Neutron star structure in strong magnetic fields

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NEUTRON STARS
WITH STRONG MAGNETIC FIELDS

- high B pulsars, a few XDINSs and RRATs, having super critical magnetic fields \( \text{Popov et al. (2006)} \)

- Soft Gamma-ray Repeaters (SGRs), Anomalous X-ray Pulsars (AXPs)

- Observations indicate common features \( \Rightarrow \) SGRs/AXPs: neutron stars with high surface field \( B \)

- \( P - \text{Pdot, magnetic braking} \Rightarrow B \sim 10^{15} - 10^{16} \, \text{G} \) \( \text{(Duncan & Thomson 1992; Thomson & Duncan 1993)} \)

- Direct measurements of the field (Ibrahim et al.)

- Virial Theorem \( \Rightarrow B_{\text{max}} \sim 10^{18} - 10^{19} \, \text{G} \)
AIM OF THE STUDY

Consistent neutron star models in a strong magnetic field

• effects of magnetic field on the dense matter Equation of State

• interaction of the electromagnetic field with matter (magnetisation)


• anisotropy of the energy momentum tensor caused by breaking of the spherical symmetry by the electromagnetic field

• to calculate the structure and observable properties of the neutron star within General Relativistic framework

Bonazzolla, Gourgoulhon, Salgado, Marck (1993)
EFFECT OF MAGNETIC FIELD ON DENSE MATTER

- Lagrangian density of a fermion system in the presence of a magnetic field

\[ \mathcal{L} = \bar{\psi}(x)(D_\mu \gamma^\mu - m)\psi(x) - \frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} \]

- Microscopic energy-momentum tensor in a symmetrized and gauge invariant form

\[ T^{\mu\nu} = \frac{1}{4\pi} (F^{\mu\alpha} F^\nu_\alpha + g^{\mu\nu} \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta}) + \frac{1}{2} \bar{\psi} (\gamma^\mu D^\nu + \gamma^\nu D^\mu) \psi \]
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- matter
- field

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- field
- matter

• thermodynamic average of the energy-momentum tensor

\[ \langle T^{\mu\nu} \rangle = \left( \frac{\varepsilon + p}{2} u^\mu u^\nu + p g^{\mu\nu} \right) + \frac{1}{2} (F^{\nu}_\tau M^{\tau\mu} + F^\mu_\tau M^{\tau\nu}) - \frac{1}{\mu_0} \left( F^{\mu\alpha} F_{\alpha}^\nu + \frac{g^{\mu\nu}}{4} F_{\alpha\beta} F^{\alpha\beta} \right) \]

The structure equations of neutron stars are obtained by solving Einstein’s field equations.

In the 3+1 Formalism, solving the Einstein’s equations (system of 2nd order PDEs) are reduced to integration of a system of coupled 1st order PDEs subject to certain conditions:

- 6 evolution equations for the extrinsic curvature
- 1 Hamiltonian constraint equation
- 3 momentum constraint equations

The formulation has been employed to construct a numerical code (LORENE) using spectral methods.

The code has been extended to include coupled Einstein-Maxwell equations describing rapidly rotating neutron stars with a magnetic field.

To incorporate magnetisation one must modify the inhomogeneous Maxwell equations.
MAGNETOSTATIC EQUILIBRIUM
(WITHOUT MAGNETISATION)

- Equations for magnetostatic equilibrium (from the conservation of energy and momentum):
  \[ \nabla_\mu T^{\mu\nu} = 0 \]

- Inhomogeneous Maxwell equations:
  \[ \frac{1}{\mu_0} \nabla_\mu F^{\nu\mu} = j_\text{free}^{\nu} \]

- Einstein-Maxwell equations
  \[ \nabla_\alpha T^{\alpha\beta} = \nabla_\alpha T^f_{\alpha\beta} - F^{\beta\nu} j_\text{free}^{\nu} \]

- First integral of fluid motion:
  \[ \ln h(r, \theta) + \nu(r, \theta) - \ln \Gamma(r, \theta) + M(r, \theta) = \text{const.} \]

- In terms of enthalpy per baryon for neutron star
  \[ h := \frac{\varepsilon + p}{n_b} \]

  and current function
  \[ M(r, \theta) = - \int_0^\Lambda \varphi(r, \theta) f(x) \, dx. \]
**Magnetostatic Equilibrium**  
*(with magnetisation)*

- In the Fluid Rest Frame, assuming perfect conductor, $E = 0$  
  \[ F_{\mu\nu} = \epsilon_{\alpha\beta\mu\nu} u^\beta b^\alpha \]
- assuming isotropic medium, the magnetisation is aligned with the magnetic field  
  \[ M_{\mu\nu} = \epsilon_{\alpha\beta\mu\nu} u^\beta m^\alpha \]
  \[ m_\mu = \frac{x}{\mu_0} b_\mu \]

- Modified inhomogeneous Maxwell equations:  
  \[ \nabla_\mu F^{\sigma\mu} = \frac{1}{1 - x} \left( \mu_0 j_\text{free}^{\sigma} + \left( F^{\sigma\mu} \nabla_\mu x \right) \right) \text{magnetisation} \]

- first integral of fluid motion:  
  \[ (\varepsilon + p) \left( \frac{1}{\varepsilon + p} \frac{\partial p}{\partial x^i} + \frac{\partial \nu}{\partial x^i} - \frac{\partial \ln \Gamma}{\partial x^i} \right) - F_{i\rho} j_\rho^{\text{free}} - \frac{x}{2\mu_0} F_{\mu\nu} \nabla_i F^{\mu\nu} = 0 \]

- it can be shown that  
  \[ \frac{x}{2\mu_0} F_{\mu\nu} \nabla_i F^{\mu\nu} = \frac{x}{\mu_0} \left( b_\mu \nabla_i b^\mu - b_\mu b^\mu u_\nu \nabla_i u^\nu \right) = b \nabla_i b = m \frac{\partial b}{\partial x^i} \]

- enthalpy per baryon for neutron star with magnetic field  
  \[ h = h(n_b, b) = \frac{\varepsilon + p}{n_b} \]

- derivative of logarithm of enthalpy  
  \[ \frac{\partial \ln h}{\partial x^i} = \frac{1}{\varepsilon + p} \left( \frac{\partial p}{\partial x^i} - m \frac{\partial b}{\partial x^i} \right) \]
MAGNETIC FIELD DEPENDENT EQUATION OF STATE

• Landau quantisation in the direction perpendicular to the magnetic field
• The critical field for electrons is $B_m^{(e)(c)} = 4.4 \times 10^{13}$ G, and for protons it is $B_m^{(p)(c)} \sim 10^{20}$ G
• Example: Quark Matter in MCFL phase (Noronha and Shovkovy 2007, Paulucci et al. 2011)
• massless 3-flavor MIT Bag model (with $B = 60$ MeV/fm$^3$) + Pairing interaction of NJL-type

- effects of Landau quantization become noticeable only for fields of $\sim 10^{19}$G.

- Magnetisation negligible
- de Haas-van Alphen oscillations

MAXIMAL DEFORMATION DUE TO MAGNETIC FIELD

$B_p = 8.16 \times 10^{17} \, G$

Magnetic field lines and enthalpy isocontours in the meridional $(x, z)$ plane for static configuration for $B_{\text{polar}}=8.16 \times 10^{17} \, G$, $M_G=2.22 \, M_{\odot}$ (including magnetic field effect in EoS)

- Stellar configurations strongly deviate from spherical symmetry
- Upon increasing magnetic field strength, the shape of the star becomes more and more elongated, finally reaching toroidal shape
Gravitational mass varies with central log enthalpy and magnetic field
Static configurations determined by different values of central log-enthalpy along constant sequences of magnetic dipole moment
Maximum gravitational mass \( M_G^{\text{max}} \) was determined by parabolic interpolation
Plot of polar magnetic mass \( M_G^{\text{max}} \) was determined by parabolic interpolation
Plot of polar magnetic moment for a neutron star of \( M_B = 1.6 \, M_{\odot} \)

**Effect of EoS(B) and M**

- **The 3 cases:**
  1. (i) without magnetic field dependence in EoS, without magnetisation: no EoS(B), no M
  2. (ii) with magnetic field dependence in EoS, without magnetisation: EoS(B), no M
  3. (iii) with magnetic field dependence in EoS, with magnetisation: EoS(B), M

- Maximal gravitational mass is an increasing function of magnetic moment

- The effects of inclusion of magnetic field dependence of the EoS and the magnetisation are negligible, contrary to the claims of several previous works

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• Observed magnetars are slowly rotating ($P \sim s$)
• We chose a sequence of neutron stars rotating at 700 Hz, close to fastest known rotating pulsar (716 Hz)
• Maximum mass increases with magnetic moment
• Effect of magnetisation and magnetic field dependence of EoS again found to be negligible

We studied the behaviour of compactness of a neutron star with baryon mass 1.6 with magnetic moment.

The compactness was found to decrease with increase in magnetic moment.

Centrifugal forces exerted by the Lorentz force on matter increases with increasing magnetic moment.

The influence of magnetic field dependence of EoS and magnetisation are negligible.

The main effect arises from the purely electromagnetic part.

\[ C = \frac{GM_G}{R_{\text{circ}} c^2} \]
SUMMARY

• In this work, we developed a self-consistent approach to determine the structure of neutron stars in strong magnetic fields, relevant for the study of magnetars.

• Taking as an example the EoS of quark matter in MCFL phase, we investigated the effect of inclusion of magnetic field dependence of the EoS and magnetisation.

• In particular, it was found that the equilibrium only depends on the thermodynamic EoS and magnetisation explicitly only enters Einstein-Maxwell equations.

• In contrast to previous studies, we found that these effects do not significantly influence the stellar structure, even for the strongest magnetic fields considered.

• The difference arises due to the fact that in previous works isotropic TOV equations were used to solve for stellar structure, whereas magnetic field causes the star to deviate from spherical symmetry considerably.