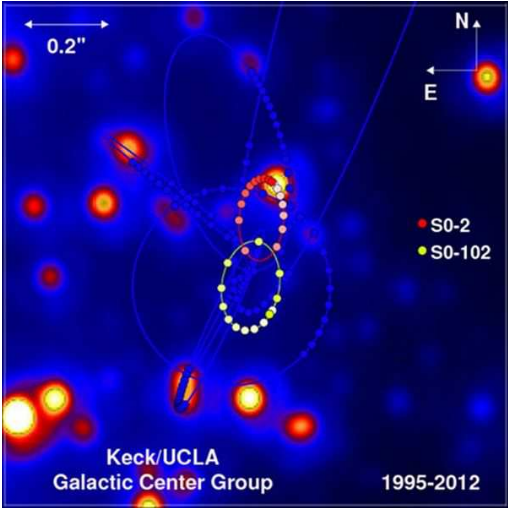
Two-body relaxation of stellar disc around an SMBH

Ladislav Šubr & Jaroslav Haas

Astronomical Institute, Charles University in Prague

Young stars in the Galactic Centre – S-stars

- spectral type B0 – B9
- $N \approx 15$
- $M_* \approx 10\, M_\odot$
- age $\approx 10 - 100\,\mathrm{Myr}$
- highly eccentric orbits; pericentres $\gtrsim 0.001\,\mathrm{pc} \approx 10^3\, R_\mathrm{g}$
- shortest period $\approx 11.5\,\mathrm{yr}$ (S0-102)

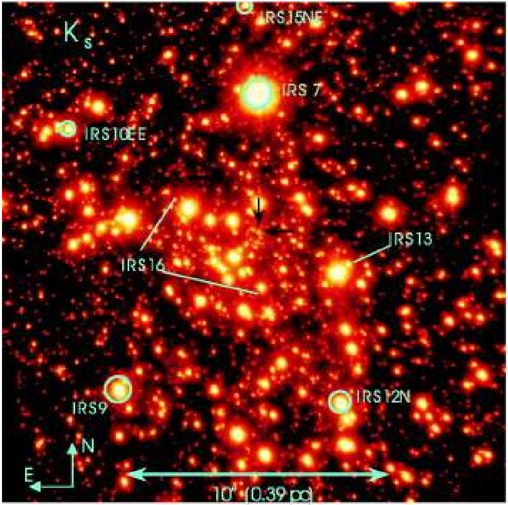


Young stars in the Galactic Centre – blue giants

- spectral types O & B, Wolf-Rayet stars, LBVs
- $N \approx 200$
- $M_* \gtrsim 20\,M_\odot$
- age $\approx 7 \pm 2$ Myr

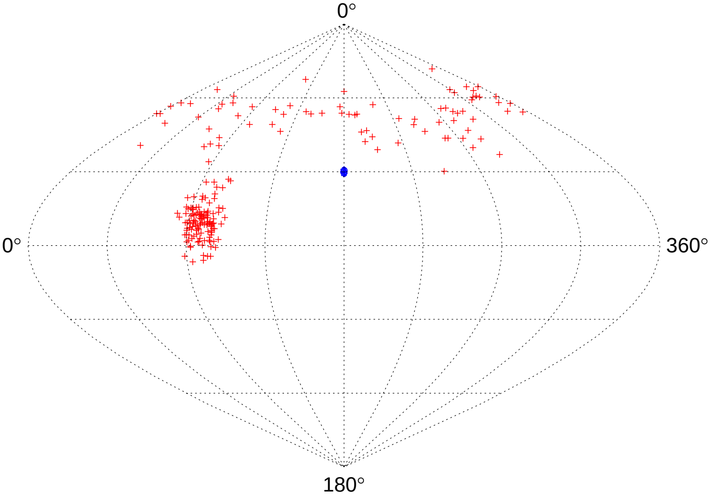
"Clockwise stellar disc" (Genzel et al. 1996, Levin & Beloborodov 2003)

- $N \approx 30$
- $0.03\,\mathrm{pc} \lesssim R \lesssim 0.5\,\mathrm{pc}$
- opening angle $\approx 15°$



Genzel et al. 2003

Warped disc of mutually interacting stars



(Haas, Šubr & Kroupa 2011; Haas, Šubr & Vokrouhlický 2011)

Two-body relaxation

Standard two-body relaxation (energy transfer)

$$
\begin{aligned}
t_{\mathrm{R}} &\approx 0.34 \frac{\sigma^3}{G^2 M_\star^2 n \ln \Lambda} \\
&\approx \frac{2}{\ln \Lambda} \left(\frac{N(R)}{10^4}\right)^{-1} \left(\frac{M_{\mathrm{BH}}}{4 \times 10^6 M_\odot}\right)^{-1/2} \left(\frac{M_{\mathrm{BH}}/M_\star}{10^6}\right)^{2} \left(\frac{R}{0.04\,\mathrm{pc}}\right)^{3/2} \mathrm{Gyr}
\end{aligned}
$$

... for $\sigma = v_{\mathrm{K}} = \sqrt{R^3/GM_{\mathrm{BH}}}$.

Vector resonant relaxation (angular momentum transfer)

$$
\begin{aligned}
t_{\mathrm{RR,v}}(R) &\approx 0.6 \left(\frac{M_{\mathrm{BH}}}{M_\star}\right) \frac{P(R)}{N^{1/2}(R)} \\
&\approx 0.3 \left(\frac{N(R)}{10^4}\right)^{-1/2} \left(\frac{M_{\mathrm{BH}}}{4 \times 10^6 M_\odot}\right)^{-1/2} \left(\frac{M_{\mathrm{BH}}/M_\star}{10^6}\right)^{1/2} \left(\frac{R}{0.04\,\mathrm{pc}}\right)^{3/2} \mathrm{Myr}
\end{aligned}
$$

Relaxation in protoplanetary discs

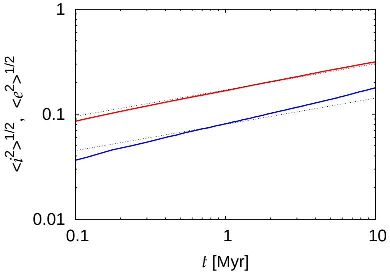
Temporal evolution of orbital eccentricities and inclinations (e.g. Stewart & Ida 2000):

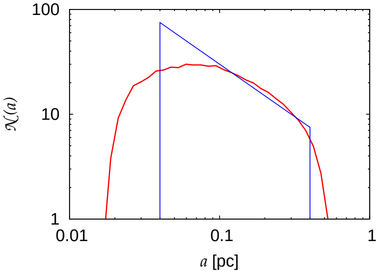
$$t(\langle e^2 \rangle) \approx 0.015 \, \frac{\langle e^2 \rangle^2}{\Omega \, \Sigma \, R^2} \left(\frac{M_{\rm BH}}{M_\star} \right)^2$$

$$t(\langle e^2 \rangle) \;\approx\; 1.2 \ln \left(\frac{R_{\rm out}}{R_{\rm in}} \right) \left(\frac{R}{0.04\,{\rm pc}} \right)^{3/2} \left(\frac{M_{\rm BH}/M_\star}{10^6} \right) \times \left(\frac{M_{\rm BH}/M_{\rm d}}{1000} \right) \left(\frac{M_{\rm BH}}{4 \times 10^6 \, M_\odot} \right)^{-1/2} \left(\frac{\langle e^2 \rangle}{0.01} \right)^2 {\rm Myr}$$

Evolution of the stellar disc due to two-body relaxation

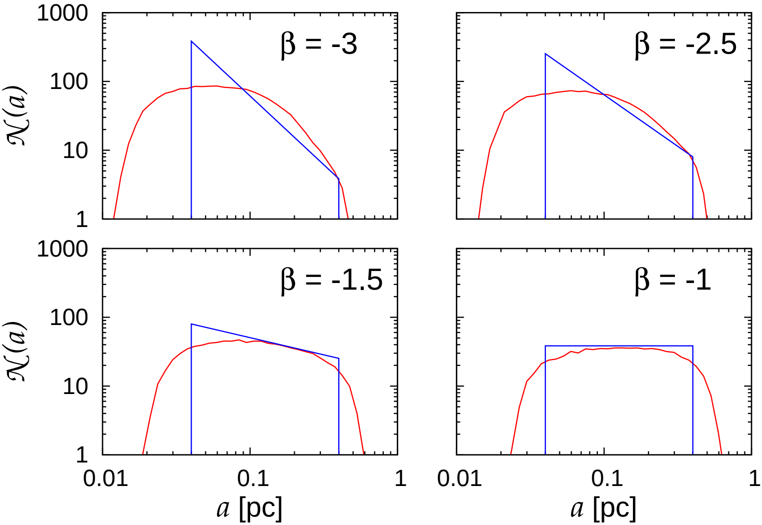
- numerical N-body model (integrator `NBODY6`)
- disc of 1200 stars, $\mathcal{N}_M \propto M_\star^{-1.5}$, $M_\star \in \langle 1\,M_\odot,\ 150\,M_\odot \rangle$
- $\mathcal{N}_a \propto a^{-1} \implies \Sigma(R) \propto R^{-2}$, $a \in \langle 0.04\,\mathrm{pc},\ 0.4\,\mathrm{pc} \rangle$



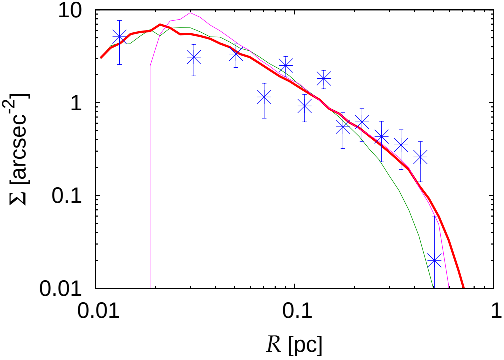


- evolution of the disc radial structure (i.e. semi-major axes)
- two-body relaxation leads to a flat profile ($\mathcal{N}_a \approx$ const.)
- Šubr & Haas, 2014)

Role of the initial disc density profile $\Sigma_0(R) \propto R^\beta$



What do we observe? – Projection!



- observational data (Do et al. 2013)
- model $\Sigma_0(R) \propto R^{-2},\ R \in \langle 0.04\,\mathrm{pc},\ 0.4\,\mathrm{pc}\rangle$
- model $\Sigma_0(R) \propto R^{-1.5},\ R \in \langle 0.03\,\mathrm{pc},\ 0.6\,\mathrm{pc}\rangle$
- model $\Sigma_0(R) \propto R^{-1.5},\ R \in \langle 0.03\,\mathrm{pc},\ 0.6\,\mathrm{pc}\rangle,\ t = 0$

Conclusions

Two-body relaxation within the thin stellar disc

- leads to change of the radial structure of the disc $\Longrightarrow$ the observed profile is compatible with the theoretical prediction about its *initial* state
- indicates a possible solution to the mystery of the origin of the S-stars

Viscous fluid approach

Thin disc approximation with viscosity determined by the dynamical friction ($\nu \propto \Sigma$)

$$\frac{\partial \Sigma}{\partial t} = \frac{A}{R}\frac{\partial}{\partial R}\left(R^{1/2}\frac{\partial}{\partial R}\left(R^3\Sigma^2\right)\right)$$