Monte Carlo Modelling of Globular Clusters

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Collaboration with Mirek Giersz (Warsaw)

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The globular cluster M4

Distance from sun	2.2kpc
Mass	63 000$M_\odot$
Core radius	0.53 pc
Half-mass radius	2.3 pc
Tidal radius	21 pc
Relaxation time (R_h)	660 Myr

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Why model globular clusters?

- Inferring the mass from surface brightness profile, radial velocities, proper motions, mass functions. Can weigh “dark components”: white dwarfs, ...
- Inferring the global mass function from local mass functions: is this the same for all clusters?
- Inferring the primordial mass function from local, present-day mass functions: correct for preferential escape of low-mass stars
- Inferring pimordial parameters of the binaries from present-day abundance and period distribution: primordial abundance, period distribution, etc
- Measuring cluster distances by comparison of radial velocities and proper motions: correct for rotation, different observational fields, different observed components, ...
- Determine the effect of dynamics on spectral energy distribution

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Modelling globular clusters

1. Static models:
   - Plummer's model (Plummer 1911)
   - King's model (King 1966, Peterson & King 1975)
   - Anisotropic models (King; Michie 1963)
   - Multi-mass models (Gunn & Griffin 1979; Meylan & Mayor et al; Pryor et al)
   - Non-parametric models (Gebhardt & Fischer 1995)
   - Schwarzschild's method (van de Ven et al 2006)
   - Jeans' equations (ref:....)

This list is by no means exhaustive

These methods can be used to solve some problems but not all

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Modelling globular clusters

2. Dynamic Evolutionary Models
   - Gas/fluid models (Angeletti & Giannone 1980 [M3])
   - Fokker-Planck models (Cohn and co-workers 1997 [M15], 1992 [N6624]; Drukier 1993, 1995 [N6397]; Phinney 1993 [M15])
   - Monte Carlo model (Giersz & H 2003 [ω Cen])
   - N-body model (? 2015 [?])

Note: N-body modelling of open clusters has just become feasible (see Hurley et al 2005 for M67; present-day mass ~2000$M_\odot$; initial mass 19 000 $M_\odot$; 50% binaries; took 1 month.)

Recent work with the Warsaw Monte Carlo code

Background (papers by M. Giersz)

1998, MNRAS, 298, 1239
Monte Carlo simulations of star clusters - I. First Results
2001, MNRAS, 324, 218
Monte Carlo simulations of star clusters - II. Tidally limited, multimass systems with stellar evolution
2006, MNRAS, 371, 484:
Monte Carlo simulations of star clusters - III. **A million-body star cluster**

The code incorporates many refinements invented by J.S. Stodołkiewicz (1982-6), which was in turn based on the method of Hénon (1971-5)

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Monte Carlo code: the basic idea

Assume spherical symmetry.

Orbit of star characterised by energy, E, and angular momentum, J.

The code repeatedly alters E, J to mimic the effects of gravitational encounters, using theory of relaxation.

The Monte Carlo code: additional dynamical processes

1) Galactic tidal field: treated as a spherically symmetric cut-off
2) Binaries: Treated by reaction cross sections

- Binary-single interactions: Spitzer 1987
- Binary-binary interactions: based on Mikkola 1983, 1984

Notes:

- There are significant differences between a tidal field and a tidal cutoff
- No triples, and so we cannot handle hierarchical triples
- Do not include physical collisions taking place during 3- and 4-body interactions
- Cross sections are thought to result in excessive binary activity (Fregeau)

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Monte Carlo code: developments in 2006

Addition of stellar evolution of binary and single stars

- Single stars: Hurley, JR, Pols, O, Tout, CA, "Comprehensive analytic formulae for stellar evolution as a function of mass and metallicity", 2000, MNRAS, 315, 543
- Binary stars: Hurley, JR, Tout, CA, Pols, O, "Evolution of binary stars and the effect of tides on binary populations", 2002, MNRAS, 329, 897

Implemented via the *McScatter* interface:

Heggie, DC, Portegies Zwart, S, Hurley, JR, 2006, "McScatter: A simple three-body scattering package with stellar evolution", NewA, 12, 20

(Also interfaces to stellar evolution package SeBa in starlab, but this is only for solar metallicity.)

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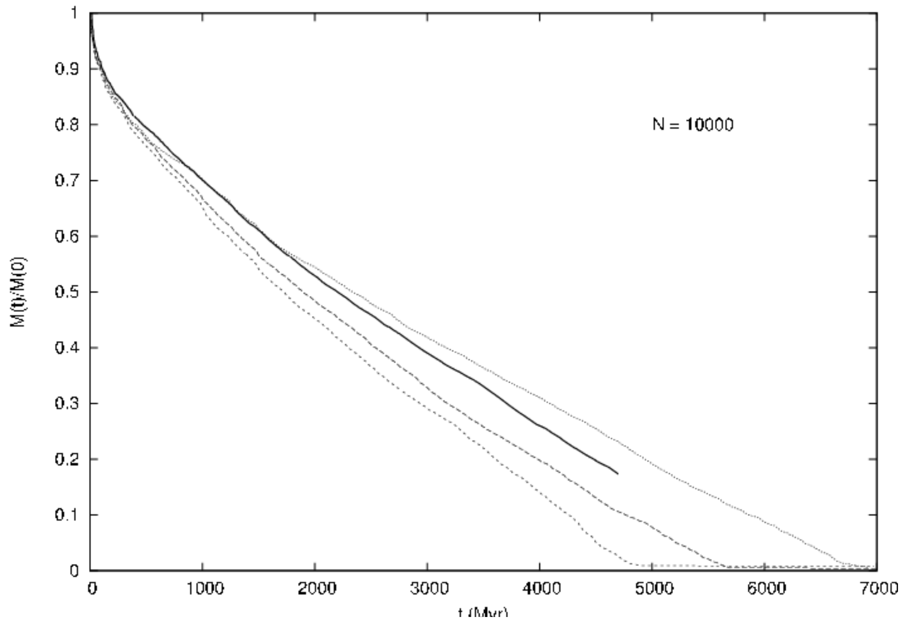
Monte Carlo code: calibration

Time unit depends on relaxation time $t_r = \dfrac{0.065\, v^3}{G^2 m^2 n \ln(\gamma N)}$

γ not well known for unequal masses.

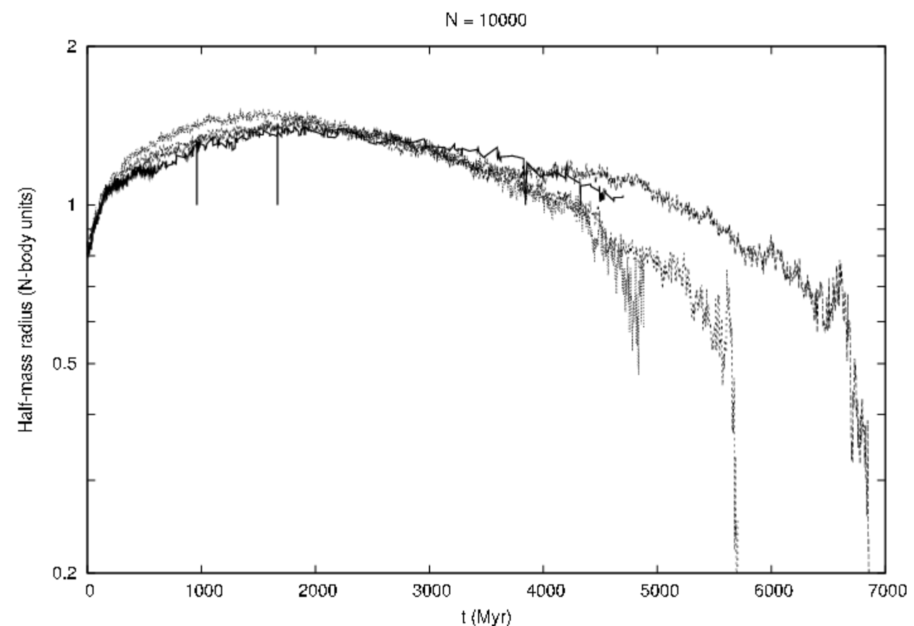
Calibration against N-body model suggests $\gamma \approx 0.005\text{-}0.01$ cf. Hénon 1975

Mass



Time (Myr)

Half-mass radius



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Monte Carlo: comparison with N-body model of M67

Source: Hurley, JR, Pols, OR, Aarseth, SJ, Tout, CA, 2005, “A complete N-body model of the old open cluster M67”, MNRAS, 363, 293

Initial model:

Number of single stars	12000
Number of binaries	12000
IMF	KTG
Initial tidal radius	32.2 pc
Initial total mass	19 340 $M_{\odot}$

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Monte Carlo: comparison with M67

Data at 4Gyr

	N-body	Monte Carlo
$M/M_\odot$	2037	1974
Binary frac	60%	48%
Half-mass rad	3.8 pc	4.3 pc
Run time	1 month	68 min

Remark:

- Low binary fraction because of excessive binary activity through use of cross sections?

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Preliminary Monte Carlo model of M4

Initial mass:	1.5 x 10^5 $M_\odot$	(2.8 x 10^5 $M_\odot$)
Initial # singles:	100 000	(180 000)
Initial # binaries:	100 000	(180 000)
Tidal radius:	66 pc	(31 pc)
IMF etc as M67		
Run time	~1 day	~1 week

Values at 12 Gyr

	This work		Observation*
Total mass	70 810$M_\odot$	49 000$M_\odot$	68 000$M_\odot$
Tidal radius	51 pc	17 pc	21 pc
Half-mass radius	12.4 pc	4.0 pc	2.3 pc
Core radius	4 pc	0.32 pc	0.53 pc

**compilation by Kristin Warnick*

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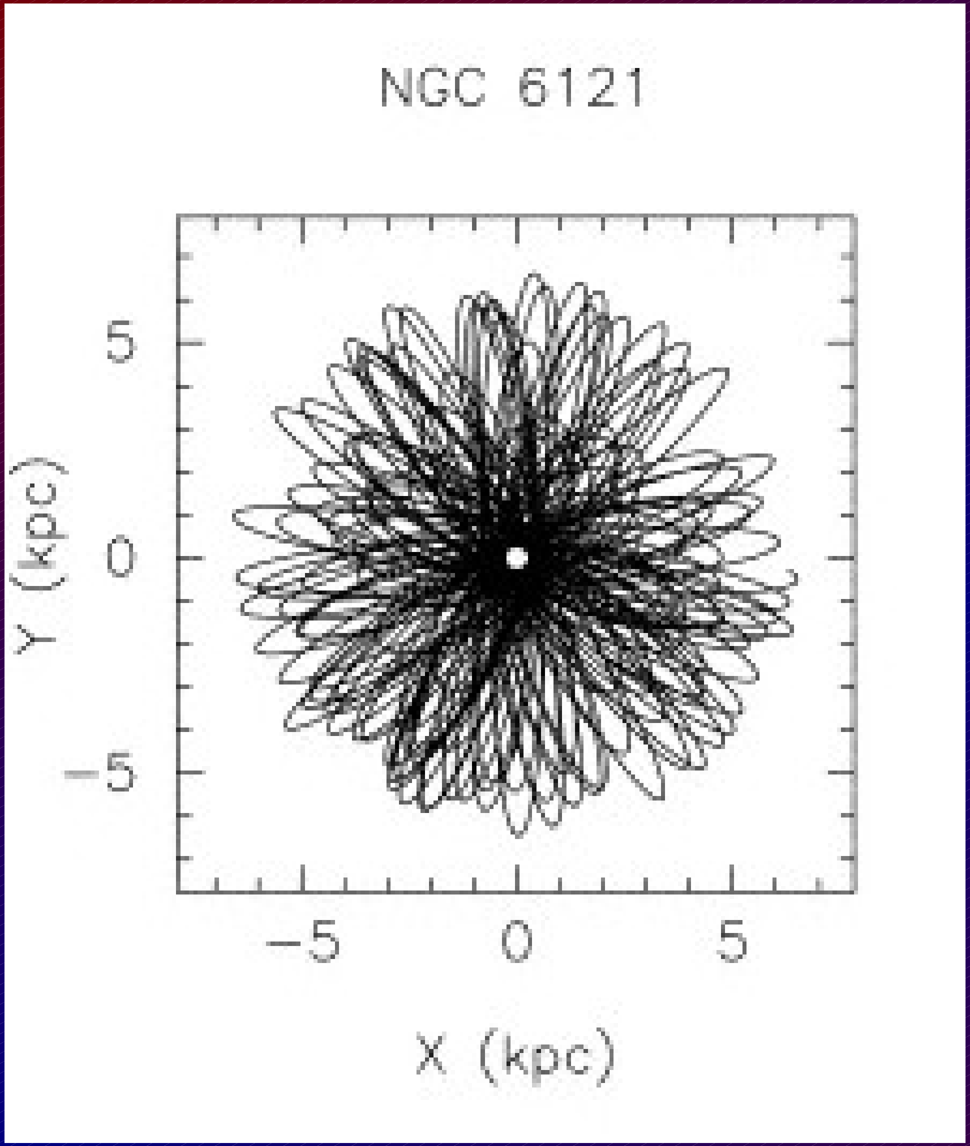
A galactic orbit for M4



From Dinescu, DI, Girard, TM, van Altena, WF. 1999, "Space Velocities of Globular Clusters. III. Cluster Orbits and Halo Substructure", AJ, 117, 1792

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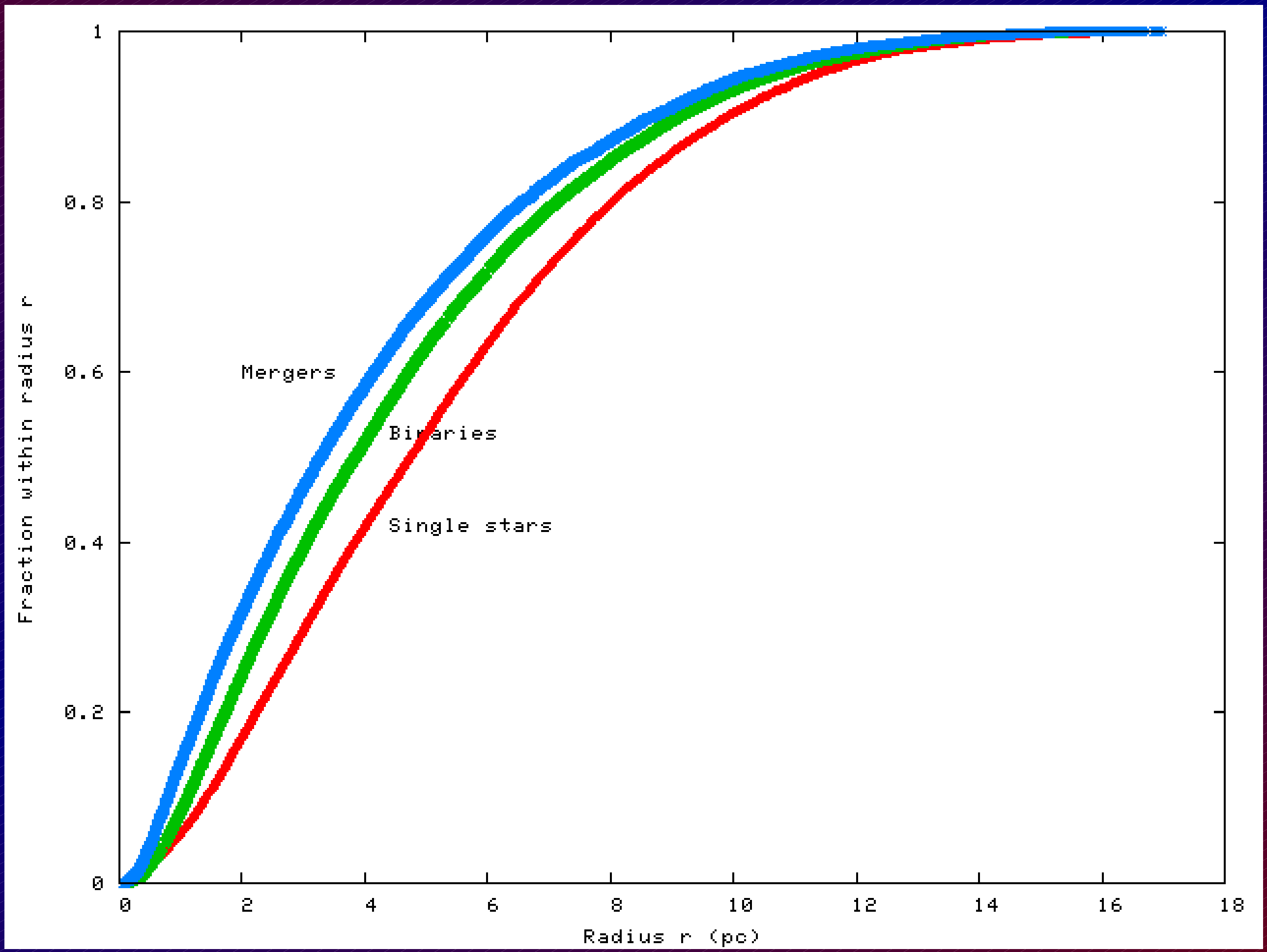
Results from the preliminary model

Mass segregation of binaries (green) compared to singles (red) and mergers (blue)

Mass fraction (<r)



Radius (pc)

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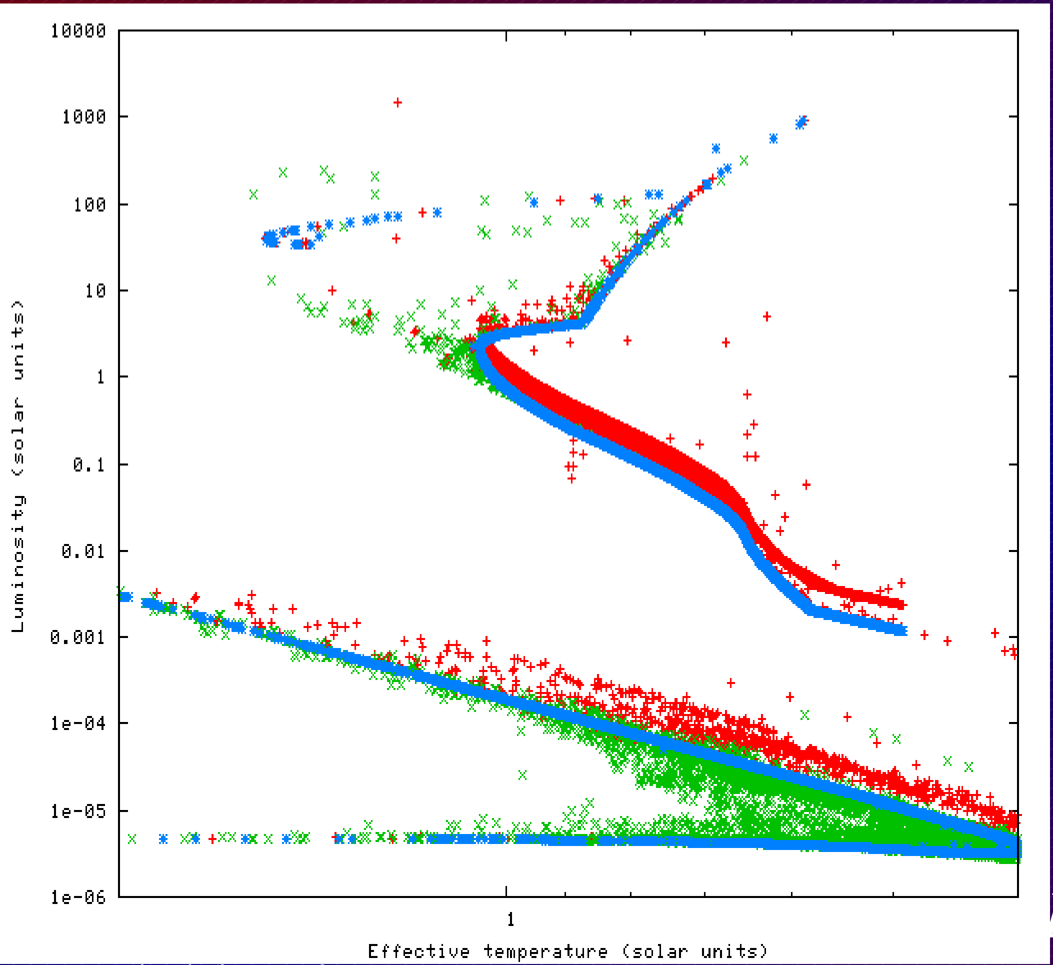
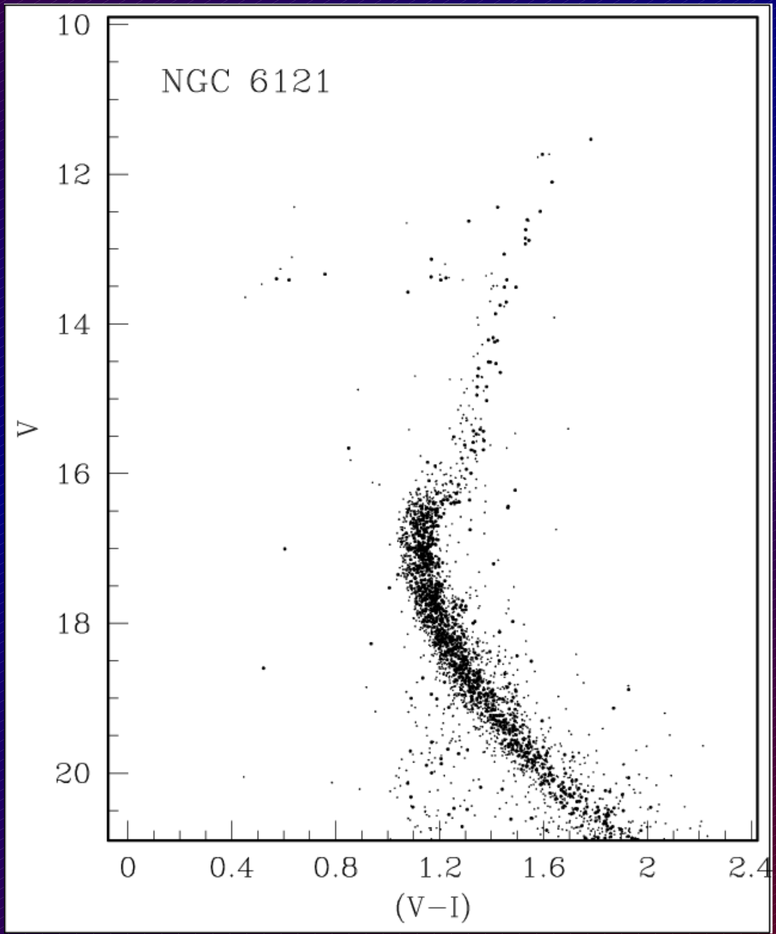
Results from the preliminary model. II

"Colour-magnitude" diagram





red: binaries;
blue: single stars;
green: mergers

From Rosenberg, A, Piotto, G, Saviane, I, Aparicio, A. 2000, "Photometric catalog of nearby globular clusters. I." A&AS, 144, 5

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Conclusions

1. Monte Carlo models can provide similar results to N-body models, with similar physics (binaries, stellar evolution, etc.), except for

- Use of a tidal cutoff
- Use of static tide (curable)
- Rotation
- Use of cross sections for triple/quad interactions (curable)
- Neglect of triples (?curable)

2. Monte Carlo models are feasible in reasonable time for globular clusters, which are too large for N-body models. Yield predictions for mass segregation, distributions of binary parameters, ...
The only comparable method is the hybrid code of Giersz & Spurzem (*"A stochastic Monte Carlo approach to modelling real star cluster evolution – III", 2003, MNRAS, 343, 781,* and references therein)

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Future work

1. General repairs

2. A collapsed core cluster: NGC 6624

3. Replace cross sections by directly integrated binary-single and binary-binary encounters (spring 2007)

And finally, a word from my collaborator.....

"Dear Douglas,
Thank you very much for sending me the pdf file with your talk ....
I am a little surprised about the results for M67 and particularly for M4.
They look very promising........
Mirek"