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Physics of the ISM

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Exercises VI

In-class problems

1 Observed line width of neutral hydrogen

The rms velocity of a cloud is given by

$$v_{\rm rms} := \sqrt{\langle v^2 \rangle} = \left[\int_0^\infty v^2 \, f(v) \, dv \right]^{1/2} , \tag{1}$$

where

$$f(v) = \left(\frac{m}{2\pi \, k \, T_{\rm k}} \right)^{3/2} e^{-\frac{m \, v^2}{2 \, k \, T_{\rm k}}} \cdot v^2 , \tag{2}$$

is the Maxwellian velocity distribution of the atoms in a gas with kinetic temperature $T_{\rm k}$ (cf. Exercises II). The measured quantity, the line width, is given in terms of the so-called "full width at half power" ($FWHP$, $\Delta v_{1/2}$). The relation between the rms velocity and the $FWHP$ is

$$\Delta v_{1/2} = \sqrt{8 \, \ln 2 \, / 3} \, v_{\rm rms} . \tag{3}$$

(a) Now work out the relation between the kinetic temperature $T_{\rm k}$ and the measured $FWHP$. Calculate this quantity for a gas with $T_{\rm k} = 1000$ K.

(b) Compare with the speed of sound in an isothermal gas, which is given by the pressure P and the density ρ via

$$c_s = \sqrt{\kappa \frac{P}{\rho}} , \tag{4}$$

where κ is the adiabatic index of the gas.

Homework

2 Gas masses

2.1 Hydrogen cloud

(a) Calculate the total HI mass of a cloud at a distance $D = 10$ kpc, with an observed hydrogen column density $N_{HI} = 10^{22}$ cm^{-2} and an angular diameter of $\theta_{\rm cl} = 5'$,

$$M_{HI} = m_H \, N_{HI} \, D^2 \, \Omega_{\rm cl} \,, \tag{5}$$

where m_H is the mass of the hydrogen atom, and

$$\Omega_{\rm cl} = \pi \, \theta_{\rm cl}^2 \,, \tag{6}$$

is the solid angle subtended by such a cloud with angular diameter $\theta_{\rm cl}$.

(b) Is this really the total gas mass, or are there other constituents that would have to be accounted for?

2.2 Galaxy disk

Assume that the above column density is present in a galaxy out to a radius of 15 kpc.

(a) Calculate the total HI mass.

(b) If this mass were concentrated at the centre of the galaxy, what would the expected rotational velocity be of any stars or gas at 15 kpc radius?