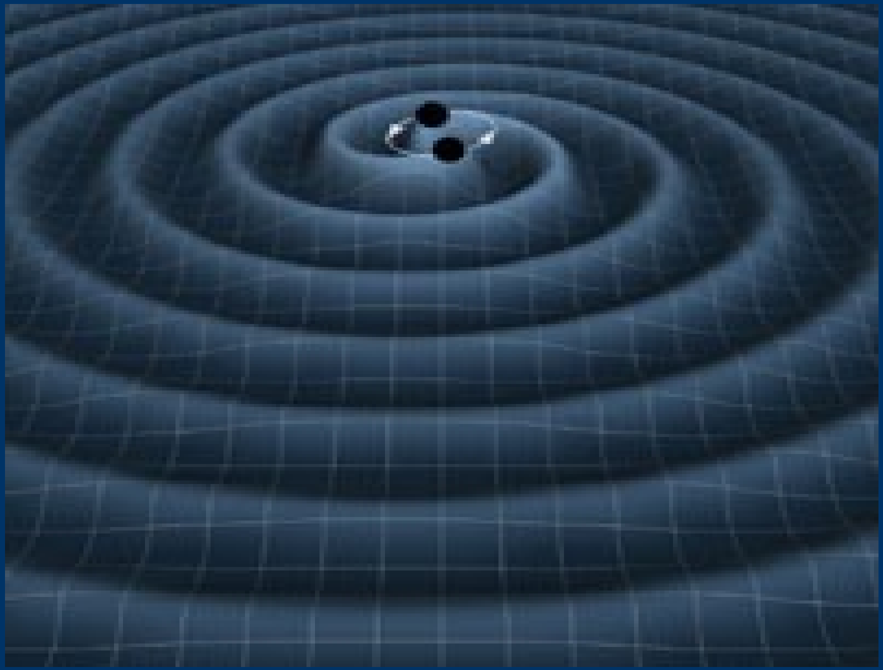
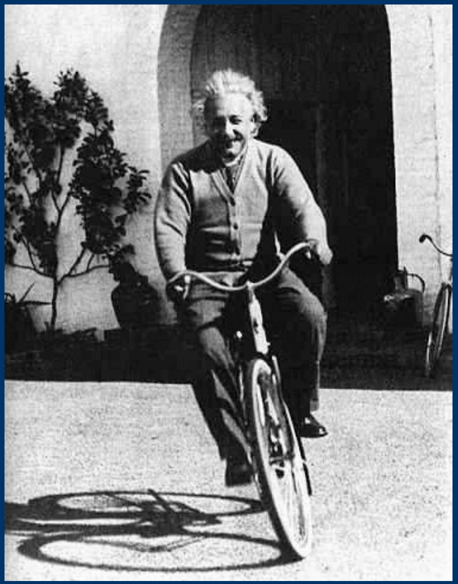
Post-Newtonian Dynamics: Numerical Studies of Binary Black Holes

Ingo Berentzen – Astronomisches Rechen-Institut @ ZAH Heidelberg





Collaborators

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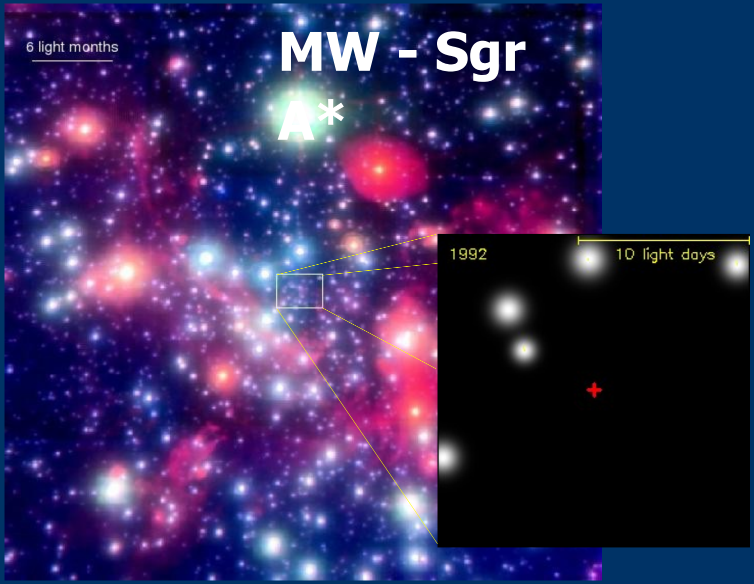
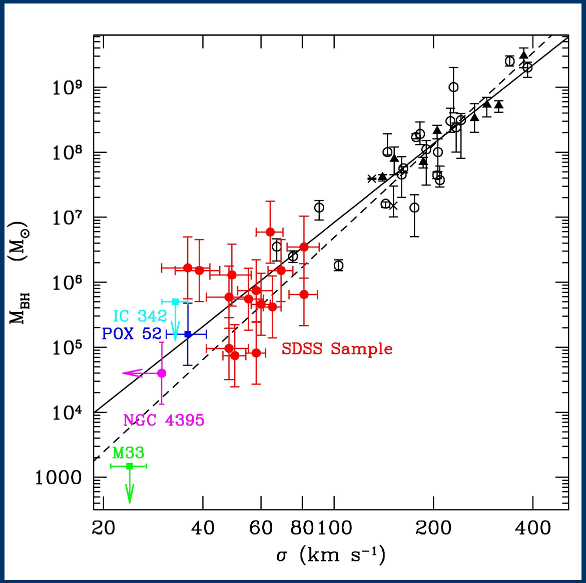
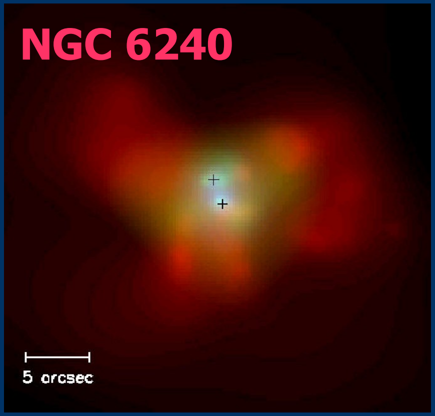
- **Volkswagen-Stiftung & Land of Baden-Württemberg**
- **SFB-439 from the Deutsche Forschungsgemeinschaft**

Black Holes in Galaxies







Numerical Modelling of Binary Black Hole Mergers

$G = M = 1$

$E_{TOT} = -\frac{1}{4}$

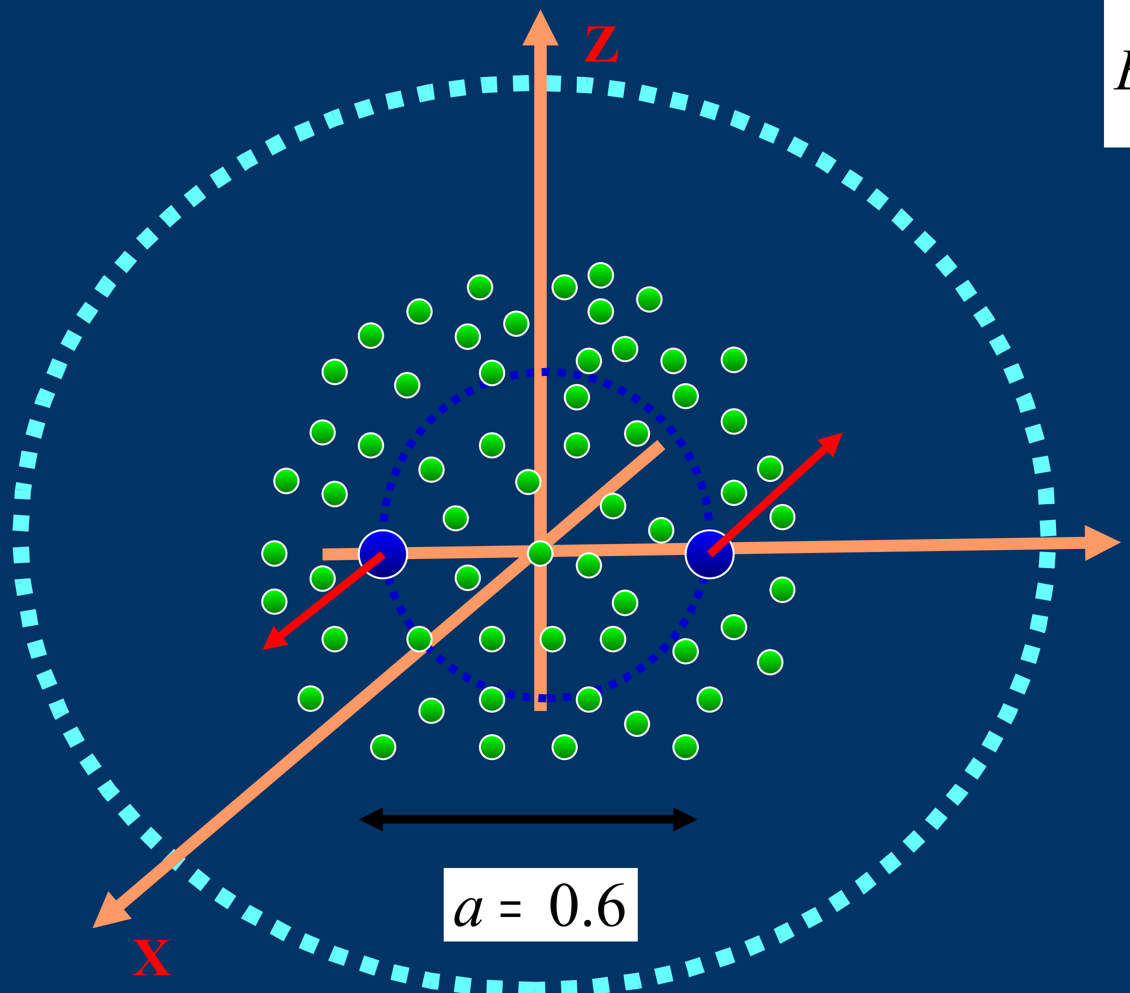
$25k$

$50k$

$100k$

$200k$

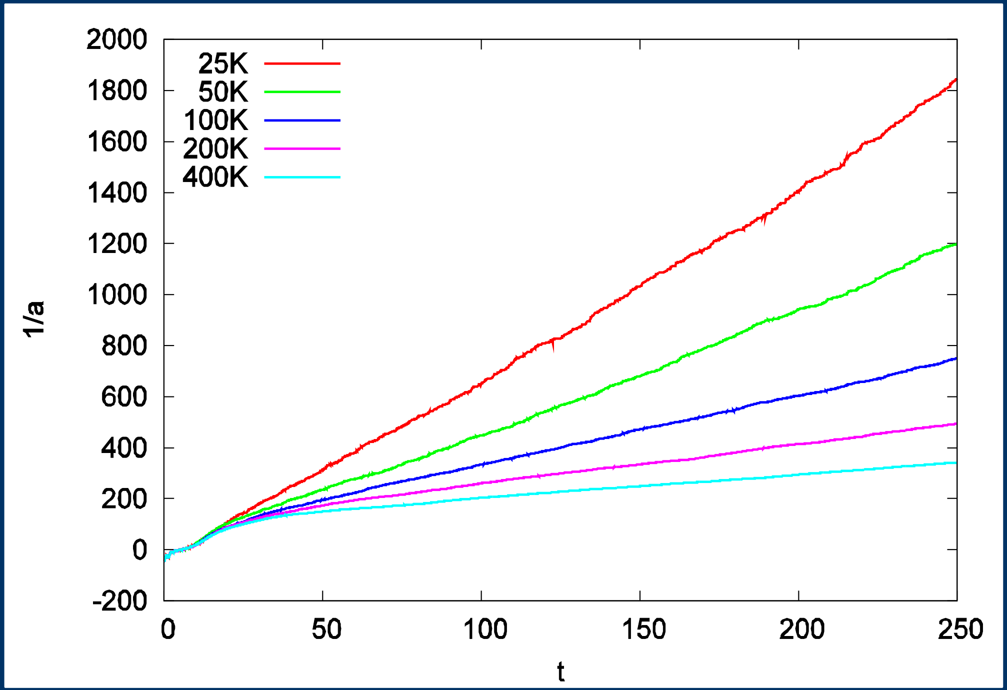
$400k$



Berczik et al. 2006, ApJ, 642, L24

Simulating BH Mergers : The Final pc Problem

Models without Rotation



Full Loss Cone

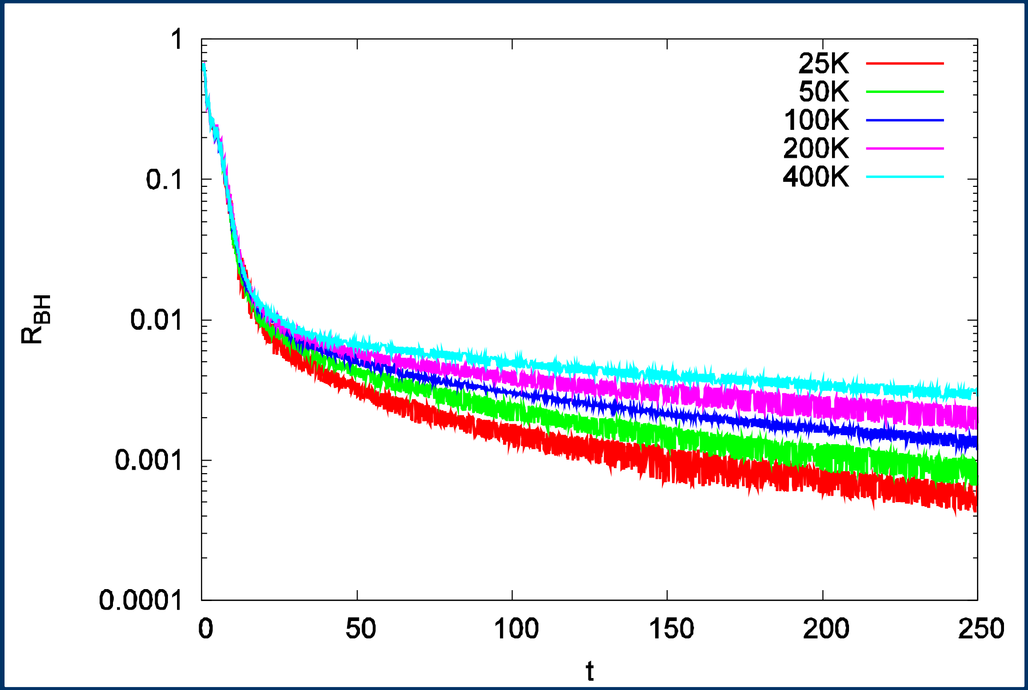
$$\frac{1}{a} \propto t$$

Empty Loss Cone

$$\frac{1}{a} \propto \frac{t}{N}$$

Berczik et al. 2006, ApJ, 642, L24

Simulating BH Mergers : The Final pc Problem



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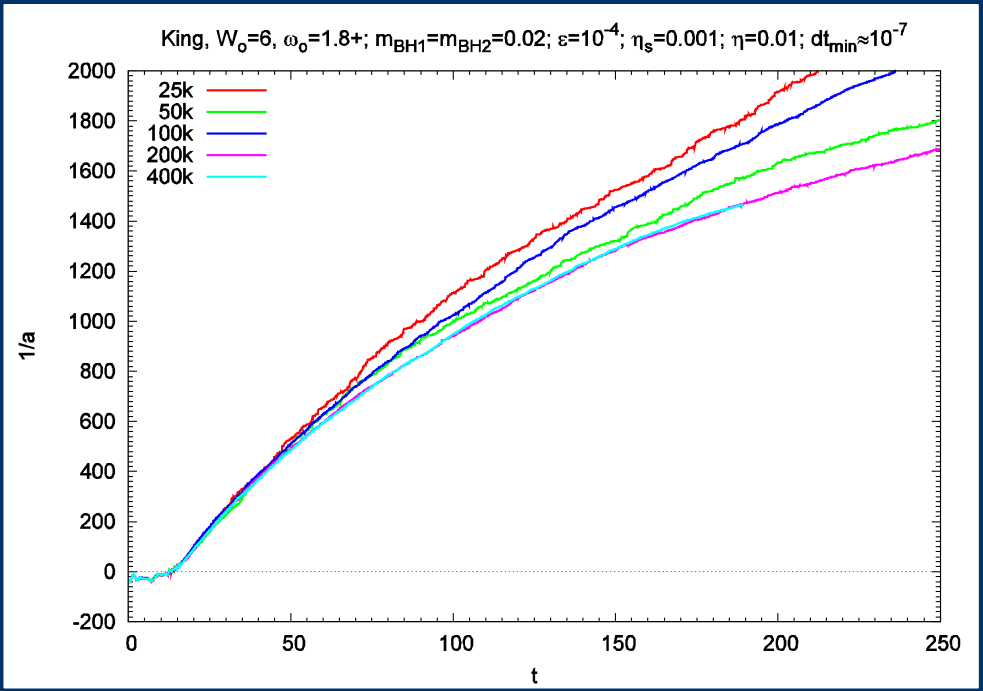
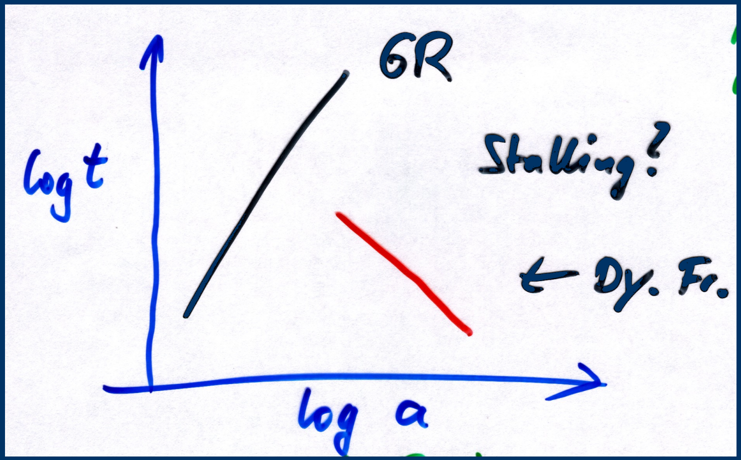
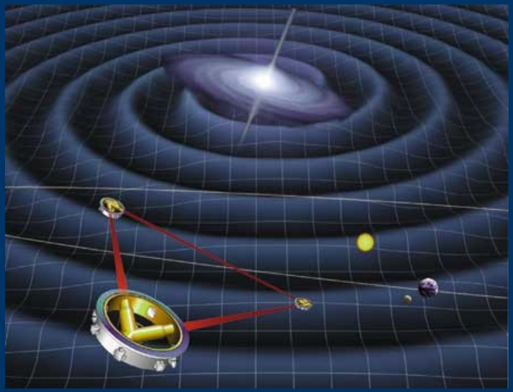
Simulating BH Mergers : Models with Rotation

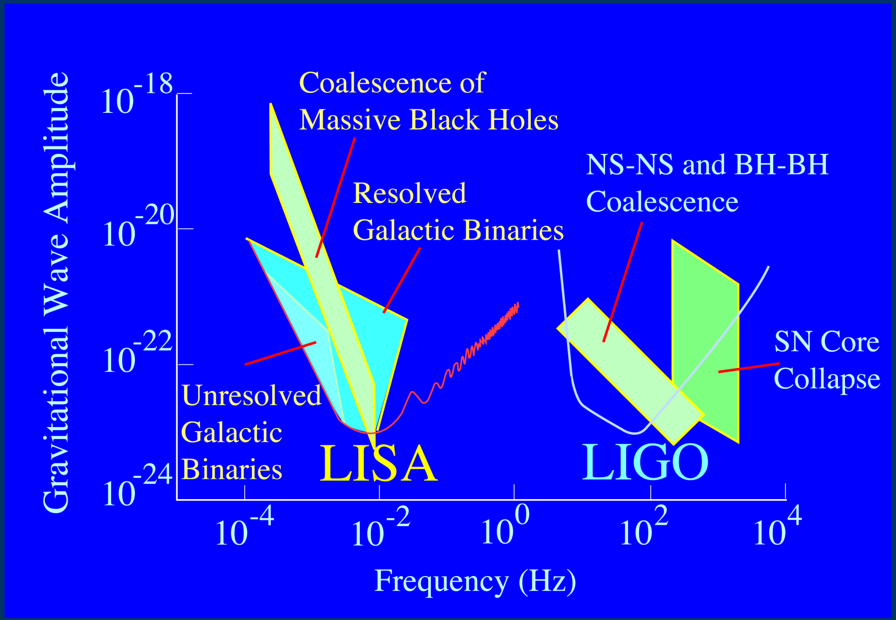


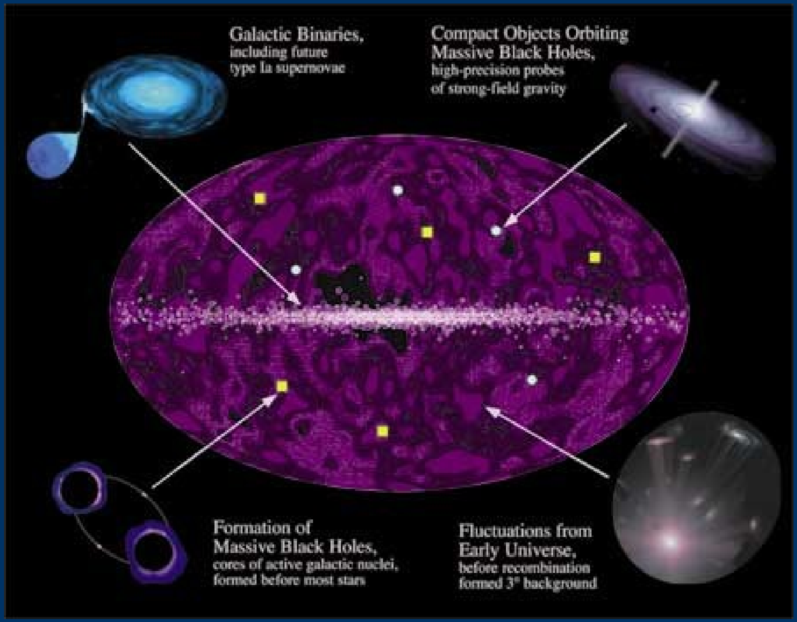
Berczik et al. 2006, ApJ, 642, L24

Binary Black Hole Merger as a Source of Gravitational Waves









Dynamics Close to the Merging Regime

General Relativity : Emission of Gravitational Waves

Approach: Post-Newtonian Dynamics

$$\boldsymbol{a} = \underbrace{\boldsymbol{a}_0}_{\text{Newt.}} + \underbrace{\underbrace{c^{-2}\boldsymbol{a}_2}_{1\mathcal{PN}} + \underbrace{c^{-4}\boldsymbol{a}_4}_{2\mathcal{PN}}}_{\text{periastron shift}} + \underbrace{\underbrace{c^{-5}\boldsymbol{a}_5}_{2.5\mathcal{PN}}}_{\text{grav. rad.}} + \mathcal{O}(c^{-6}),$$

$$\boldsymbol{a}_2 = \frac{Gm_2}{r^2} \Big\{ \boldsymbol{n} \Big[-v_1^2 - 2v_2^2 + 4v_1v_2 + \frac{3}{2}(nv_2)^2 + 5\left(\frac{Gm_1}{r}\right) + 4\left(\frac{Gm_2}{r}\right) \Big] + (v_1 - v_2)[4nv_1 - 3nv_2] \Big\}$$

$$\boldsymbol{a}_5 = \frac{4}{5}\frac{G^2 m_1 m_2}{r^3} \times \Big\{ (v_1 - v_2)\Big[-(v_1 - v_2)^2 + 2\left(\frac{Gm_1}{r}\right) - 8\left(\frac{Gm_2}{r}\right) \Big] + \boldsymbol{n}(nv_1 - nv_2)\Big[3(v_1 - v_2)^2 - 6\left(\frac{Gm_1}{r}\right) + \frac{52}{3}\left(\frac{Gm_2}{r}\right) \Big] \Big\}$$

$$\begin{aligned}
\boldsymbol{a}_4 = \frac{Gm_2}{r^2} \Big\{ \boldsymbol{n} \Big[& -2v_2^4 + 4v_2^2(v_1v_2) - 2(v_1v_2)^2 \\
& + \frac{3}{2}v_1^2(nv_2)^2 + \frac{9}{2}v_2^2(nv_2)^2 - 6(v_1v_2)(nv_2)^2 \\
& - \frac{15}{8}(nv_2)^4 + \left(\frac{Gm_1}{r}\right)\Big(-\frac{15}{4}v_1^2 + \frac{5}{4}v_2^2 - \frac{5}{2}v_1v_2 \\
& + \frac{39}{2}(nv_1)^2 - 39(nv_1)(nv_2) + \frac{17}{2}(nv_2)^2 \Big) \\
& + \left(\frac{Gm_2}{r}\right)\Big(4v_2^2 - 8v_1v_2 + 2(nv_1)^2 \\
& - 4(nv_1)(nv_2) - 6(nv_2)^2\Big) \Big] \\
& + (v_1 - v_2)\Big[v_1^2(nv_2) + 4v_2^2(nv_1) - 5v_2^2(nv_2) \\
& - 4(v_1v_2)(nv_1) + 4(v_1v_2)(nv_2) - 6(nv_1)(nv_2)^2 \\
& + \frac{9}{2}(nv_2)^3 + \left(\frac{Gm_1}{r}\right)\left(-\frac{63}{4}nv_1 + \frac{55}{4}nv_2\right) \\
& + \left(\frac{Gm_2}{r}\right)(-2nv_1 - 2nv_2) \Big] \Big\} \\
& + \frac{G^3 m_2}{r^4}\boldsymbol{n}\left(-\frac{57}{4}m_1^2 - 9m_2^2 - \frac{69}{2}m_1m_2 \right),
\end{aligned}$$

Previous *PN* Studies (incomplete list)

Using *PN*2.5 only

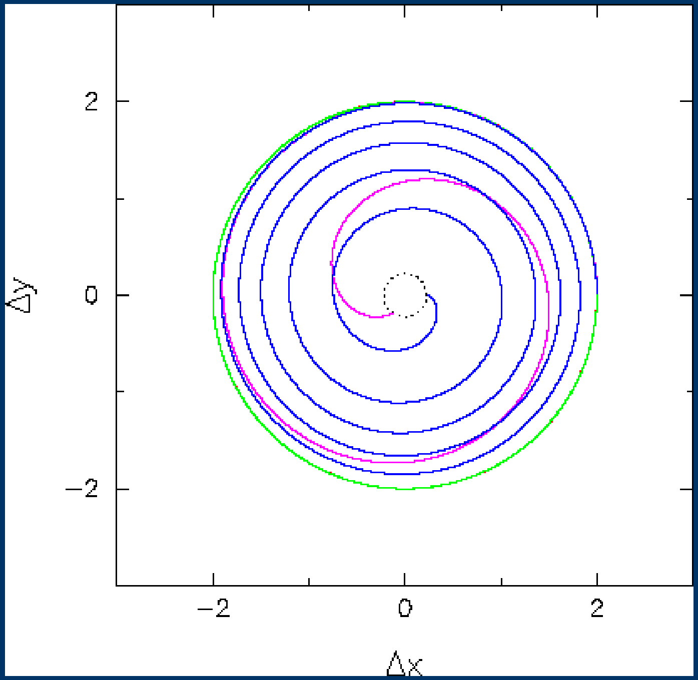
Lee, 1993, ApJ, 418, 147
Iwasawa, Funato & Makino, 2006, ApJ, 651, 1059
O'Leary, 2006, ApJ, 637, 937
Gültekin, Miller & Hamilton, 2006, ApJ, 640, 156
etc.

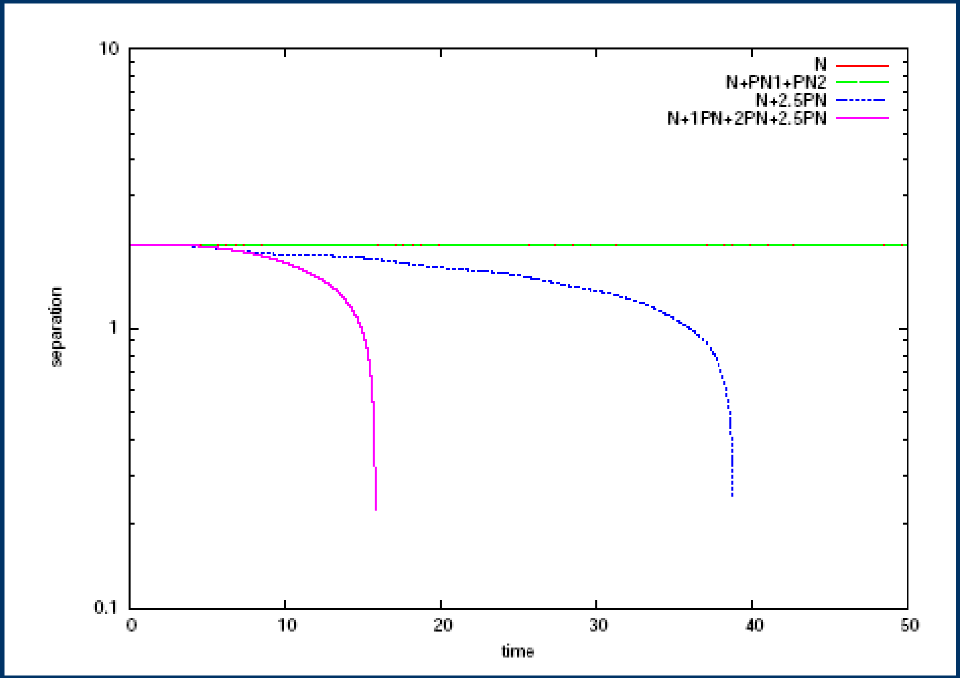
Using *all PN* terms up to 2.5

Valtonen, Mikkola & Pietila, 1995, MNRAS, 273, 751
Kupi, Amaro-Seoane & Spurzem, 2006, MNRAS, 371, L45

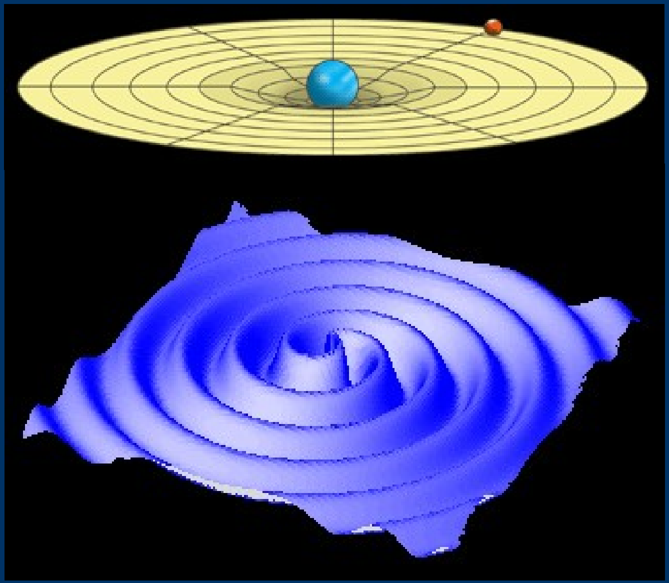
The *PN* Two-Body Problem

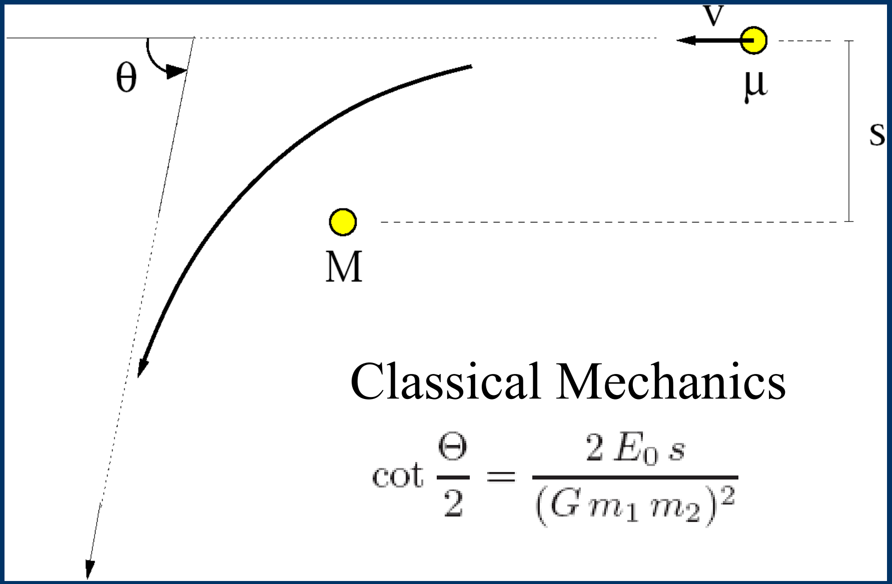
Inspiral of Binary BHs

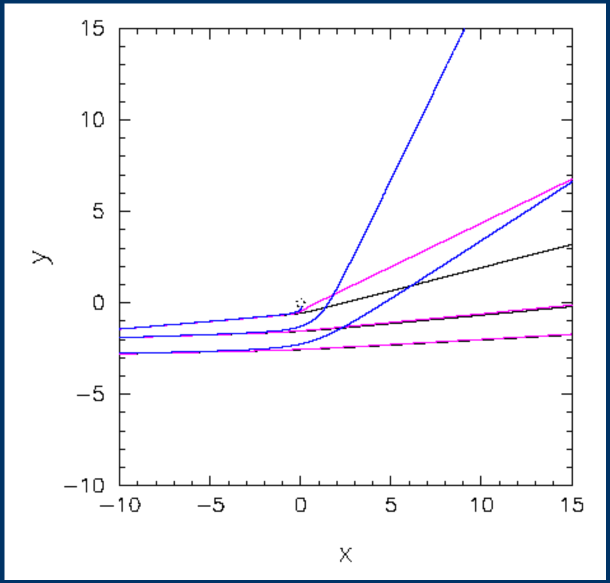


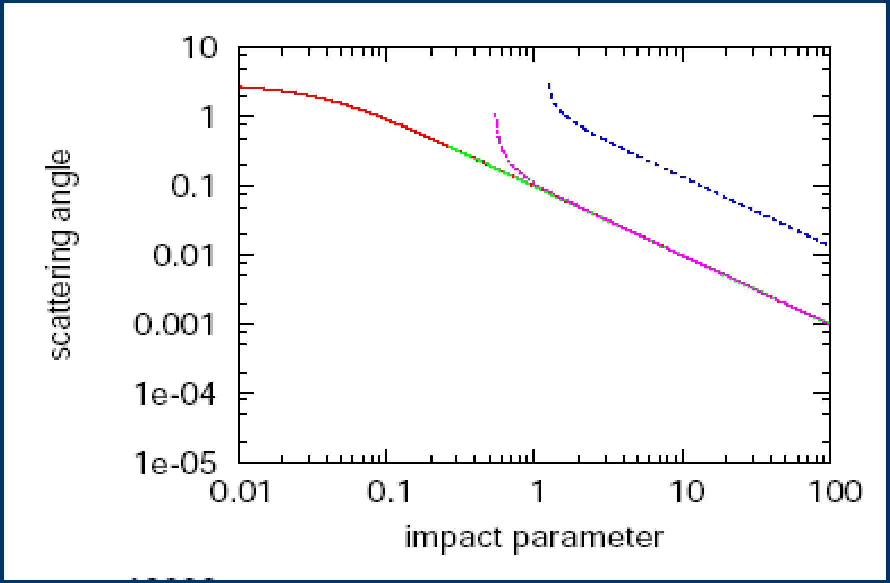


The *PN* Two-Body Problem
Scattering Experiments









Units & Initial Conditions

$G \equiv 1$
$M = 10^8$ Mo
$R = 1$pc

$T = 1.491\ 10^{-3}$ Myr
$v = 655.8$ km/s
$c \equiv 457.14$

$M_{BH1} = 0.03$
$R_{BH1} = 2.87\ 10^{-7}$

$M_{BH2} : M_{BH1} = 1:1,\ 1:2,\ 1:10,\ 1:100$

$s_{min} = 2.8\ 10^{-6}$
$s_{max} = 2.8\ 10^{-3}$

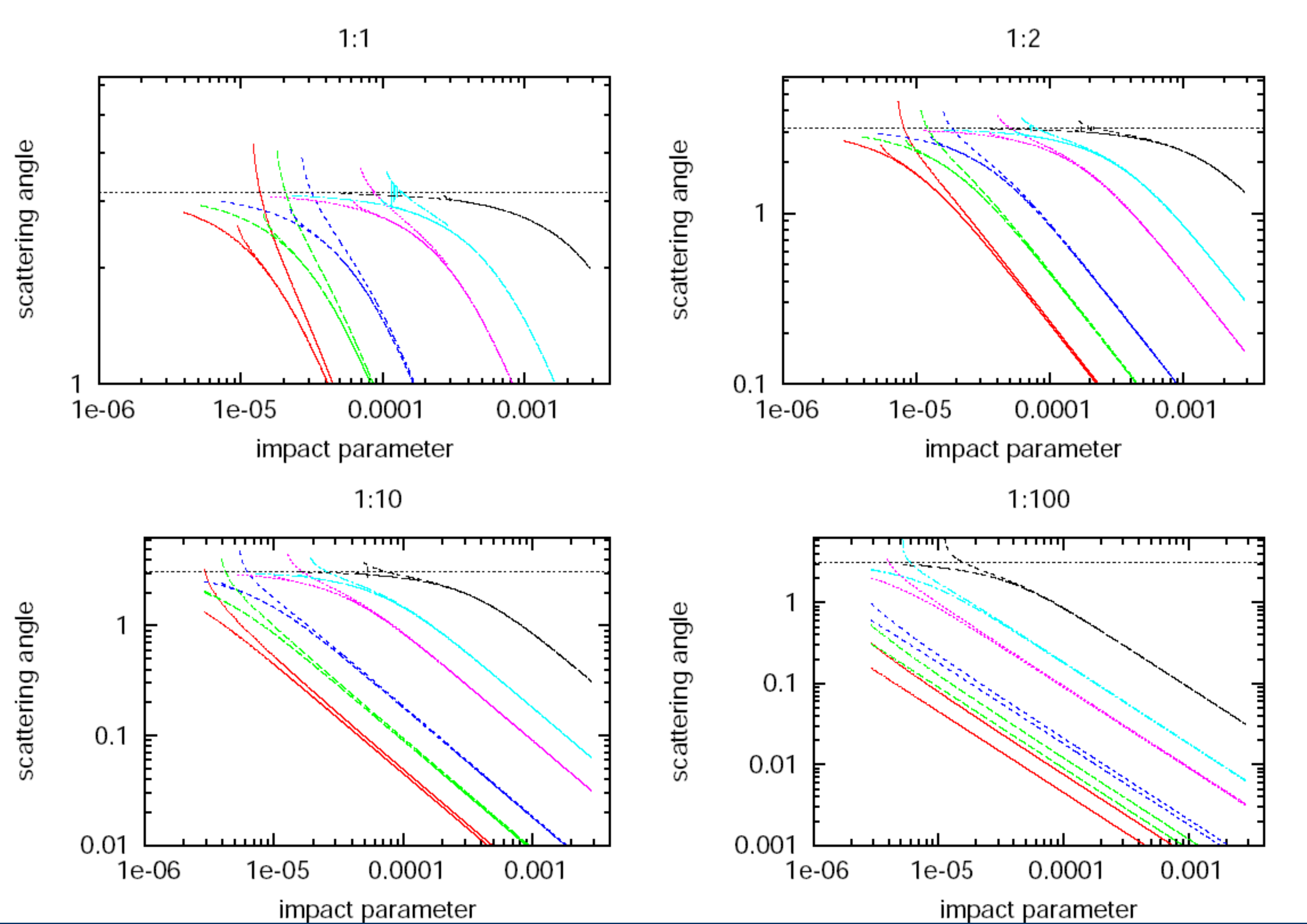
100.000 trajectories

$E = \{0.1, 0.5, 1, 5, 10, 20\}$ →

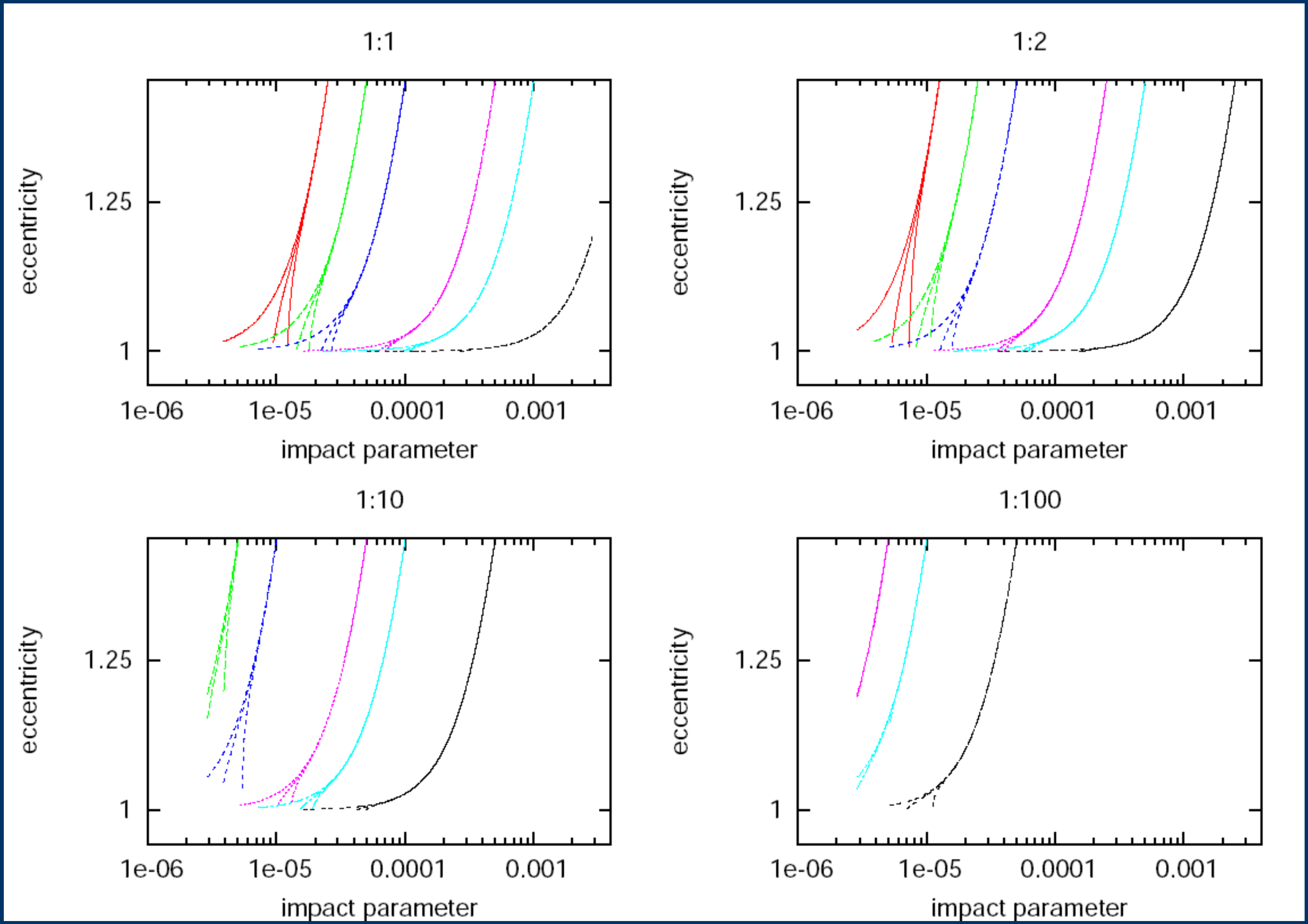
I.) N
II.) N+*PN*2.5
III.) N+*PN*1+*PN*2+*PN*2.5

All calculations were performed with JUMP @ NIC Jülich

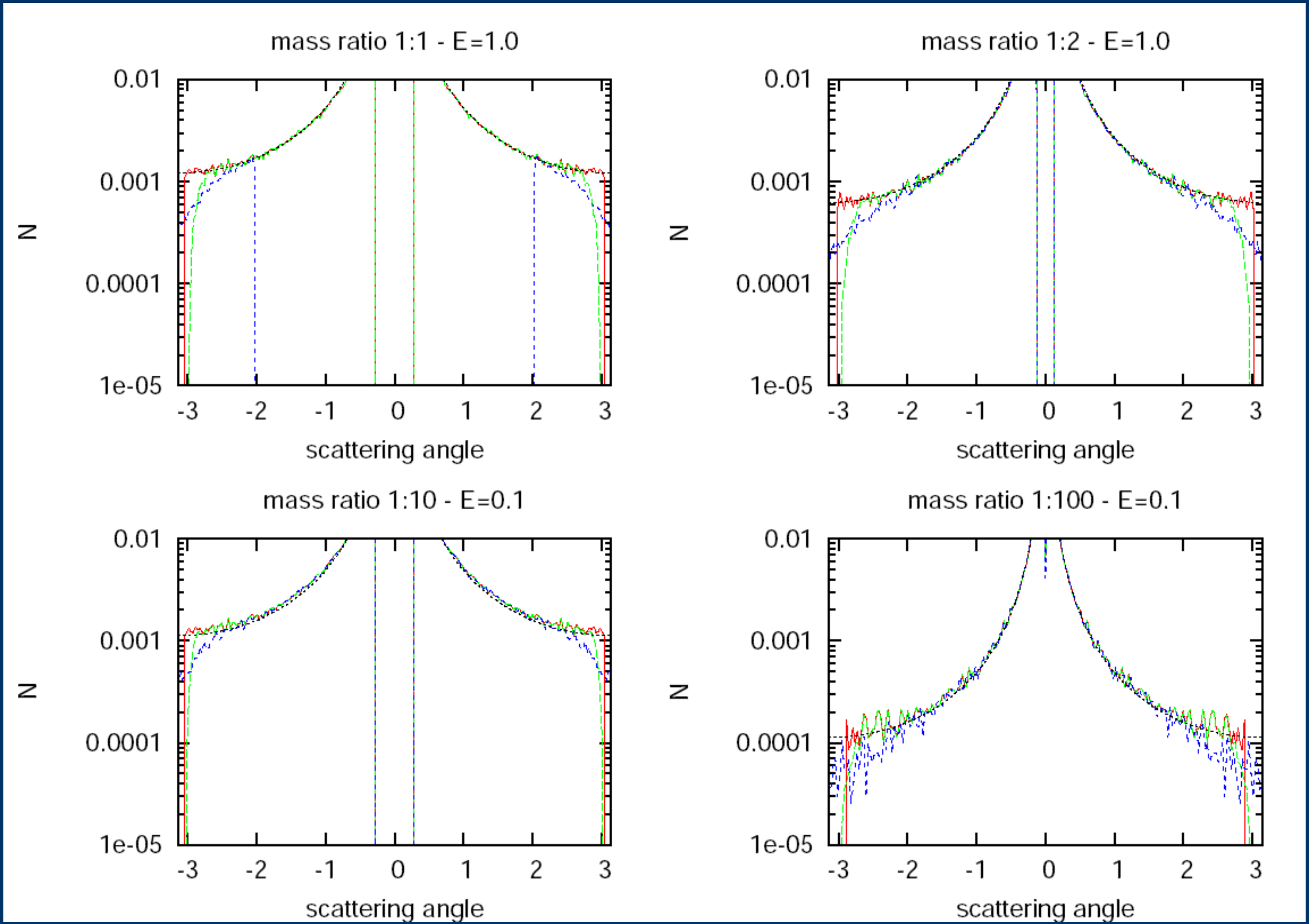
PN Two-Body Problem – Scattering Angles



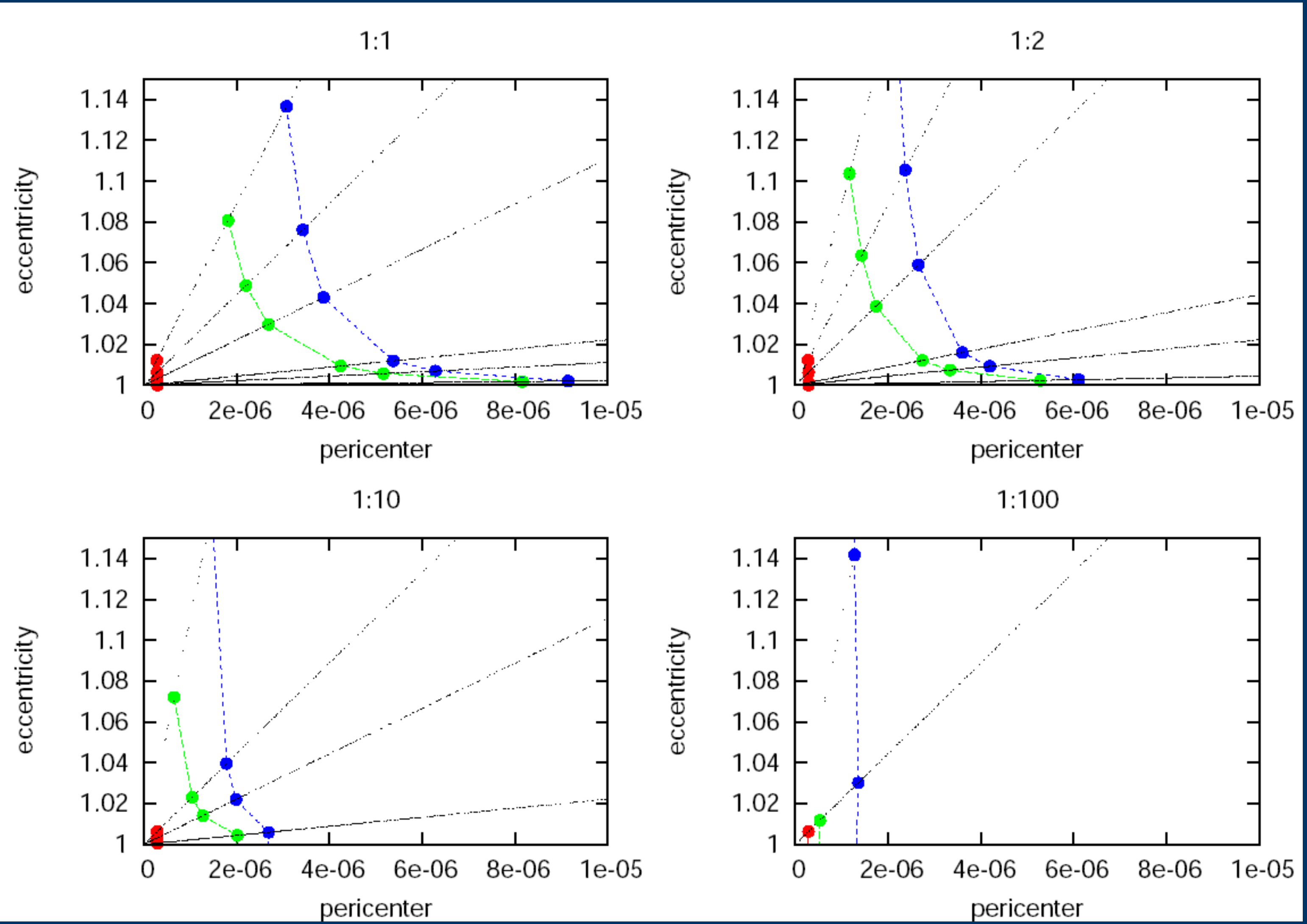
PN Two-Body Problem – Eccentricities



PN Two-Body Problem – Cross Sections



PN Two-Body Problem – Merging Regimes for Hyperbolic Orbits



Summary

- Preliminary Results from the *PN* Two-body Problem:

  - *PN* terms affect the cross sections
    → increase the rate of collisions/captures
  - "Full" *PN* treatment results in higher merger rates than just using *PN*2.5 only
  - Timescale for inspirals decreases with *PN*
    → *PN*2.5 can only give an upper limit
  - Trajectories are modified when using full *PN*
    → *important for GW Waveforms*

Outlook

- Binary mergers in Galactic Centers & Globular Clusters

  - high N experiments using the ARI Titan cluster

    → *work in progress*

- Two-body Problem:

  - Inner-most stable orbit (ISCO)

  - Scattering experiments / Bound orbits
    → cross-sections, merger times/rates
    → possible sources for GW detections
    → *work in progress*

Thank you for your attention !

A simple toy-code

Hermite Integration Scheme

$$\vec{a}_i = \sum \frac{G m_j}{[(\vec{x}_j - \vec{x}_i)^2 + \epsilon^2]^{3/2}} (\vec{x}_j - \vec{x}_i)$$

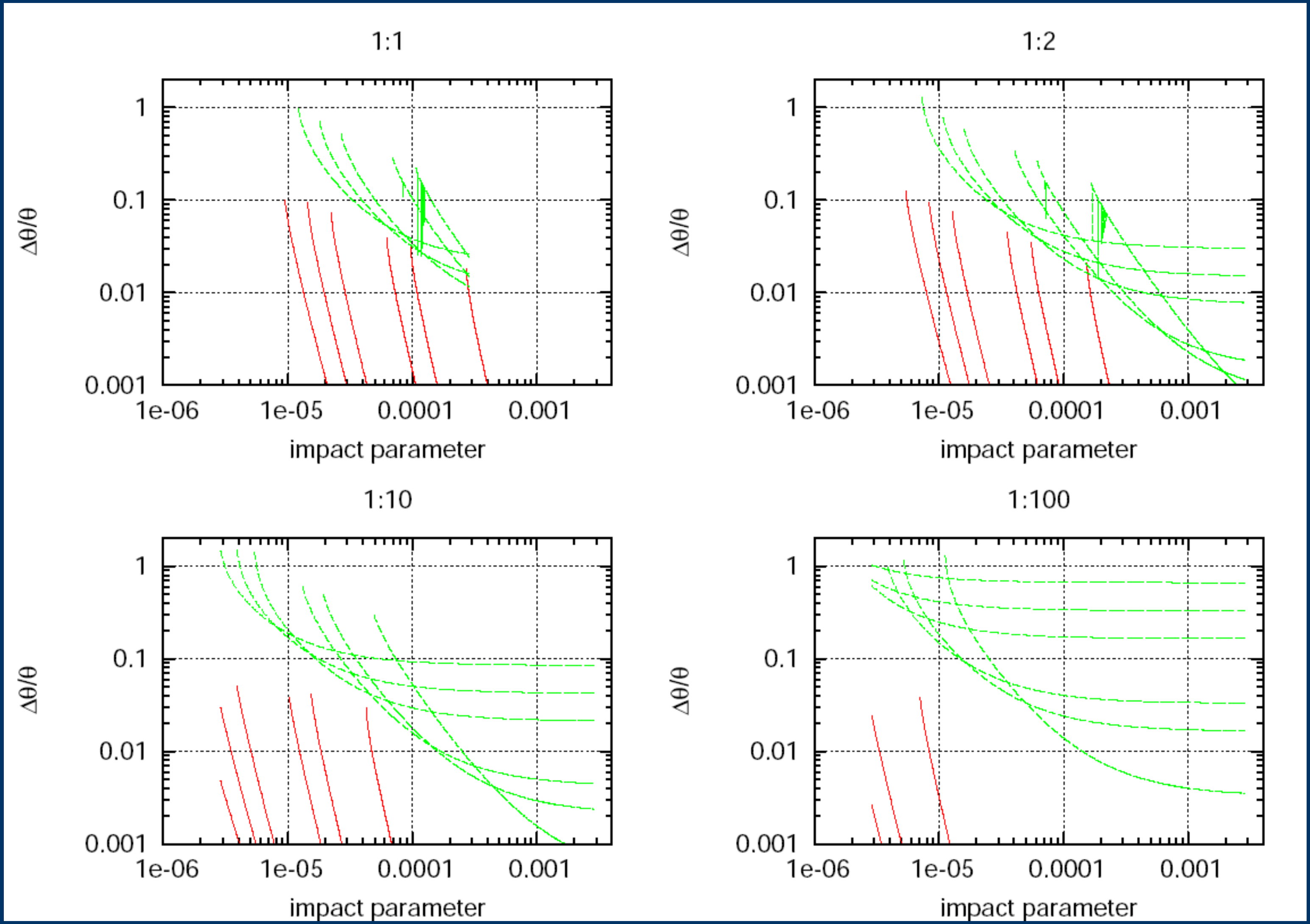
$$\dot{\vec{a}}_i = \sum \frac{G m_j}{[(\vec{x}_j - \vec{x}_i)^2 + \epsilon^2]^{3/2}} (\dot{\vec{x}}_j - \dot{\vec{x}}_i) - 3 \sum \frac{G m_j (\vec{x}_j - \vec{x}_i)(\dot{\vec{x}}_j - \dot{\vec{x}}_i)}{[(\vec{x}_j - \vec{x}_i)^2 + \epsilon^2]^{5/2}} (\vec{x}_j - \vec{x}_i)$$

$$\begin{aligned} \Delta t &= t - t_j \\ \mathbf{x}_p &= \frac{\Delta t^4}{24}\mathbf{a}_0^{(2)} + \frac{\Delta t^3}{6}\dot{\mathbf{a}}_0 + \frac{\Delta t^2}{2}\mathbf{a}_0 + \Delta t \mathbf{v}_0 + \mathbf{x}_0 \\ \mathbf{v}_p &= \frac{\Delta t^3}{6}\mathbf{a}_0^{(2)} + \frac{\Delta t^2}{2}\dot{\mathbf{a}}_0 + \Delta t \mathbf{a}_0 + \mathbf{v}_0, \end{aligned}$$

Hierarchical Time-Step Scheme

Supports MPI

PN Two-Body Problem – Scattering Angles



PN Two-Body Problem – Merging Timescales