Universität Bonn Summer Term 2009

Physics of the ISM

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Exercises II

In-class problems

1 Continuum Radiation

1.1 Thermal particles

The first exercise deals with thermal free-free radiation (or bremssstrahlung) from ionized gas (HII regions). In deriving the radiation from an ensemble of thermal electrons (i.e. electrons having thermal velocities), we have to average over velocity and over its reciprocal value. Assuming that the velocity distribution function is Maxwellian, i.e.

$$f(v)\,dv = 4\pi\left(\frac{m_e}{2\pi k T_e}\right)^{\frac{3}{2}} e^{-\frac{m_e v^2}{2kT_e}}\,v^2\,dv\ , \tag{1}$$

(a) Show that

$$\int_0^\infty f(v)\,dv = 1 \tag{2}$$

(b) Show that

$$\langle v\rangle = \sqrt{\frac{8kT}{\pi \cdot m_e}}\,. \tag{3}$$

(b) Show that

$$\left\langle \frac{1}{v}\right\rangle = \sqrt{\frac{2m_e}{\pi k T}}\,. \tag{4}$$

1.2 Relativistic particles

Charged particles emit electromagnetic radiation when accelerated. The energy radiated per unit of time and unit frequency into unit solid angle is

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c}\,\frac{|\vec{n}\times(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}|^2}{(1-\vec{n}\cdot\vec{\beta})^5}\ , \tag{5}$$

where

$$\vec{n}\cdot\vec{\beta} = \beta\,cos\theta\ , \tag{6}$$

Here, θ is the angle between the velocity vector and the line-of-sight.

1.2.1 Synchrotron radiation

Particles moving at relativistic speeds in a magnetic field emit synchrotron radiation, as they are forced into helical paths about the magnetic field by virtue of the Lorentz force. The radiated power per unit solid angle then is

$$\frac{dP}{d\Omega} = \frac{e^2 \dot{v}^2}{4\pi c^3} \frac{\left[1 - \dfrac{\sin^2\theta \cos^2\phi}{\gamma^2 (1 - \beta\cos\theta)^2}\right]}{(1 - \beta\cos\theta)^3} \,, \tag{7}$$

where as before θ is the angle between the velocity vector and the line-of-sight, while ϕ is the angle along a circle perpendicular to the velocity vector. Making use of $\beta \approx 1$, or $\gamma \gg 1$, and considering only small angles (the forward beam of a relativistic particle is very narrow),

(a) show that the angle between the directions where $\frac{dP}{d\Omega}$ drops to zero is $\theta_0 = \frac{2}{\gamma}$;

(b) show that $P_\Omega \sim \gamma^6$ in the direction $\theta = 0°$.

Homework

2 Continuum radiation

2.1 Free-free radiation

Assume you observe the radio continuumk spectrum of an HII region having a diameter of $d = 50$ pc and a number density of free electrons and of protons (or positively charged nuclei) of $n_i = n_e = 100\ \mathrm{cm}^{-3}$.

(a) Calculate the emission measure EM.

(b) Assuming an electron temperature of $T_e = 8000$ K, at which frequency do you expect to see a turnover in the spectrum, i.e. the frequency above which the emission becomes optically thin?

(c) Calculate the average seperation of the protons.

(d) Calculate the mean velocity of the electrons.

(e) Calculate the mean velocity of the protons.