Universität Bonn Summer Term 2009

Physics of the ISM

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Exercises III

In-class problems

1 Continuum Radiation Cont'd

1.1 Relativistic particles

Charged particles emit electromagnetic radiation when accelerated. The energy radiated per unit of time and unit frequency into unit solid angle is

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c} \frac{|\vec{n} \times (\vec{n} - \vec{\beta}) \times \dot{\vec{\beta}}|^2}{(1 - \vec{n} \cdot \vec{\beta})^5} , \tag{1}$$

where

$$\vec{n} \cdot \vec{\beta} = \beta \, cos\theta , \tag{2}$$

Here, θ is the angle between the velocity vector and the line-of-sight.

1.1.1 Linear accelerator

The radiation from a linearly accelerated particle as derived from Eqn. (**??**) reads

$$\frac{dP}{d\Omega} = \frac{e^2 \dot{v}^2}{4\pi c^3} \frac{\sin^2\theta}{(1 - \beta\cos\theta)^5} . \tag{3}$$

Making use of $\beta \approx 1$, or $\gamma \gg 1$,

(a) calculate θ_{max};

(b) show that $\theta_{max} \approx \frac{1}{2\gamma}$;

(c) show that $\frac{dP}{d\Omega} \sim \gamma^8$;

(d) show that $P \sim \gamma^6$.

1.1.2 Inverse-Compton radiation

Assume that the strength of the magnetic field in the lobes of a radio galaxy is $B = 4\ \mu$G. Calculate the redshift beyond which Inverse-Compton dominate over synchrotron losses.

Homework

2 Continuum radiation

2.1 Synchrotron radiation

You observe a radio galaxy which exhibit a break in the spectrum of its lobes at 10 GHz. Assume we know the strength of the magnetic field to be $B = 4\mu$G. Calculate the time elapsed since the last acceleration of particles in "hot spots", i.e.

$$t_{1/2} = 8.24 \times 10^9 \left(\frac{B}{\mu\text{G}}\right)^{-2} \left(\frac{E}{\text{GeV}}\right)^{-1} \text{yrs} . \tag{4}$$

Here, the "critical" frequency is related to the energy E and strength of the magnetic field B via

$$v_c = \frac{3}{4\pi} \frac{eBE^2}{m_0^3 c^5} . \tag{5}$$

The rest mass of the electron m_0 enters as the 3rd power. What would be the corresponding critical frequency of protons?