# Yoshihide Kozai's Trip to the Galactic Centre

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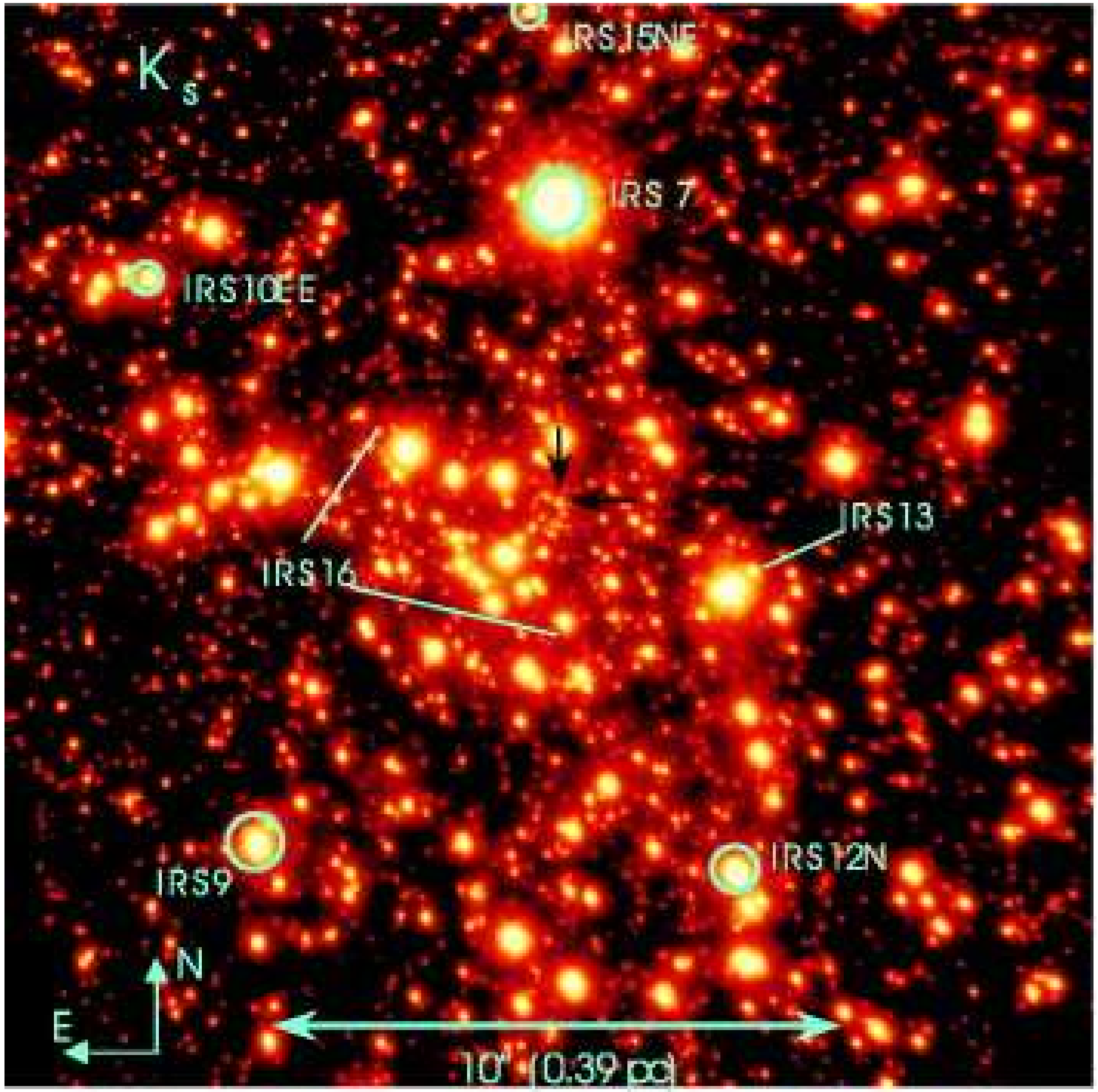
# A beautiful place to visit . . .



# Central parsec

- Sgr A*
  - supermassive black hole with mass $\approx 3.5 \times 10^6 M_\odot$
- S-stars (Ghez et al. 2003, 2005, Genzel et al. 2003)
  - young stars less than $1''$ off Sgr A* $\Rightarrow$ origin?
- Discs (Levin & Beloborodov 2003, Genzel et al. 2003, Paumard et al. 2006)
  - two well defined, almost perpendicular structures
  - about 50 young stars observed only $1''$–$13''$ off Sgr A*
  - WR-stars, LBV, approximately 6 Myr old
  - origin: in situ or infall?
- Spherical cluster (Genzel et al. 2003)
  - old stars with density $\rho(r) \sim r^{-1.4}$

# IRS 13E

- Stellar concentration in the counter-clockwise disc $3.5''$ off Sgr A*
  - at least 12 bright stars
  - total mass $\sim 10^3\,M_\odot$ (Paumard et al. 2006)
- Mailard et al. (2004): total mass at least $10^3\,M_\odot$, to survive tidal forces of Sgr A* $\Rightarrow$ intermediate mass black hole?
- Schödel et al. (2005): total mass at least $10^4\,M_\odot$, to survive rapid movements of the inner stars ($\sim$ 100 km s$^{-1}$)
- Nayakshin et al. (2006): how massive discs would have a destructive influence on each other?
- Haas & Šubr (2006): how massive IRS 13E ($\approx$ counter-clockwise disc) would have a destructive influence on the clockwise disc?

# Model of the system

- Dominant Sgr A*: Newtonian central body

$$\phi_\bullet\left(r\right) = -\mathcal{G}\frac{M_\bullet}{r}$$

- Perturbations:
  - IRS 13E: “averaging” technique → homogenous ring

$$\phi_{\mathrm{r}}\left(R, z\right) = -2\mathcal{G}\lambda\sqrt{\frac{R_{\mathrm{r}}}{R}}kK\left(k\right)$$

  - spherical cluster → Poisson equation:

$$\rho\left(r\right) = \rho_0\left(\frac{r}{r_0}\right)^{-\alpha} \quad \Rightarrow \quad \phi_{\mathrm{c}}\left(r\right) = \mathcal{G}\frac{4\pi\rho_0 r_0^\alpha}{\left(2-\alpha\right)\left(3-\alpha\right)}r^{2-\alpha}$$

- Stars in the clockwise disc: test particles

# System parameters and integrals of motion

- Which values of the system parameters would lead to destructive oscillations of inclination of stellar orbits in the clockwise disc?
- Many parameters . . .
  - initial orbital elements: $a$, $e$, $i$, $\omega$, $\Omega$
  - mass of the ring $M_\mathrm{r}$ and spherical cluster $M_\mathrm{c}$
- Axial symetry: arbitrary initial $\Omega$
- “Averaging” technique: $a$, $c_1$ and $\bar{\phi}_\mathrm{perturbation}$ are integrals of motion

$$c_1 = \sqrt{1 - e^2}\cos i$$

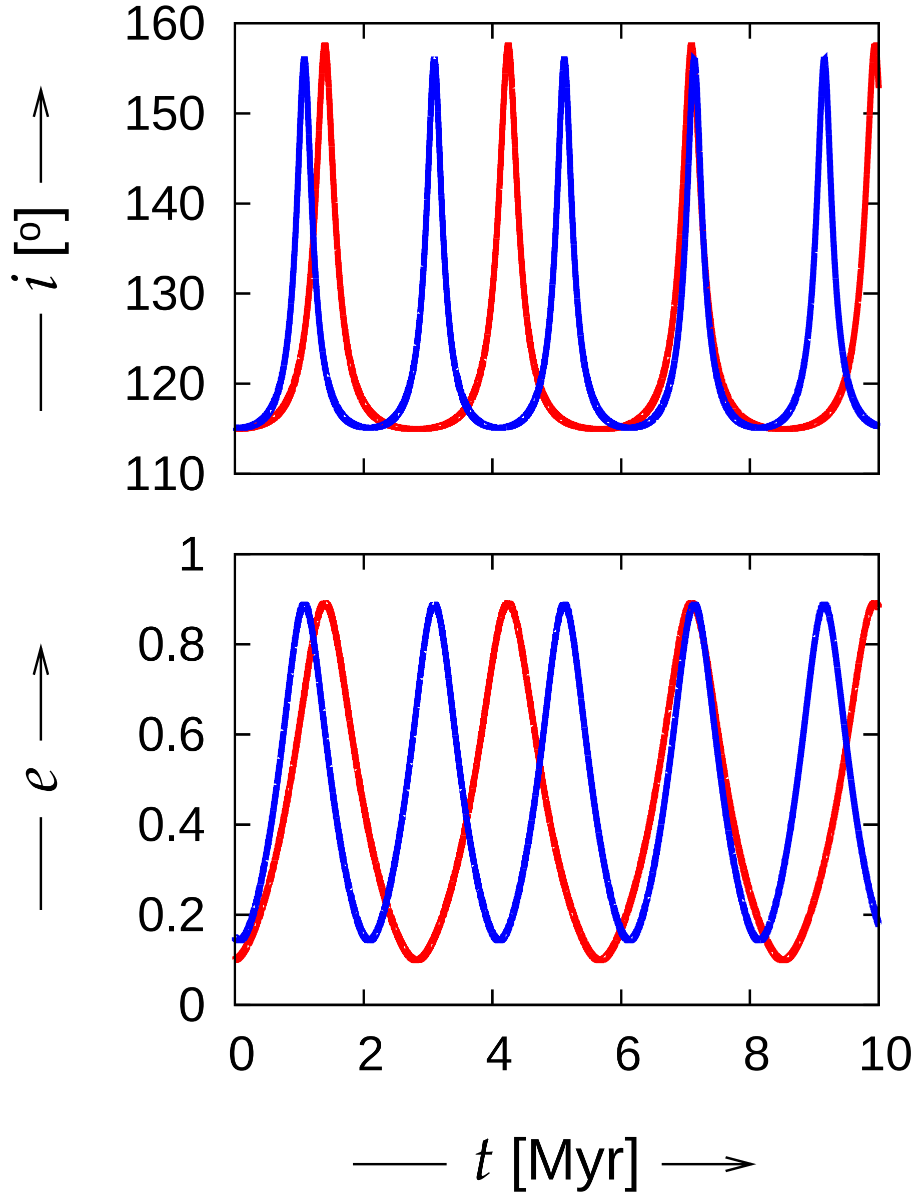
- $\bar{\phi}_\mathrm{perturbation}$: evolutionary diagrams
- Remaining parameters to go through: $a$, $c_1$, $M_\mathrm{r}$, $M_\mathrm{c}$

# Some Kozai oscillations

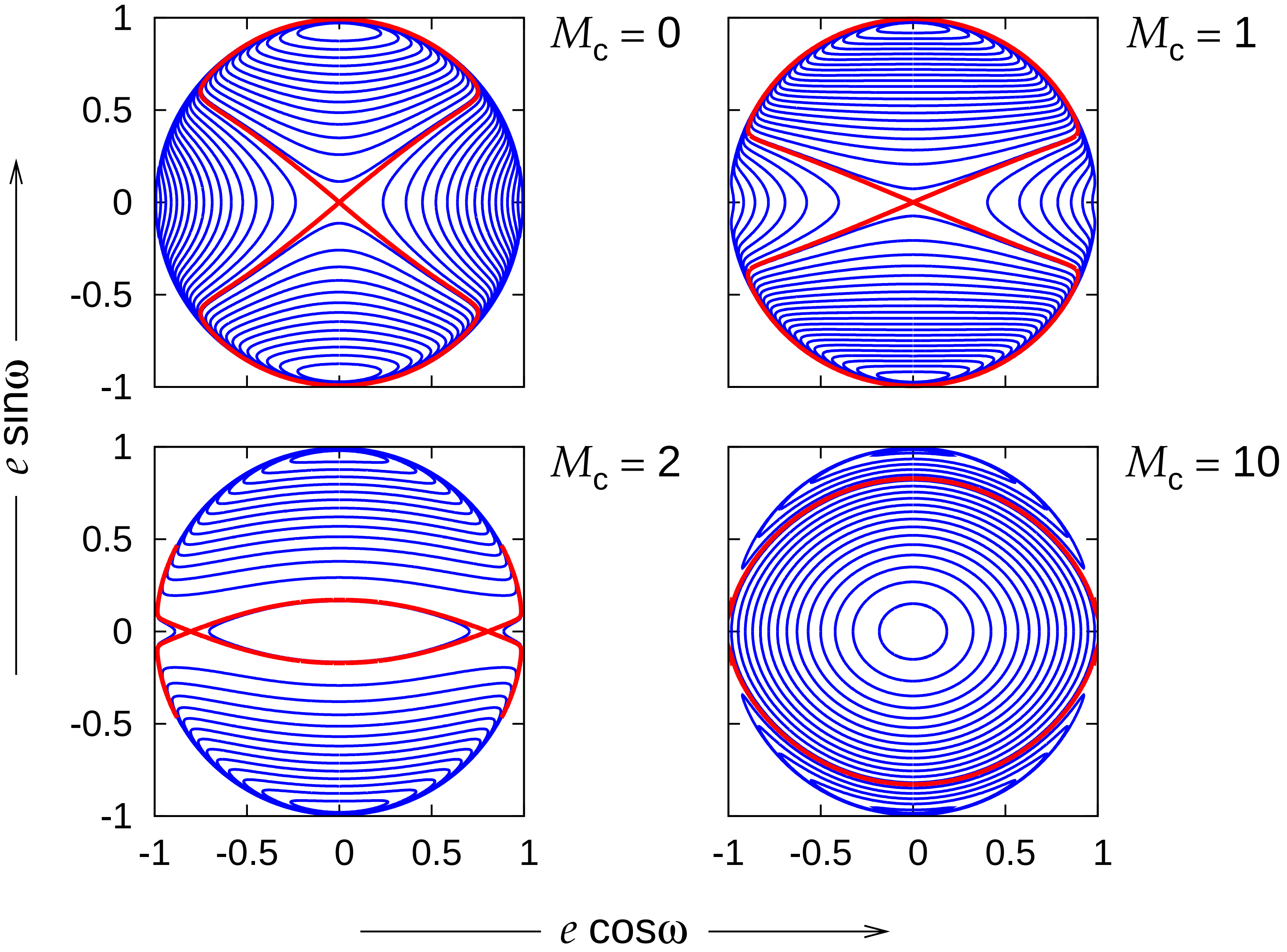
$$M_\mathrm{r} = 10^4\,M_\odot$$
$$M_\mathrm{c} = 10^3\,M_\odot$$

$a$ = 0.047 pc
$i$ = 115°
$e$ = 0.15
$\omega$ = 105°

$a$ = 0.042 pc
$i$ = 115°
$e$ = 0.10
$\omega$ = 85°



# Some evolutionary diagrams

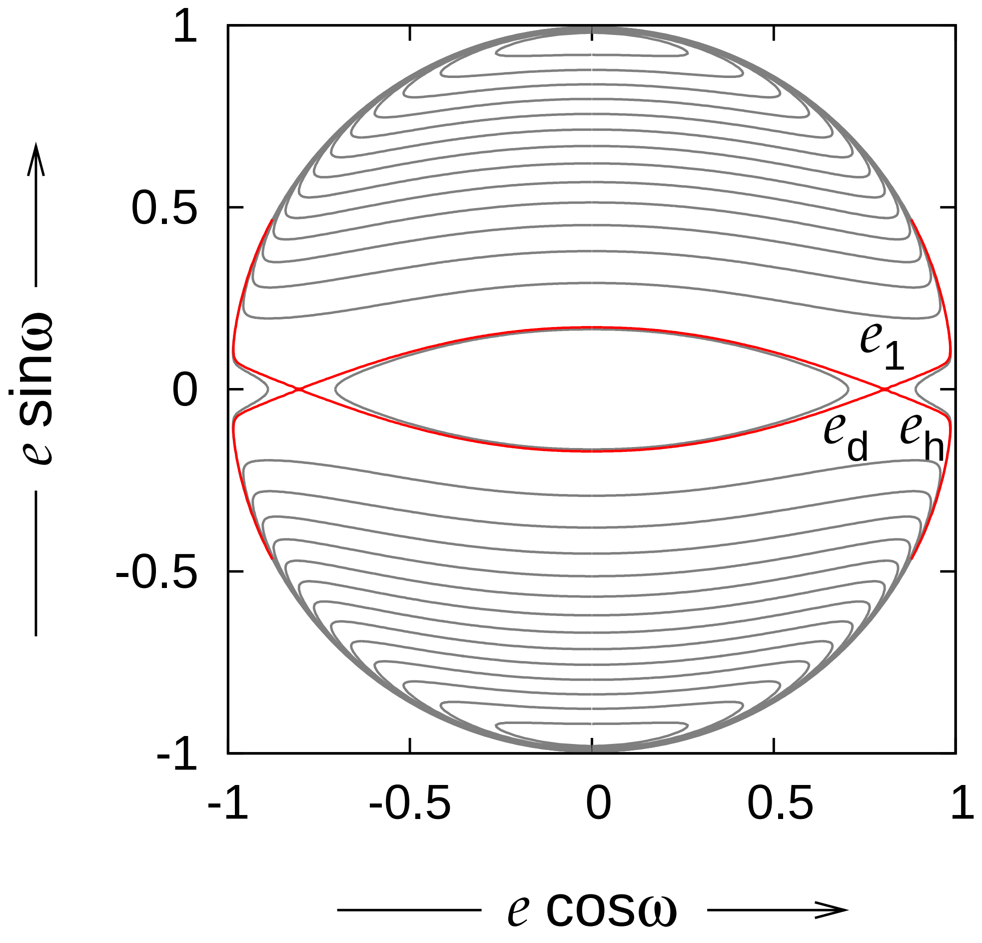


# Clockwise disc stability criteria

- Amplitude of the oscillations
  - too small ($\lesssim 20\,^\circ$) $\Rightarrow$ stability
  - possible only in the inner rotational region $\rightarrow$ initial eccentricity $e_{\mathrm{stab}}$, which guarantees such oscillations
- Period of the oscillations
  - too long ($\gtrsim$ 6 Myr) $\Rightarrow$ stability
  - the fastest oscillations in the librational region $\rightarrow$ period $P_{\mathrm{lib}}$
- Key problem: find both $e_{\mathrm{stab}}$ and $P_{\mathrm{lib}}$ for all possible values of $a$, $c_1$, $M_{\mathrm{r}}$ and $M_{\mathrm{c}}$

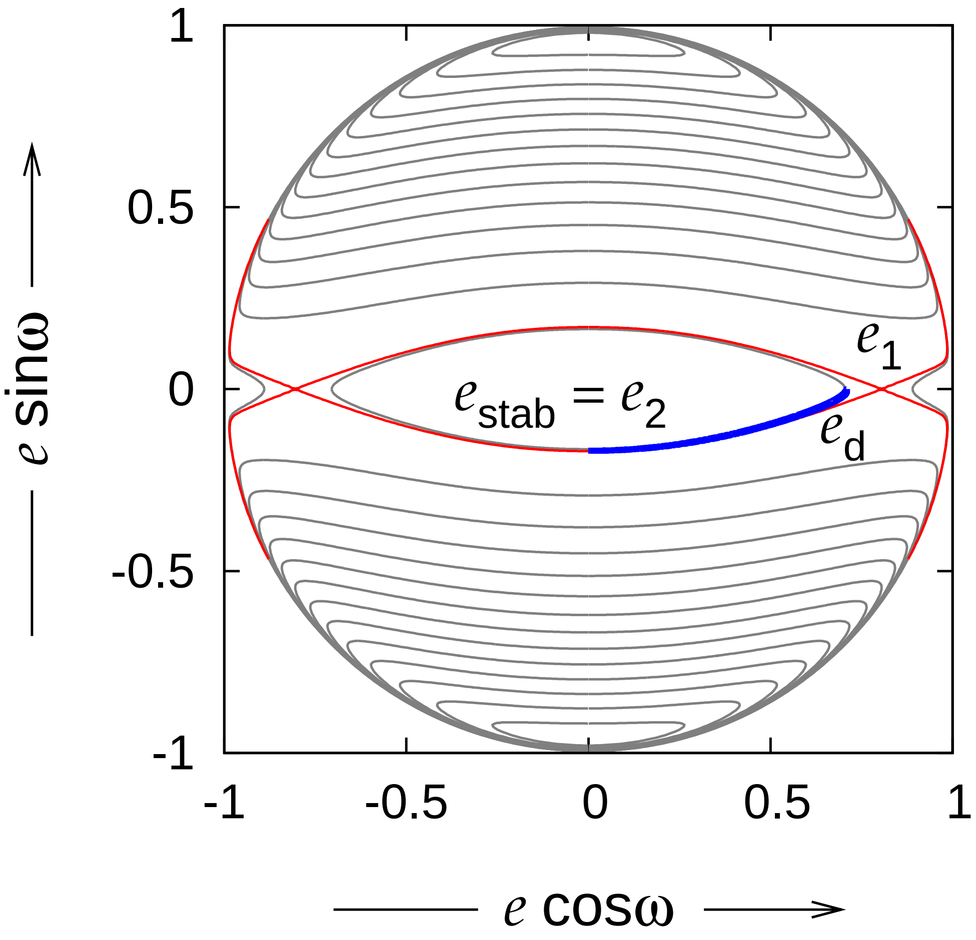
# Algorithm – looking for $e_1$

- $e_1 \equiv$ eccentricity of cross points on separatrix
- Interval halving: $e_{\rm d} < e_1 < e_{\rm h}$



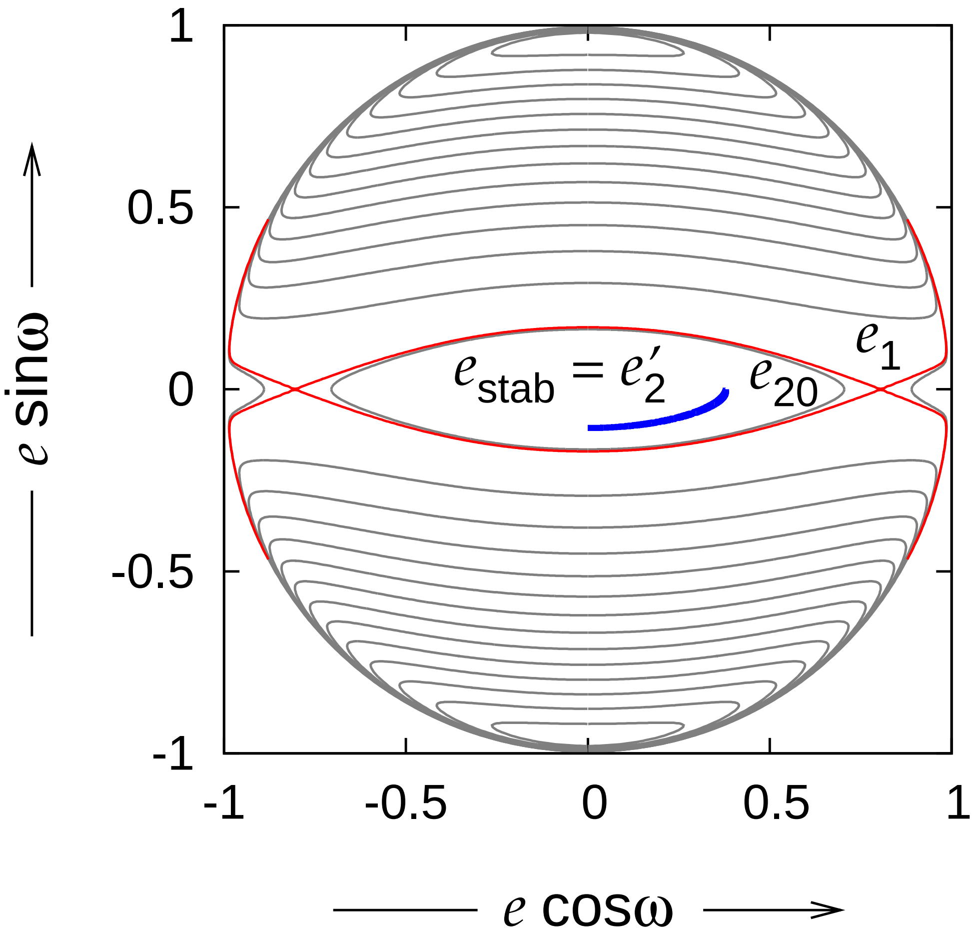
# Algorithm – looking for $e_{\mathrm{stab}}$

- Circular orbit referential
- Destruction of the disc: $\min i < i_{\mathrm{destr}} \equiv i_{\mathrm{circ}} - 20^\circ$
- If $e_1 < e_{20} \Rightarrow e_{\mathrm{stab}} = e_2$, $e_{20}^2 \equiv 1 - (c_1/\cos i_{\mathrm{destr}})^2$, $e_2 \equiv$ minimal eccentricity on separatrix ($\approx$ evolutionary track starting with $e_{\mathrm{d}}$)
  - whole inner rotational region stable



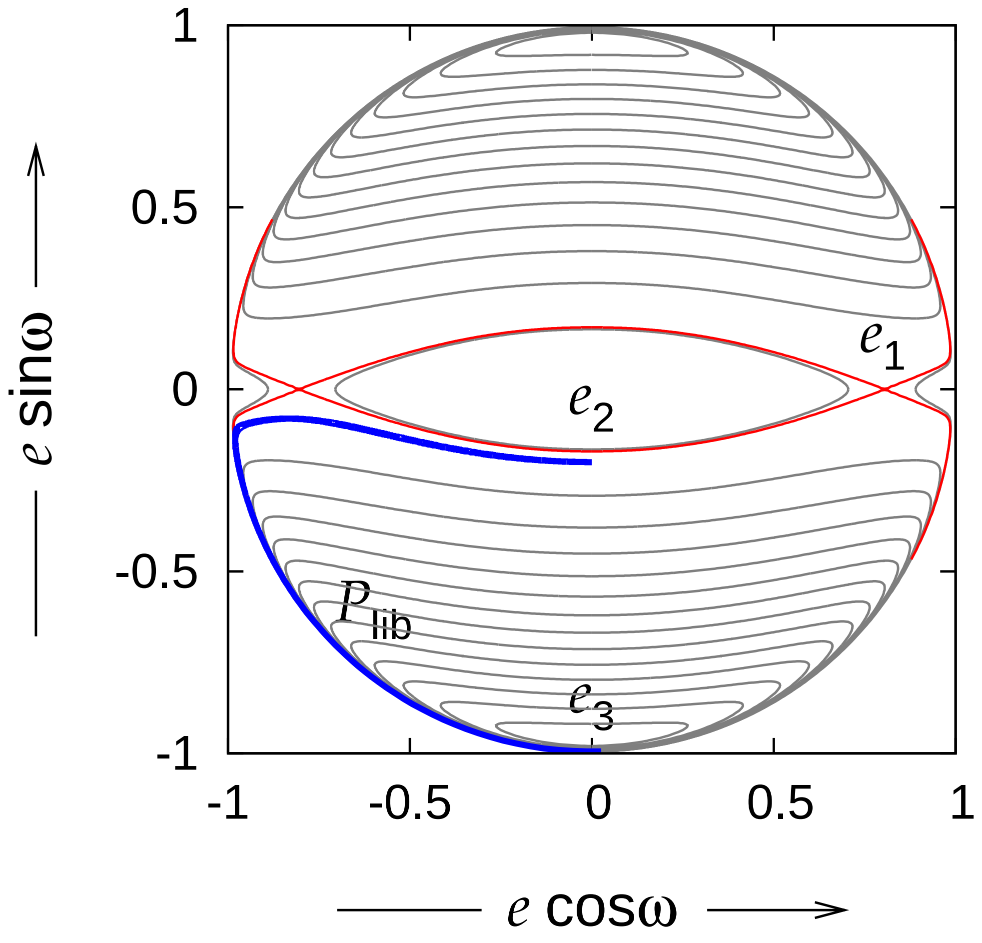
# Algorithm – looking for $e_{\rm stab}$

- If $e_1 > e_{20} \Rightarrow e_{\rm stab} = e'_2$, $e'_2 \equiv$ minimal eccentricity on evolutionary track starting with $e_{20}$
  - only part of the inner rotational region stable



# Algorithm – looking for $P_{\mathrm{lib}}$

- Maximal oscillations on separatrix ($\approx$ evolutionary track starting with $e_2 + \delta$)
- $e_3 \equiv$ maximal eccentricity on separatrix



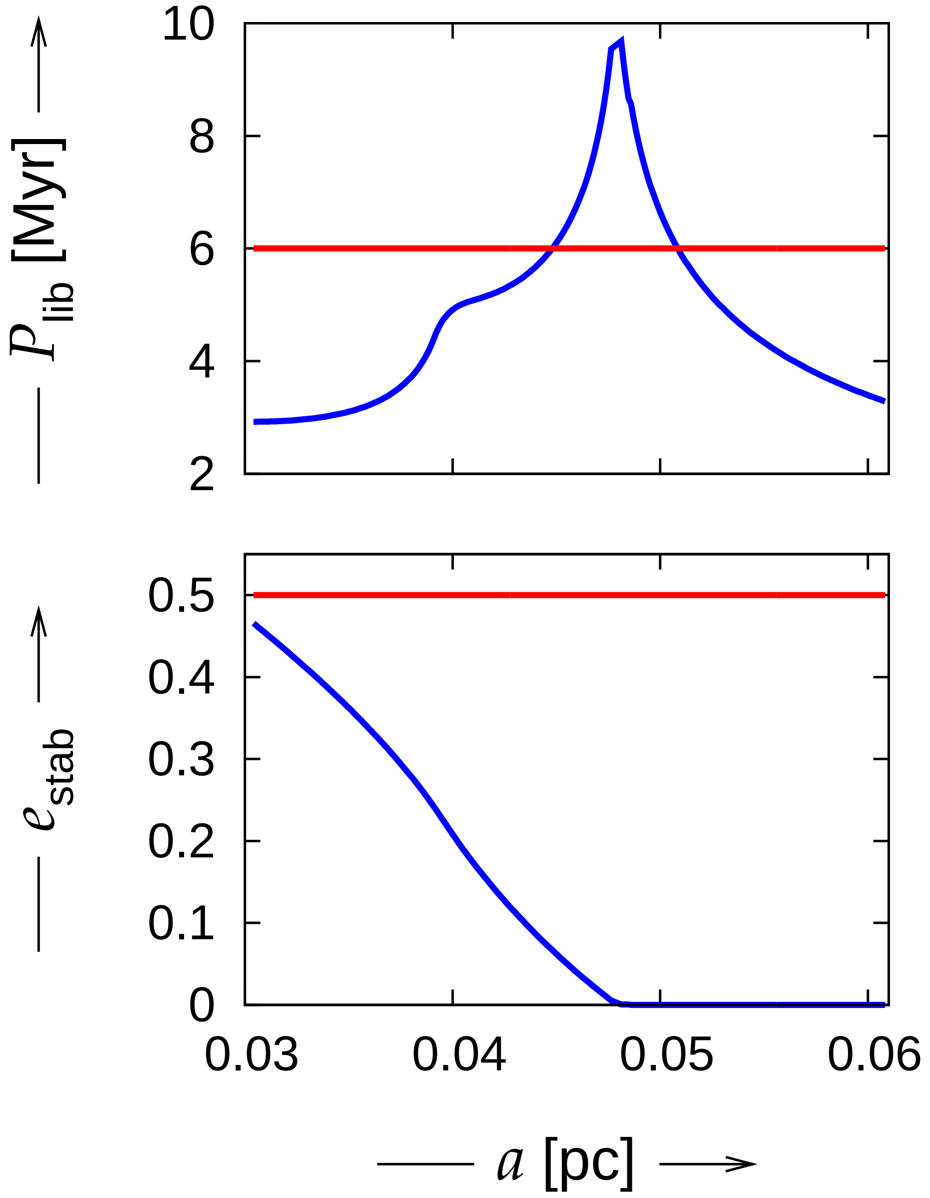
# Global stability

- Global criteria:

$$\begin{aligned} P_{\rm lib}\,(a) &> 6\ \text{Myr} \\ e_{\rm stab}\,(a) &> 0.5 \end{aligned}$$

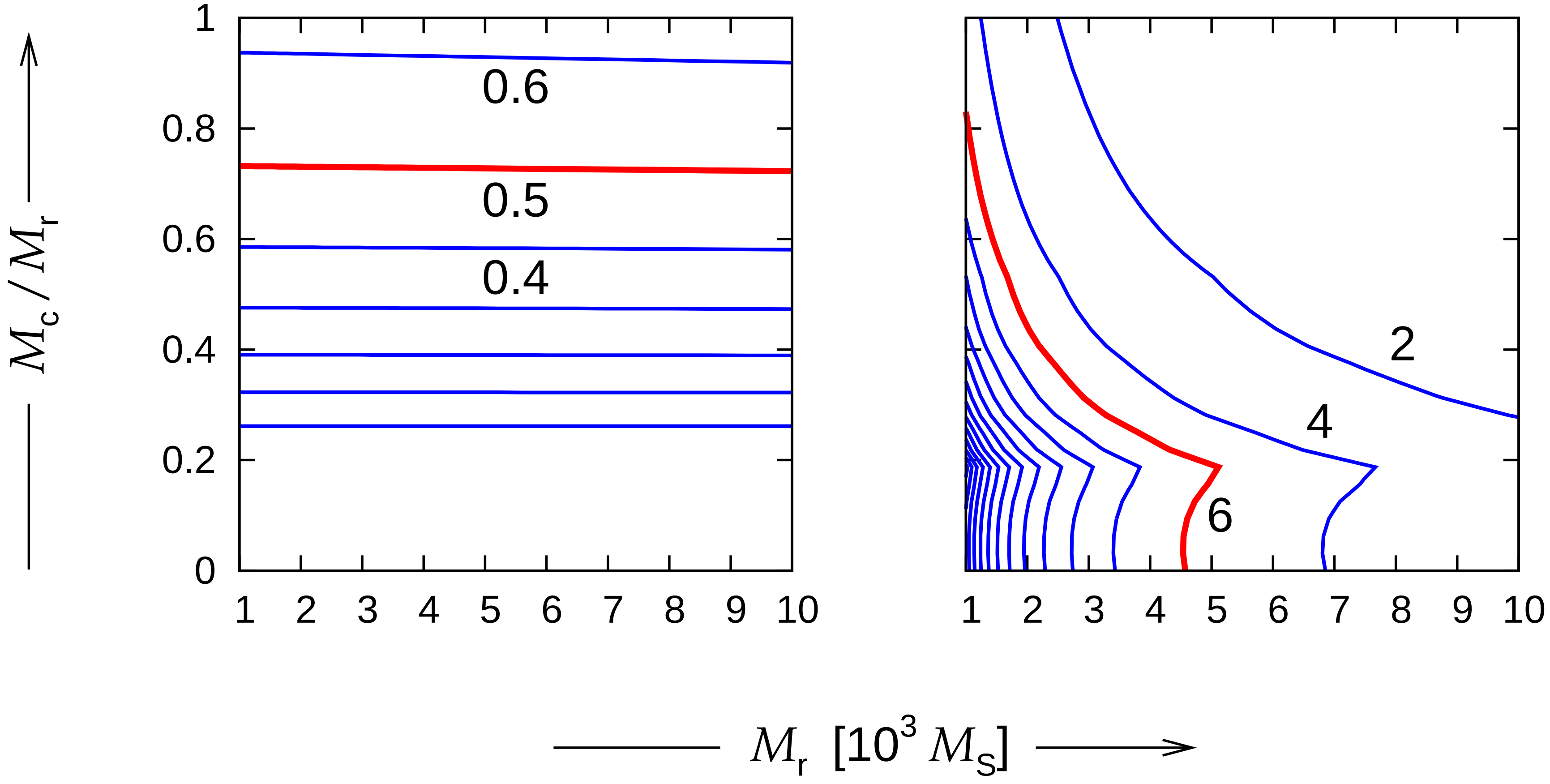
- At least one fulfilled $\forall a$
  $\Rightarrow$ whole system stable
- An example of unstable configuration:

$$\begin{aligned} M_{\rm r} &= 10^4\,M_\odot \\ M_{\rm c} &= 2\times10^3\,M_\odot \\ c_1 &= 0.2 \end{aligned}$$



# Stability as a function of masses

- Isocontours of $\min_{\forall a} e_{\text{stab}}$ and $\min_{\forall a} P_{\text{lib}}$ for $c_1 = 0.02$



# Results – two stability regions

- $M_\mathrm{c}/M_\mathrm{r} \lesssim 0.2$ and $M_\mathrm{r} \lesssim 4.5\times10^3\,M_\odot$
  - too slow oscillations
  - really possible only for small masses of the spherical cluster
  - compatible with Nayakshin et al. (2006): $M_\mathrm{r} \lesssim 5.5\times10^3\,M_\odot$

    $\Rightarrow$ Yoshihide Kozai in the Galactic Centre!
- $M_\mathrm{c}/M_\mathrm{r} \gtrsim 0.75$
  - a new region
  - too small oscillations
  - with the observed stellar density in the spherical cluster is the system stable for $M_\mathrm{r} \lesssim 6\times10^4\,M_\odot$!!

    $\Rightarrow$ very stabilizing influence

# Conclusions

- Used test particle method is very good
  - compatible results with $N$-body models (Nayakshin et al. 2006)
  - possible inclusion of more system parameters
- Presence of a star cluster in IRS 13E with total mass of order of magnitude $10^3\,M_\odot$ from the point of view of its influence on observed configuration can not be excluded
- No observational evidence (proper motion of Sgr A*, accretion) for black hole with mass $\gtrsim 10^4\,M_\odot$ in the centre of IRS 13E exists
- Inside out?
  - the same method
  - evaluating influence of the clockwise disc on an orbiting star cluster

    $\Rightarrow$ counter-clockwise disc, IRS 13E and “diagonal feature”?