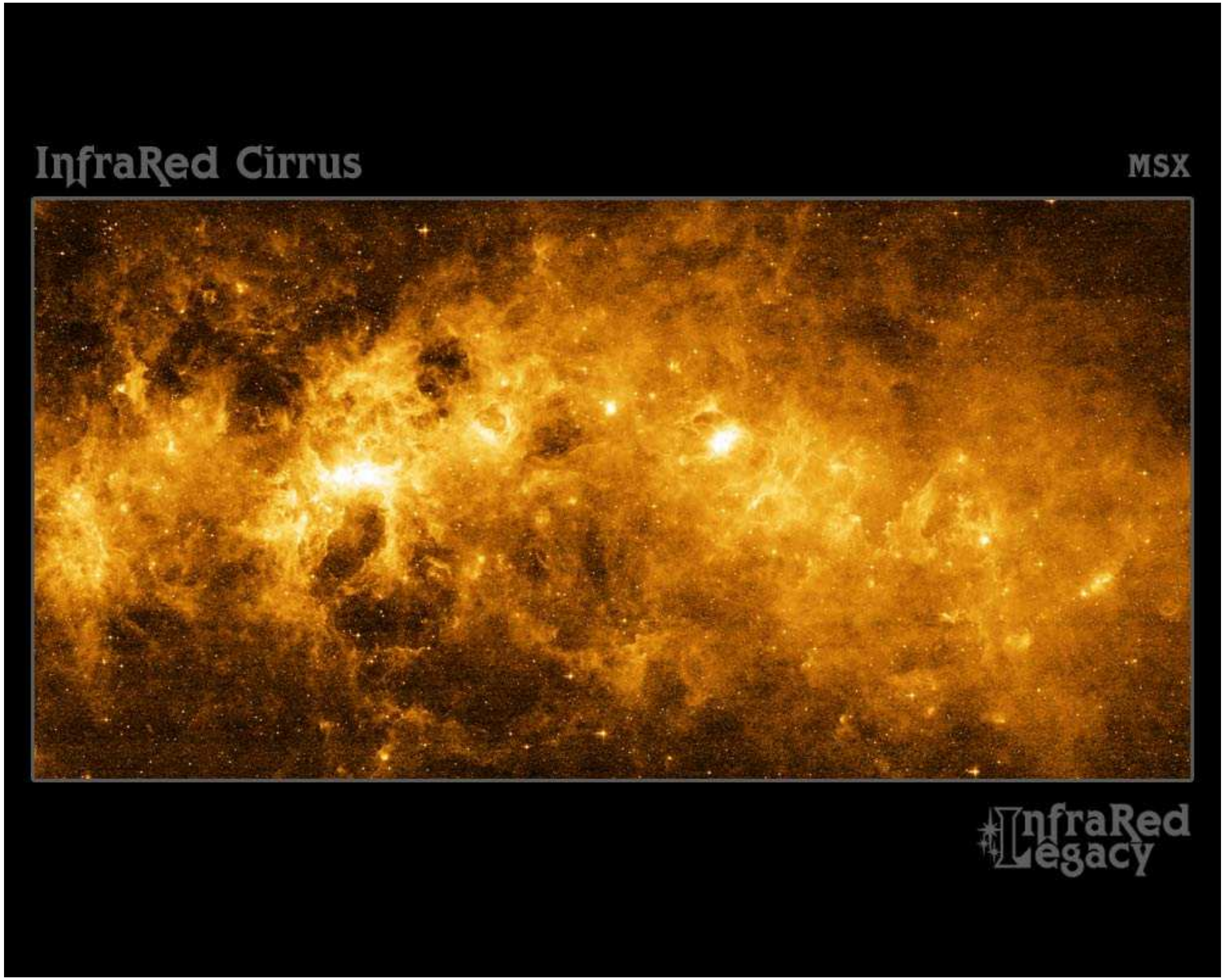
Neutral gas regions (radio-range)

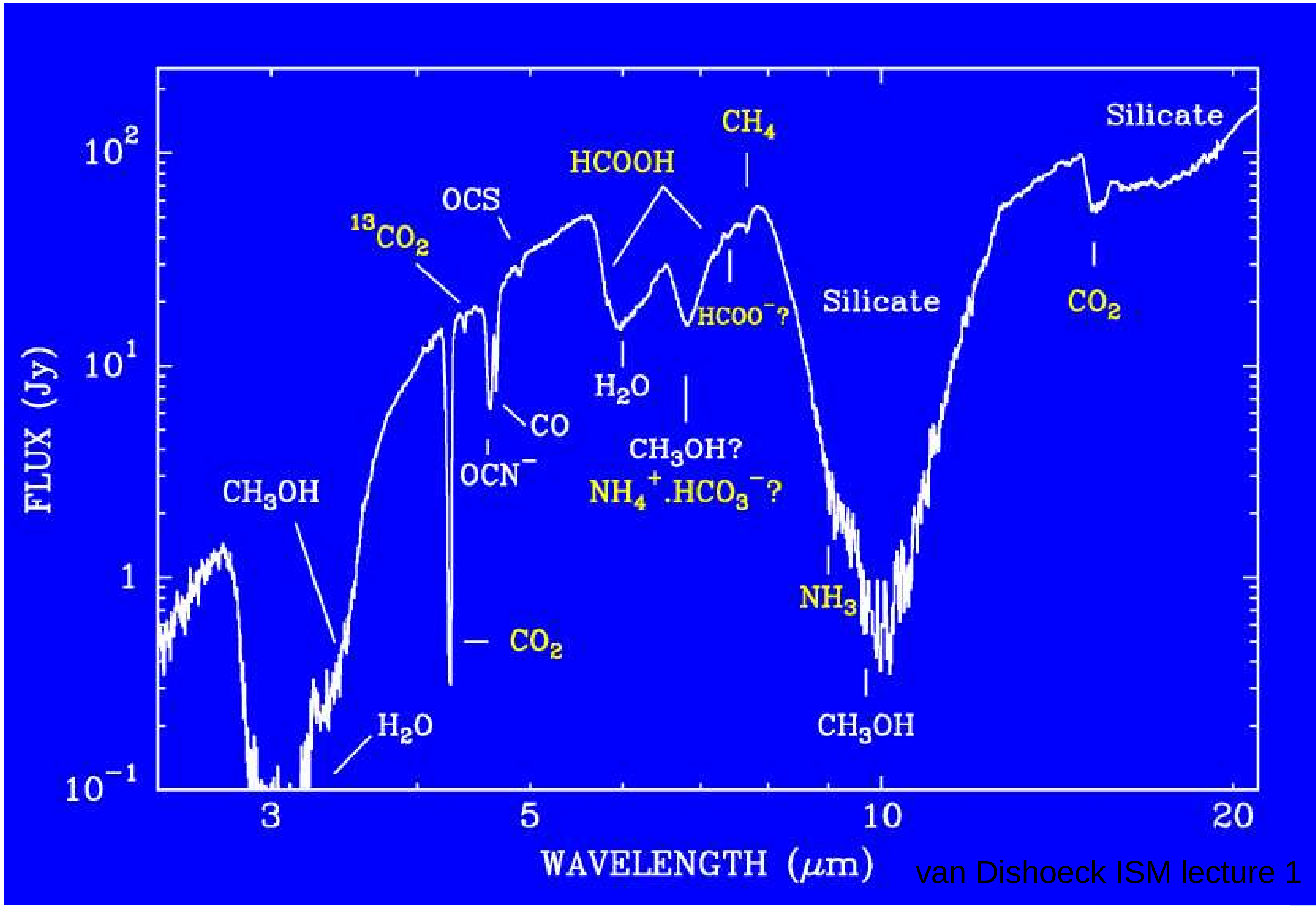
- Dusty cirrus clouds
  - $T \approx 80$ K
  - $n \approx 1$ cm^{-3}
- Warm neutral gas
  - $T \approx 6000$ K
  - $n \approx 0.05 \ldots 0.2$ cm^{-3}



Neutral gas regions (infrared range)



Neutral gas regions (infrared range)



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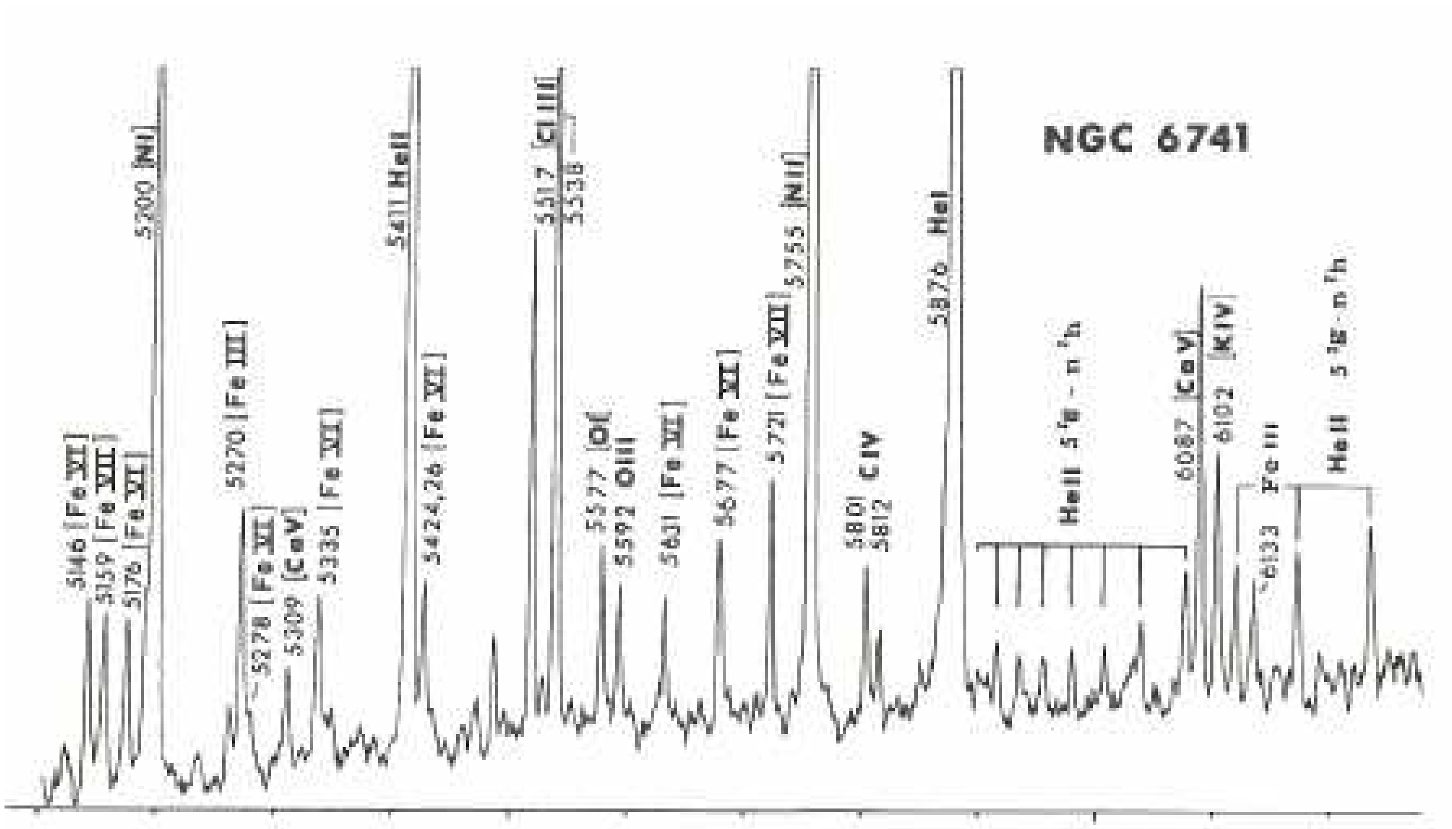
Ionized gas regions

- HII regions are envelopes of early-type stars

  – $T \approx 10^4$ K

  – $n \approx 0.1 \ldots 10^4$ cm^{-3}

Ionized gas regions



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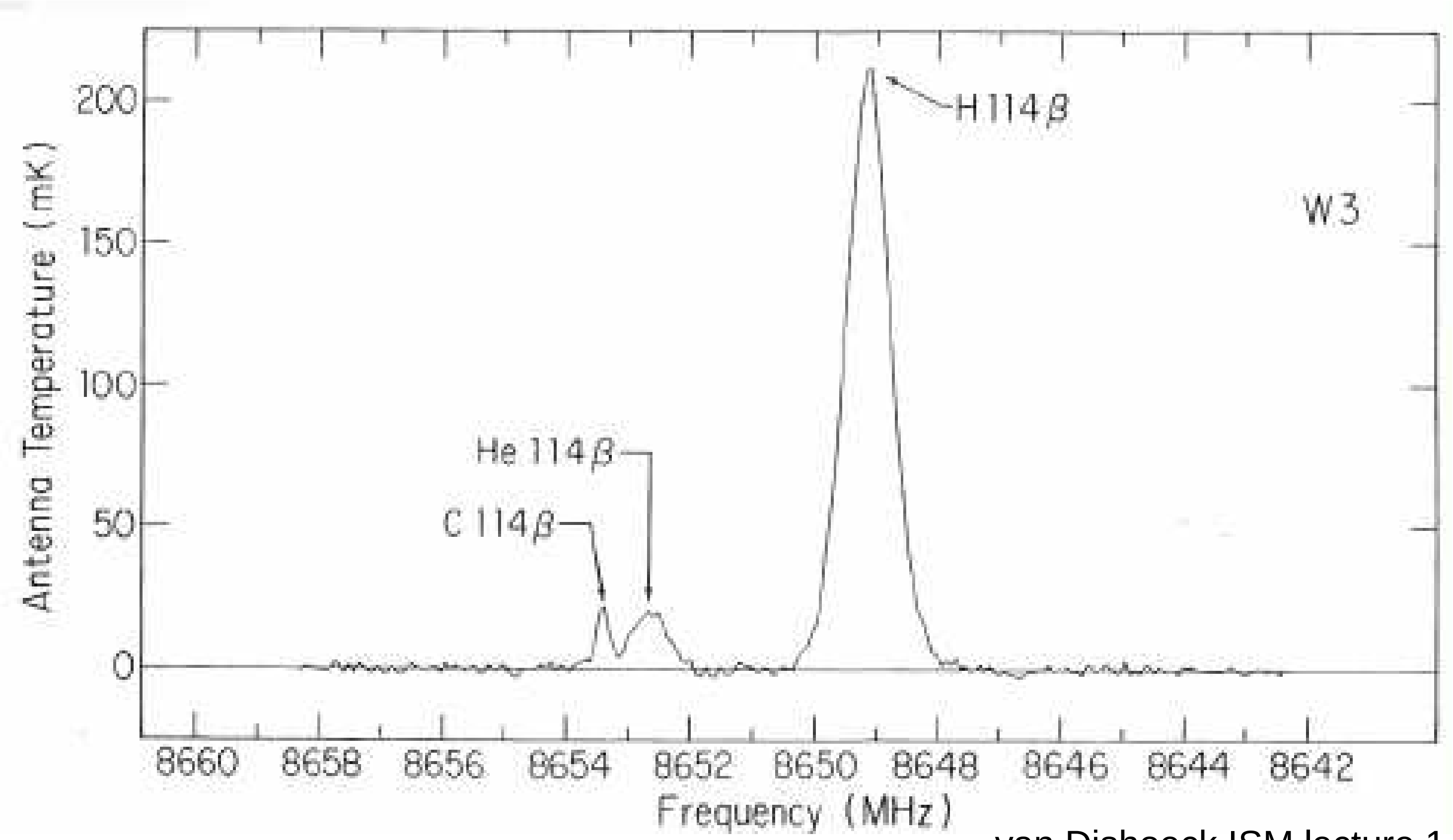
Ionized gas regions



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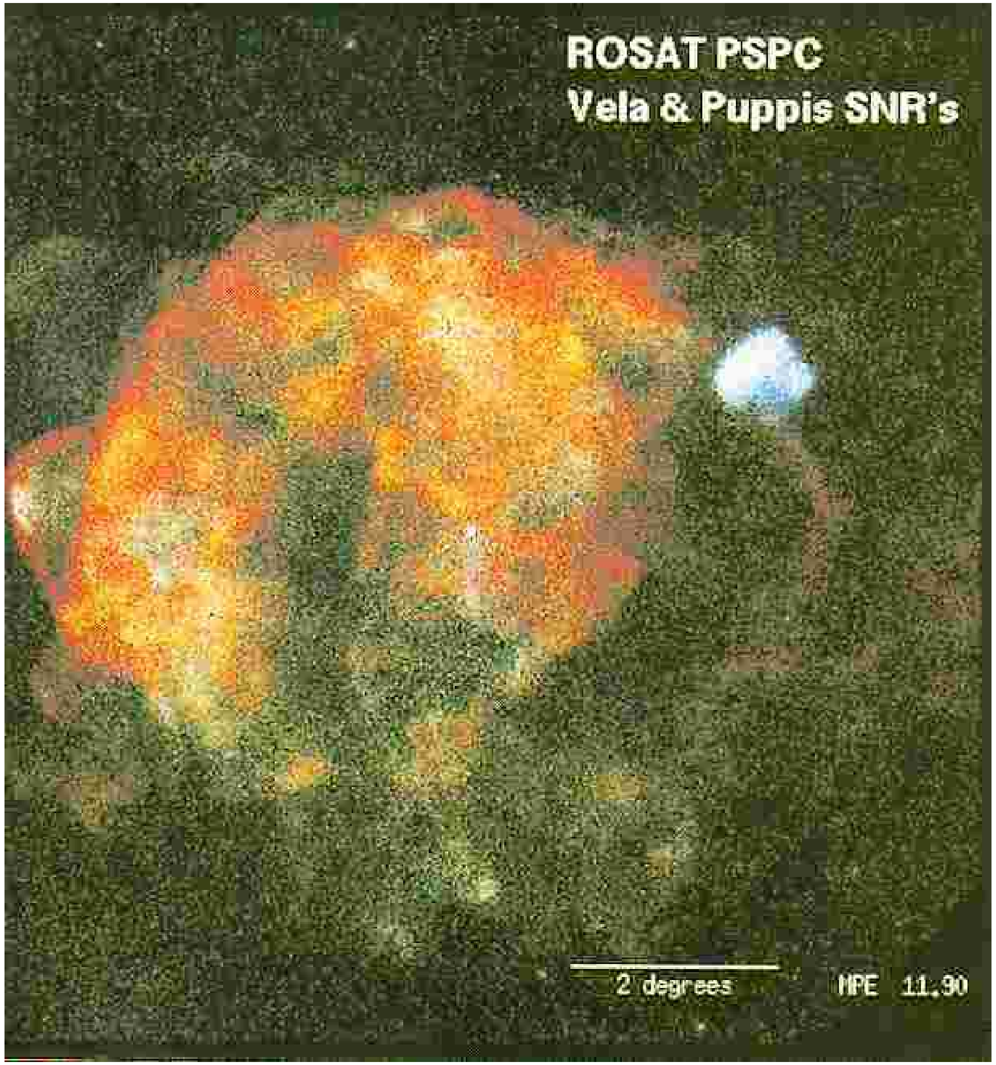
Ionized gas regions

- Coronal Gas
  - $T \approx 10^6$ K
  - n ≈ 0.005 cm^{-3}



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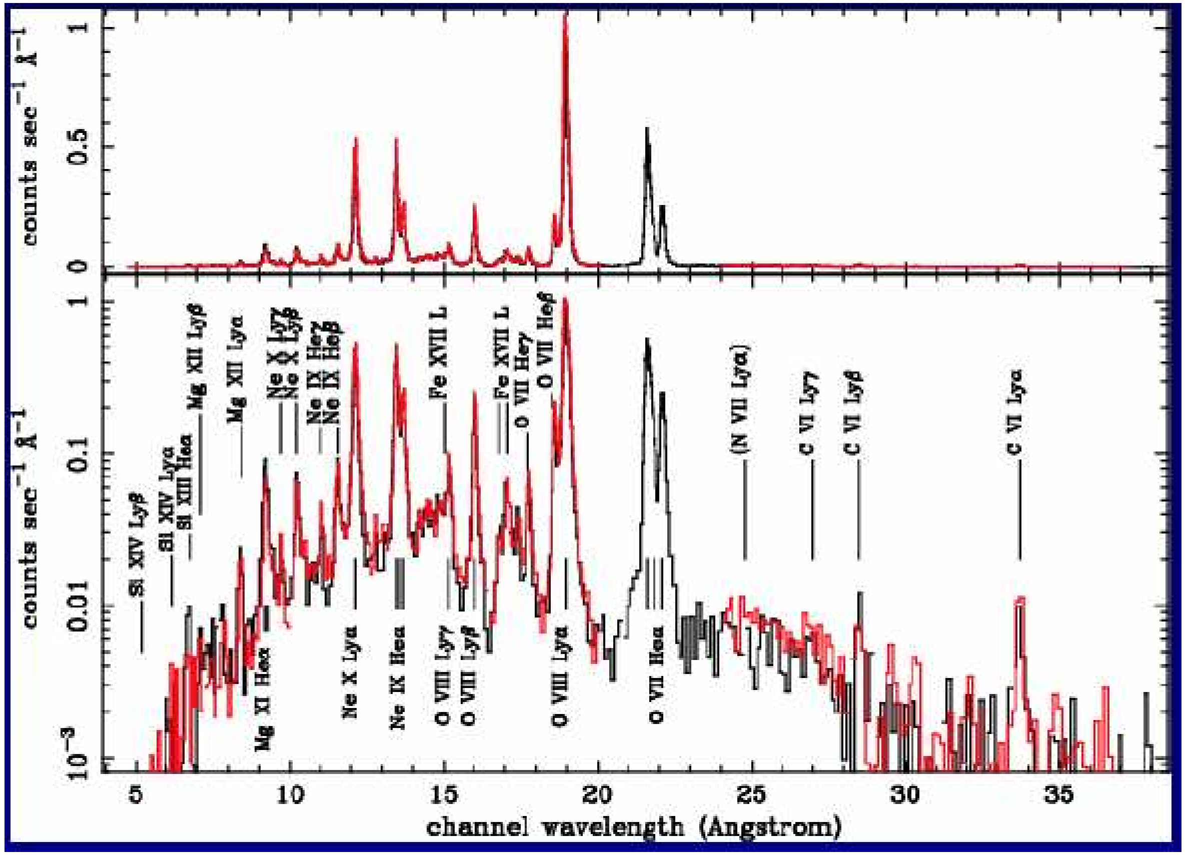
Ionized gas regions



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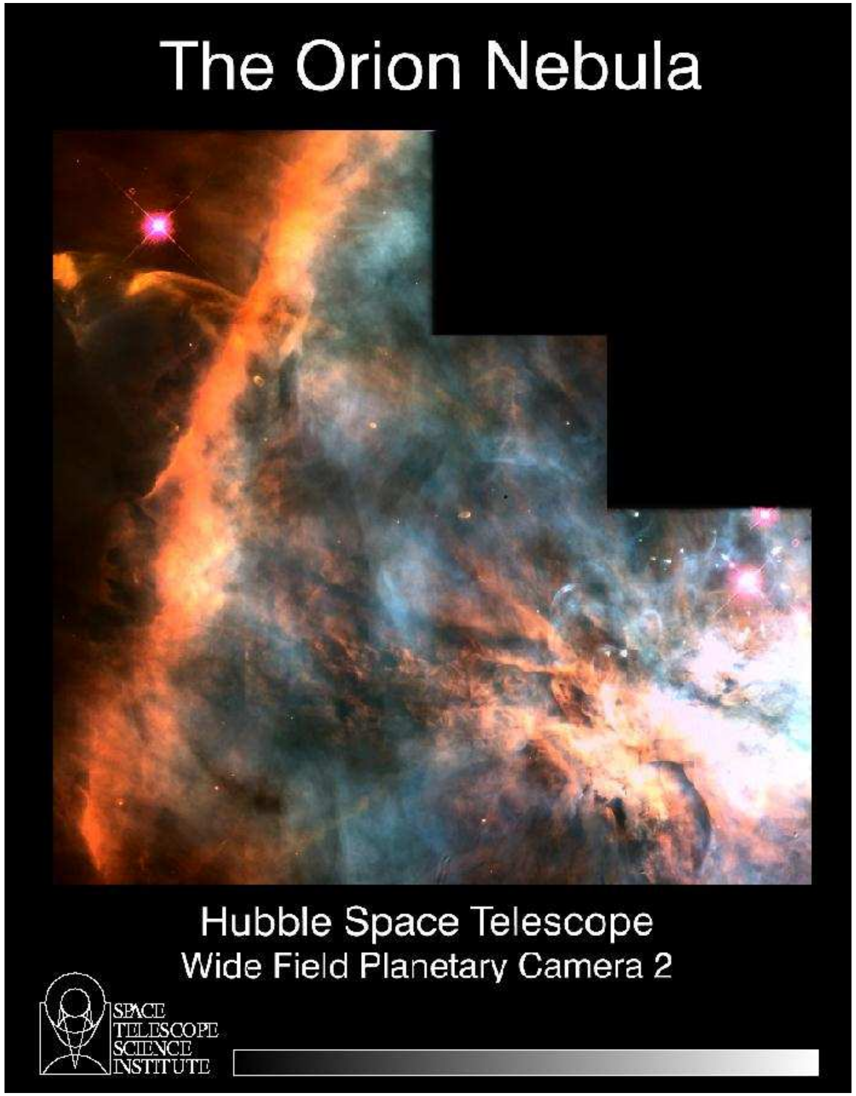
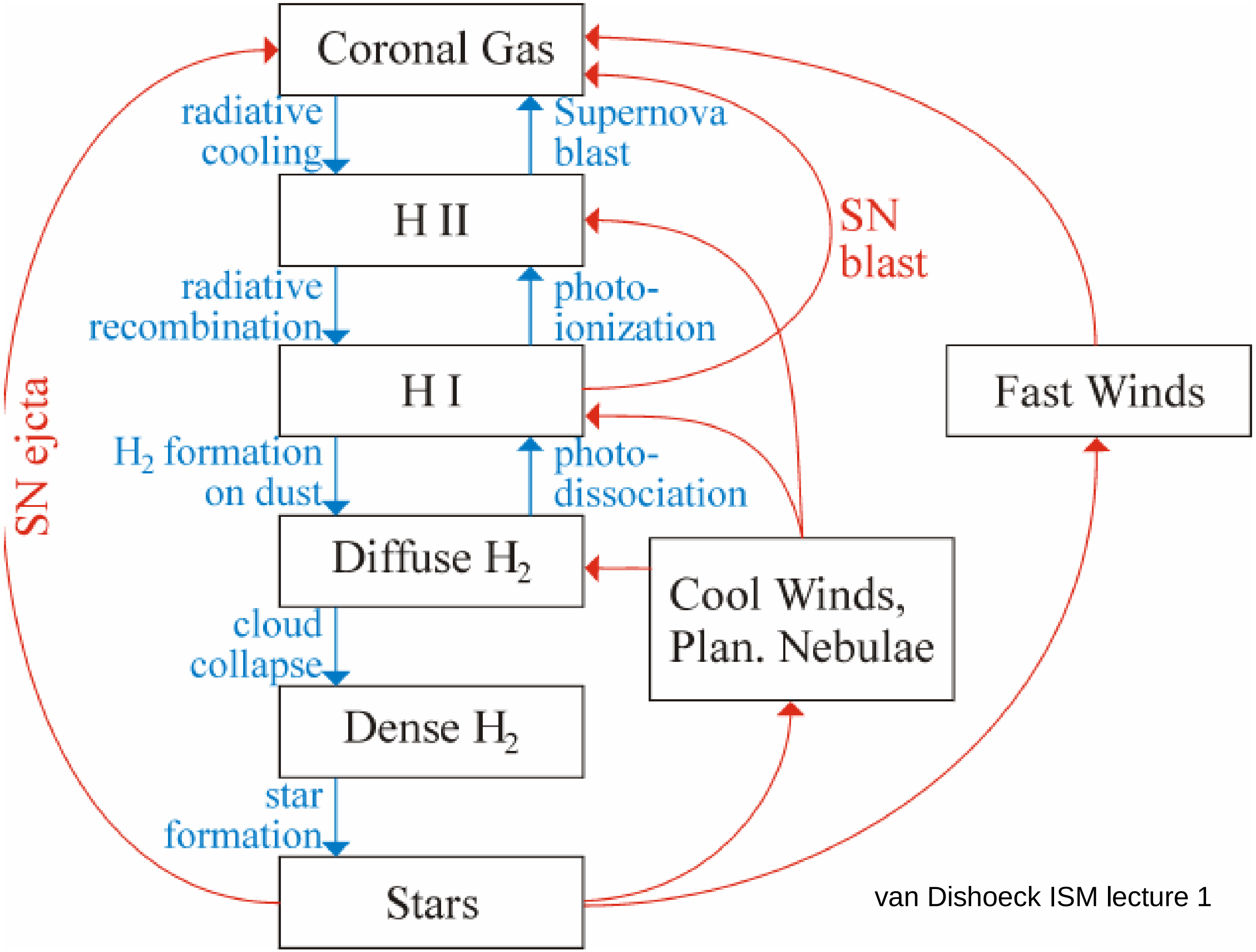
Phase transitions: Orion nebula



Phase transitions



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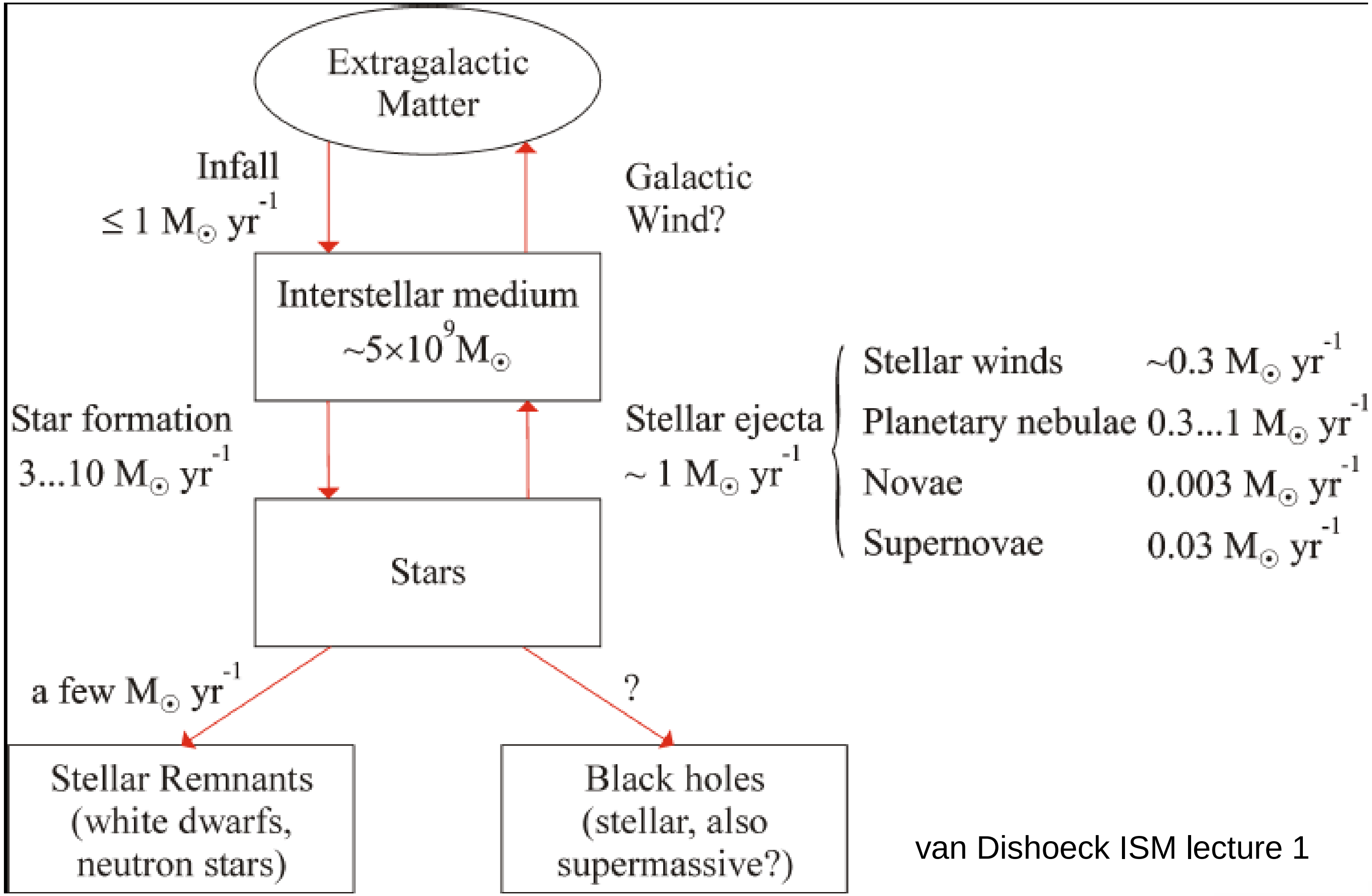
Phase transitions: M82

Blue: X-ray/Chandra
Red: Infrared/Spitzer
Visible light/HST

www.wikipedia.org

Phase transitions



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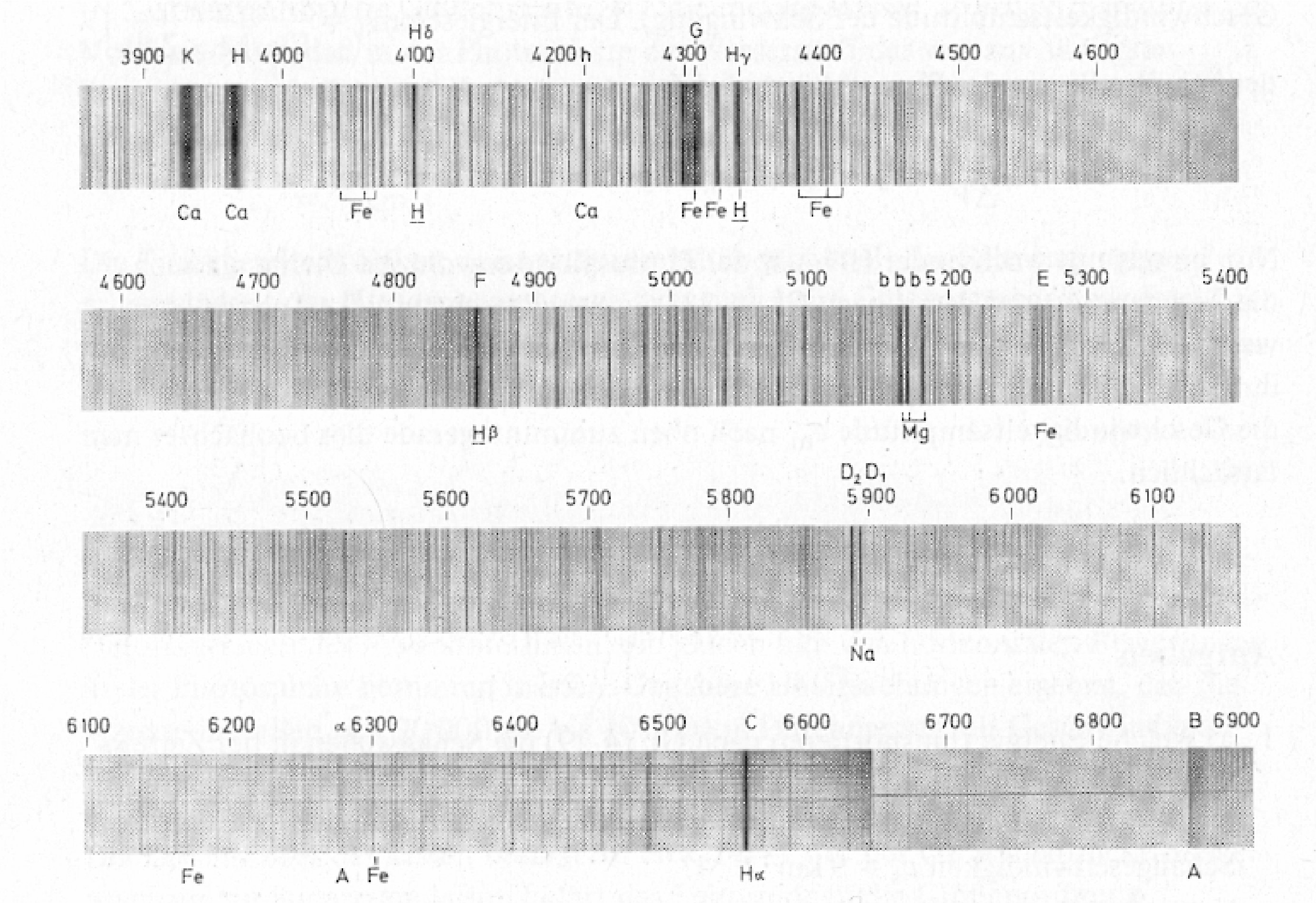
Physical quantities

Atoms and Ions

- The dominant part of baryonic matter in the universe is in neutral or partly ionized state (Warm Hot Intergalactic Medium, 80% of all baryons)
- Line emission and absorption of atoms and ions are the dominant sources of photon emission and absorption
- Basic knowledge of atomic physics is necessary to understand the processes in the universe

Fraunhofer lines



Fraunhofer lines of the Sun, from 390 nm to 690 nm

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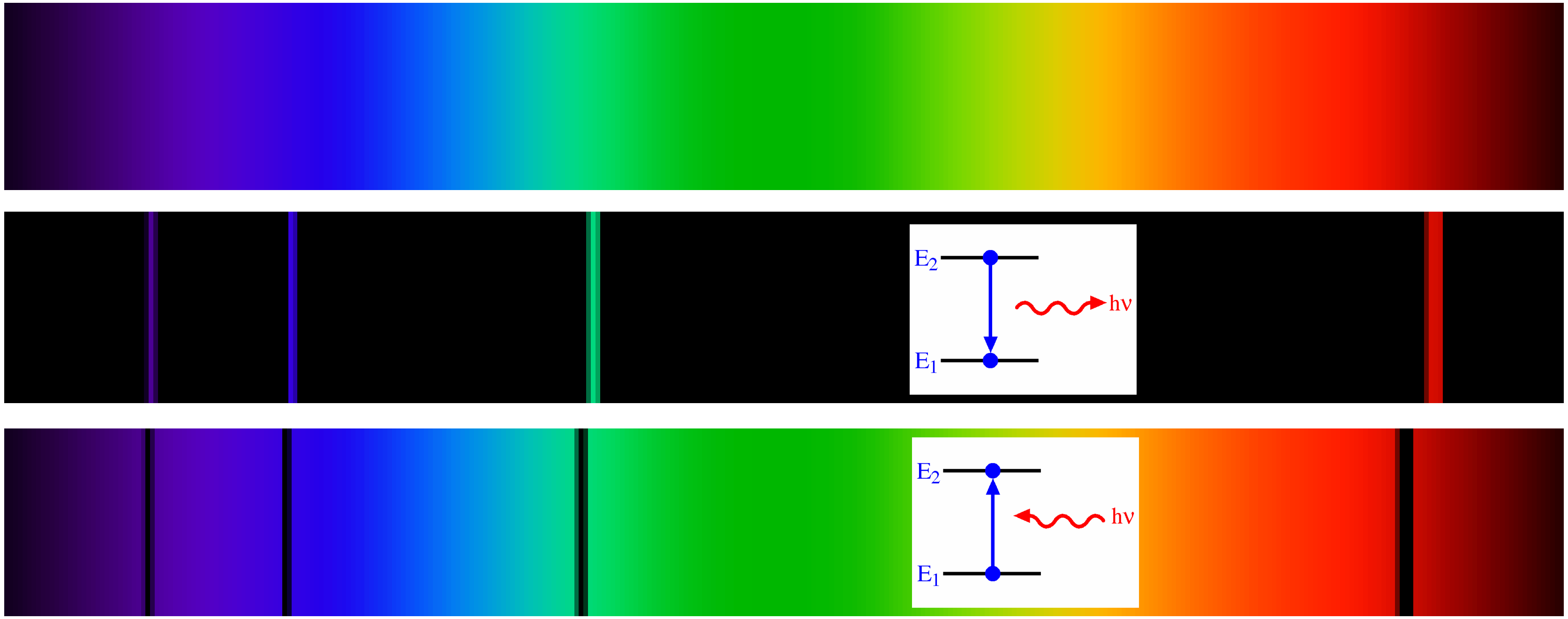
Fraunhofer lines

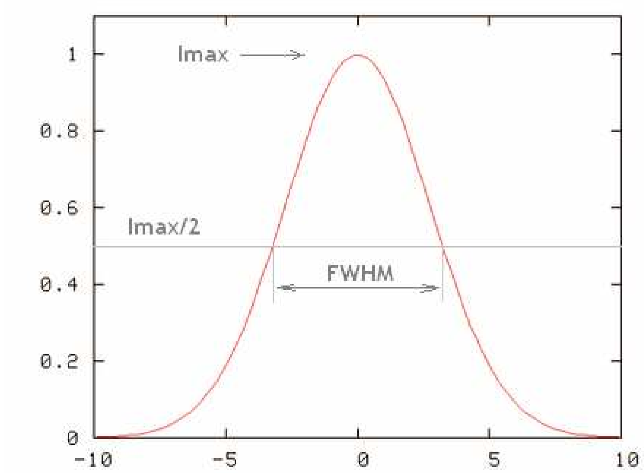
- Fraunhoferlines can be observed in laboratory experiments. A continuous light source is observed through a gas. Using a spectrometer, absorption lines are observed, which are unique for individual chemical elements.
- If the continous light source is switched off, the absorption lines change into emission lines in the spectrum.
- Fraunhofer lines originate via resonance absorption. The gas attenuates the continous light emission only at those wavelengths (energies) which corresponds exactly to the spacing of the energy levels.

Linien radiation

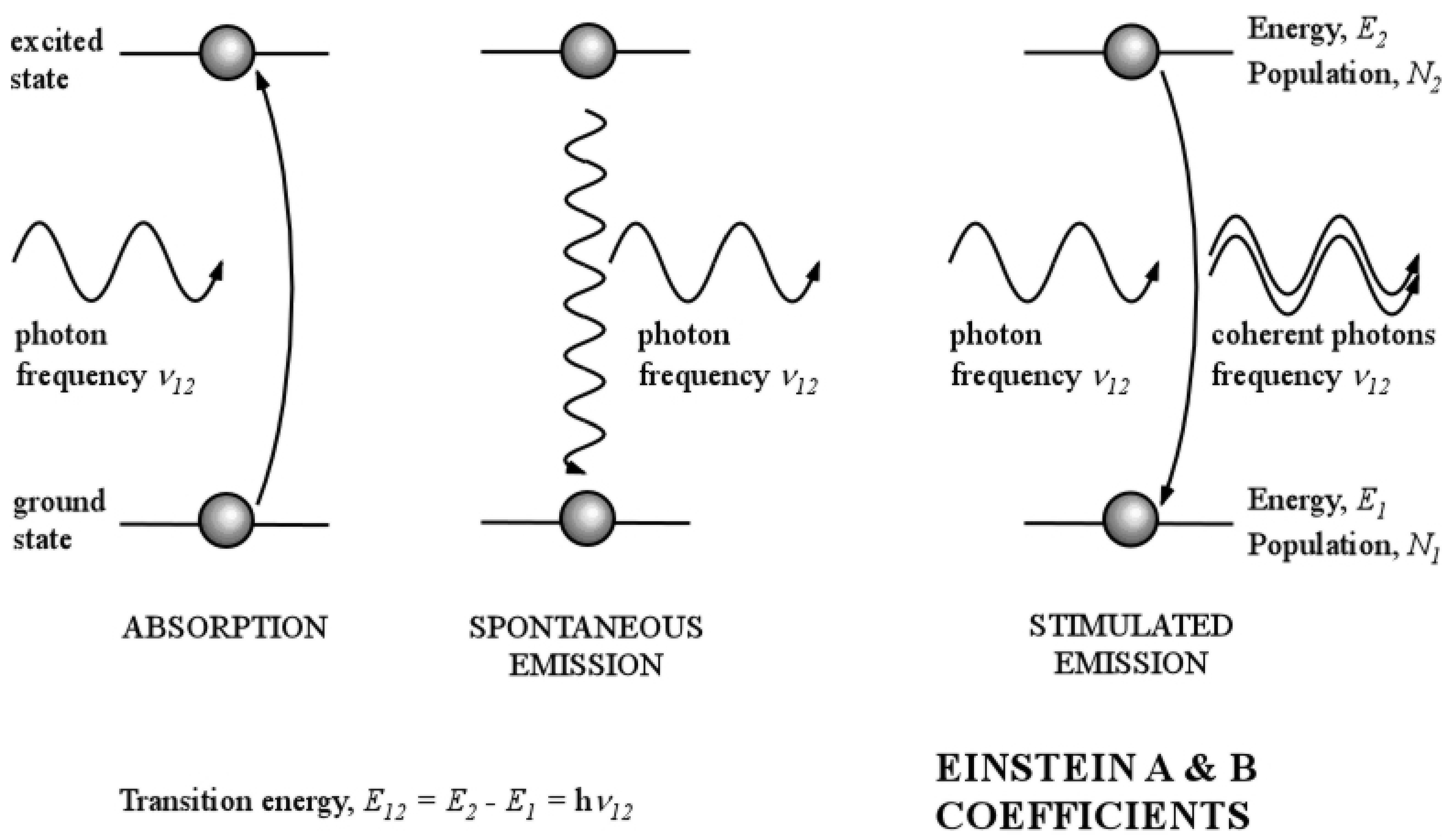


http://de.wikipedia.org

Top: continous spectrum
Middle: emission spectrum of the gas
Bottom: absorption spectrum of the gas

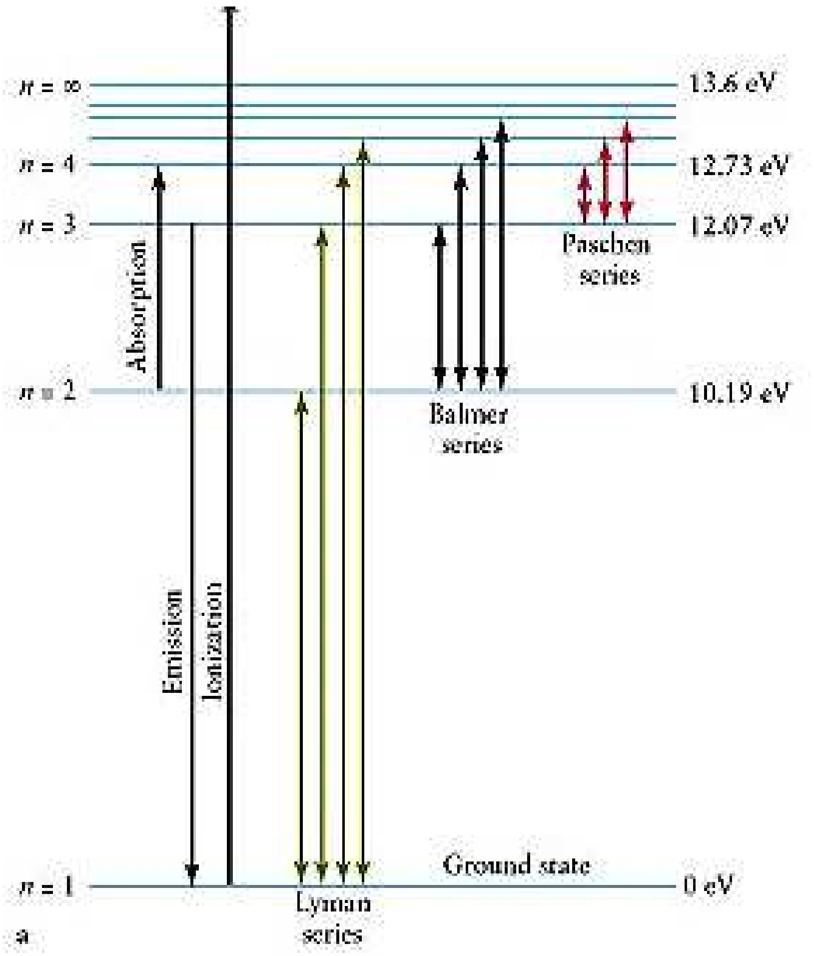


Line radiation processes

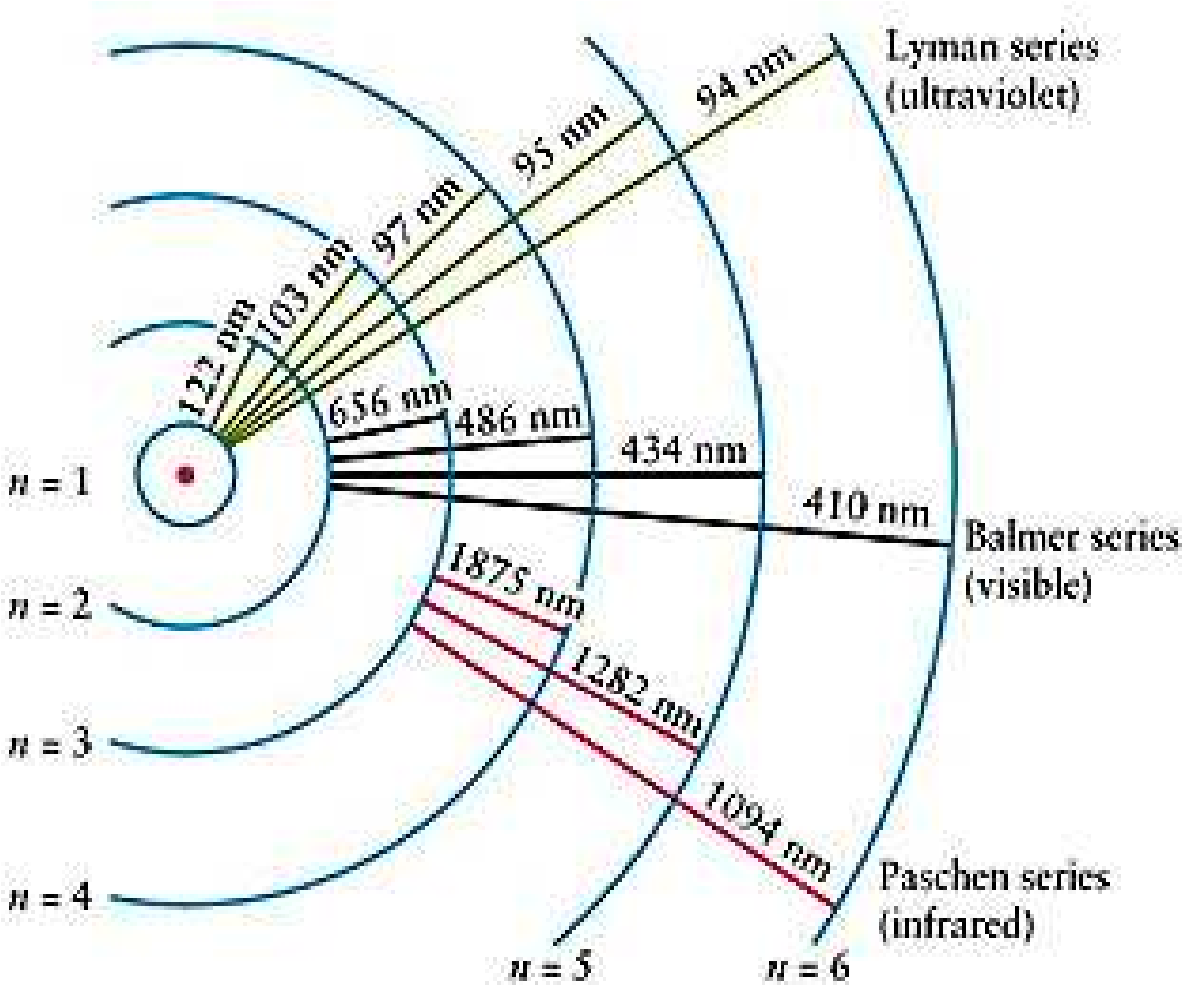


http://www.homepages.ucl.ac.uk/~ucapphj/EinsteinAandB.jpg

Line radiation: hydrogen



Line radiation: hydrogen



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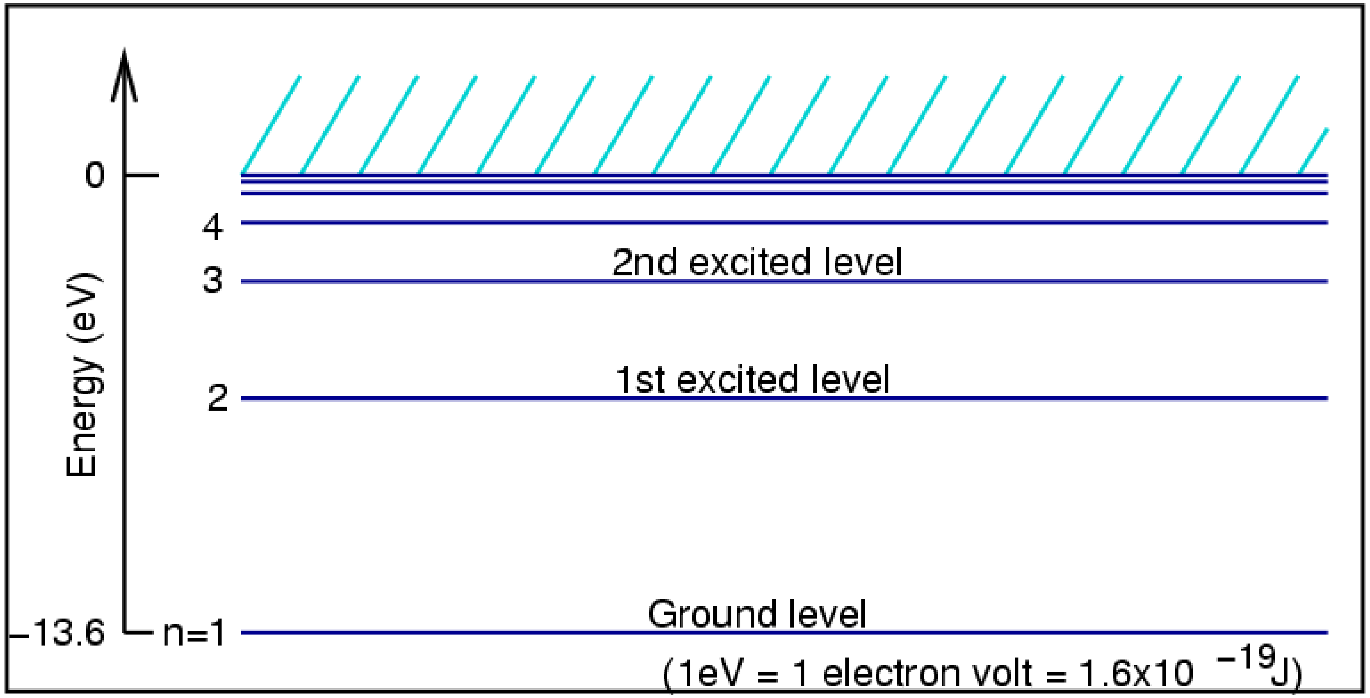
Line radiation: hydrogen

Energy levels in Hydrogen:

$$E_n = -\frac{2\pi^2 \mu e^4}{\hbar^2} \cdot \frac{1}{n^2} \propto -\frac{1}{n^2}$$

($n \in \mathbb{N}$; Balmer formula)



http://pulsar.sternwarte.uni-erlangen.de/wilms/teach

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Line radiation: line width

- The natural line with is determined by the time-energy uncertainty due to the Heisenberg uncertainty principle

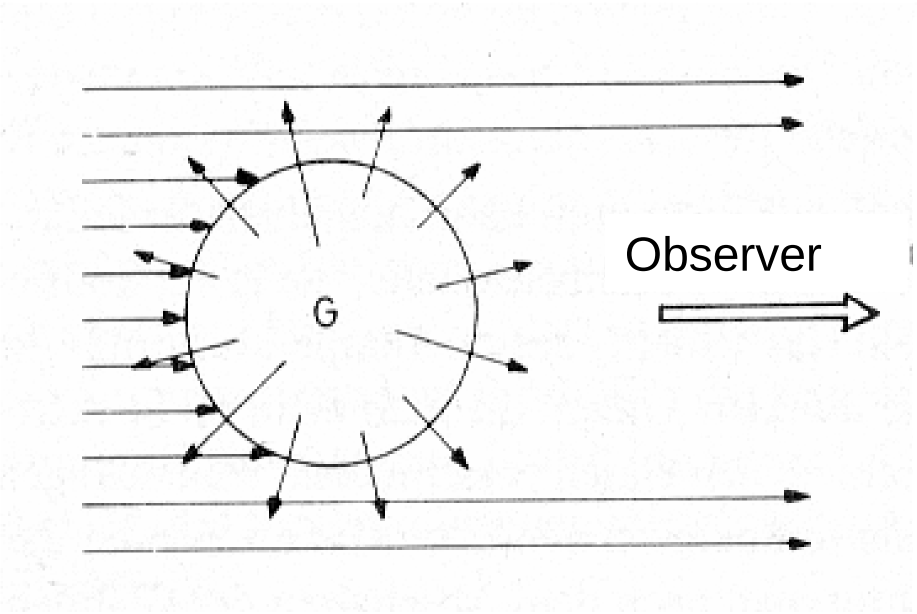
$$\frac{\hbar}{2} \leq \Delta E \cdot \Delta t$$

- This relates the life time of an excited state with the energy uncertainty of the transition. The natural line width can be describe by a Lorentzian profile

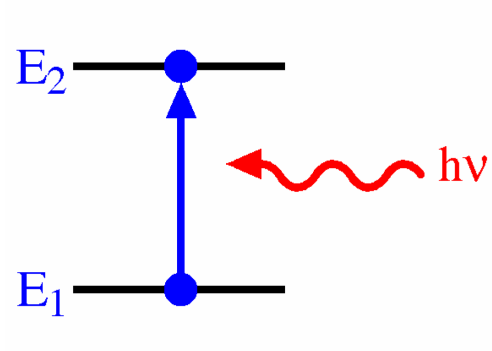
$$L(x,\gamma) = \frac{\gamma}{\pi \cdot \left(x^2 + \gamma^2\right)}$$

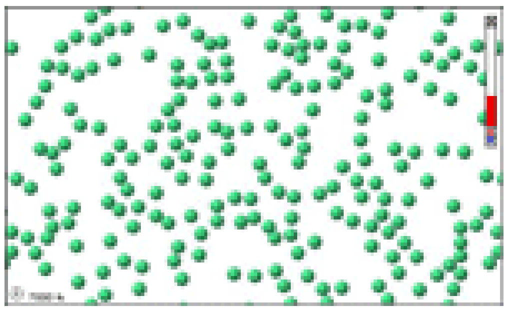
- γ determines the FWHM and the amplitude $\approx 1/\gamma\pi$

Linien radiation: line broadening



Radiation from a background source is absorbed by the gas cloud G. The absorbend photon energy is emitted isotropically. The total amount of absorbed energy is relased after the absorption processes but without any preferred direction.





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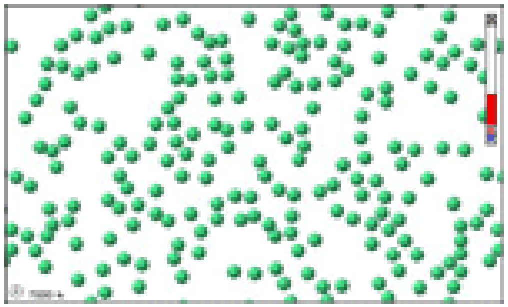
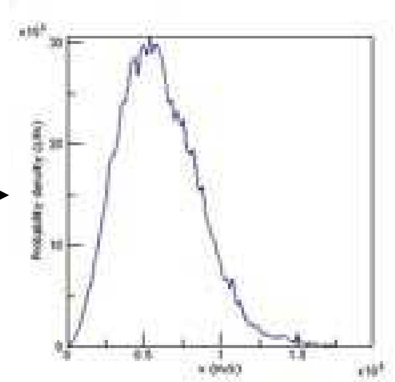
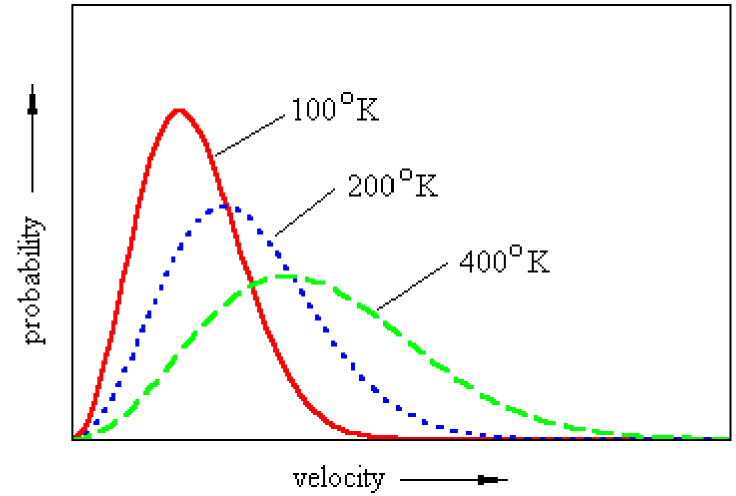
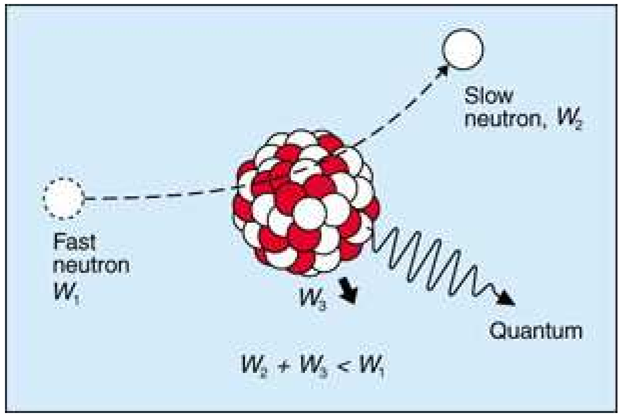
Line radiation: line broadening





Maxwell-Boltzmann distribution





Inelastic scattering

Line radiation: line broadening

- An absorption line is produced wenn a atom absorbs an amount of energy $\Delta E = h \cdot c / \lambda$ at a wavelength λ.
- The electron which is bound to the electronic state E_m is excited after the absorption to the higher energy state E_k.

$$E_k - E_m = \frac{h \cdot c}{\lambda}$$

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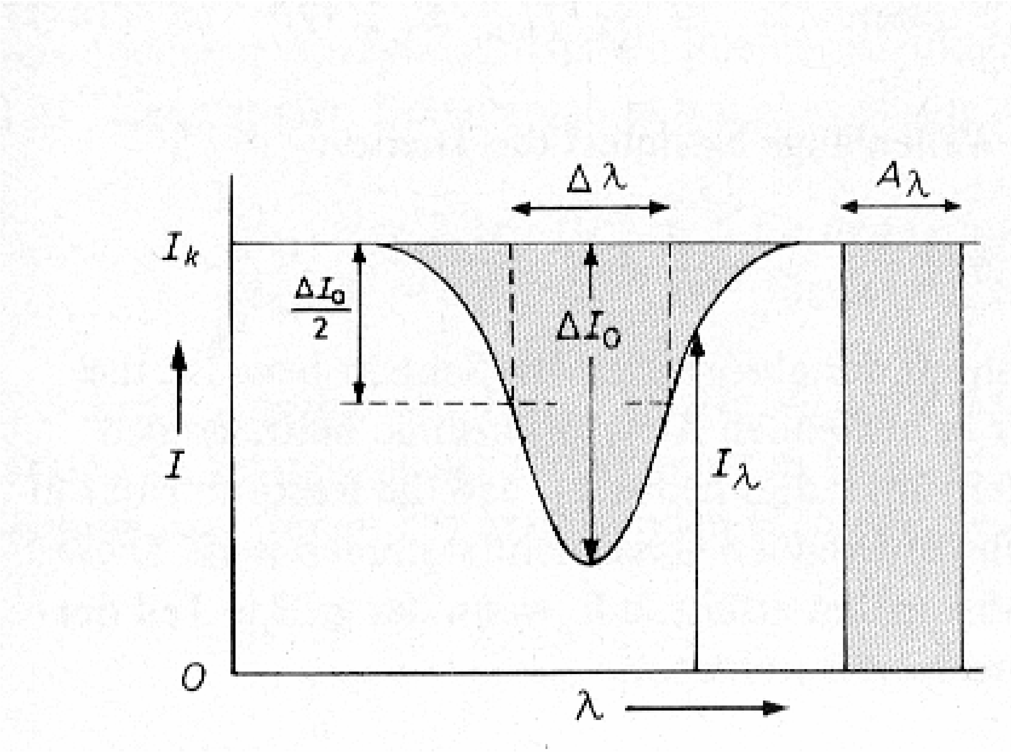
Equivalent width



Definition of the full-width half maximum equivalent width Δλ. I_K denotes the intensity of the continuum at the wavelenght of the absorption line. I_0 is the intensity minimum and I_λ is the residual intensity at the wavelength λ. A_λ is the corresponding equivalent width, which has the same area as the the absorption line.

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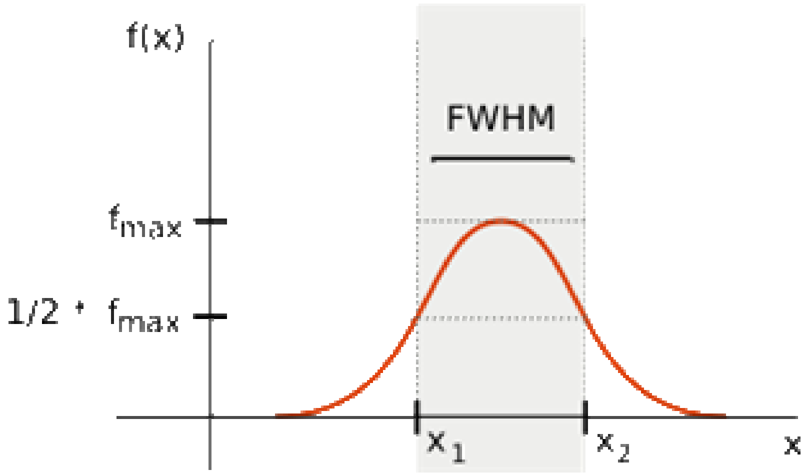
Full-width half maxium



The advantage of use the full-width half maximum definition is, that we apply the assumption, that the absorbing/emitting gas atoms are following a Maxwell-Boltzmann distribution.

According to this assumption we find

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left[\frac{(x-x_0)^2}{2\sigma^2}\right]}$$

$$FWHM = \sqrt{8\ln 2}\cdot\sigma \approx 2.35482\cdot\sigma$$

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Maxwell-Boltzmann FWHM

$$P(\upsilon)d\upsilon = \sqrt{\frac{m}{2\pi kT}} e^{-\left(\frac{m\upsilon^2}{2kT}\right)} d\upsilon$$

$$P(f)df = \sqrt{\frac{mc^2}{2\pi kTf_0^2}} e^{-\left(\frac{mc(f-f_0)^2}{2kTf_0^2}\right)} df$$

$$\sigma_f = \sqrt{\frac{kT}{mc^2}} f_0$$

$$\Delta f_{FWHM} = \sqrt{\frac{8kT \ln 2}{mc^2}} f_0$$

Line radiation: line broadening

- We apply the non-relativistic Doppler formula
- For a gas with the moleculare mass M* we can calculate the average velocity dispersion of the gas atoms
- Applying the Doppler shift formula, we can calculate the average line width due to thermal motion.
- A typical value for the interstellar medium is

$$\frac{\Delta\lambda}{\lambda} = \frac{\vartheta}{\mathrm{c}}$$

$$\overline{\vartheta_r} = \sqrt{\frac{2 \cdot \mathrm{R} \cdot T}{M^*}}$$

$$\Delta\lambda_D = \frac{\lambda}{\mathrm{c}} \sqrt{\frac{2 \cdot \mathrm{R} \cdot T}{M^*}}$$

$$\Delta\lambda_D = \frac{5{,}491 \cdot 10^{-7}\,\mathrm{m}}{3 \cdot 10^{8}\,\mathrm{m} \cdot \mathrm{s}^{-1}} \sqrt{\frac{2 \cdot 8{,}314 \cdot 10^{3}\,\mathrm{J} \cdot \mathrm{K}^{-1}\mathrm{kmol}^{-1} \cdot 6 \cdot 10^{3}\,\mathrm{K}}{47{,}9\,\mathrm{kg} \cdot \mathrm{kmol}^{-1}}}$$

$$\Delta\lambda_D = 0{,}0026\,\mathrm{nm}$$

Line shape

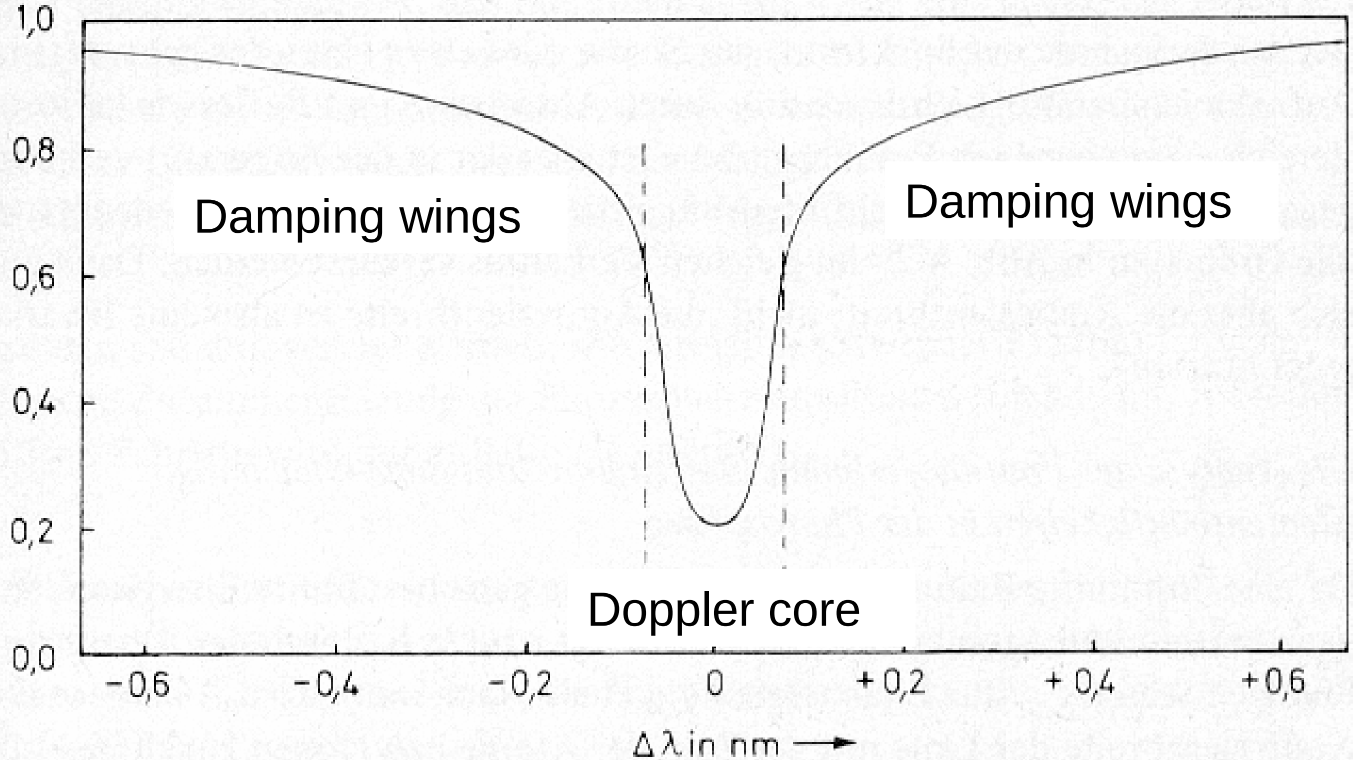

Abb. 4.28 Profil der Wasserstoff-Linie H_α (Wellenlänge $\lambda = 656{,}3$ nm) aus dem Utrechter photometrischen Atlas des Sonnenspektrums (Beispiel einer „starken" Linie)

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Line radiation: line broadening

- As shown in the viewgraph before, we have two processes which lead to the broadening of an absorption/emission line.
  - Thermal broadening (temperature *T*)
  - Pressure broadening (temperature *T* and volume density *n*)
- Thermal and pressure broadeing are different in their physical behaviour.
  - Thermal broadening implies a folding of the Lorentzian profile (natural line width) with the Maxwell-Boltzmann velocity distribution.
  - Pressure broadening leads to a shift of the atomic energy levels due to the interaction process. In gerenal, the interaction with ambient gas atoms is on shorter time scale than the spontaneous emission. Here, the volume density *n* is of prime importance, *T* is of second interest.

$$\Delta I \propto e^{-\frac{(\lambda-\lambda_0)^2}{\Delta\lambda_D}}$$

$$\Delta I \propto \frac{1}{(\lambda-\lambda_0)^2+\Delta\lambda_D^2}$$

Evaluation the strength of an absorption line

- A photon of the wavelenght λ will be absorbed by a ion which has the ionzation degree i will move the excited electron from orbit m ($E_{i,m}$) to orbit k ($E_{i,k}$).
- The „strenght" of the absorption line is proportional to the path length through the absorbing medium *H*.
- The strenght of the absorption line is also proportional to the number of ions in the same energetic state $E_{i,m}$
- Finally, the strength of absorption is a function of the propability that the absorption events occurs quantum mechanically. This propability is describe by the oscillator stength $f_{m,k}$

$$A_\lambda \approx f_{m,k} \cdot N_{i,m} \cdot H$$

Evaluation the strength of an absorption line

1. Determination of A_λ.
2. The gas atom, its ionization state i , energy levels $E_{i,m}$ and $E_{i,k}$ and $f_{m,k}$ are known, accordingly A_λ yields directly $N_{\mathrm{i,m}} \cdot H$
3. Using the **Boltzmann equation** one can calculate the population of the Ions at a certain energy level ($E_{i,m}$)

$$\frac{N_{i,m}}{N_{i,1}} = e^{-\left(\frac{E_{i,m} - E_{i,1}}{\mathrm{k}T}\right)}$$

Evaluation the strength of an absorption line

- We know from quantum mechanics, that most of the energy levels show up with a degeneracy in the orbital and magnetic quantum number. These „hidden“ energy levels are well known as fine-structure levels. According to this degeneracy, we have to apply som weightening to the number of energy levels.

$$\frac{N_{i,m}}{N_{i,1}} = \frac{g_{i,m}}{g_{i,1}} \cdot e^{-\left(\frac{E_{i,m}-E_{i,1}}{\mathrm{k}T}\right)}$$

Fine-structur

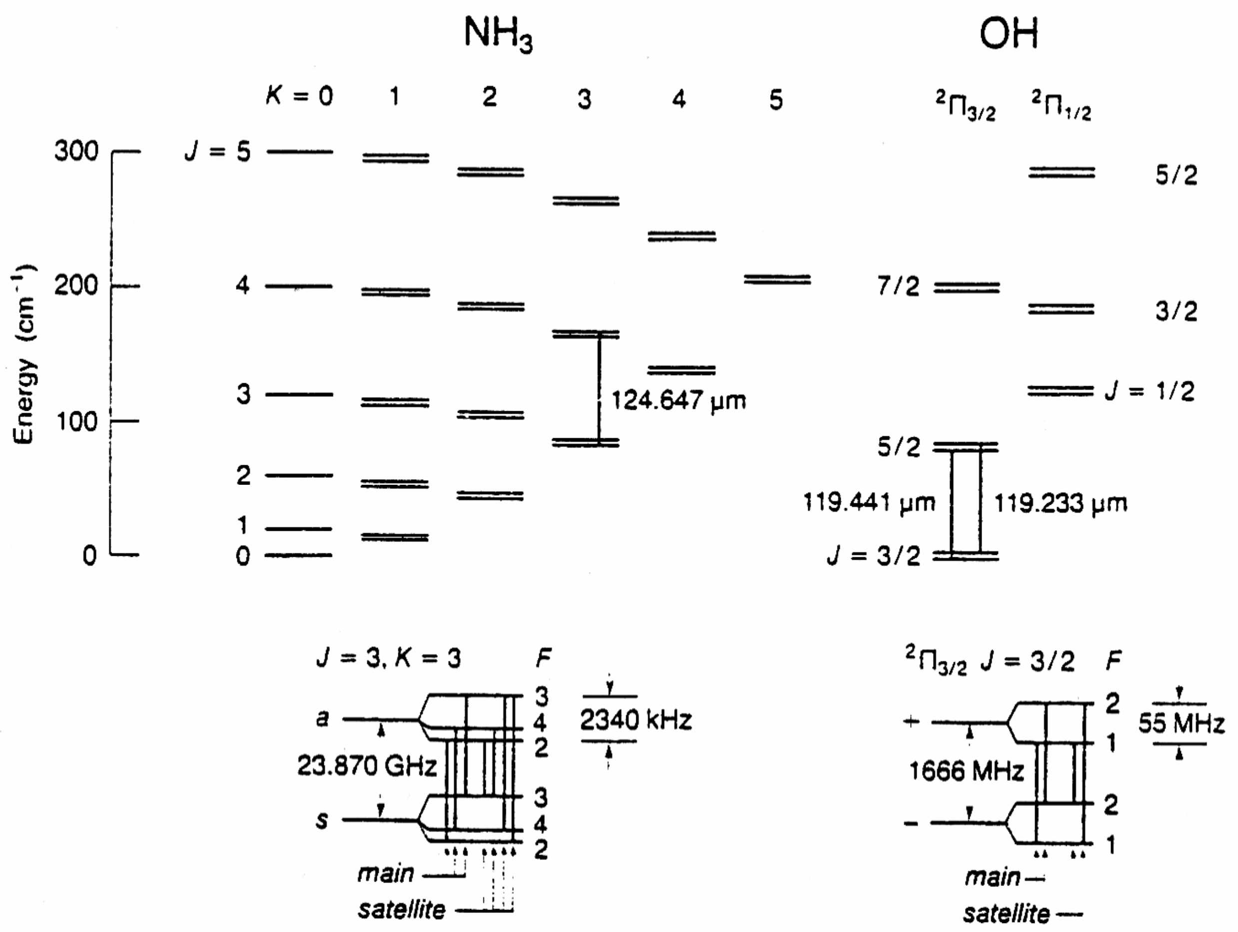

Fig. 5. Rotational levels of NH_3 (left) and OH (right) (adapted from Watson 1982)

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Evaluation the strength of an absorption line

- Finally, we need the number of elements which are within the excited state $E_{i,m}$
- The population of elements in the state is a function of the temperature. It follows the Boltzmann equation but deals with the ions.
- The equation is the Saha equation

$$\frac{N_{1,1} \cdot N_e}{N_{0,1}} = 2 \cdot \frac{g_{1,1}}{g_{0,1}} \left(\frac{\sqrt{2\pi m_e \mathrm{k} T}}{\mathrm{h}} \right)^3 \cdot e^{-\left(\frac{E_{1,1} - E_{0,1}}{\mathrm{k} T} \right)}$$

Evaluation the strength of an absorption line

- Using the Saha and the Boltzmann equation, we can calculate
  - Temperature
  - Elektron density
  - Element abundance
  - Elektron pressure
- Accordingly, we can determine the physical state of a gas!