1 Continuum Radiation Cont’d

1.1 Relativistic particles

Charged particles emit electromagnetic radiation when accelerated. The energy radiated per unit of time and unit frequency into unit solid angle is

\[
\frac{dP}{d\Omega} = \frac{e^2}{4\pi c} \frac{|\vec{n} \times (\vec{n} - \vec{\beta}) \times \vec{\beta}|^2}{(1 - \vec{n} \cdot \vec{\beta})^5},
\]

(1)

where

\[
\vec{n} \cdot \vec{\beta} = \beta \cos \theta,
\]

(2)

Here, \( \theta \) is the angle between the velocity vector and the line-of-sight.

1.1.1 Linear accelerator

The radiation from a linearly accelerated particle as derived from Eqn. (??) reads

\[
\frac{dP}{d\Omega} = \frac{e^2 \dot{v}^2}{4\pi c^3} \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^5}.
\]

(3)

Making use of \( \beta \approx 1 \), or \( \gamma \gg 1 \),

(a) calculate \( \theta_{\text{max}} \);

(b) show that \( \theta_{\text{max}} \approx \frac{1}{2\gamma} \);

(c) show that \( \frac{dP}{d\Omega} \sim \gamma^8 \);

(d) show that \( P \sim \gamma^6 \).

1.1.2 Inverse-Compton radiation

Assume that the strength of the magnetic field in the lobes of a radio galaxy is \( B = 4 \mu \text{G} \). Calculate the redshift beyond which Inverse-Compton dominate over synchrotron losses.
Homework

2 Continuum radiation

2.1 Synchrotron radiation

You observe a radio galaxy which exhibit a break in the spectrum of its lobes at 10 GHz. Assume we know the strength of the magnetic field to be $B = 4 \mu G$. Calculate the time elapsed since the last acceleration of particles in "hot spots", i.e.

$$t_{1/2} = 8.24 \times 10^9 \left( \frac{B}{\mu G} \right)^{-2} \left( \frac{E}{\text{GeV}} \right)^{-1} \text{ yrs}.$$  \hfill (4)

Here, the “critical” frequency is related to the energy $E$ and strength of the magnetic field $B$ via

$$v_c = \frac{3}{4\pi} \frac{eBE^2}{m_0 c^5}.$$  \hfill (5)

The rest mass of the electron $m_0$ enters as the 3rd power. What would be the corresponding critical frequency of protons?