Individual Orbits in Large N-Body Simulations

Does chaotic behaviour increase/decrease with time?

N=1000 particles

Total Integration Time: 512 N-body time units

$t_{rel} \approx 37$

QuickTime™ and a
GIF decompressor
are needed to see this picture.

20 (N-body length units)

Equal mass Plummer sphere: $\rho(\mathbf{r},0)=3/4\pi M R^{-3}[1+(r/R)^2]^{-5/2}$

Goal: We propose to track each stellar orbit individually during the simulation and to give an estimate of its chaoticity for each instant t.

Question: Initial conditions being provided, at what time of the integration do we encounter a maximum/minimum of irregular orbits?

Overview

- Scope of the Analysis
- Method: How to analyse a given N-body system?
- The Method at Work: From small N systems to large N-body simulations
- Conclusion and Outlook

Scope of the Analysis

- Definitions and Assumptions:
  - Sensitivity to Initial Conditions; Any system with N>2 is chaotic (Miller 1964)
  - We discuss **physical** chaos ($\neq$ numerical). Responsible mechanism: ‘cumulative effect of the interaction of near neighbours’ (Goodman et al 1993). Regular motion vs irregular motion due to close encounters.
- We aim to understand:
  - The global orbital content of a given N-body system and its evolution with respect to time, N and the IMF.
  - BH formation: How do orbits of massive stars evolve in dense stellar clusters? In how far will a BH influence the orbital behaviour in its immediate surroundings?
  - Core Collapse?
  - High-velocity X-ray sources

Method: How to analyse an N-body system?

- Two independent methods of time series analysis.
- Method 1: Continuous Wavelet Transform Information Entropy (CWaTIE)
  - CWaTIE ∝ "State of disorder of the system"
  - "The amount of information required per time unit to specify the state of the system"
  - CWaT formalism:

$$T_{\Psi}(a,\tau) = a^{-0.5} \int_{-\infty}^{+\infty} f(t_i)\, \overline{\Psi}_m\!\left(\frac{t-\tau}{a}\right) dt$$

$$where \quad \Psi_m(t) = \frac{d^m}{dt^m} e^{\frac{-t^2}{2}} \quad (DOG\ wavelet)$$

--> CWaTIE: Shannon 1948, Quiroga et al 1999

Method (cont'd)

- Method 2: Second Order Fractal Correlation Dimension (CorrDi).
  - Correlation Integral technique (Grassberger and Procaccia 1983)

$$C(\varepsilon) = 1/P \sum_{i,\, j=1,\, j>i}^{P} \Theta\left(\varepsilon - \left|\vec{X}_i - \vec{X}_j\right|\right) \propto \varepsilon^{D_2}$$

  - Kaplan-Yorke conjecture (Kaplan and Yorke 1979):

$$D_2 \leq D_{KY} = j + \frac{\Sigma_{i=1}^{j} \lambda_i}{\left|\lambda_{j+1}\right|}$$

with $\lambda_1 > \lambda_2 > \ldots > \lambda_k$ the ordered spectrum of Lyapounov exponents and j the largest integer such that $\Sigma\lambda_i > 0$

The Method at Work: From small N systems to large N-body simulations

- N=2. Unperturbed binary motion.
- N=3. The Pythagorean problem. Planar configuration consisting of 3 particles at rest placed at the vertices of a Pythagorean triangle.
- N=7. The outer asteroid belt object 522 Helga. Stable chaos. (Milani et al 1997)
- Towards large N models…?

Unperturbed Binary

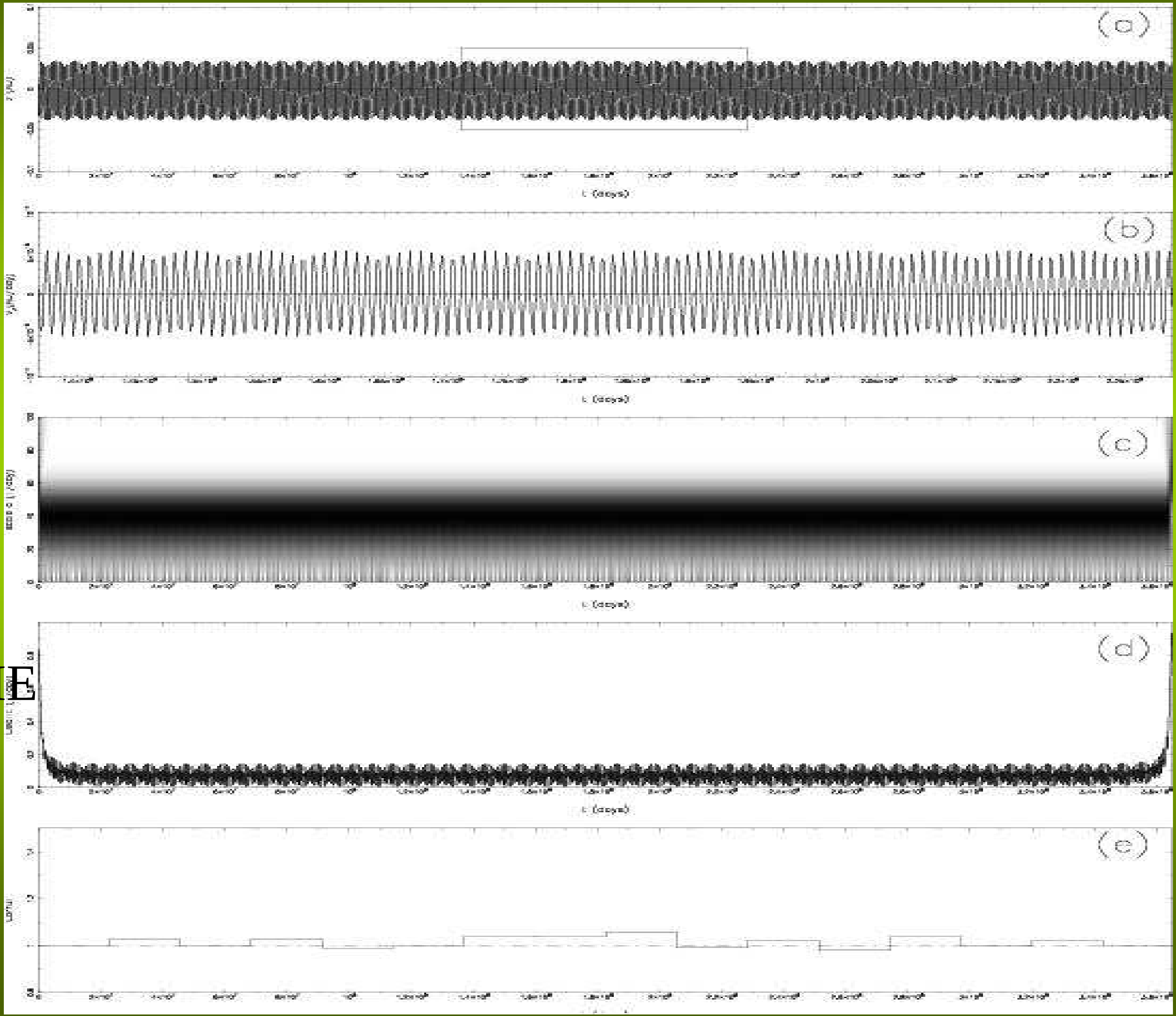
z

v_z (zoom)

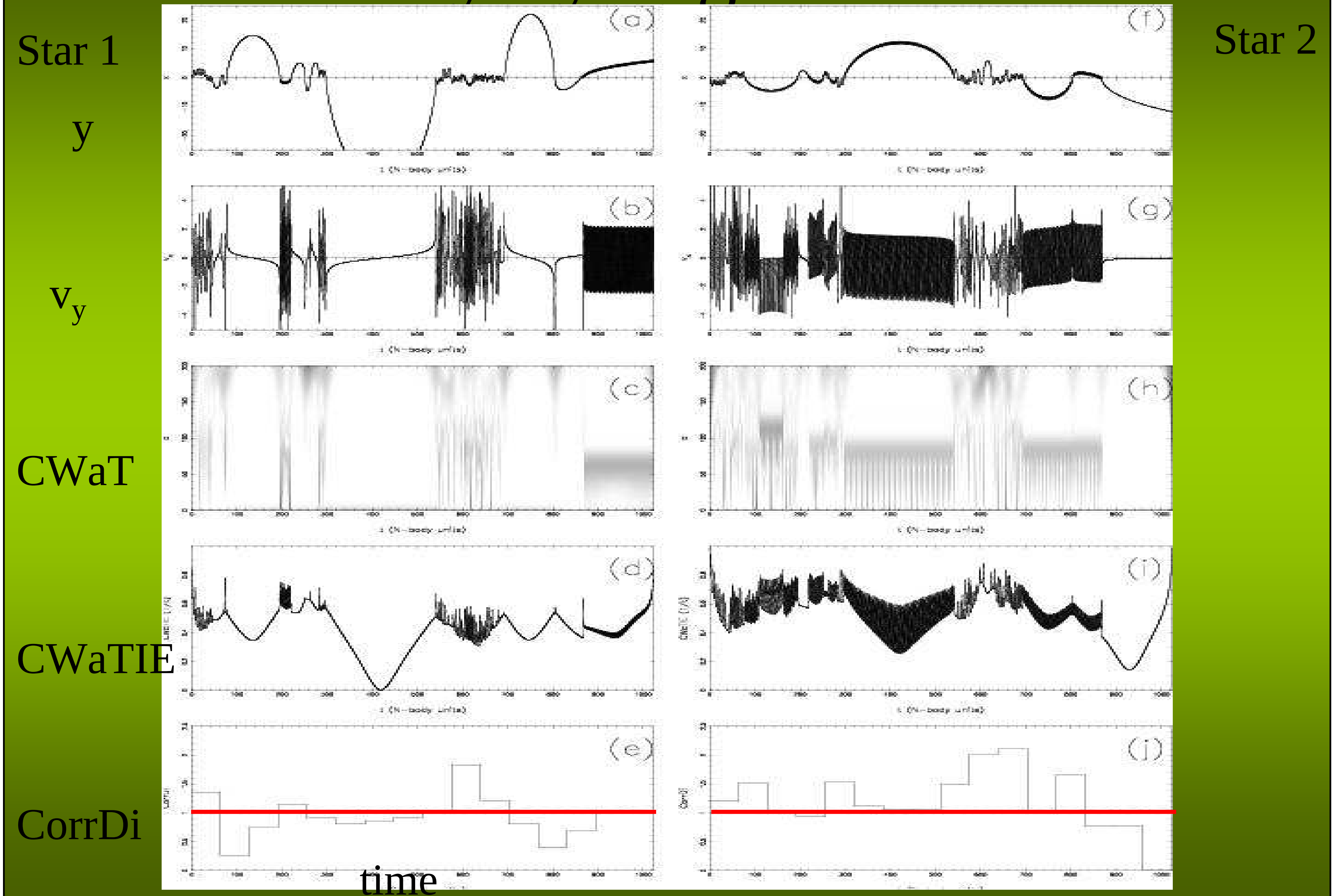
CWaT

CWaTIE

CorrDi



Three-Body Pythagorean Problem



The asteroid 522 Helga (7 bodies)

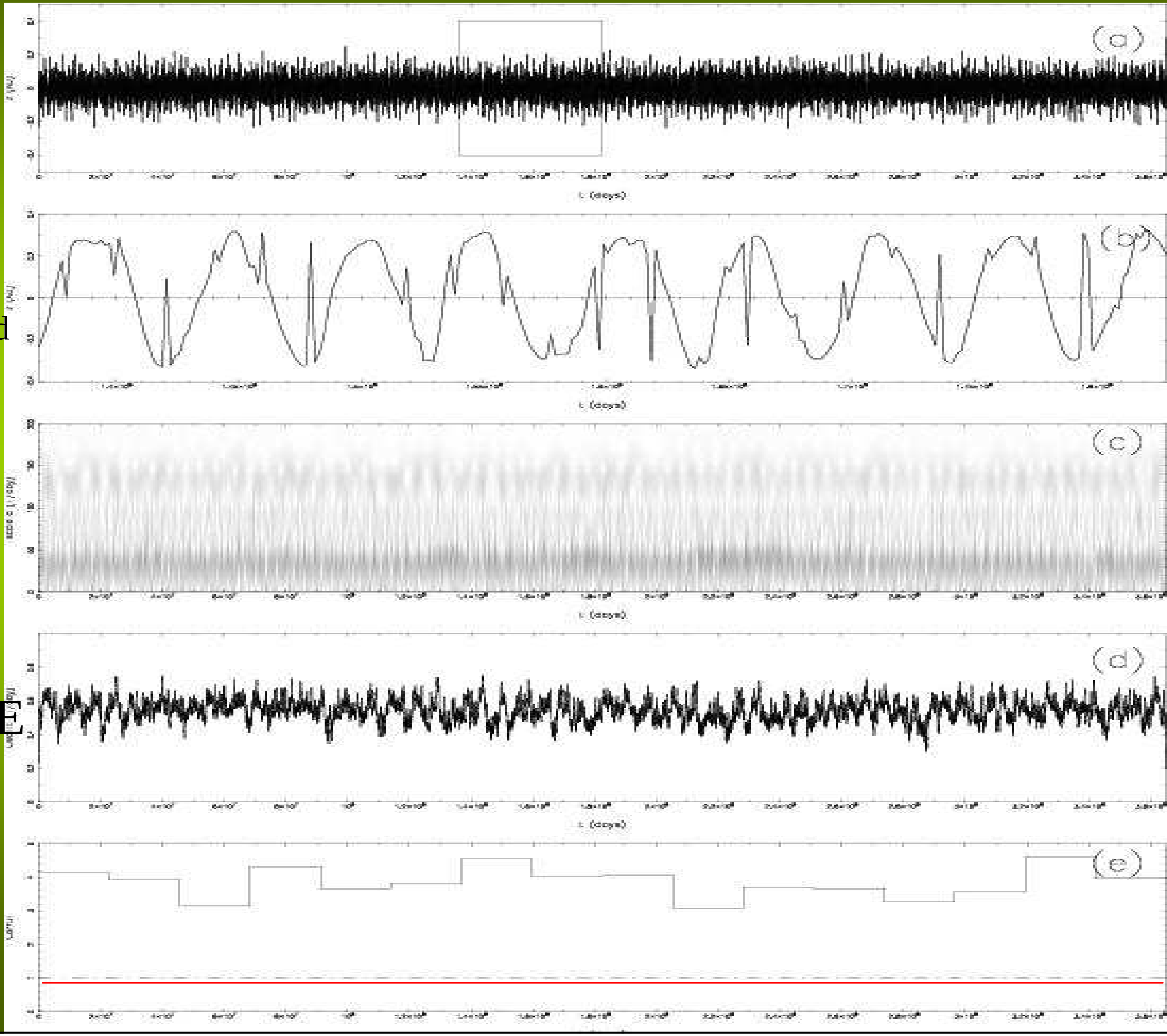
z

$V_{z\ zoomed}$

CWaT

CWaTIE

CorrDi

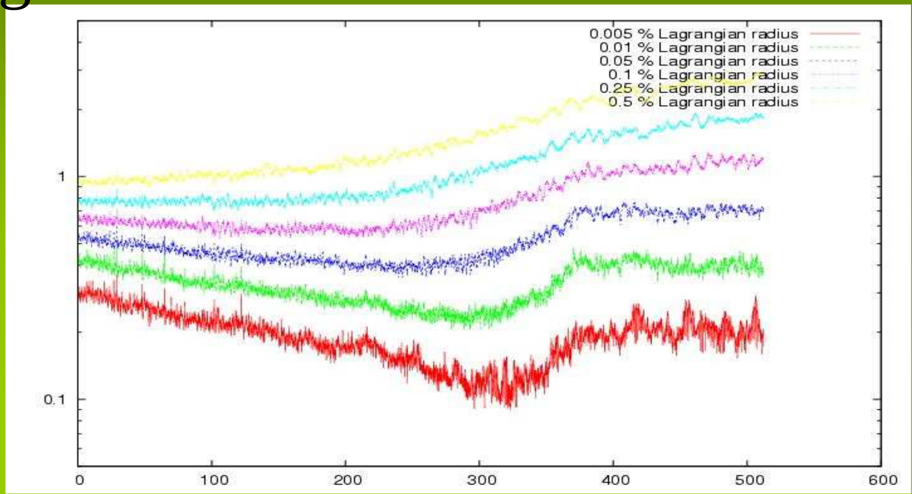


Conclusion

- We developed a tool for estimating the chaoticity of the orbital content of a given N-body simulation with respect to integration time.
- Small N problems are accurately analyzed. Ultmatively the method shall be applied to large stellar cluster simulations.
- Work still in progress!

Outlook: What we want do to in the end…

Fig. 1



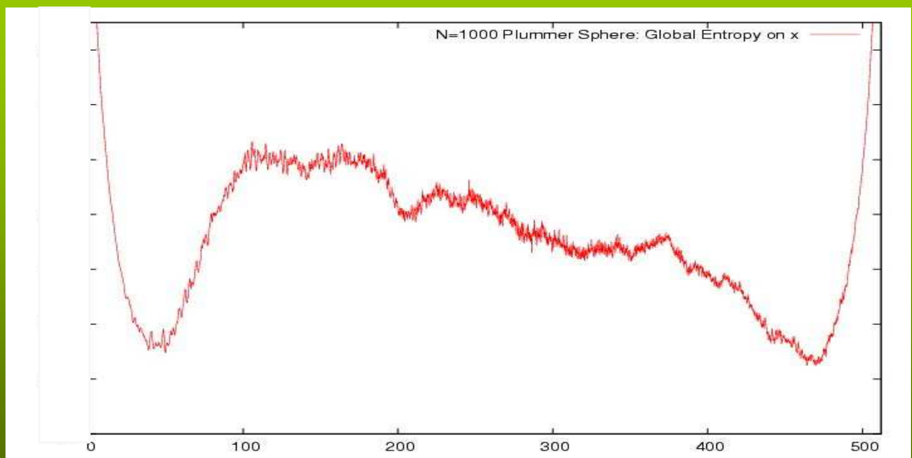


Fig. 2

- N=1000 Equal Mass Plummer Sphere
- Figure 1: Lagrangian radii vs time
- Figure 2: CWaTIE
- Is it too early to conclude? Redo the exercise only for bound particles? Or with resolved time steps for central particles?

Thank You!