Universität Bonn Summer Term 2009

Physics of the ISM

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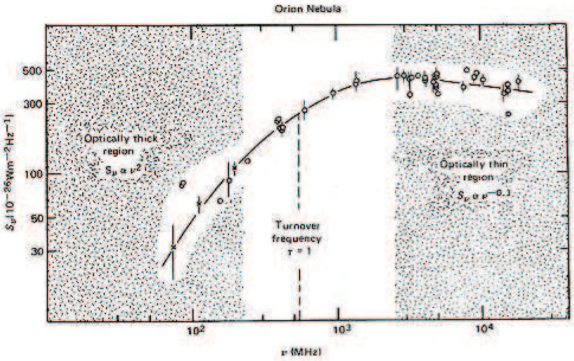
Exercises VIII

In-class problems

1 HII regions

1.1 Emission measure

In this exercise, we investigate some properties of the Orion nebula, named Ori A by radio astronomers. This is a template HII region with young massive stars ionizing it. The figure on the right exhibits the radio continuum spectrum of Ori A, which is thermal free-free emission. Thermal absorption at low radio frequencies is clearly visible. Use this spectrum to calculate the emission measure EM of the Orion nebula via the optical depth:



$$\tau_\nu = 3.01 \cdot 10^{-2} \left(\frac{EM}{10^6 \text{ cm}^{-6} \text{ pc}}\right) \left(\frac{\nu}{\text{GHz}}\right)^{-2} \left(\frac{T_e}{10^4 \text{K}}\right)^{-3/2} g_{\text{ff}} \ , \tag{1}$$

where g_{ff} is the Gaunt factor:

$$g_{\text{ff}} = 11.8 + \ln \left[Z^{-1} \left(\frac{\nu}{\text{GHz}}\right)^{-1} \left(\frac{T_e}{10^4 \text{K}}\right)^{1.5} \right] \ , \tag{2}$$

where Z is the charge number ($Z = 1$ for ionized hydrogen). In the calculation, assume $T_e = 10^4$ K.

1.2 Strömgren radius

Although the Orion nebula cannot be considered as spherical, we can nevertheless compute its dimensions from its rough angular diameter ($\theta_{Ori\,A} = 20'$) and distance ($D = 500$ pc). If we make the further assumption that $< n_e > = < n_e^2 >^{1/2}$, then we can determine the number density n_e of free electrons. This assumption is quite all right for an estimate, such as to be done here. We can then proceed to calculate the excitation measure

$$U = r_{\text{HII}} \cdot n_e^{2/3} \tag{3}$$

and the total rate of ionizing Lyman continuum photons

$$\dot{N}_{Lyc} = \alpha(T) \cdot \frac{4}{3} \, \pi \, r_{\text{HII}}^3 \cdot n_e^2 \ , \tag{4}$$

where $\alpha(T) = 3.76 \cdot 10^{-13}$ s^{-1} cm^3 is the recombination coefficient at $T = 10^4$ K. Finally, calculate the total mass of ionized gas in the Orion nebula (do not forget to account for 10% of helium)

Homework

2 Radio recombination lines

2.1 Electron temperature

The ratio of the brightness temperatures of a radio recombination line and the thermal free-free continuum, if emitted from the same volume (with the same emission measure!) is given by

$$\frac{T_L}{T_C}\left(\frac{\Delta v}{\text{km s}^{-1}}\right) = 6.978 \cdot 10^3 \left(\frac{\nu}{\text{GHz}}\right)^{1.1}\left(\frac{T_e}{10^4\text{K}}\right)^{-1.15}\frac{1}{1+n_{He^+}/n_{H^+}}\ , \tag{5}$$

In the early days of the Effelsberg 100-m telescope, simultaneous measurements of the thermal free-free continuum and recombination line emission were carried out at 5 GHz (e.g. H109α). The observed ratio of the brightness temperatures in the centre of the Orion nebula were measured to be $T_L/T_C = 0.25$, the observed line width (FWHM) was $\Delta v = 25.7\text{km s}^{-1}$. Calculate the electron temperature.

2.2 Rydberg formula

The Rydberg formula allows to quickly calculate the frequency of transitions in Rydberg (i.e. hydrogen-like) atoms:

$$\nu_{n,n+\Delta n} = Z^2\ R\ c \cdot \left[\frac{1}{n^2} - \frac{1}{(n+\Delta n)^2}\right]\ , \tag{6}$$

with

$$R = R_\infty \cdot \left(1 - \frac{m_e}{M}\right)\ , \tag{7}$$

where m_e is the mass of the electron and M the mass of the atom. For large quantum numbers n the above formula can be approximated by

$$\nu_{n,n+\Delta n} \approx 2\ Z^2\ R\ c \cdot \frac{\Delta n}{n^3}\ , \tag{8}$$

Such approximations can be dangerous if their validity is not checked prior to an experiment. Assume you wish to observe the H109α and H166α lines of hydrogen. For each observation, you have tuned the receiver such that the respective recombination line should appear in the centre of the bandpass (see figure on the right). In order to calculate the corresponding observing frequencies, you have quickly used the approximate formula. The bandwidth is 50 MHz. Will you see the line(s)?