Some insights into galaxy bias with a toy model

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Cosmology/Lens seminar
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• simulation slice from $z=0$; LCDM

• gray: dark matter

• dots: B-V colors of galaxies

Credit: J. Colberg and A. Diaferio; GIF simulations (1998)
modes of density fluctuations (random fields):

\[
\tilde{\delta}(k) = \int d^3x \, \delta(x) e^{-ix \cdot k}.
\]

complete second-order statistics of fluctuations:

\[
\langle \tilde{\delta}_m(k) \tilde{\delta}_m(k') \rangle = (2\pi)^3 \delta_D(k + k') P_m(k) ;
\]

\[
\langle \tilde{\delta}_m(k) \tilde{\delta}_g(k') \rangle = (2\pi)^3 \delta_D(k + k') P_{gm}(k) ;
\]

\[
\langle \tilde{\delta}_g(k) \tilde{\delta}_g(k') \rangle = (2\pi)^3 \delta_D(k + k') \left( P_g(k) + \bar{n}_g^{-1} \right),
\]

Biasing functions (linear stochastic bias):

\[
b(k) = \sqrt{\frac{P_g(k)}{P_m(k)}} ; \quad r(k) = \frac{P_{gm}(k)}{\sqrt{P_g(k) P_m(k)}}.
\]
Bias measured with lensing (on the sky inside apertures)

more recent:
Buddendiek et al., 2016, A&A

GaBoDS

deprojected (ongoing work)

z~0.3

RCS/VIRMOS-DESCART

Simon et al., 2007, A&A
GaBoDS
details: e.g. Simon et al., 2007, A&A, 861
In simulation: Millennium Simulation + SAMs
(by colour and redshift; flux limit $r < 25$ mag)


SAMs: Guo et al. (2011); dark matter: Springel et al. (2005)
Halo-model inspired *toy model* that features:

- unclustered matter halos;
- single-mass halos with same density profile (mass $m_0$);
- optionally mix of satellite and central galaxies or just satellite galaxies;
Ingredients of model:

\[ \langle N \rangle \] mean galaxy number in halo;

\[ \langle N(N - 1) \rangle \] mean number of galaxy pairs in halo;

\[ u_m(x) \] radial matter density profile;

\[ u_g(x) \] radial galaxy density profile;

\[ n_h \] halo number density;

\[ \tilde{u}(k) := \frac{\int_0^\infty dx \, x \, k^{-1} \, u(x) \, \sin(kx)}{\int_0^\infty dx \, x^2 \, u(x)} \] Fourier transform of radial profile (normalised!)

\[ P_g(k) = \frac{n_h}{\bar{n}_g^2} \tilde{u}_g^{2p}(k) \langle N(N - 1) \rangle = \frac{\langle N(N - 1) \rangle}{n_h \langle N \rangle^2} \tilde{u}_g^{2p}(k) \]

\[ P_m(k) = \frac{m_0^2 n_h}{\bar{\rho}_m^2} \tilde{u}_m^2(k) = \frac{\tilde{u}_m^2(k)}{n_h} \]

\[ P_{gm}(k) = \frac{n_h m_0}{\bar{\rho}_m \bar{n}_g} \tilde{u}_m(k) \tilde{u}_g^q(k) \langle N \rangle = \frac{\tilde{u}_m(k) \tilde{u}_g^q(k)}{n_h} \]

uses \( \bar{\rho}_m = n_h m_0 \), \( \bar{n}_g = n_h \langle N \rangle \), \( n(m) = n_h \delta_D(m - m_0) \), and

\[ p = \begin{cases} 
1 & \langle N(N - 1) \rangle > 1 \\
1/2 & \text{otherwise}
\end{cases} \]

\[ q = \begin{cases} 
1 & \langle N \rangle > 1 \\
0 & \text{otherwise}
\end{cases} \]

controls impact of central galaxies
Correlation factor for scale $k$:

$$r(k) = \frac{P_{gm}(k)}{\sqrt{P_g(k) P_m(k)}} = \frac{\tilde{u}_g^{q-p}(k) \langle N \rangle}{\sqrt{\langle N(N - 1) \rangle}} = \tilde{u}_g^{q-p}(k) \left(1 + \frac{\Delta \sigma_N^2}{\langle N \rangle^2}\right)^{-1/2}$$

where we have introduced the excess variance

$$\Delta \sigma_N^2 := \sigma_N^2 - \sigma_N^2 |\text{Poisson}\rangle$$

$$= \langle N^2 \rangle - \langle N \rangle^2 - \langle N \rangle = \langle N(N - 1) \rangle - \langle N \rangle^2$$

$\Delta \sigma_N^2 = 0$ variance of $N$ as in Poisson statistic;

$\Delta \sigma_N^2 < 0$ sub-Poisson variance;

$\Delta \sigma_N^2 > 0$ super-Poisson variance
We learn about two ways to produce $r(k) > 1$:

1. dominating central galaxies; for $q-p = -1/2$, $r(k)$ can become arbitrarily large for $k \gg 1$:
   \[ r(k) \propto \tilde{u}_g^{-1/2}(k) \]

2. no/negligible central galaxies and sub-Poisson variance; hence $p = q = 1$ and $\Delta \sigma^2_N < 0$;

Note: impact of 2. becomes small if $\Delta \sigma^2_N << \langle N \rangle^2$
Bias factor for scale $k$:

$$b(k) = \sqrt{\frac{P_g(k)}{P_m(k)}} = \frac{\tilde{u}_g^p(k)}{\tilde{u}_m(k)} \sqrt{\langle N(N-1) \rangle} = \frac{\tilde{u}_g^q(k)}{\tilde{u}_m(k)} \frac{1}{r(k)}$$

1. even without central galaxies and identical radial profiles we do not necessarily have $b(k) = r(k) = 1$; yet we find $b(k)r(k) = 1$;

2. only a Poisson variance in addition to 1. ensures $b(k) = r(k) = 1$; truly unbiased galaxies require a Poisson-like variance of $N$;

3. for satellite-dominated halos, $b(k)$ reflects the difference between matter and galaxy radial profile; hence $q = 1$ and $r(k) = \text{const}$;
• Central galaxies or a sub-Poisson variance of $N$ can make $r(k) > 1$ because

1. $P_g(k)$ is defined in excess of Poisson shot noise of discrete galaxies. If galaxy sampling is actually sub-Poisson, this over-corrects the shot noise power (Seljak 2000; Guzik & Seljak 2001).

2. Putting one galaxy always at the halo centre is not a Poisson sampling of the halo density profile; so is non-Poisson $\Delta\sigma_N^2$.

• Is $r(x) > 1$ also possible for the real-space biasing functions (shot noise only at zero lag)?

$$b(x) = \sqrt{\frac{\xi_g(x)}{\xi_m(x)}}; \quad r(x) = \frac{\xi_{mg}(x)}{\sqrt{\xi_g(x)\xi_m(x)}}.$$

References:

TBD
-> transform three power spectra of toy model to real-space correlation functions via (back Fourier transform)

\[ \xi(x) = [P](x) := \frac{1}{2\pi^2 x} \int_0^\infty dk \, k \, P(k) \sin(kx), \]

and you get real-space counterparts of \( b(k) \) and \( r(k) \):

\[
\begin{align*}
  b(x) &= \frac{[\hat{u}_m \cdot \hat{u}_g^q](x)}{||[\hat{u}_m^2](x)||} \frac{1}{r(x)}; \\
  r(x) &= \frac{[\hat{u}_m \cdot \hat{u}_g^q](x)}{\sqrt{[\hat{u}_g^{2p}](x) [\hat{u}_m^2](x)}} \left( 1 + \frac{\Delta \sigma^2_N}{\langle N \rangle^2} \right)^{-1/2}
\end{align*}
\]

Interestingly, \( r(x) \) depends also on matter density profile.
Assume: identical density profiles \( u_m(x) = u_g(x) \)

- without central galaxies — \( p = q = 1 \) — we have

\[
b(x) \, r(x) = 1 \, ; \, r(x) = \left( 1 + \frac{\Delta \sigma_N^2}{\langle N \rangle^2} \right)^{-1/2}
\]

- with centrals dominating and Poisson variance — \( p = 1/2, \, q = 0, \) and \( \Delta \sigma_N^2 = 0 \) — we have

\[
b(x) \, r(x) = \frac{[\tilde{u}_m](x)}{[\tilde{u}_m^2](x)} \, ; \, r(x) = \sqrt\frac{[\tilde{u}_m](x)}{[\tilde{u}_m^2](x)} \geq 1
\]
Summary and conclusions

• biasing functions of linear stochastic bias fully capture the differences in distributions of galaxies vs. matter for two-point statistics; can be measured with lensing;

• their definition assumes Poisson shot-noise for galaxies; can give seemingly curious values $r > 1$ if sampling is actually sub-Poisson;

• demonstrated with toy model that $r(k) > 1$ or $r(x) > 1$ can arise through central galaxies or a sub-Poisson variance of galaxy numbers inside matter halos;

• even if galaxies perfectly trace matter, we still have $b = r = 1$ for all scales if sampling variance is not Poisson;