ASTRONOMISCHES RECHEN-INSTITUT
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Chaos and Escape within the Tidal Approximation

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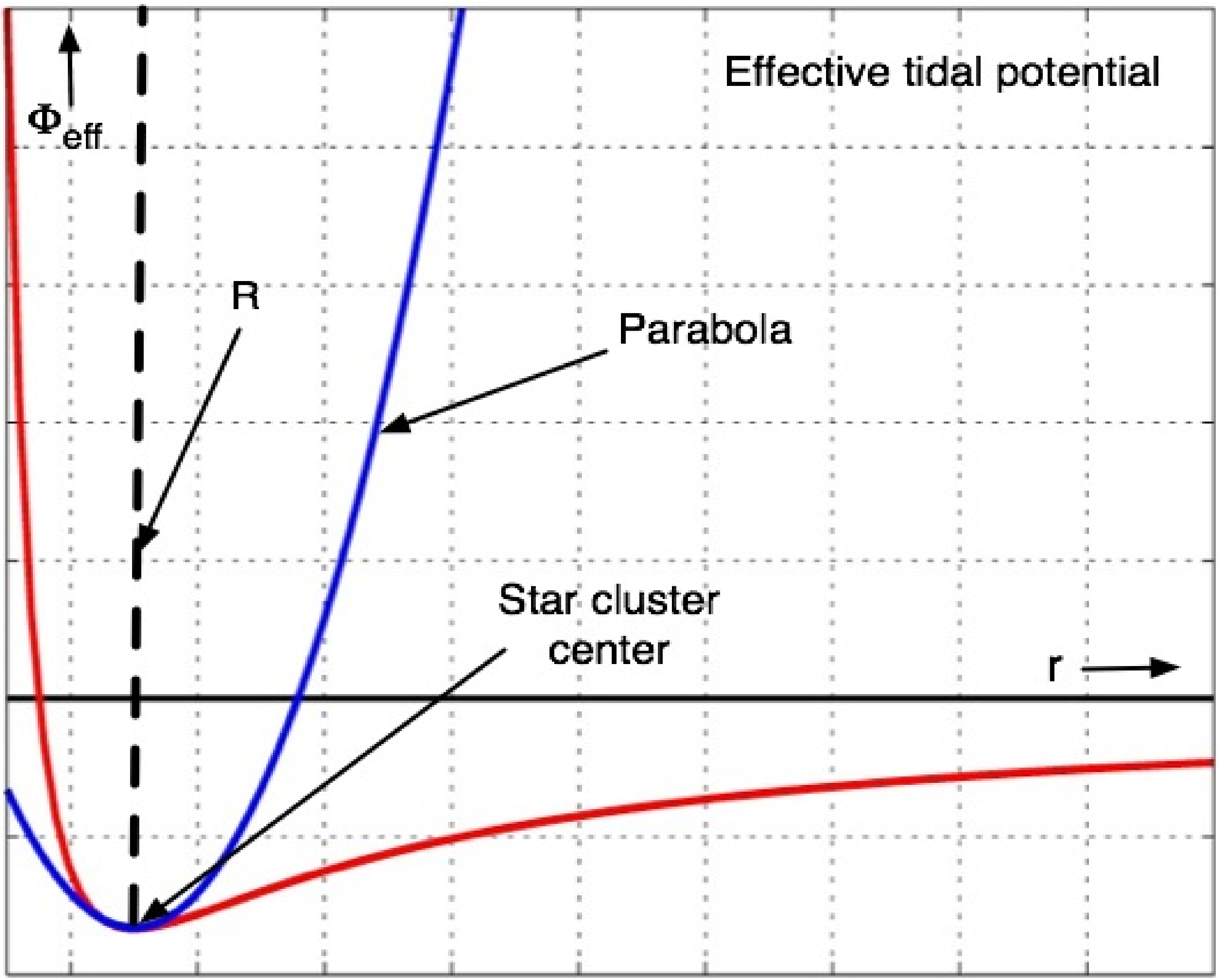
Epicyclic approximation of tidal potential



The „Tidal approximation“

$$\Phi_{eff} \simeq \Phi_{cl}(x, y, z) + \frac{1}{2}\mu^2 x^2 + \frac{1}{2}\nu^2 z^2 + const$$

Effective potential

$$\vec{\omega}_0 = (0, 0, \omega_0)$$
$$\kappa^2 = -4B(A-B)$$
$$\mu^2 = \kappa^2 - 4\omega_0^2 = -4A(A-B)$$
$$\nu^2 = 4\pi G\rho_g + 2(A^2 - B^2)$$

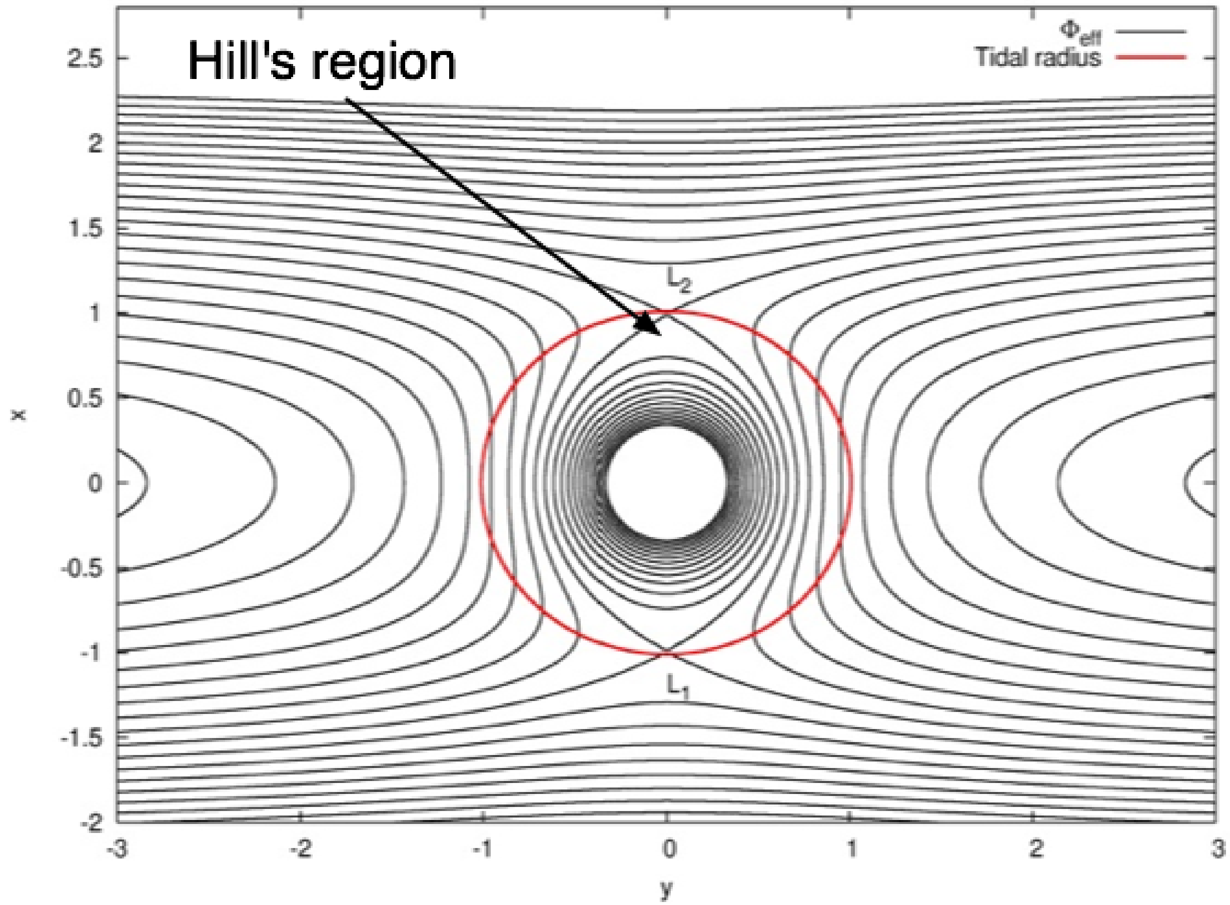
A,B: Oort's constants
κ,ν: Epicyclic frequencies
ρ_g: Local Galactic density

$$\dot{\vec{x}} = -\nabla\Phi_{eff} - 2(\vec{\omega}_0 \times \dot{\vec{x}})$$

$$\ddot{x} = f_x + 2(A-B)\dot{y} + 4A(A-B)x$$
$$\ddot{y} = f_y - 2(A-B)\dot{x}$$
$$\ddot{z} = f_z - [4\pi G\rho_g + 2(A^2 - B^2)]z$$

Equations of motion in the rotating frame

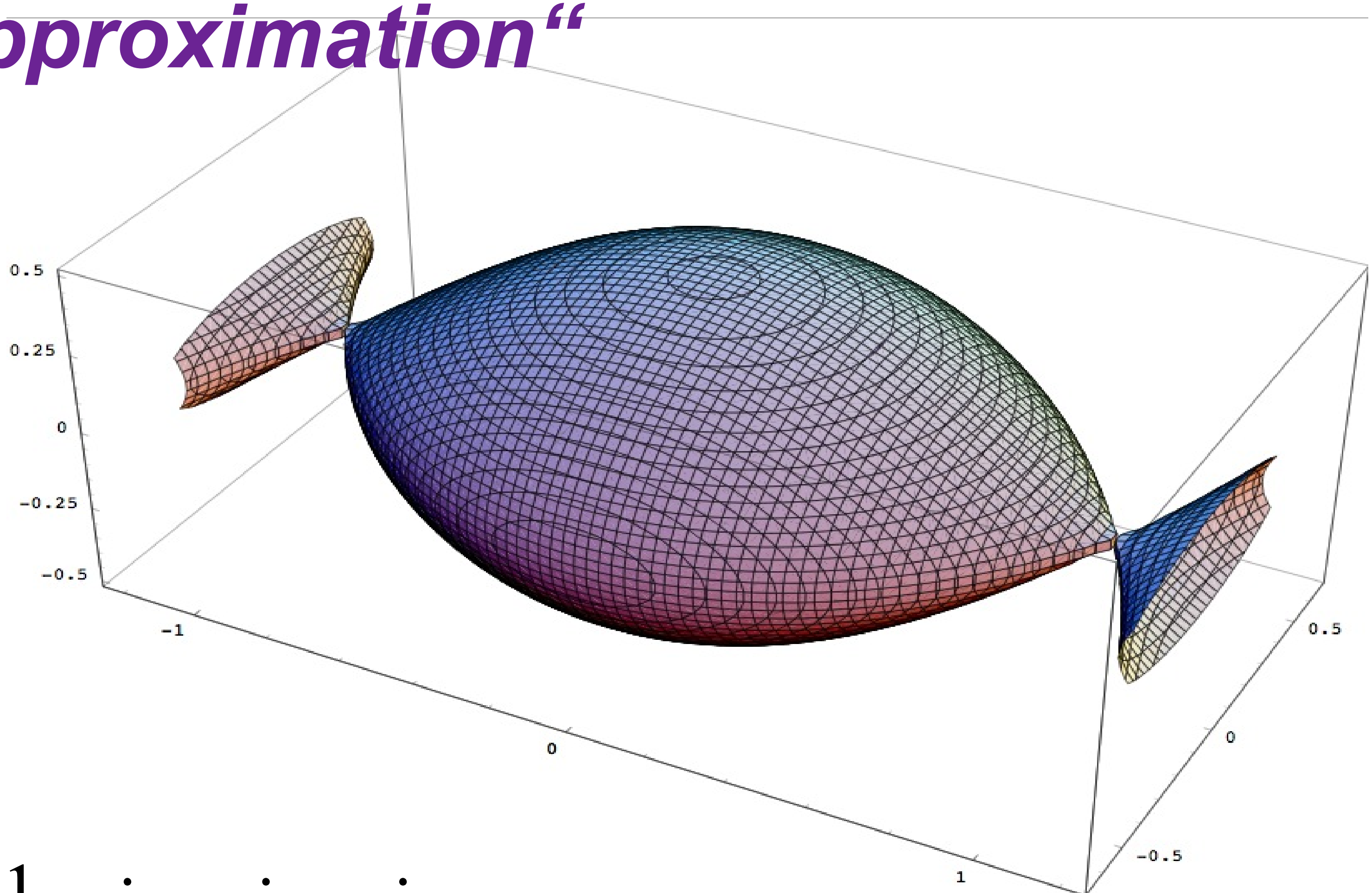
Equipotential lines of Φ_{eff} (z=0 plane)



$$r_t = \left[\frac{GM}{4A(A-B)} \right]^{1/3} = R \left(\frac{M}{3M_g} \right)^{1/3}$$

Tidal radius (King 1962)

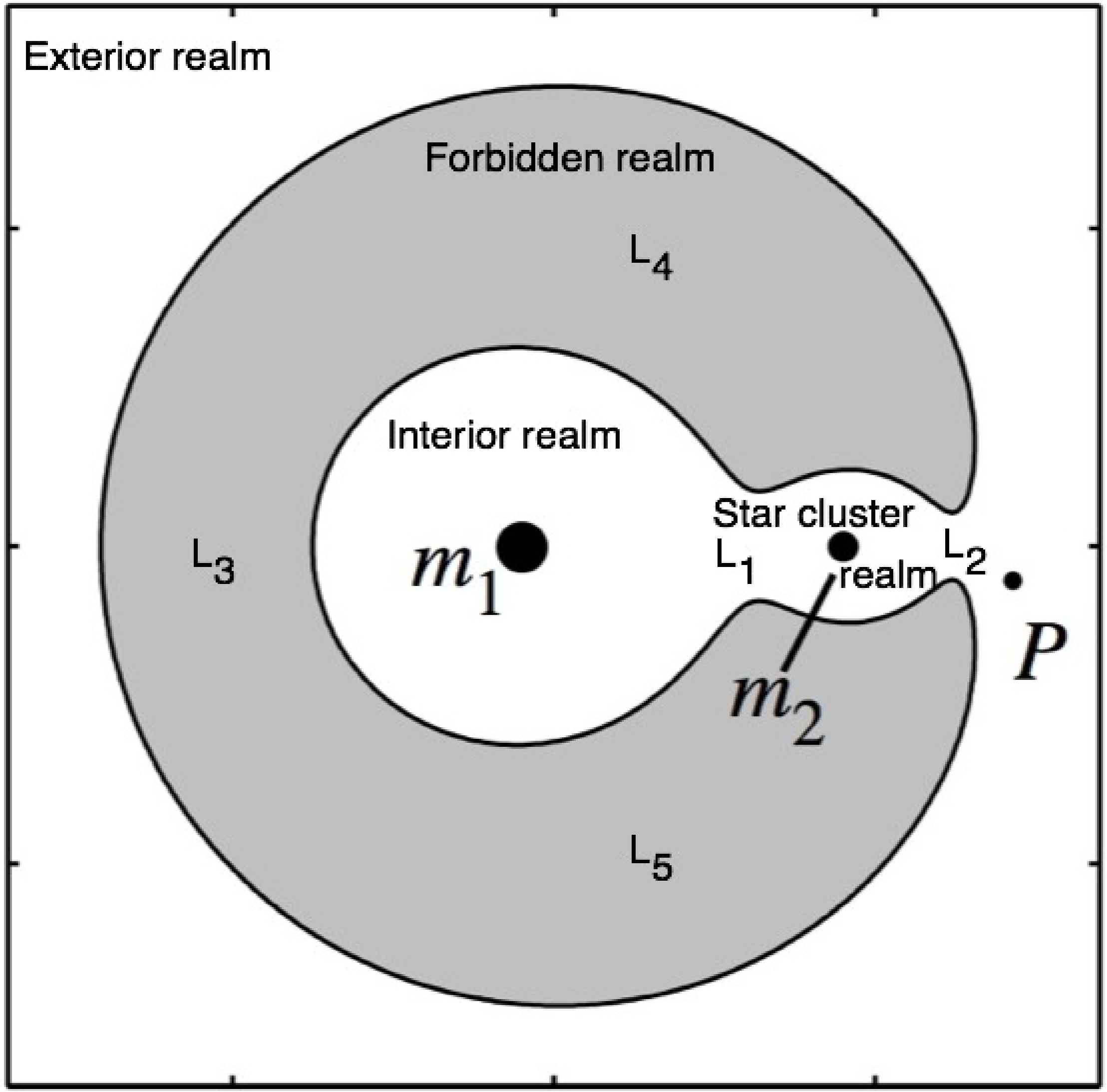
The tidal boundary within the „Tidal Approximation"



$$C=\frac{1}{2}(\dot{x}^2+\dot{y}^2+\dot{z}^2)+\Phi_{eff}$$ Jacobi's integral

$$C_L=-\frac{3GM}{2r_t}$$ Critical Jacobi constant (Wielen 1972)

The global picture: Realms in the restricted 3-body problem



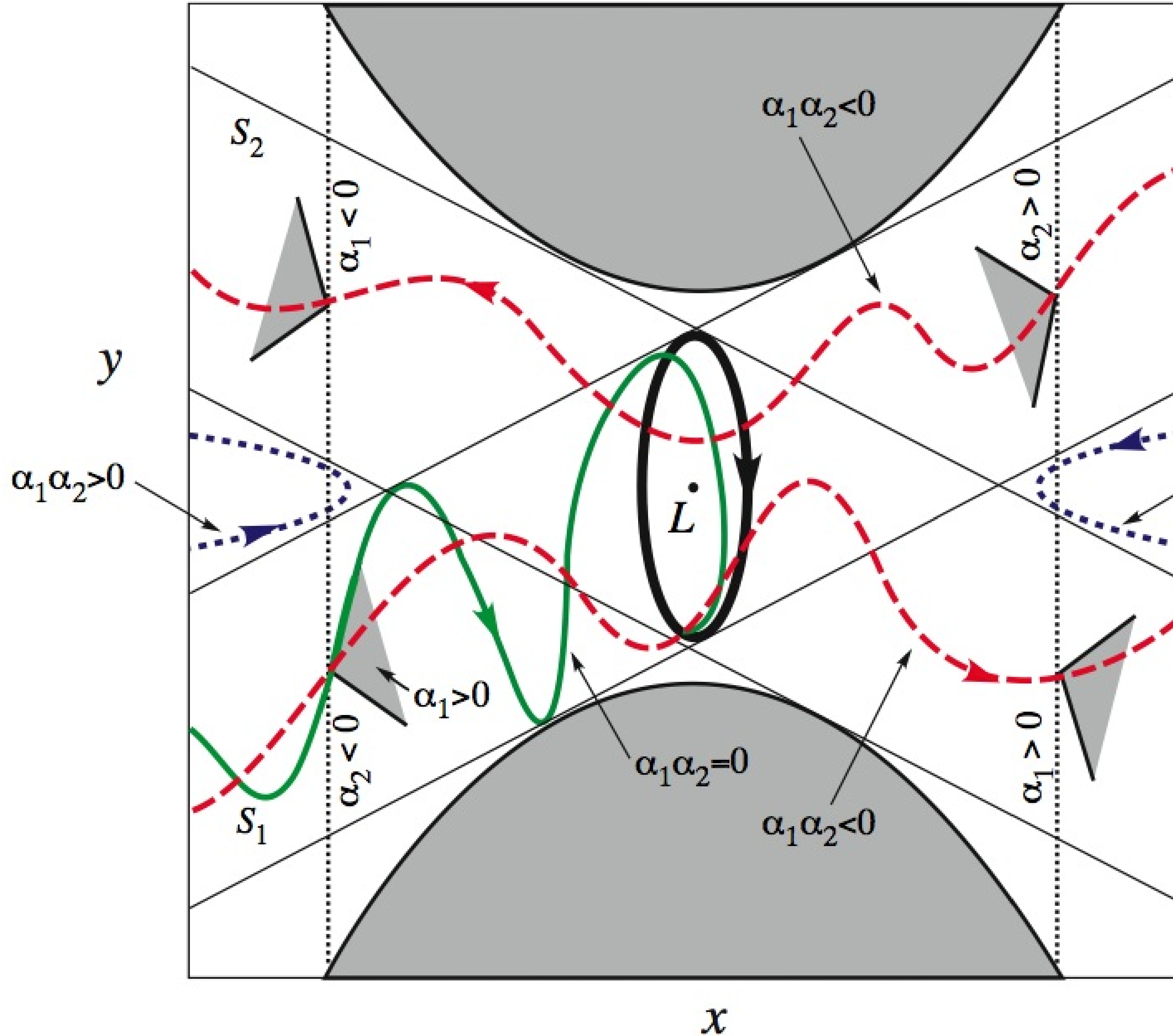
m_1: Galaxy mass

m_2: Star cluster mass

P: Location of a star

$$C > C_L$$

Orbits in the neck region



Lyapunov orbit

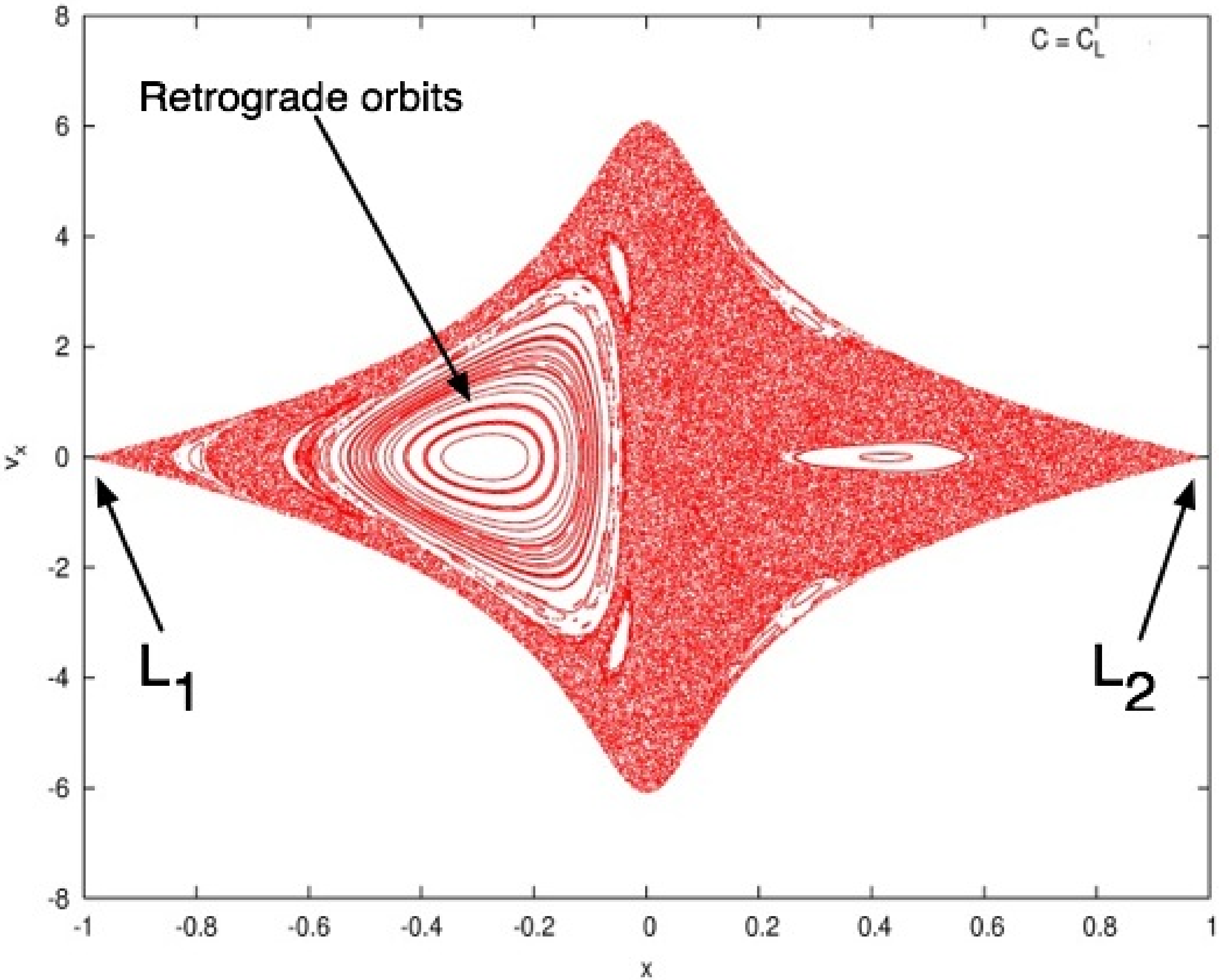
Asymptotic orbits

Transit orbits

Non-transit orbits

Conley (1968)
Ross (2004)

Poincaré section of orbits at $C = C_L$



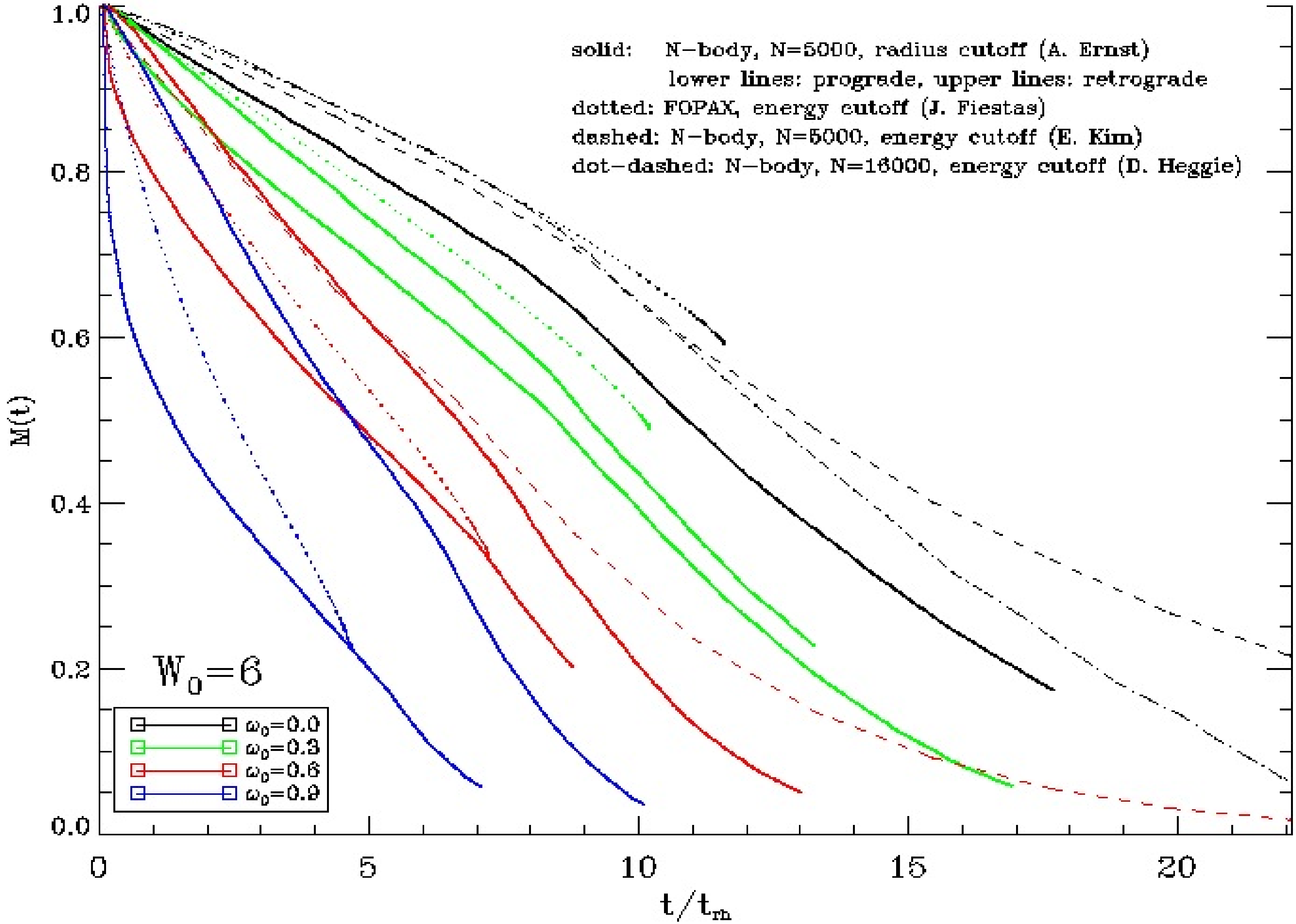
At the moment of crossing $y = 0$ with $v_y > 0$

Particles in retrograde „quasiperiodic“ orbits can cannot reach L_1 / L_2!

- The full range of initial conditions is covered.
- For some regions a „third integral“ exists.

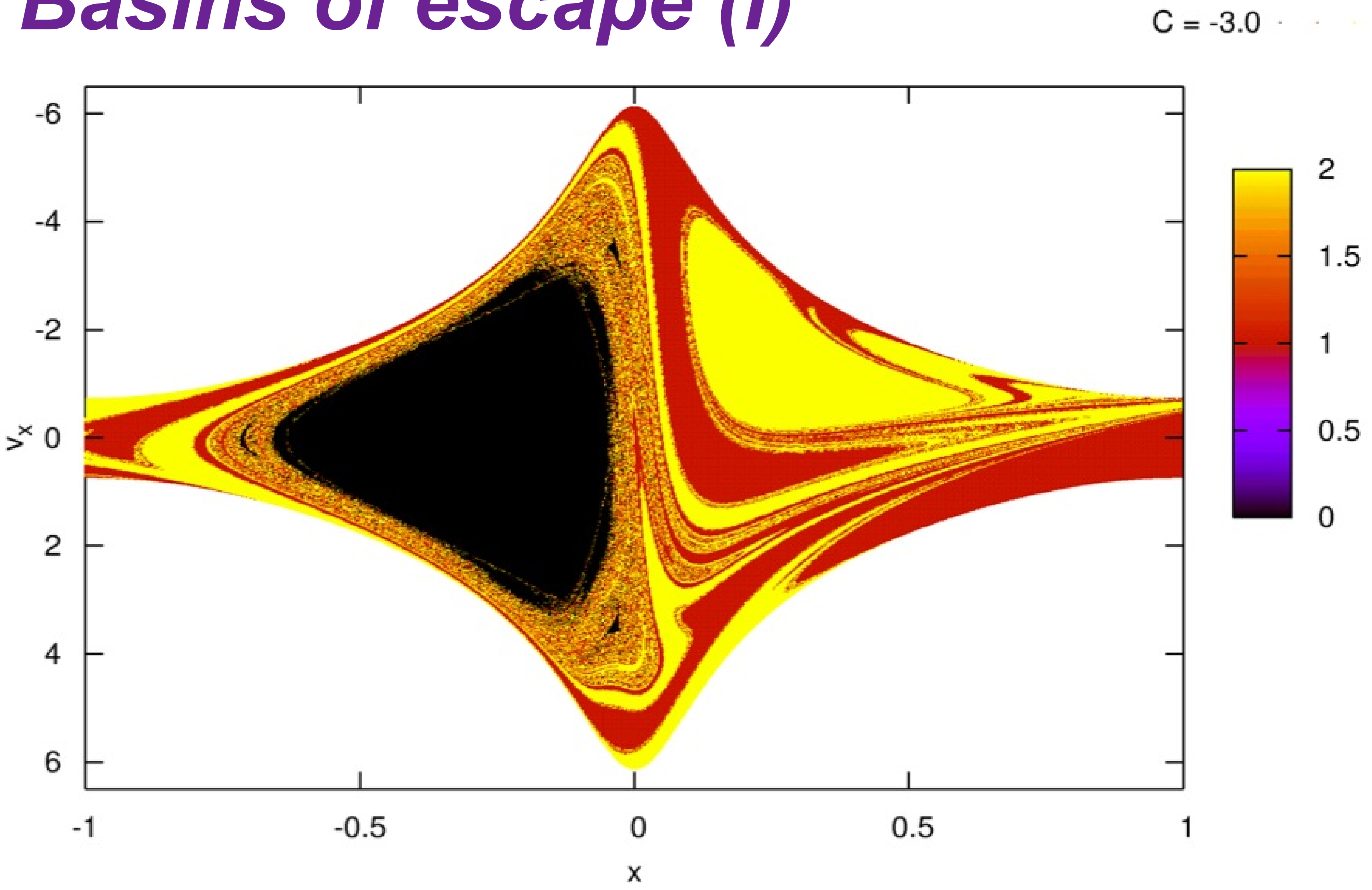
Fukushige & Heggie (2000)

Prograde and retrograde rotating star clusters



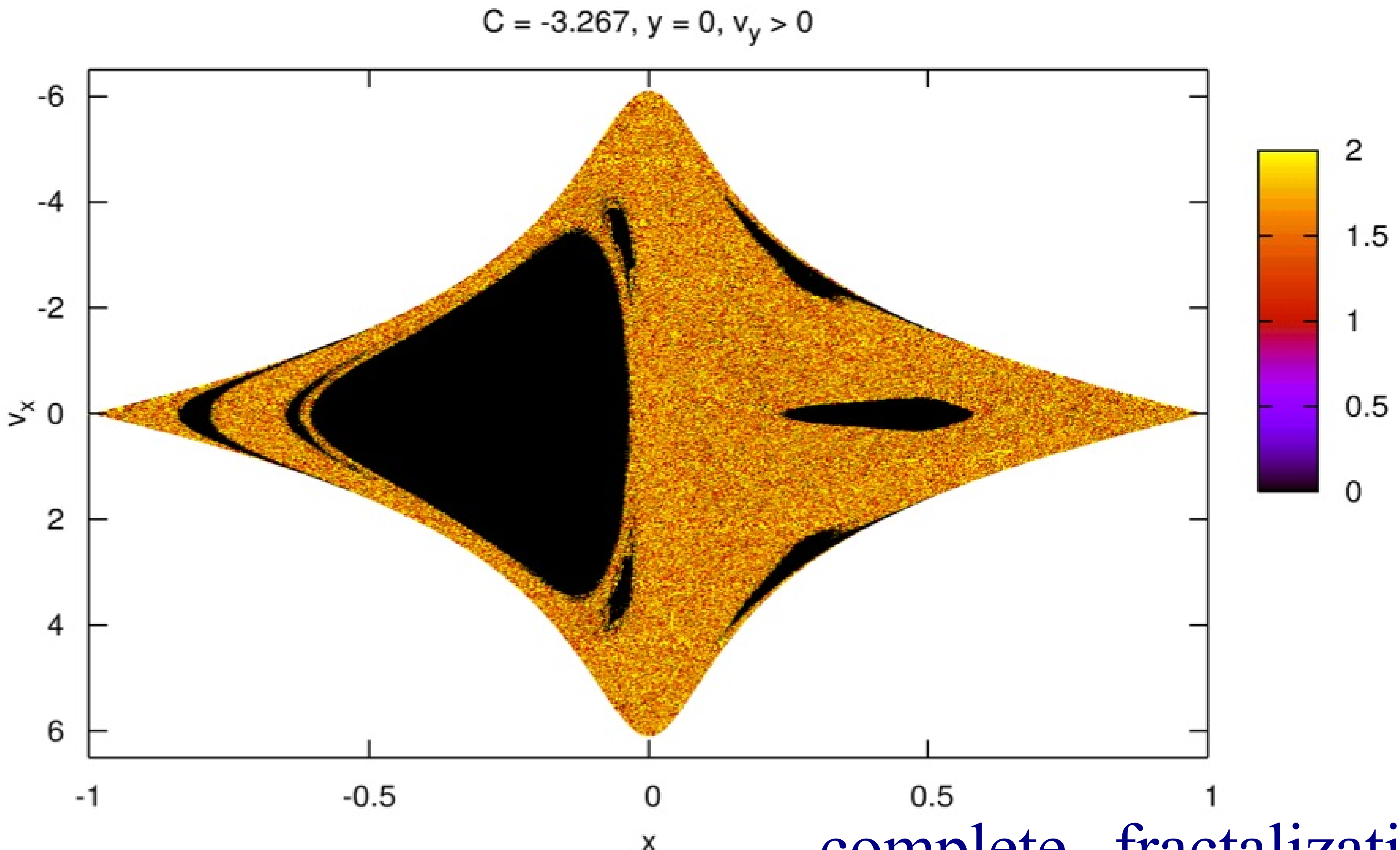
A. Ernst, P. Glaschke, J. Fiestas, A. Just, R. Spurzem,
submitted to MNRAS (2006)

Basins of escape (I)



$C_L = -3.267567369$ ...ordered regions exist...

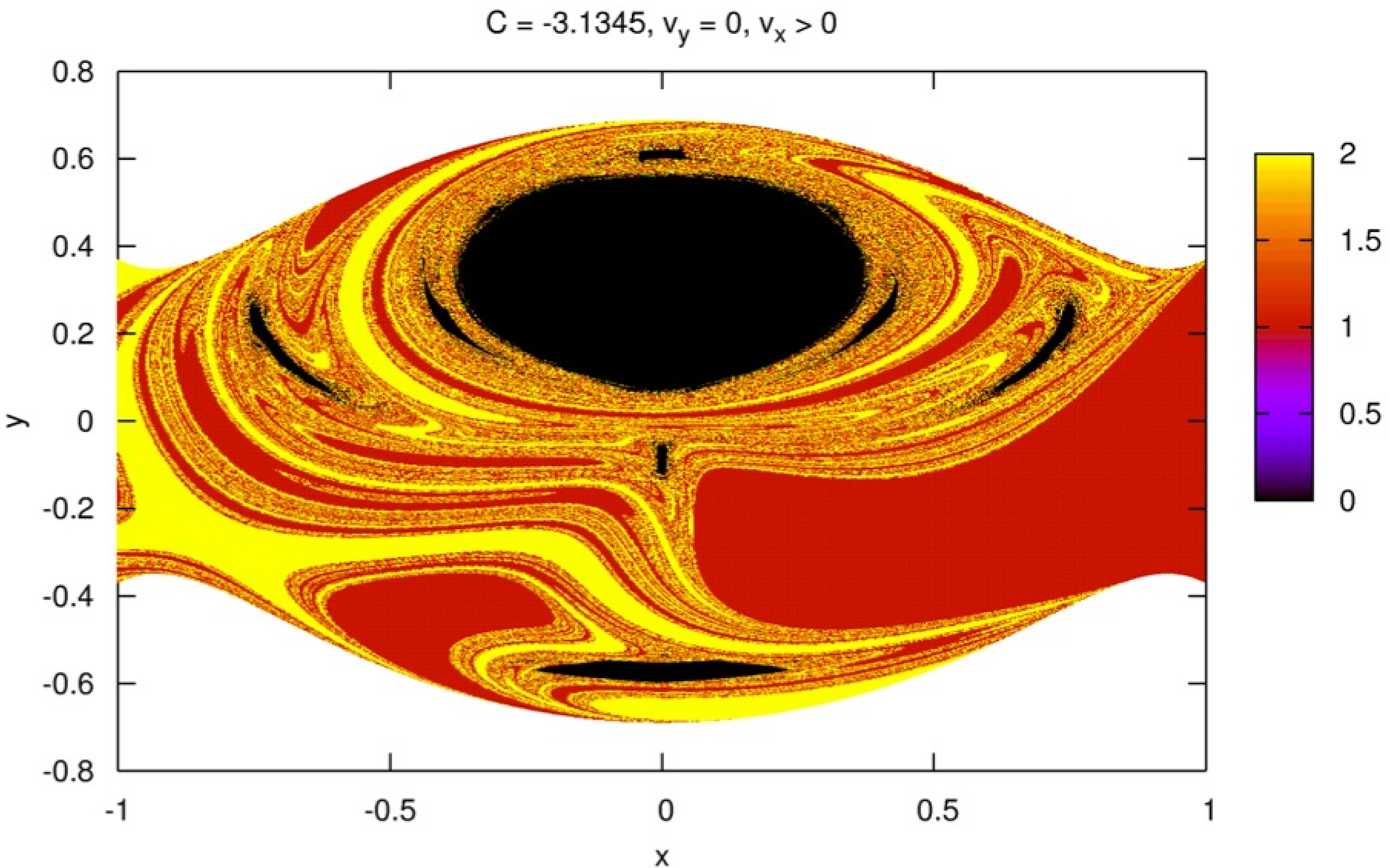
Basins of escape (II)



$C_L = -3.267567369$

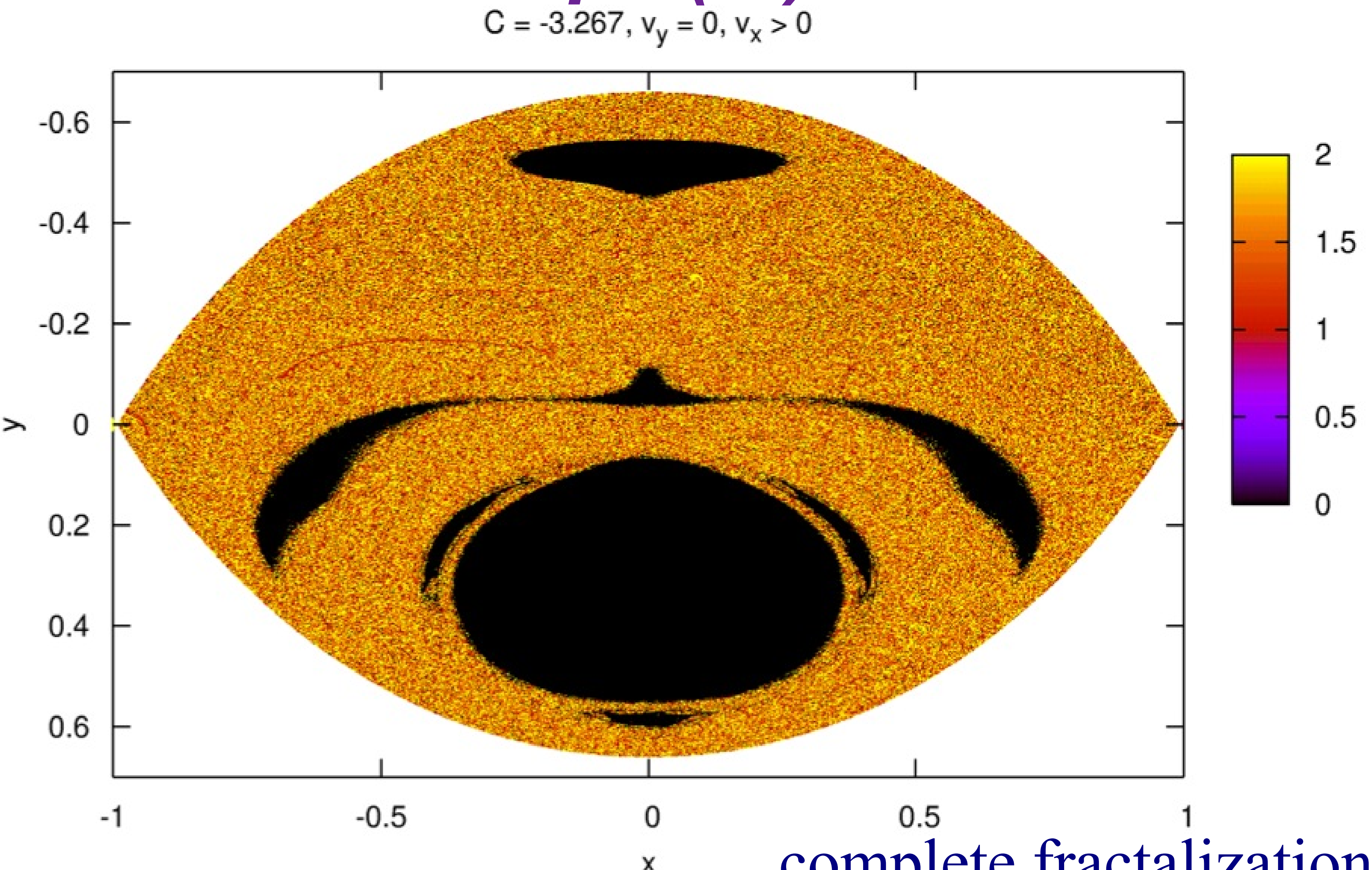
...complete „fractalization“ of phase space...

Basins of Escape (III)



$C_L = -3.267567369$ ...ordered regions exist...

Basins of Escape (IV)



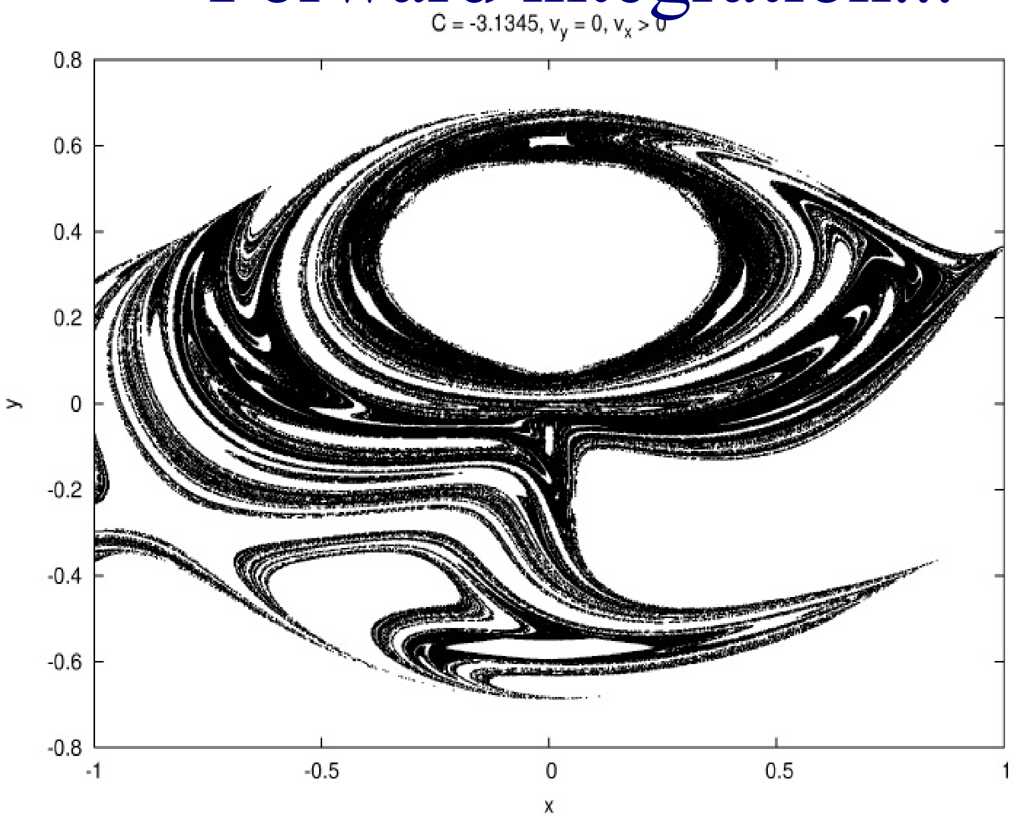
$C_L = -3.267567369$

...complete fractalization of phase space...

Stable and unstable manifolds of the chaotic invariant set (I)

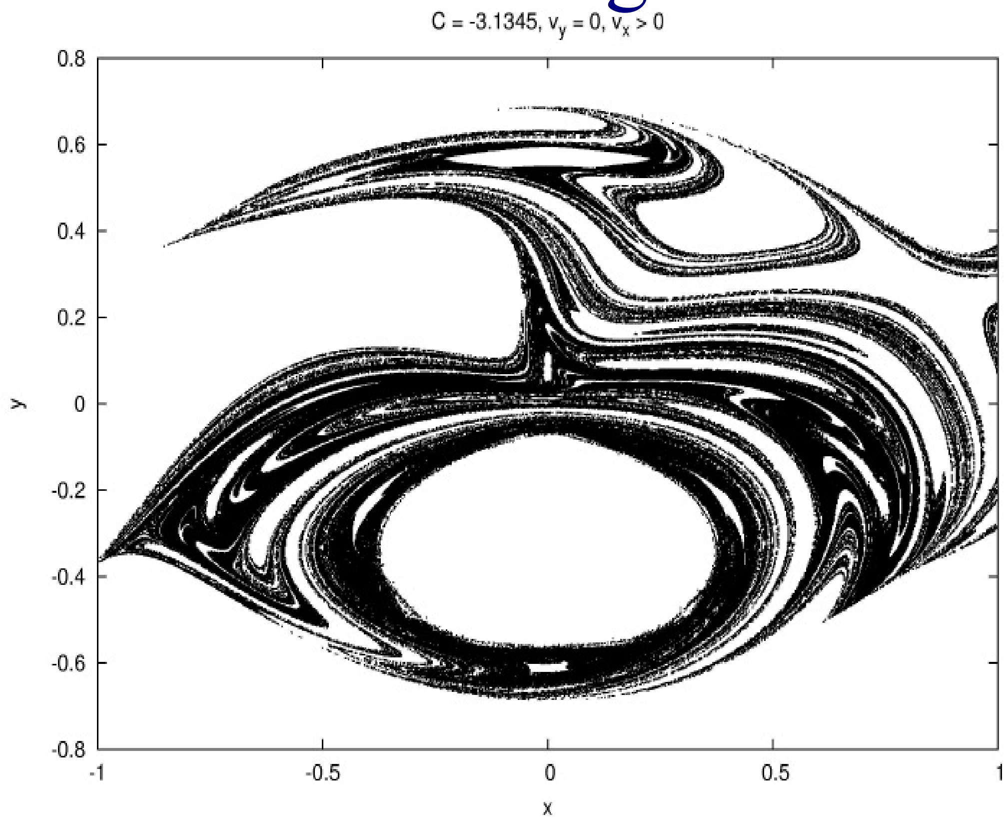
The fractal basin boundaries:

Forward integration...



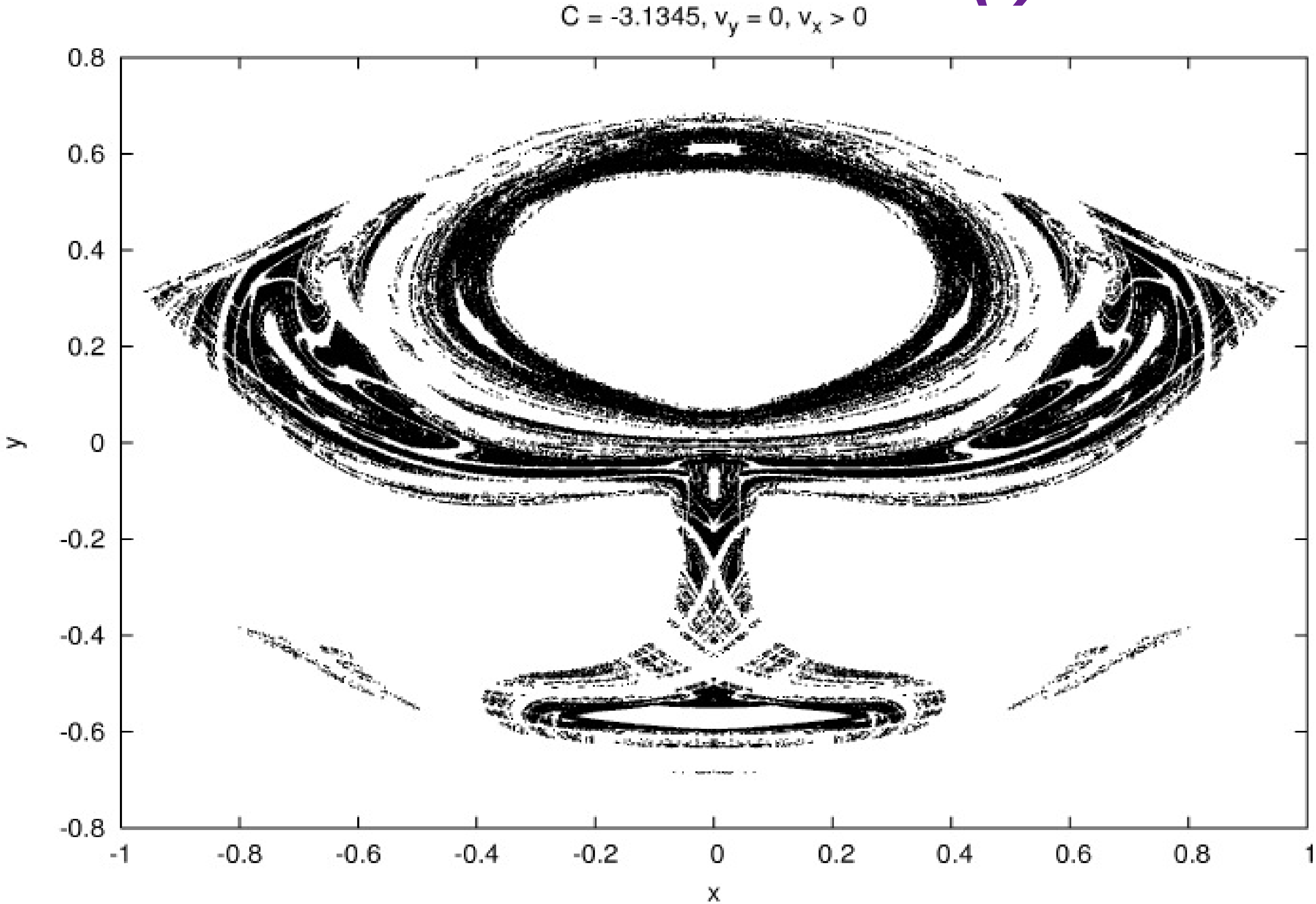
Stable manifold...

Backward integration...



Unstable manifold...

The chaotic invariant set (I)



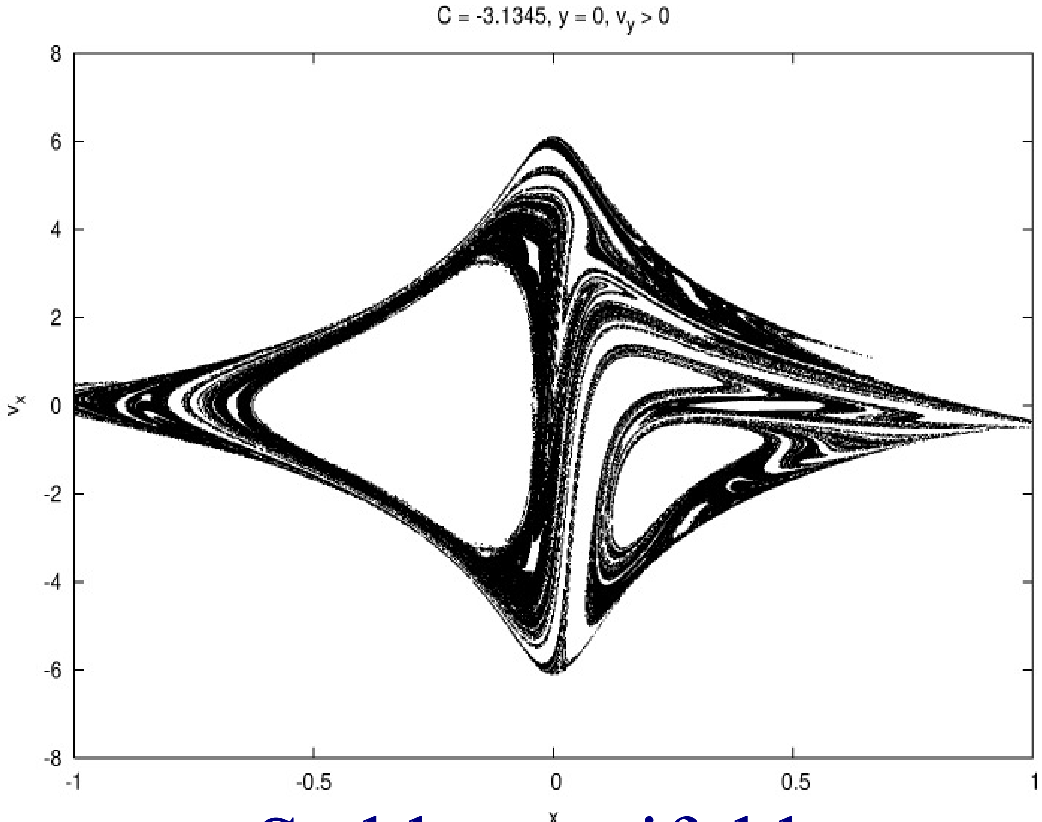
$Cantor\ set\ of\ non-escaping\ chaotic\ orbits$

$for\quad t\to\infty$ and $t\to-\infty$

Stable and unstable manifolds of the chaotic invariant set (II)

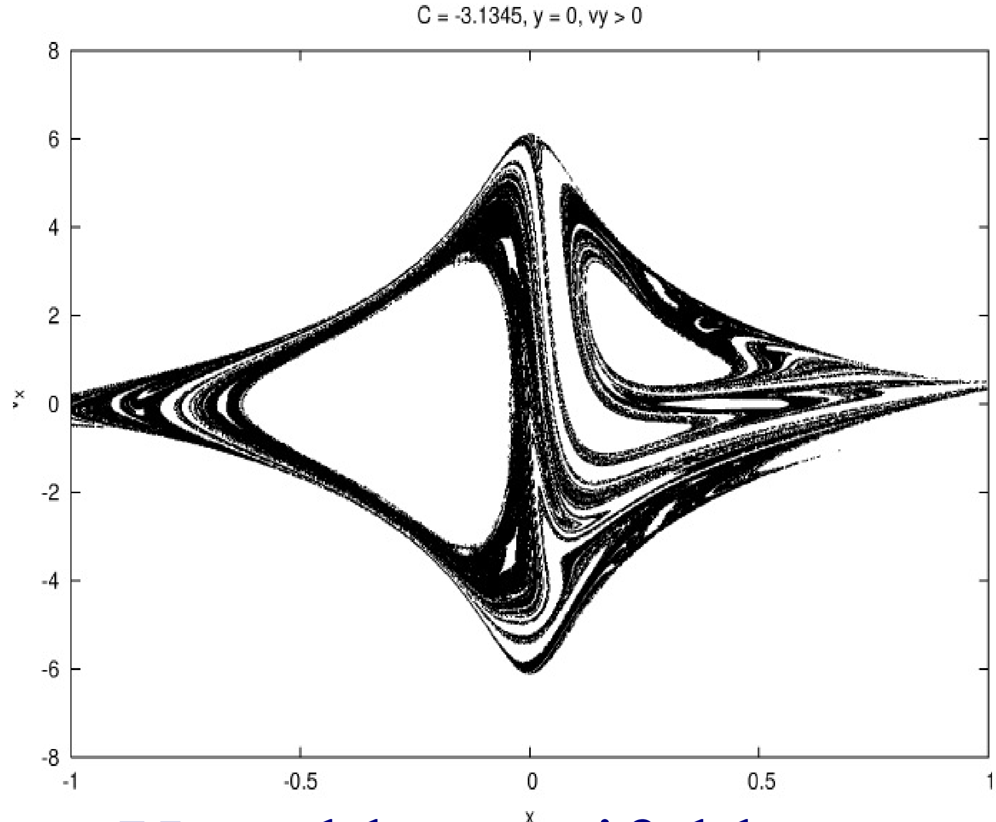
The fractal basin boundaries:

Forward integration...



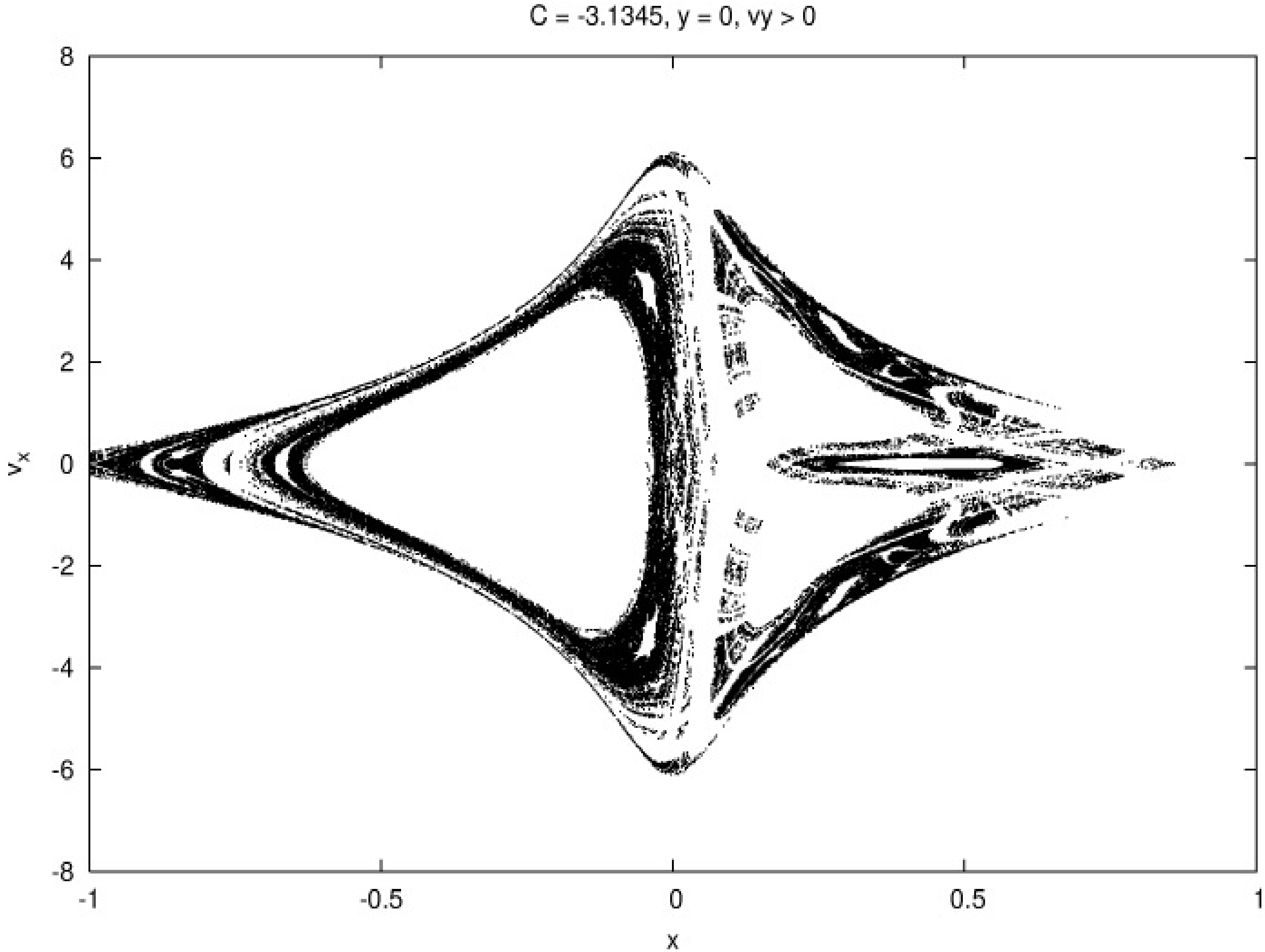
Stable manifold...

Backward integration...



Unstable manifold...

The chaotic invariant set (II)



Cantor set of non−escaping chaotic orbits

for $t \to \infty$ and $t \to -\infty$

Conclusions

- The escape process in tidally limited star clusters is chaotic!
- The non-escaping orbits are subject to a „third integral“ (i.e. quasiperiodic) or lie on the fractal basin boundaries (i.e. chaotic)!
- At the critical Jacobi constant the chaos is maximal and the numerical orbit integration erroneous!

Thank you for your attention!