In-class problems

1 Observed line width of neutral hydrogen

The rms velocity of a cloud is given by

\[ v_{\text{rms}} := \sqrt{\langle v^2 \rangle} = \left[ \int_0^{\infty} v^2 f(v) \, dv \right]^{1/2}, \]  

(1)

where

\[ f(v) = \left( \frac{m}{2\pi k T_k} \right)^{3/2} e^{-\frac{mv^2}{2k T_k}} \cdot v^2, \]  

(2)

is the Maxwellian velocity distribution of the atoms in a gas with kinetic temperature \( T_k \) (cf. Exercises II). The measured quantity, the line width, is given in terms of the so-called “full width at half power” (FWHP, \( \Delta v_{1/2} \)). The relation between the rms velocity and the FWHP is

\[ \Delta v_{1/2} = \sqrt{8 \ln 2} \frac{v_{\text{rms}}}{3}. \]  

(3)

(a) Now work out the relation between the kinetic temperature \( T_k \) and the measured FWHP. Calculate this quantity for a gas with \( T_k = 1000 \) K.

(b) Compare with the speed of sound in an isothermal gas, which is given by the pressure \( P \) and the density \( \rho \) via

\[ c_s = \sqrt{\kappa \frac{P}{\rho}}, \]  

(4)

where \( \kappa \) is the adiabatic index of the gas.
Homework

2 Gas masses

2.1 Hydrogen cloud

(a) Calculate the total H\(_i\) mass of a cloud at a distance \(D = 10\ \text{kpc}\), with an observed hydrogen column density \(N_{\text{HI}} = 10^{22}\ \text{cm}^{-2}\) and an angular diameter of \(\theta_{\text{cl}} = 5'\),

\[
M_{\text{HI}} = m_H \ N_{\text{HI}} \ D^2 \Omega_{\text{cl}},
\]  

(5)

where \(m_H\) is the mass of the hydrogen atom, and

\[
\Omega_{\text{cl}} = \pi \ \theta_{\text{cl}}^2,
\]  

(6)

is the solid angle subtended by such a cloud with angular diameter \(\theta_{\text{cl}}\).

(b) Is this really the total gas mass, or are there other constituents that would have to be accounted for?

2.2 Galaxy disk

Assume that the above column density is present in a galaxy out to a radius of 15 kpc.

(a) Calculate the total H\(_i\) mass.

(b) If this mass were concentrated at the centre of the galaxy, what would the expected rotational velocity be of any stars or gas at 15 kpc radius?