Does precision matter?

Tjarda Boekholt and Simon Portegies Zwart

Leiden Observatory

boekholt@strw.leidenuniv.nl

spz@strw.leidenuniv.nl

Introduction

Numerical **solutions** to the N-body problem **diverge** ([1], [2]) from the true solution, because of discretization and round-off errors. The general consensus in the N-body community however, is that **statistical results** of an ensemble of N-body simulations are **accurate**, even though individual simulations are not ([3], [4]).

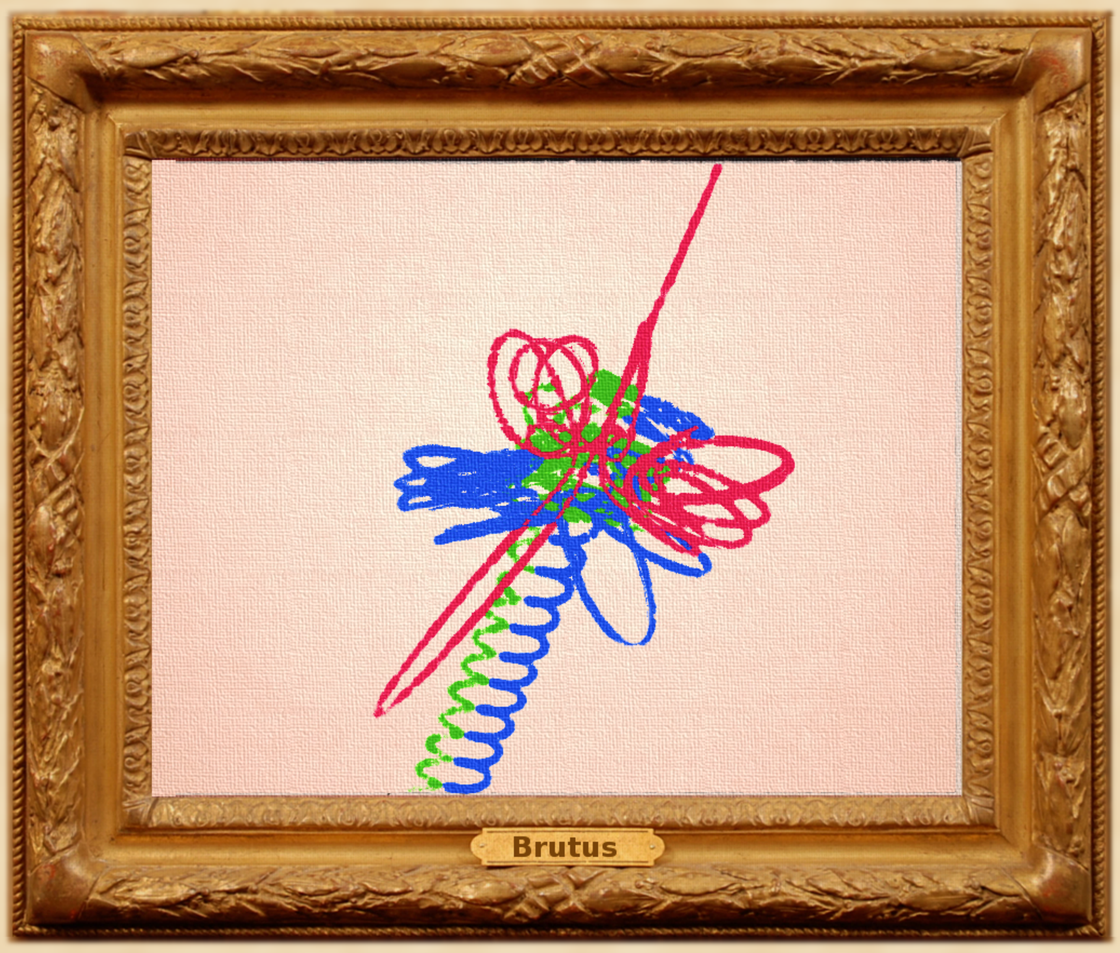
We test this hypothesis by making a direct comparison between an ensemble consisting of approximate solutions and an ensemble consisting of true solutions ([5], [6]).

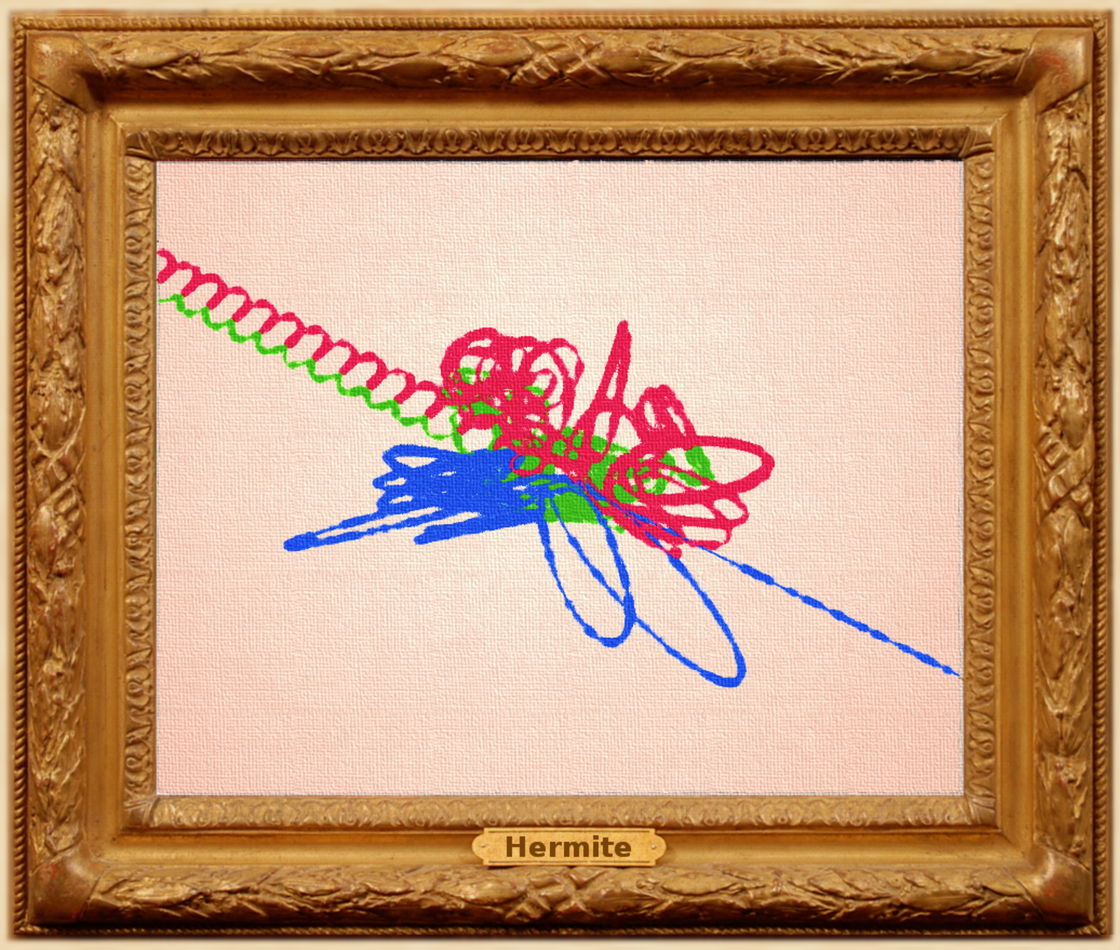
Method

In order to obtain true solutions, we wrote an N-body code called **Brutus***, that uses **arbitrary-precision arithmetic** to control the round-off error and the **Bulirsch-Stoer method** to control the discretization error.

By systematically decreasing the Bulirsch-Stoer tolerance parameter and increasing the word-length, Brutus is able to obtain converged solutions, which are true up to the first specified number of decimals.

* available as a community code in Amuse (www.amusecode.org)



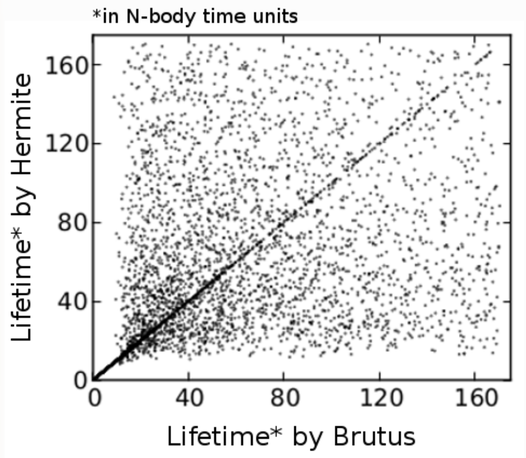


Experiment

In the two paintings above we illustrate the **influence of precision** on the numerical solution. Both simulations started with the exact same initial condition consisting of three stars. In time, Hermite starts to diverge from the true solution, ending up on a completely different trajectory in phase-space. Note for example that the direction and identity of the escaping star are different (see direction and color of the straight line going out of the frame).

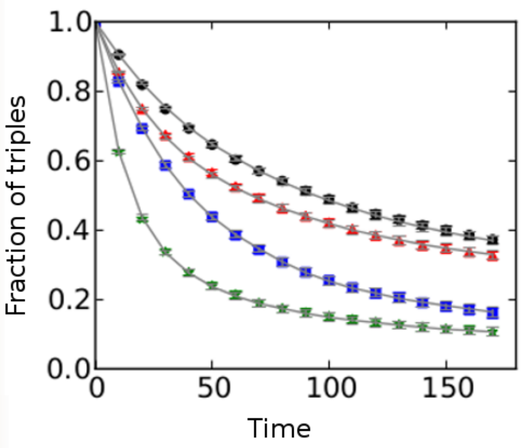
Watch the animation!

Results



We perform 10000 similar experiments, but each time starting with random initial conditions.

In this scatter diagram, we plot the **lifetime** of the **triple interaction** as given by Hermite vs. that by Brutus.



Even though there is a lot of scatter, the global **statistical distributions** of the lifetimes are in **agreement** between Hermite (curves) and Brutus (data points).

(The different colors correspond to different mass ratios and angular momentum).

Conclusion

Precision does matter if you are interested in solving a specific N-body problem. Due to the presence of exponential divergence, your numerical solution will quickly diverge from the true solution.

Precision matters less if you are interested in global statistics over an ensemble of N-body simulations. As long as each individual simulation conserves energy to better than ten percent, then **global statistical distributions are preserved under divergence of solutions** ([5], [6]).

References

[1] Miller, R.H.: Irreversibility in Small Stellar Dynamical Systems. ApJ 140, 250 (1964)
[2] Dejonghe, H., Hut, P.: Round-Off Sensitivity in the N-Body Problem. The Use of Supercomputers., (1986)
[3] Goodman, J. et al.: On the Exponential Instability of N-Body Systems. ApJ 415, 715 (1993)
[4] Valtonen, M. et al.: Statistical Approach to the Three-Body Problem. Conf. Series, vol. 316, p. 45 (2004)
[5] Portegies Zwart, S., Boekholt, T.: On the Minimal Accuracy Required for Simulating Self-gravitating Systems by Means of Direct N-body Methods. ApJ vol. 785 (2014)
[6] Boekholt, T., Portegies Zwart, S.: On the reliability of N-body simulations (In preparation)