High-Precision Integrator for Black Hole Dynamics

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Agenda

Stellar systems around super-massive black holes provide an interesting field of dynamics.

The galactic centre presents numerous dynamical mechanisms not fully understood or investigated yet, among them:

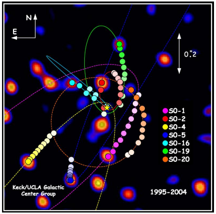
- Inspiralling of massive objects
- Creation of hyper-velocity stars
- Formation of S-stars in the central region
- Kozai effect as a source of gravitational radiation

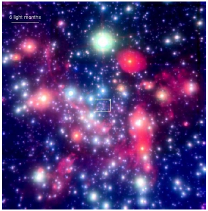
To further analyse the processes in the vicinity of a super-massive black hole, a suitable high precision integrator needs to be chosen.

Stellar systems around super-massive black holes are a unique environment.

Dynamically, stellar systems around SMBHs behave like a planetary system:

- SMBH dominates motion
- Stars move along weakly perturbed Keplerian orbits





However, they also resemble star clusters:

- Large number of similar mass stars
- Wide ranges of eccentricities and central distances

Various integration schemes for planetary systems are available.

Symplectic integrators (e.g. leapfrog) yield a very good energy conservation for nearly circular orbits.
They have constant (global) timesteps.

Leapfrog-type integrators do not conserve e.g. direction of pericenter.

Mikkola and Tanikawa (1999) found a time-symmetric adaptive timestep mechanism.

Weakly perturbed Keplerian orbits can be calculated by solving Kepler's equation.

Kepler's equation solves the two-body motion exactly.

$$H = \frac{p^2}{2m} - \frac{\mu}{r} \qquad\qquad MT = E - e \sin E$$

However, for N-body systems, kinetic energy is no longer the sum of squares of momenta $\frac{p^2}{2m}$ *relative to moving center*.

- ⇒ Introduce Jacobi coordinates for orbit calculation.
- ⇒ Transform variables back and forth for perturbations.

Saha and Tremaine (1994) found a mechanism for an MVS integrator with individual, but non-adaptive timesteps.

Symplectic integrators fail for eccentric orbits and do not allow for individual adaptive timesteps.

Non-symplectic integration schemes for large-N systems.

Standard integrators, such as

- Predictor-corrector-schemes (e.g. Hermite)
- Runge-Kutta methods and their variants
- Other high-accuray schemes (e.g. Bulirsch-Stoer)

are mainly used for integration of large systems like star clusters.
They are not symplectic and produce a secular energy error, but they allow for adaptive individual timesteps.

N-body integrators are unable to differentiate dominating massive objects from small perturbers. Timesteps chosen are therefore

- too small for perturbations, or
- too large for orbital motion.

Let's join the two...

Idea: Combination of

- High precision of Keplerian orbital motion and
- High speed and flexibility of N-body integrators.

Problem: Need to fix the center to use cartesian coordinates for Kepler's equation.

Assumption: Perturbations of large number of statistically evenly distributed low-mass stars will cancel each other out, so consider super-massive black hole as fixed.

Fixed center assumption invalid for major sources of gravitational waves, e.g. inspiralling intermediate-mass black holes?

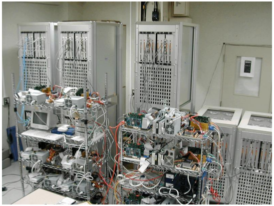
Composite Kepler-Hermite integrator for central and perturbing forces.

Algorithm:

- Direct solution of Kepler's equation for central force
- Standard Hermite integration of perturbing forces

Special treatment for close encounters:

- Define neighbour sphere for potentially strong perturbers
- Check neighbours for fast approaches and close encounters

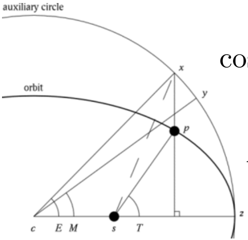


Special-purpose GRAPE hardware used for force calculation and neighbour determination.

Kepler's equation and approach checks done on host computer.

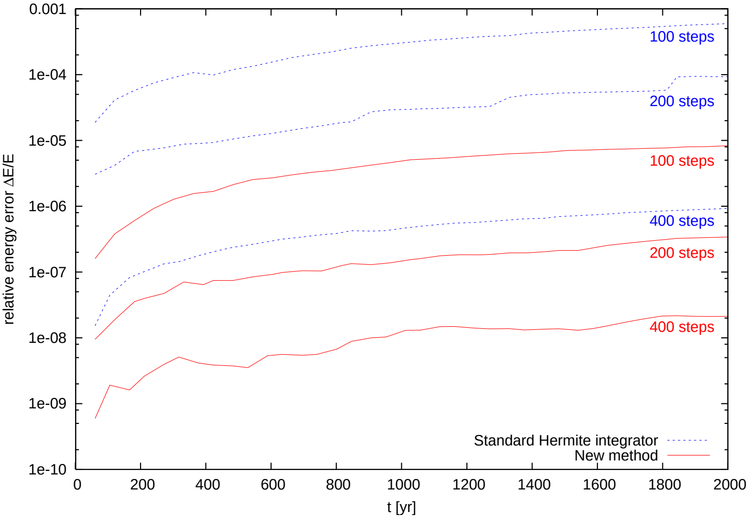
Split up the forces into two parts.

$$
\begin{aligned}
\vec{r}' = \; & \vec{r} + \dot{\vec{r}}\Delta t & + \frac{1}{2!}\ddot{\vec{r}}\Delta t^2 + \frac{1}{3!}\vec{r}^{(3)}\Delta t^3 + \dots \\
= \; & \vec{r} + \dot{\vec{r}}\Delta t & + \frac{1}{2!}\ddot{\vec{r}}_K\Delta t^2 + \frac{1}{3!}\vec{r}_K^{(3)}\Delta t^3 + \dots \\
& & + \frac{1}{2!}\ddot{\vec{r}}_P\Delta t^2 + \frac{1}{3!}\vec{r}_P^{(3)}\Delta t^3 + \dots
\end{aligned}
$$

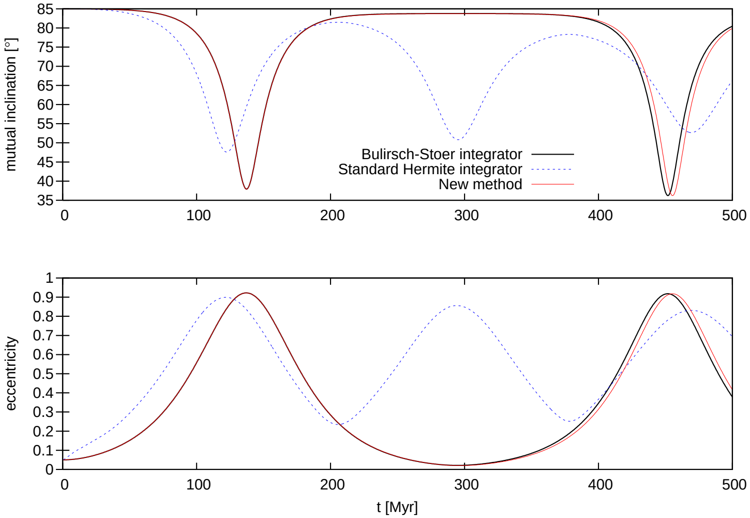


$$
\begin{aligned}
\cos E &= \frac{a - r}{ae} \\
M &= E - e \sin E \\
M_1 &= M_0 + \omega \Delta t \\
\vec{r} &= \vec{a}\,(\cos E - e) + \vec{b} \sin E
\end{aligned}
$$

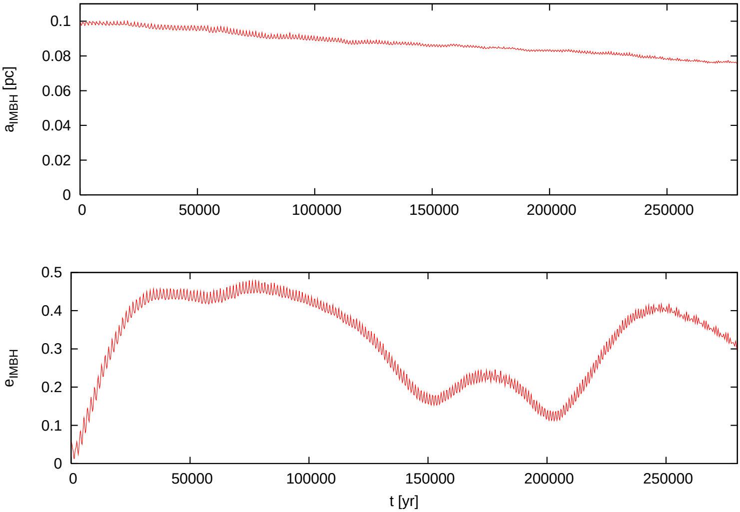
Results: Energy conservation



Results: Kozai mechanism



Example: Inspiral of an IMBH



More work ahead...

Software

- Physical collisions/mergers and tidal disruption
- Gravitational wave emission

Applications

- Stay tuned!

References

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