Escape from isolated, equal-mass star clusters

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MODEST14, Bad Honnef, 3 June 2014

Michel Hénon (1931-2013) at IAU Symposium 69, Besançon, France, 1974

His contributions to collisional stellar dynamics

1. Theory of relaxation (1958–1960)
2. Escape from isolated star clusters (1960–1969)
3. Self-similar solutions of the Fokker-Planck equation (1961–1965)
4. The Monte Carlo method (1967–1975)

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- Hénon units are defined so that

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- Fortunately these are the same as N-body units, and so only the name changes

Escape from an isolated cluster (equal masses)

1. Ambartsumian (1938)
   - Tail of Maxwellian above escape speed is replenished every relaxation time T_r
   - $\dfrac{dN}{dt} = -\dfrac{\xi_1 N}{T_r}$ (ξ_1 constant $\simeq 0.0069$)
2. Hénon (1960)
   - General formula for cluster described by distribution function $f(E)$
   - Structure $\dfrac{dN}{dt} = -\dfrac{\xi_2 N}{T_r \ln\Lambda}$ ($\ln\Lambda$ is Coulomb logarithm) $\Lambda \propto N$
3. Hénon (1965)
   - Self-similar solution of Fokker-Planck equation
   - $\dfrac{dN}{dt} = 0$
4. These are estimates of the escape rate from *two-body* encounters

Reconciling the results

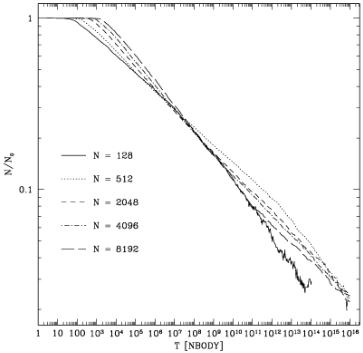
Summary:

$$\dot{N} = \begin{cases} -\xi_1 N/T_r & \text{Ambartsumian 1938} \quad (1a) \\ -\xi_2 N/(T_r \ln\Lambda) & \text{Hénon 1960} \quad (1b) \\ 0 & \text{Hénon 1965} \quad (1c) \end{cases}$$

- Hénon (1960) showed that, as a star diffuses towards the escape energy, its period grows very quickly, and therefore its rate of relaxation decreases sufficiently rapidly that the escape energy cannot be reached in finite time. Therefore (1a) is wrong.
- A derivation of the Fokker-Planck equation from a more general master equation neglects terms which are of order $\log N$ smaller than the Fokker-Planck terms. Therefore escape is also neglected. Thus (1b), (1c) are consistent.
- We shall come back to (1a)

Empirical results

Baumgardt, Hut, Heggie 2002



Why does the escape time scale *decrease* as N increases?

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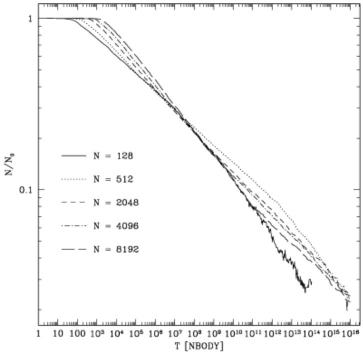

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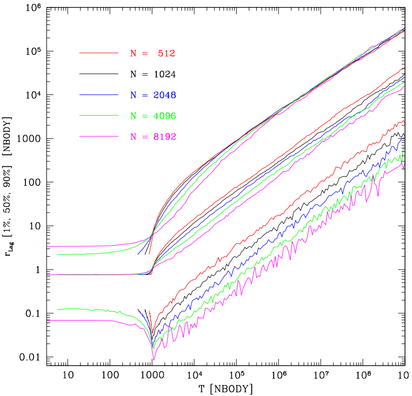


Complication: escape rate affected by post-collapse expansion, which slows down all dynamical processes.

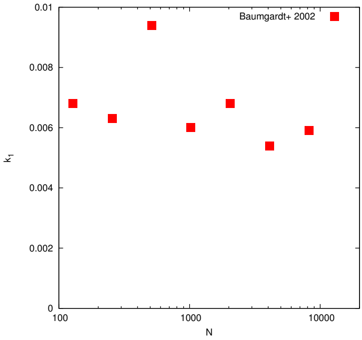
Hénon's homological model (1965)[1]

- Isotropic, isolated model
- Expands with constant mass
- Radius $R \propto t^{2/3}$
- Energy evolves like $\dot{E} = -\frac{\zeta E}{Tr}$ (ζ constant)
- Energy supplied by binary-single (i.e. 3-body encounters) within the core
- Approximately applicable to expanding isolated N-body models, but –
  - Not quite constant mass
  - Models with different N do not have exactly the same structure (Baumgardt et al 2002)
  - Models not isotropic (ibid)

[1] Trans. F Renaud (2011)

The escape rate

- Assume $\dot{N} = -k_1 \frac{N}{T_r}$ (Ambartsumian), $\dot{R} = k_2 \frac{R}{T_r}$ (Hénon)
- Assuming Coulomb logarithm constant, $N = N_0 \left(1 + \frac{k_1(t - t_0)}{\nu T_{r0}}\right)^{-\nu}$ where $\nu = \frac{2k_1}{3k_2 - k_1}$
- Suppose half stars escape at $T_{1/2}$. Then $k_1 \simeq \nu 2^{1/\nu} \frac{T_{r0}}{T_{1/2}}$.
- Baumgardt+ (2002) give $T_{1/2}, \nu$ for $N = 128 - 8192$



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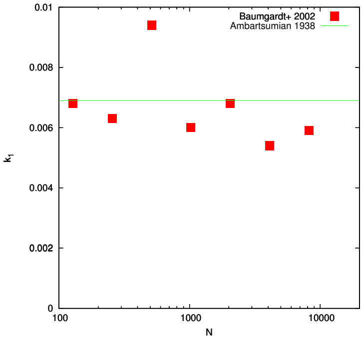

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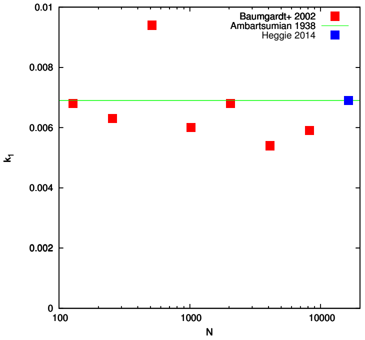

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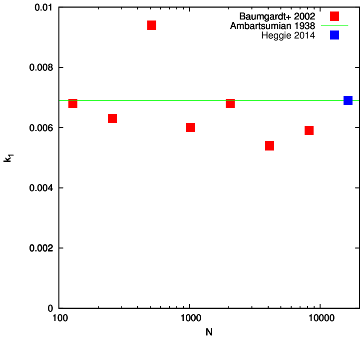

"The relatively close agreement .... is clearly somewhat fortuitous" – Lyman Spitzer Jr

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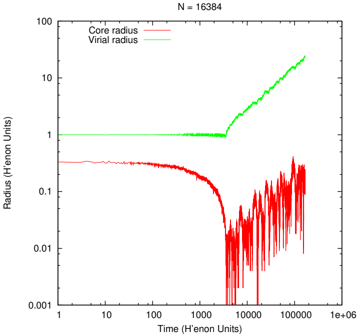
"Hénon's Principle" (1975)

- You can calculate the luminosity of a star without knowing anything about the source of stellar energy (Eddington 1926)
- Similarly you can calculate the energy generated in the core of a star cluster without knowing what generates it (Hénon 1975)
- The core adjusts so that the energy released equals that of the self-similar solution, i.e. $\dot{E} = -\frac{\zeta E}{Tr}$

- Energy source must exist (otherwise core collapse)
- Energy source must be centrally concentrated
- May be true only in time-averaged sense (gravothermal oscillations)



Three-body escape

- Assume post-collapse expansion is powered by interactions involving hard binaries in the core, where the potential is ϕ_c
- By the time its energy is of order $|m\phi_c|$, where m is individual stellar mass, it has ejected a few stars, and then ejects itself
- Hence heating rate is related to escape rate by $\dot{E} = m\dot{N}\varepsilon\phi_c$ (Goodman 1984), where $\varepsilon \sim 1$
- By "Hénon's Principle", which states that $\dot{E} = -\frac{\zeta E}{Tr}$, we get $\dot{N} = -\frac{\zeta E}{T_r m\varepsilon\phi_c}$
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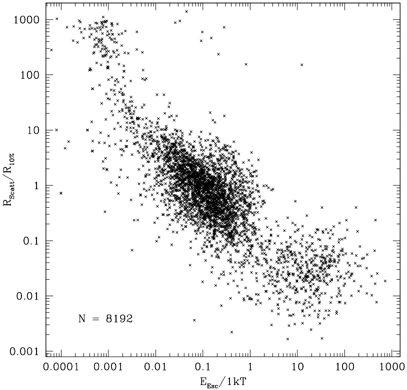
However, is it true that $E \propto Nm\phi_c$?

Where escapers are created



Radius (units of 10% Lagrangian radius) against escaper energy (units 1kT)
Baumgardt+ 2002

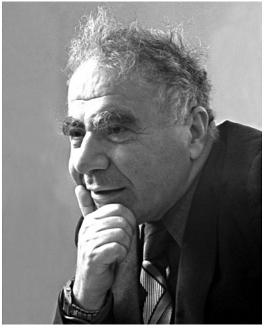
Three mechanisms of escape

1. Three-body interactions (high-energy, low radius)
2. Two-body interactions (intermediate energy and radius)
3. Induced escapers (low energy, large radius)
    - Weakly bound particles
    - Escape because of the decreasing potential well, and/or
    - Escape because of recoil of the cluster from energetic escapers

Summary and comments

What is the escape rate from an isolated star cluster?

- Our best estimate has not substantially changed in 76 years, though we do not know why Ambartsumian's estimate is correct (Hénon's 1960 argument), despite understanding much more about the problem
- Theoretical improvements would require attention to issues such as
  1. departures from homological evolution
  2. anisotropy
- Numerical improvements depend on attention to several details, e.g.
  1. The velocity of the density centre
  2. The required accuracy of very long simulations



V.A. Ambartsumian

http://heyhoheyho.blogspot.co.uk

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- By "Hénon's Principle", which states that $\dot{E} = -\dfrac{\zeta E}{Tr}$, we get $\dot{N} = -\dfrac{\zeta E}{T_r m\varepsilon\phi_c}$
- In steady post-collapse expansion, central part of cluster nearly isothermal, and core population $N_c \propto N^{1/3}$ (Goodman 1987). Hence $\dfrac{E}{m\phi_c} \propto \dfrac{1}{\ln N}$, and so $\dot{N} \sim -\dfrac{N}{T_r \ln \Lambda}$
- However, for $N \gtrsim 7000$, post-collapse expansion is not steady, binary activity may occur near minimum core radius of gravothermal oscillations. Hence three-body escape rate even slower.