100 million stars and one massive black hole

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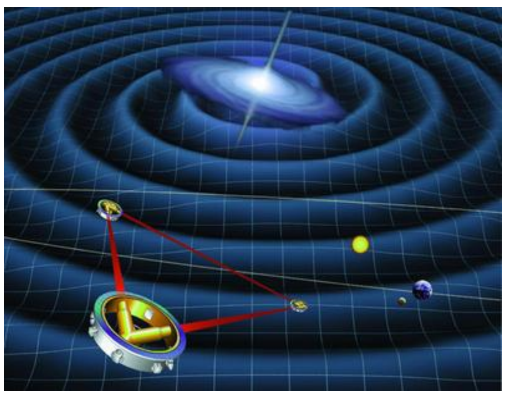
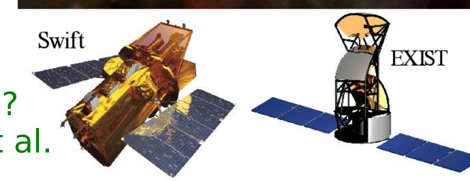
Dynamics of galactic nuclei

- Stellar contribution to MBH growth
  - Stellar gas (winds, collisions, tidal disruptions)
  - Whole stars (plunges through horizon)
- Consequences of MBH presence
  - Structure of cluster (density cusp, velocity raise, color profile)
  - Luminosity through continuous accretion of stellar gas
  - X/UV accretion flares following tidal disruptions
  - GW emission by captured stars





Tidal flares already detected?
See Komossa et al. Gezari et al.

Requires to follow long-term, collisional (“relaxational”) stellar dynamics with very high number of objects. ⇒ Monte Carlo code

But assumes spherical symmetry, isolation...

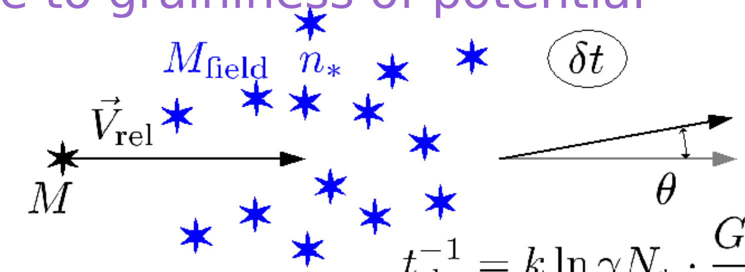
- Most important relaxational effect: mass segregation

Freitag, Kalogera & Amaro-Seoane, in prep

The physics of dense clusters

- "Collisional" = 2-body relaxation

  E, J drift and diffusion due to graininess of potential
  - Core collapse
  - Mass segregation

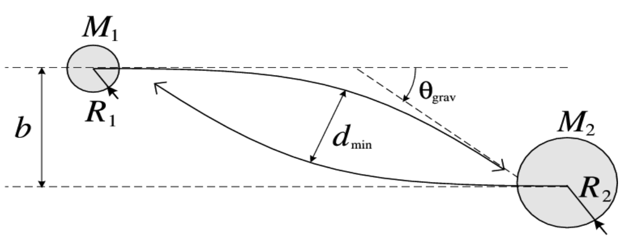


$$\langle\theta\rangle = 0$$
$$\langle\theta^2\rangle = \left(\frac{\pi}{2}\right)^2 \frac{\delta t}{t_{\rm rlx}}$$

$$t_{\rm rlx}^{-1} = k \ln\gamma N_* \cdot \frac{G M M_{\rm field} n_*}{V_{\rm rel}^3} \approx \frac{\ln\gamma N_*}{N_*} t_{\rm orb}^{-1}$$

$$t_{\rm rlx} \simeq 5\times 10^7\,{\rm yr}\, \frac{10^6\,{\rm pc}^{-3}}{n_*} \left(\frac{\sigma_v}{30\,{\rm km\,s}^{-1}}\right)^3 \left(\frac{{\rm M}_\odot}{M_*}\right)^2$$

- "Collisional" = Direct collisions between stars



$$t_{\rm coll}^{-1} = S_{\rm coll} V_{\rm rel} n_* \quad S_{\rm coll} = \pi (R + R_{\rm field})^2 \left(1 + \frac{2G(M + M_{\rm field})}{(R + R_{\rm field}) V_{\rm rel}^2}\right)$$

$$t_{\rm coll} \simeq 2.1\times 10^{12}\,{\rm yr}\, \frac{10^6\,{\rm pc}^{-3}}{n_*} \frac{\sigma_v}{30\,{\rm km\,s}^{-1}} \frac{{\rm R}_\odot}{R_*} \frac{{\rm M}_\odot}{M_*}$$

- Stellar evolution
- Interactions with a central (I)MBH
  - Tidal disruptions UV/X Flares
  - Plunges through horizon
  - Captures by emission of gravitational waves EMRBs for LISA

WANTED: Million-star cluster dynamics method

How to follow the evolution of systems containing 10^6 to 10^8 stars over millions to billions of years?

- Direct approach: *N*-body
  - Newtonian gravity without approximation. "Just" solve Newton equations for *N* particles $$\frac{d^2\vec{X}_i}{dt} = -G\sum_{j\neq i} m_j \frac{\vec{X}_i - \vec{X}_j}{\left|\vec{X}_i - \vec{X}_j\right|^3}$$
  - No spatial symmetry or dynamical equilibrium assumed
  - Very time consuming $T_{\mathrm{CPU}}/t_{\mathrm{rlx}} \propto N^3 \Rightarrow N < 10^6$
- Continuum approaches: Fokker-Planck & Gas codes
  - Follow evolution of DF or "fluid" of stars
  - Very fast
  - Very approximate (difficulties with mass spectrum, stellar evolution, collisions...)
- The best of both worlds?
  The Hénon **Monte Carlo** scheme... (Hénon 71a,b; 73)

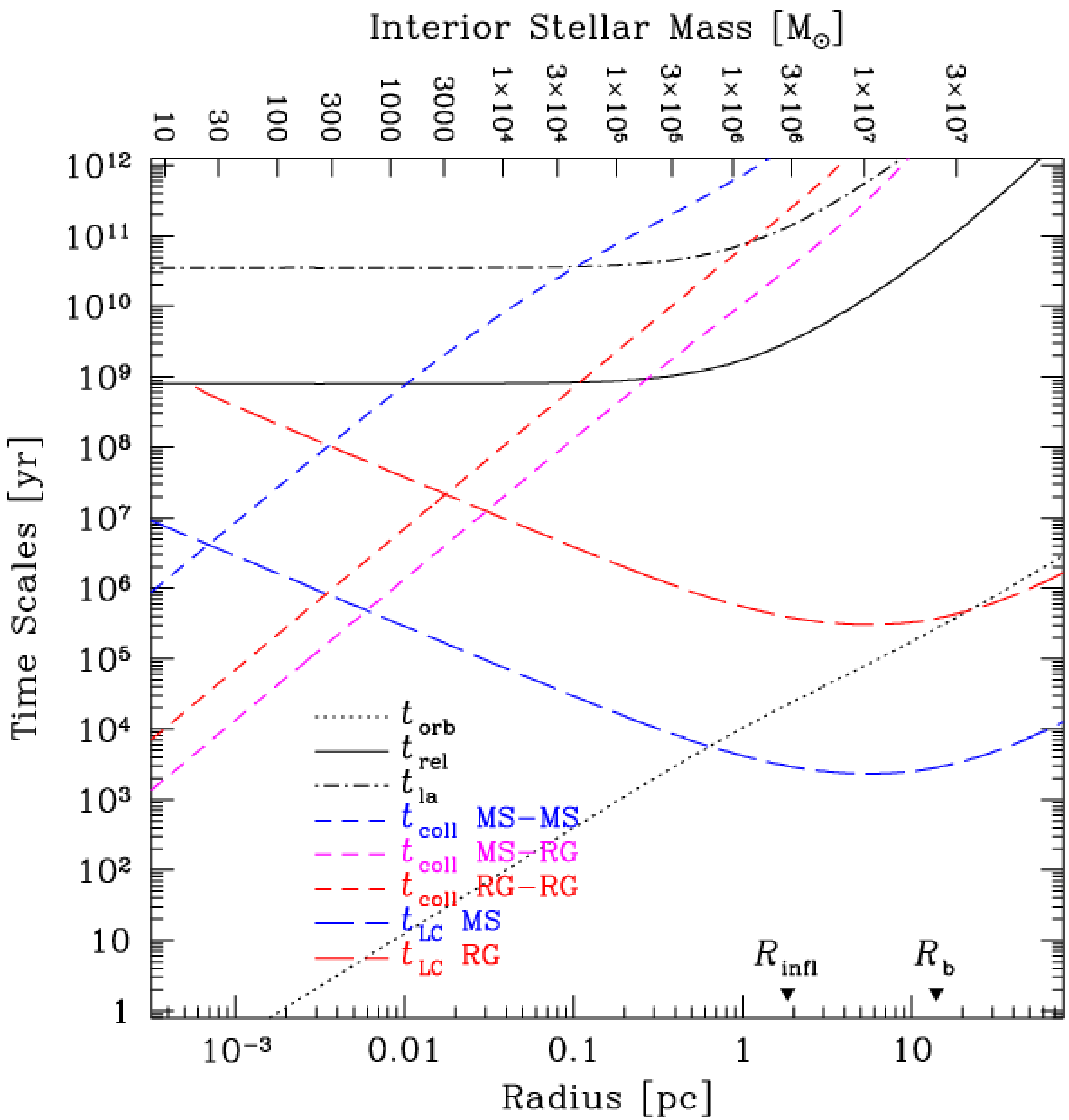
The Monte Carlo stellar dynamics method

`ME(SSY)**2` "Monte carlo Experiments with Spherically SYmmetric Stellar SYstems"

Freitag & Benz 2001, 2002

- Uses 3 assumptions: $dn_* = f(\vec{X},\vec{V})d^3X\,d^3V = 8\pi^2 F(E,J)R^2 dR V_t dV_t dV_r$
  - Spherical symmetry $E = \frac{1}{2}\vec{V}^2 + \Phi(R) \quad J = R\cdot V_t$
  - Dynamical equilibrium
  - Diffusive 2-body relaxation (Chandrasekhar; Fokker-Planck)
- Represents the cluster with particles
  - 1 particle = 1 spherical shell (given orbital and stellar prop.)
  - 1 particle = many stars (possibly) ⇒ No limit on N_*
  - Local time steps $\delta t \leq f_{\delta t}\cdot \min(T_{\rm rlx}, T_{\rm coll}, \ldots)$
- Allows rich physics
  - Cluster (+central object) self-gravity; *V*-anisotropy; Any *M*-spectrum
  - 2-body relaxation; Stellar collisions (use SPH data); Stellar evolution
  - "Loss-cone processes": Tidal disruptions; Plunges; GW-captures
- Fast $T_{\rm CPU}/t_{\rm rlx} \propto N\ln N \Rightarrow N \approx 10^4 - 10^7$

Time scales in Sgr A* nucleus



Use M-sigma relation to scale to other M_{BH}:

$$M_{\rm BH} \simeq M_{100} \left(\frac{\sigma}{100\,{\rm km\,s^{-1}}} \right)^4.$$

$$R = R|_{\rm MW} \left(\frac{M_{\rm BH}}{3.5 \times 10^6\,{\rm M_\odot}} \right)^{1/2}$$

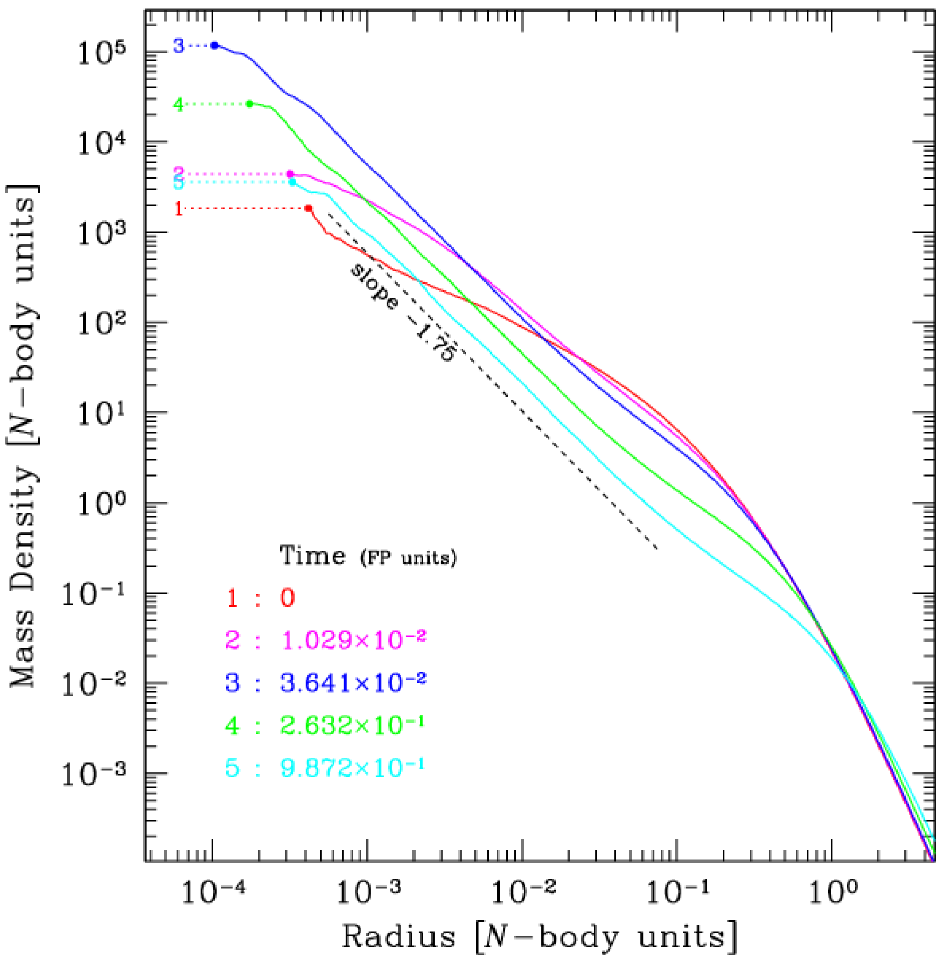
$t_{rlx}(R_{infl})>10$ Gyr for $M_{BH}>10^7$ $M_\odot$

Time scale for relax.
to replenish loss cone

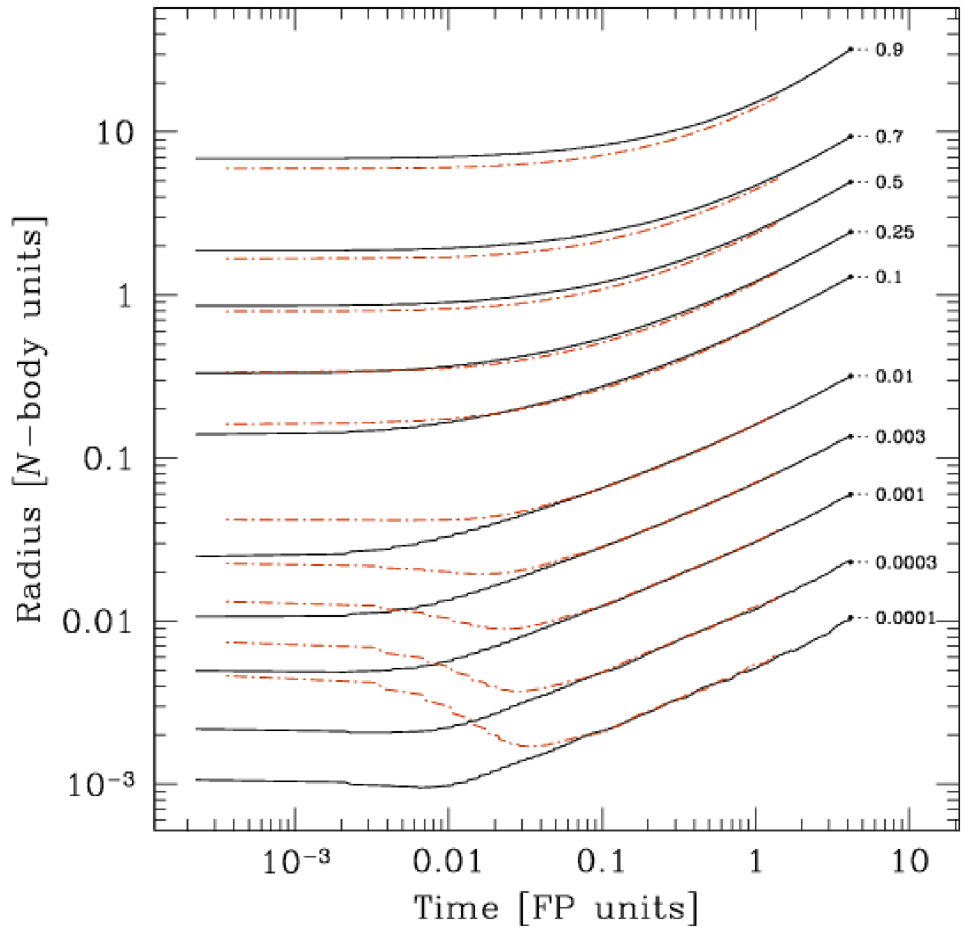
$$t_{\rm LC} = \theta_{\rm LC}^2 t_{\rm rlx} \simeq (1 - e_{\rm LC}) t_{\rm rlx}$$

Relaxational evolution of single-mass model

- 2-body relaxation + tidal disruptions
- All stars have same mass; $M_{BH}=0.05M_{clust}$



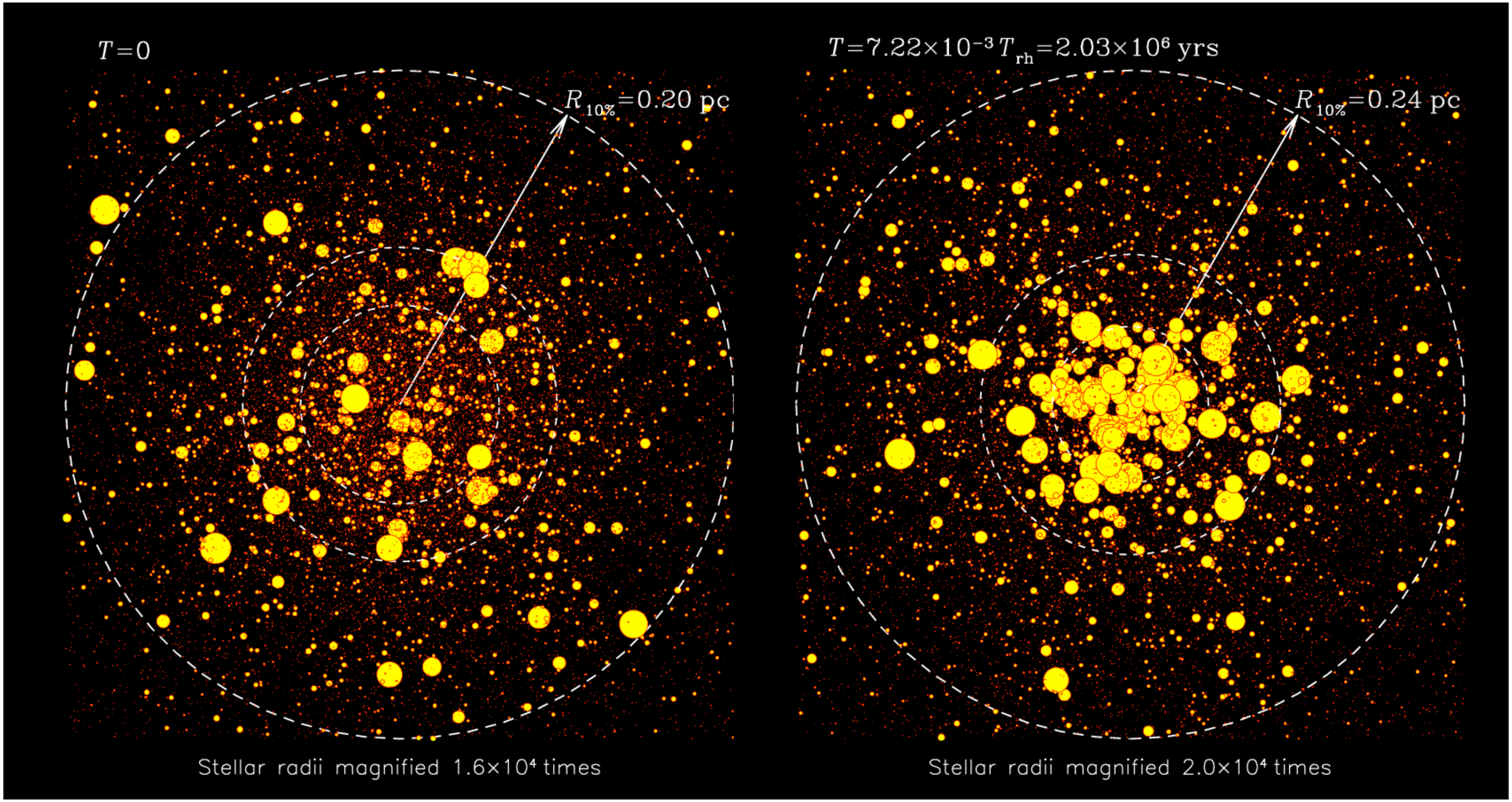
Development of Bahcall-Wolf density profile and expansion



Convergence of evolution for 2 models with different initial central density profiles

Mass segregation without a MBH

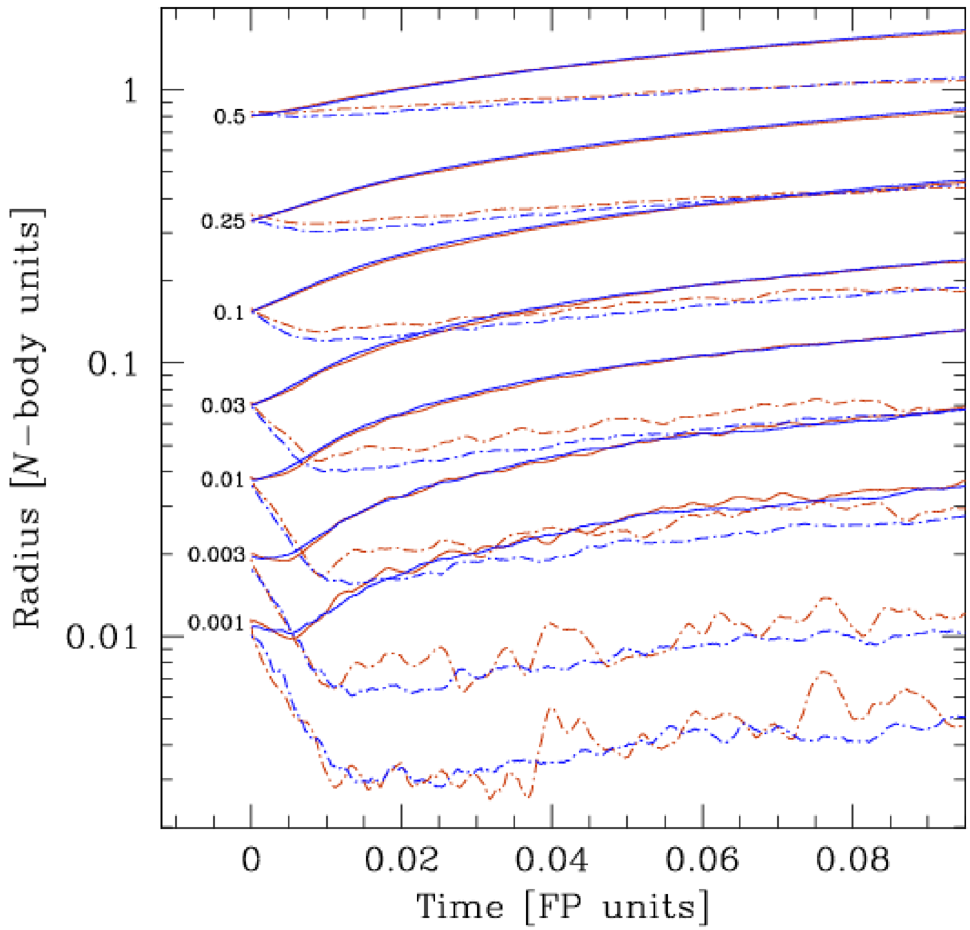
Initial conditions | Core collapse

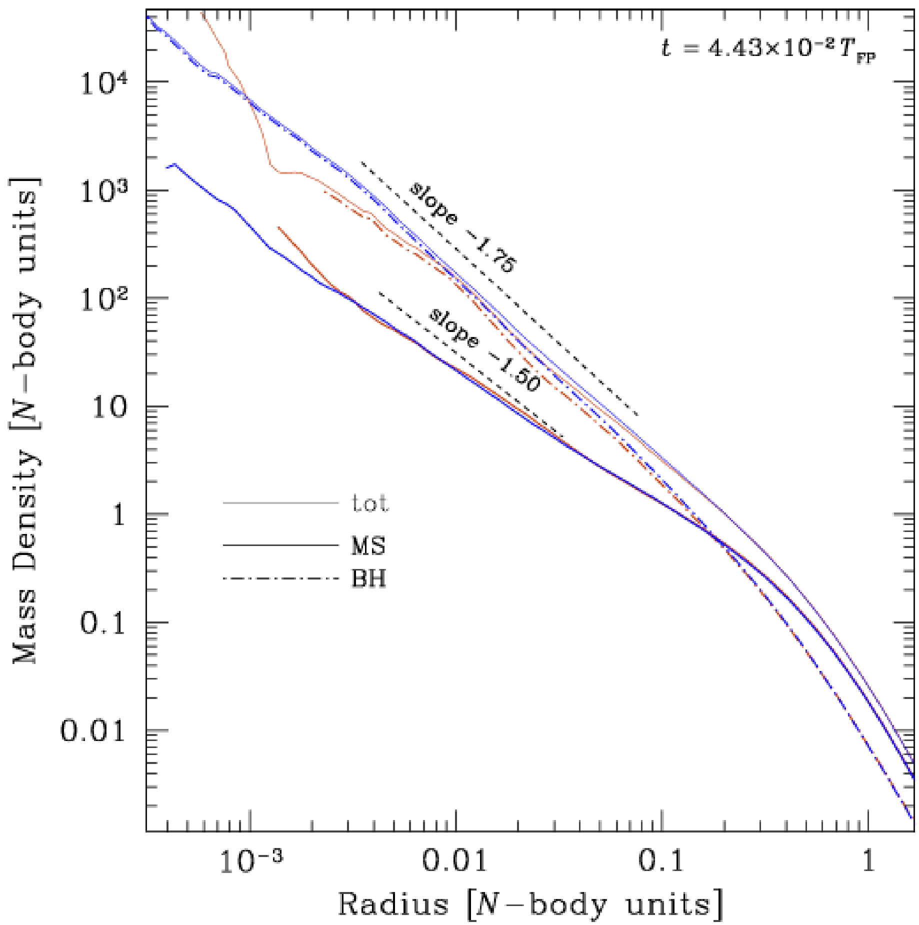


Gürkan, Freitag & Rasio 2004; Freitag, Rasio & Baumgardt 2005

Relaxational evolution of 2-component model

- 2-body relaxation + tidal disruptions
- 5% of stars are 10x more massive; $M_{BH}=0.1M_{clust}$
- Comparison with 64k N-body run by Pau Amaro-Seoane

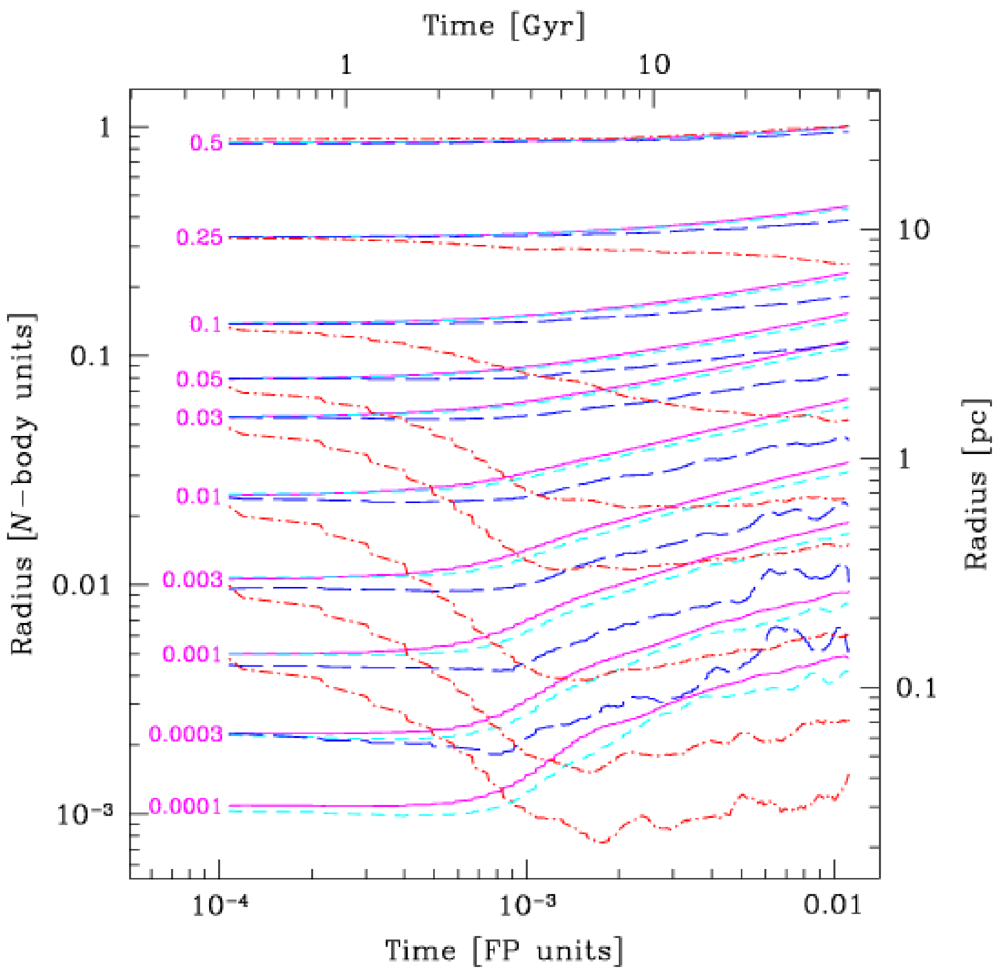


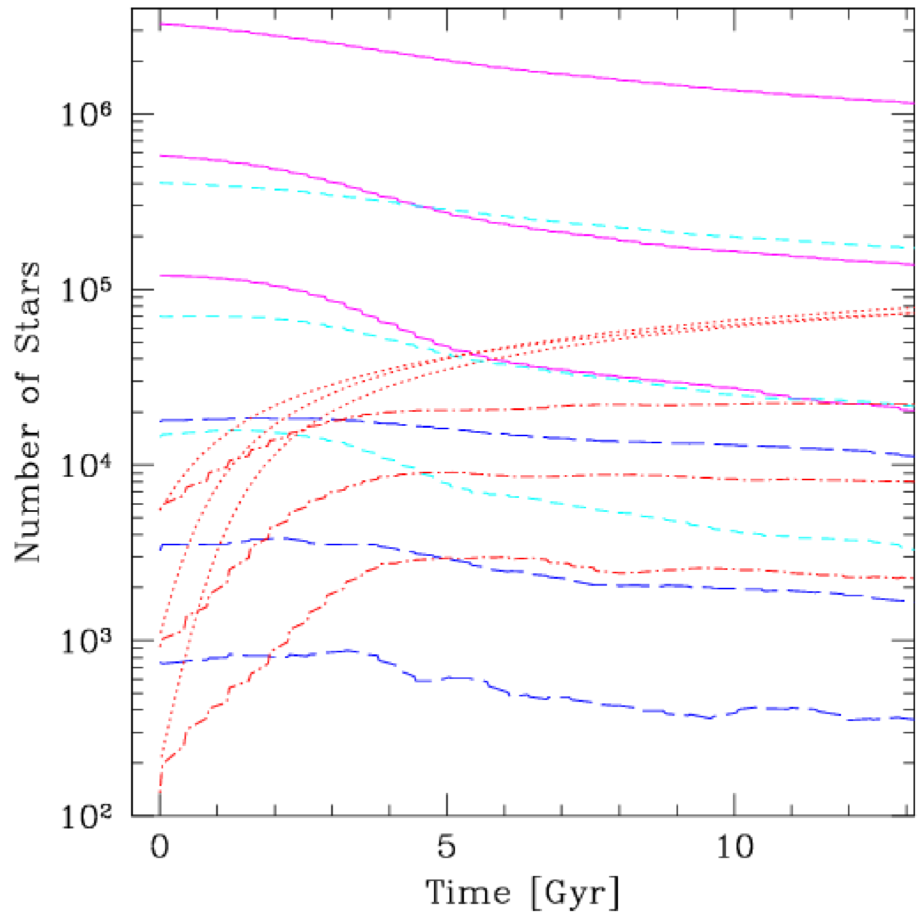


Relaxational evolution of Sgr A* model

- 2-body relaxation + tidal disruptions
- Full realistic stellar population (10 Gyr old); $M_{BH}=0.05 M_{clust}$

Lagrange radii for MS stars, WDs, NSs, stellar BHs



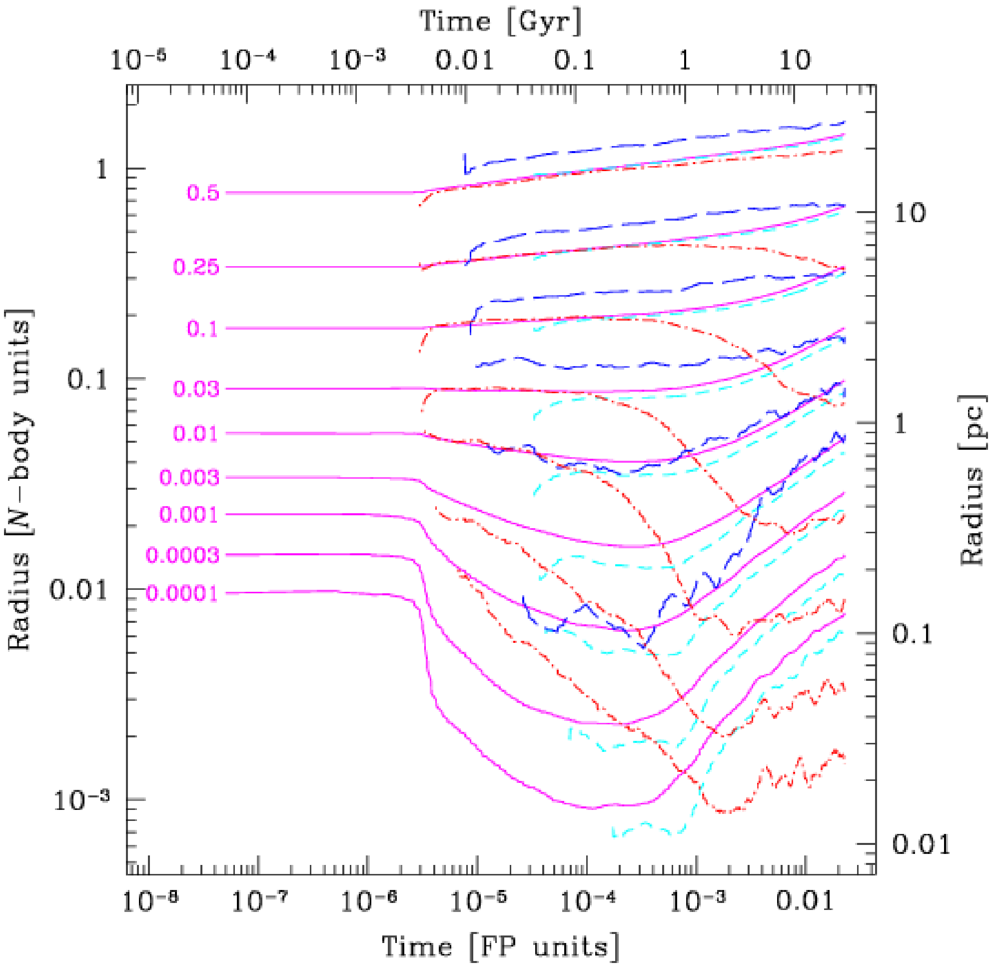


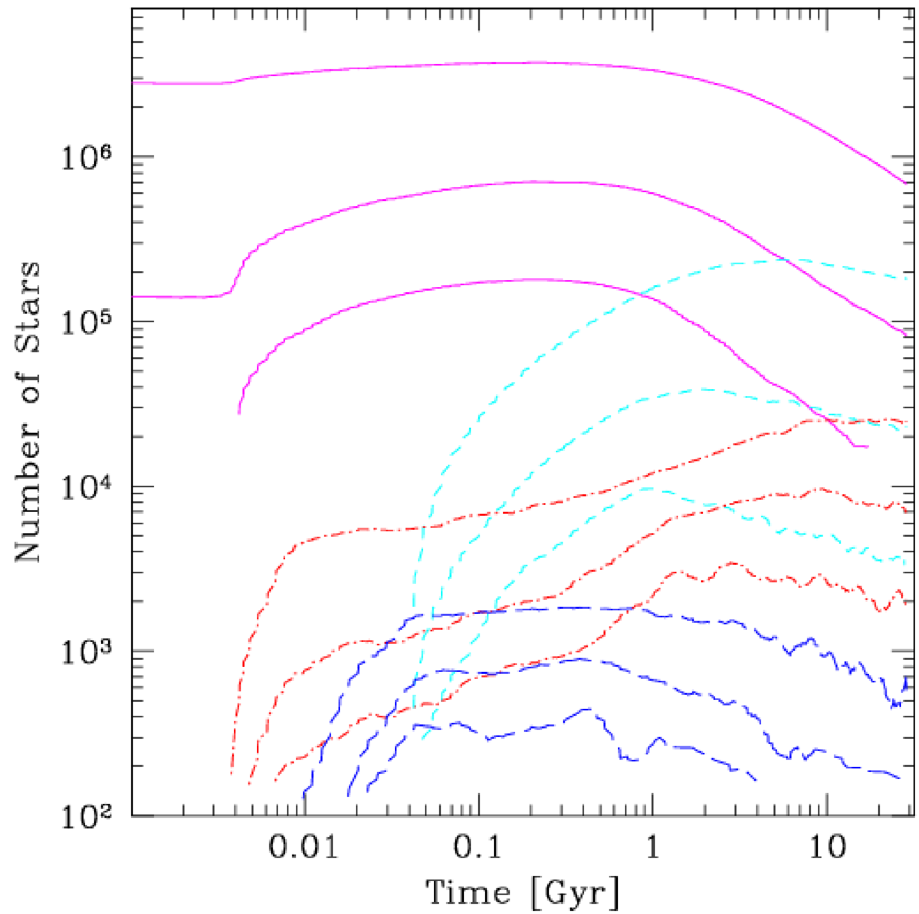
Number of stars within 0.1, 0.3, 1 pc of MBH
comparison with dyn. friction for stellar BHs (dotted lines)

Evolution of Sgr A* model

- 2-body relaxation, tidal disruption
- Stellar evolution; Partial accretion of stellar mass loss
- $M_{BH}(0)\simeq 0$; $M_{BH}(10\ \mathrm{Gyr})=0.05M_{clust}$
- Fine tuned to be compatible with MW nucleus around 10 Gyr

Lagrange radii for MS stars, WDs, NSs, stellar BHs





Number of stars within 0.1, 0.3, 1 pc of MBH

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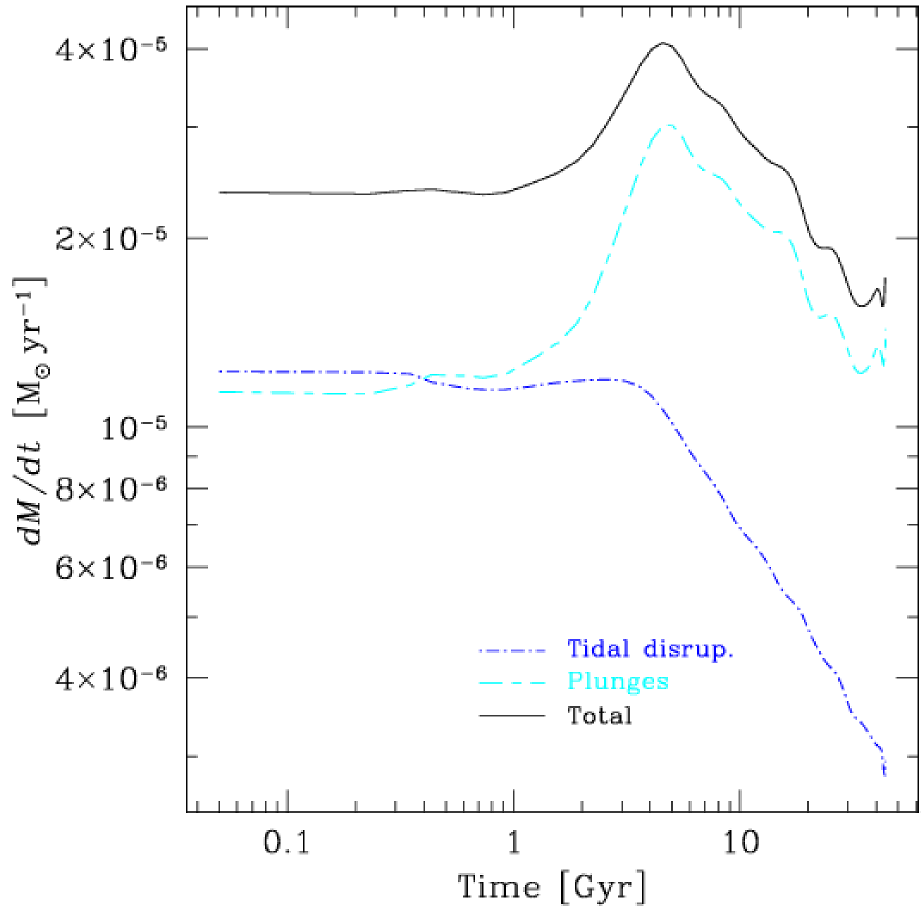
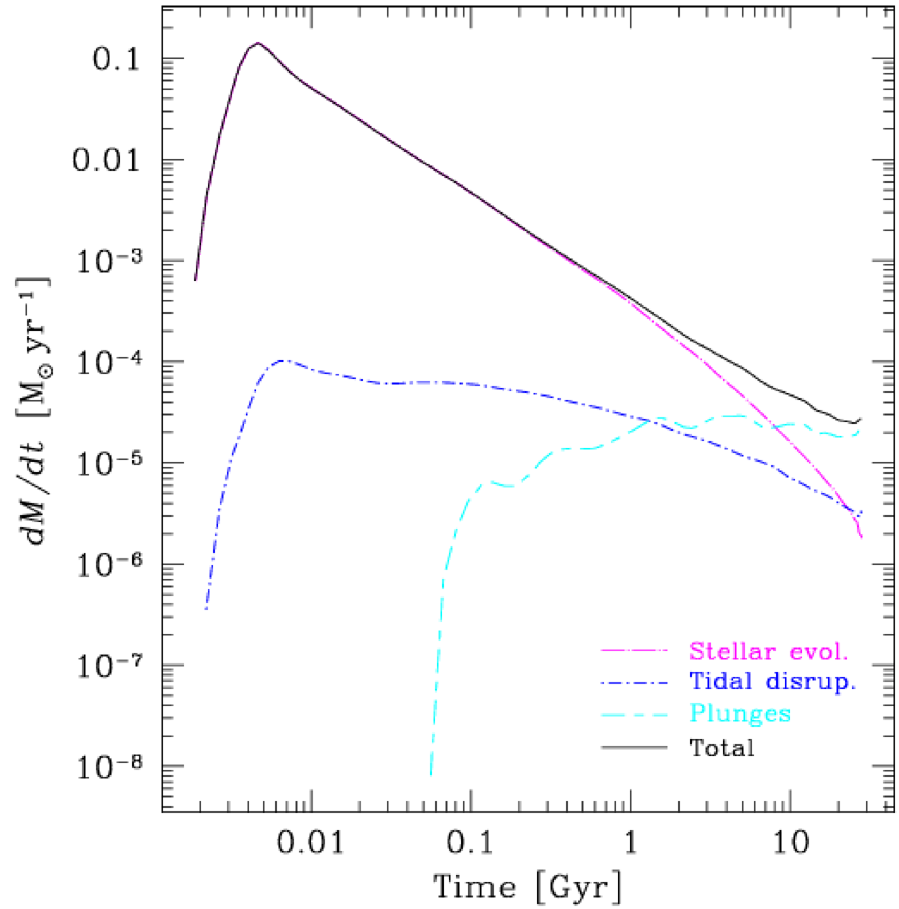
Stellar accretion onto MBH

Large initial MBH, no stellar evolution



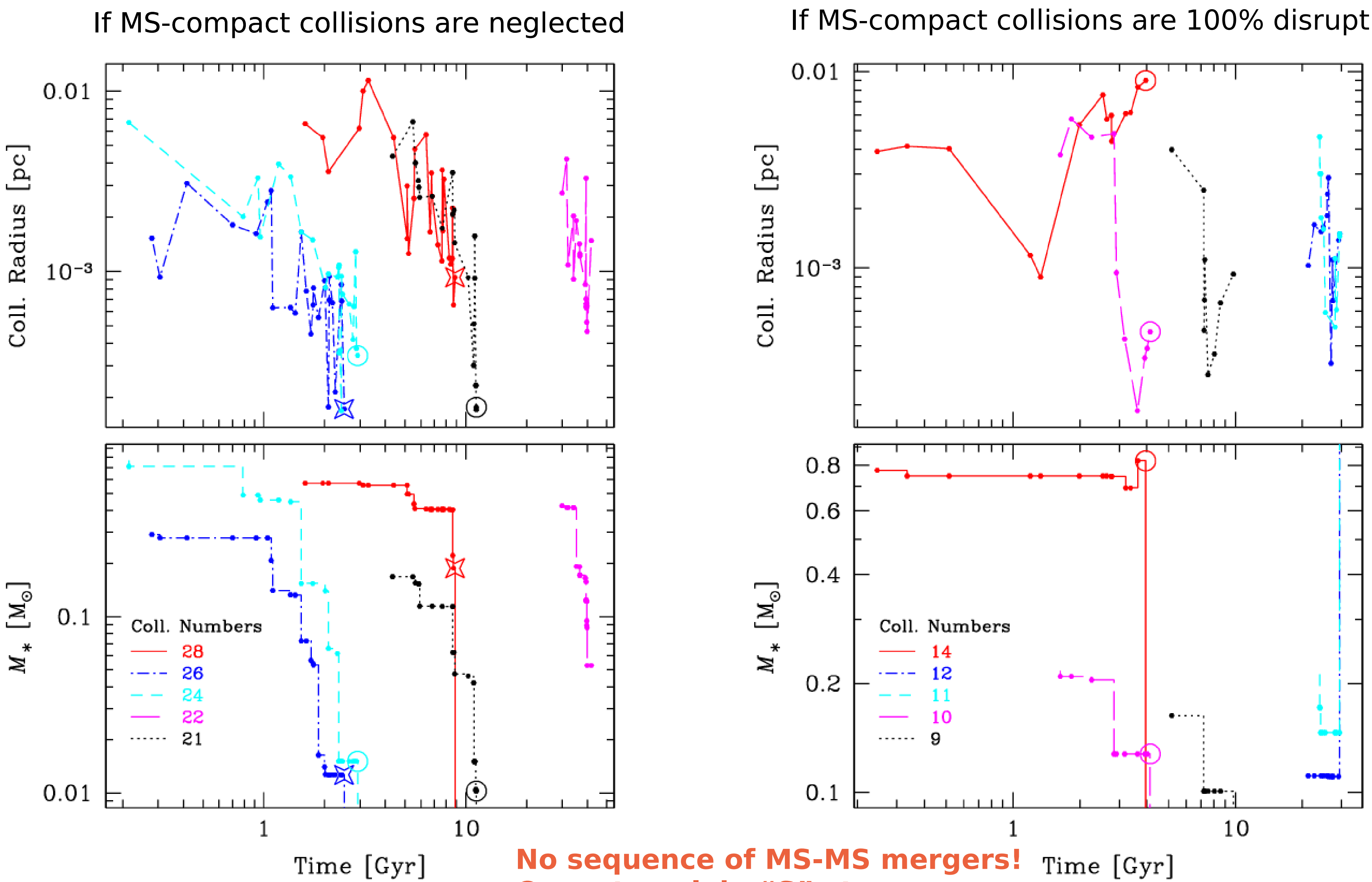
Small initial MBH, stellar evolution



Approx. same rates at 10 Gyr

Collisions in galactic nuclei

SgrA* model



No sequence of MS-MS mergers!
Cannot explain "S" stars

The possible future...

- Dynamics of isolated galactic nuclei
  - Systematic investigation of mass-segregation (in progress)
  - Survival and dynamical role of compact binaries
  - More work on collisions, tidal destructions/peeling (giants)
  - Resume work on captures for LISA
  - Use *N*-body methods when possible (tests, calibration)
- Galactic nuclei in a cosmological context
  - Use MC code to study shrinkage of binary MBH (loss-cone replenishment)
  - Combine MC simulations with cosmological merger trees

Develop new tools:

- ☆ Fast “external potential” MC code for cusp around (I)MBH
- ☆ Hybrid non-spherical MC/N-body code ??

Future application: Extreme Mass-Ratio Binaries for LISA

- Stellar mass object spiraling into 10^5-10^7 $M_\odot$ MBH
  - Only compact objects (extended stars disrupted early)
  - Stellar BH detectable to 3 Gpc
- EMRBs will allow "geo"desic mapping of space-time
  - Establishes MBH existence; measures mass and spin
- Theoretical difficulties are plenty! (Gair et al. 2004)
  - "Local" density of MBHs in LISA mass range
  - Rate of captures & "initial" orbital parameters
    - Literature: 10^{-8} – 10^{-4} yr^{-1} per galaxy

(Hils & Bender 95; Sigurdsson & Rees 97; Freitag 01, 03; Ivanov 02; Sigurdsson 03 [review]; Hopman & Alexander 05)

    - Controlled by 2-body relaxation
  - Orbital evolution & waveform calculation

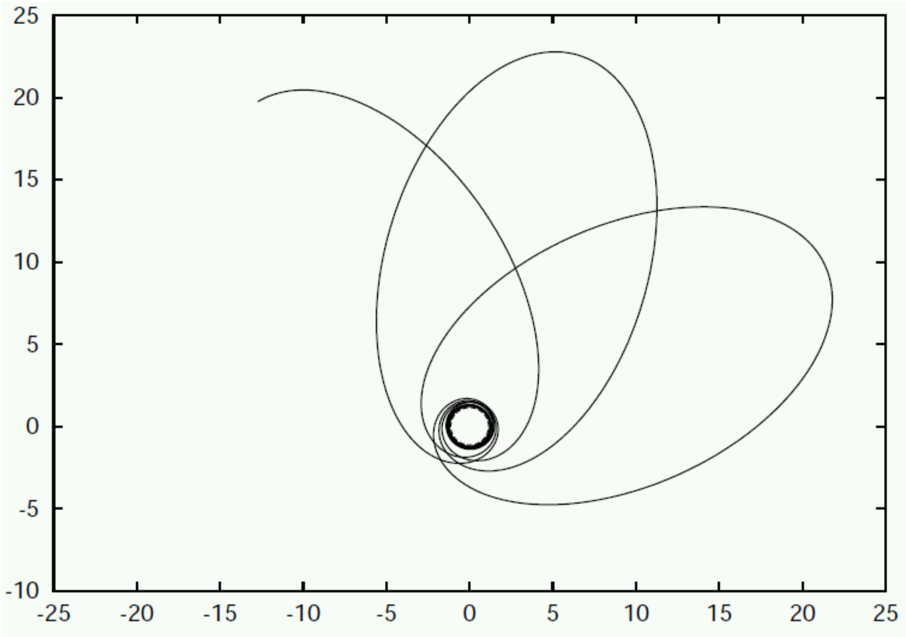
(Glampedakis & Kennefick 02; Glampedakis et al. 02; Lousto 05)

    - Full GR required; not done yet but m/M<<1 helps
    - "Zoomwhirl" orbits => complex GW signals
  - LISA signal processing; Detection strategies
    - Low S/N => match-filtering
    - High-D parameter space => exhaustive search impossible

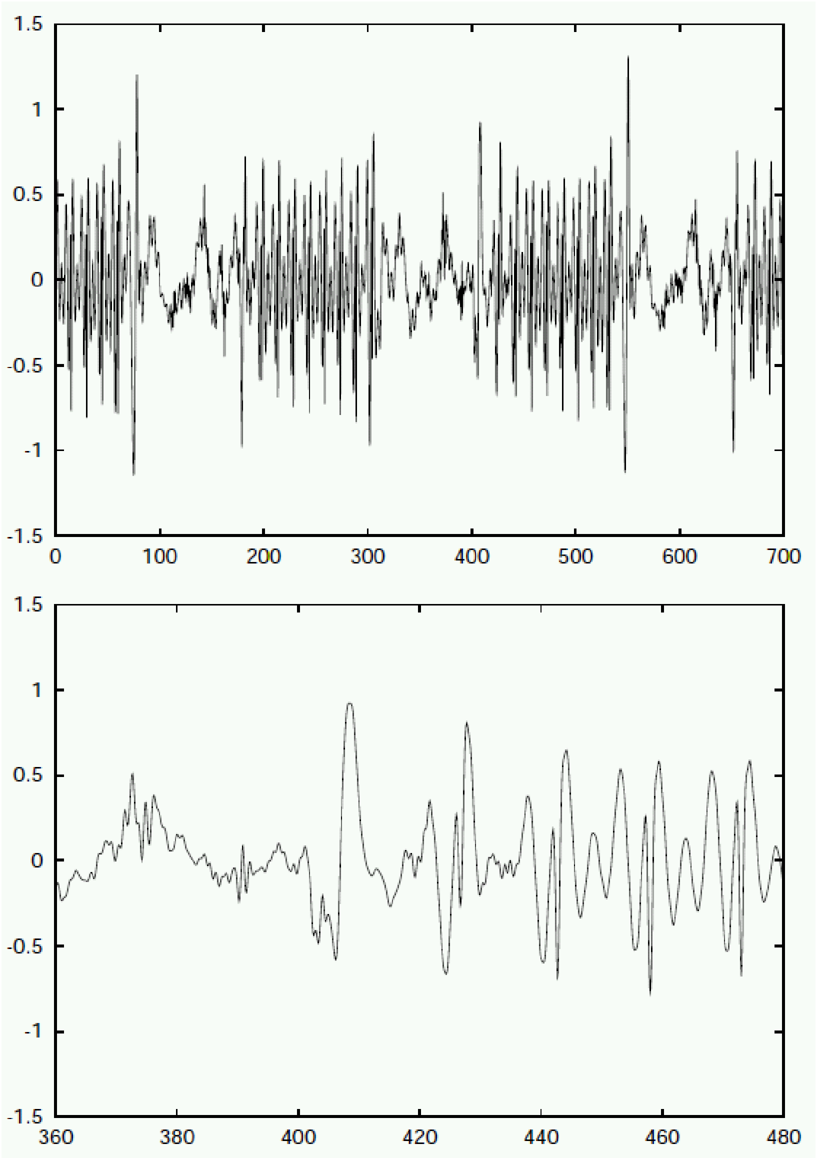
(Barack & Cutler 04; Gair & Wen 05;Wen & Gair 05)

Orbits around a Kerr MBH

Glampedakis et al. 2002



“Zoomwhirl” orbit



GW signal emitted by particle on“zoomwhirl” orbit

Predicting rates and orbital parameters of EMRB inspirals

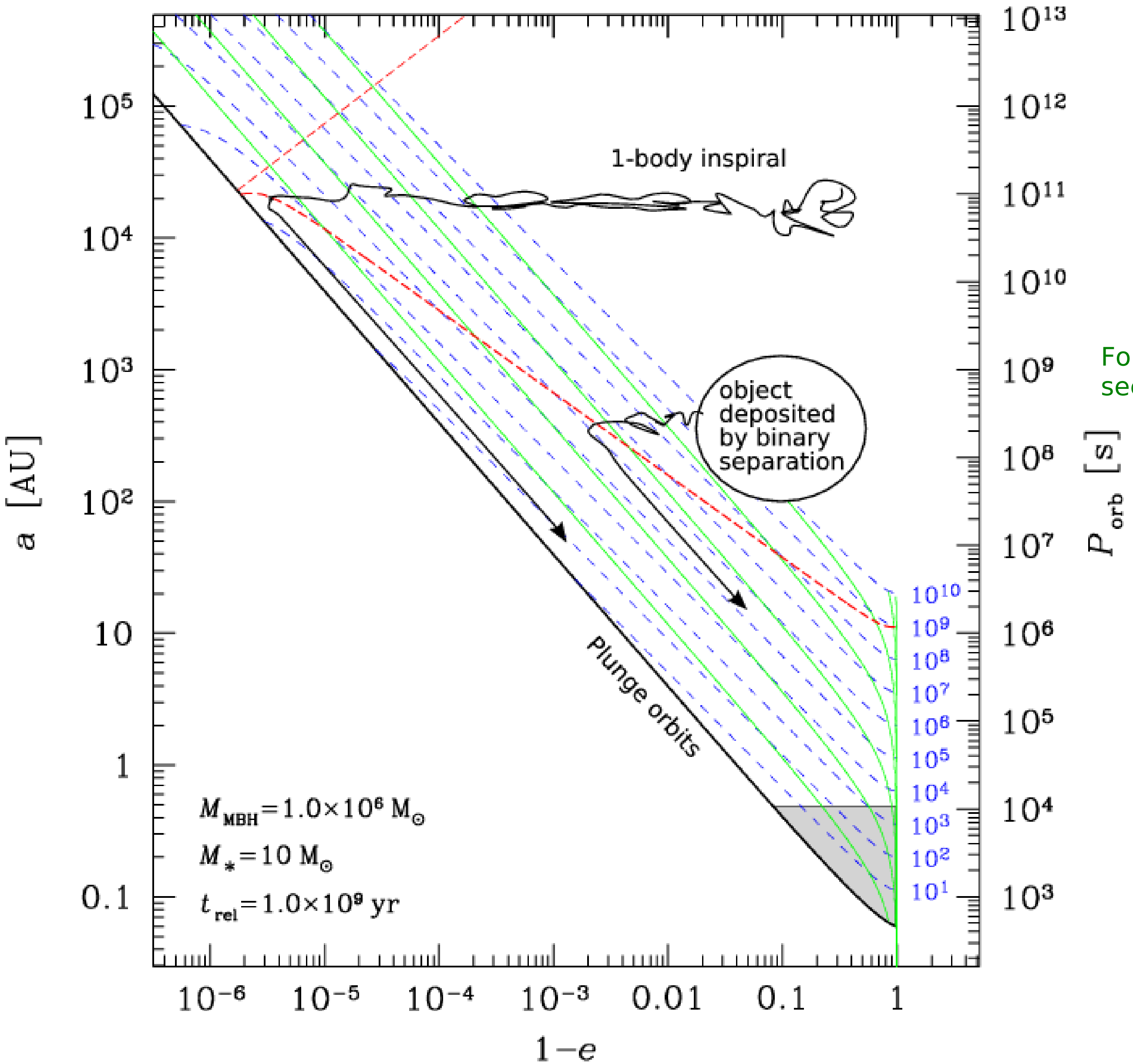
- Clean LISA inspiral requires $t_{\rm GW} < (<<?)\ t_{\rm rlx,peri}$

$$t_{\rm GW} \simeq \frac{2^{1/2}24}{85}\frac{c^5}{G^3 M_{\rm BH}^2 M_*}(1-e)^{7/2}a^4 \qquad t_{\rm rlx,peri} \approx (1-e)t_{\rm rlx}$$

$$\simeq 3.2\times10^6\,{\rm yrs}\cdot\left(\frac{M_{\rm BH}}{10^6\,{\rm M}_\odot}\right)^2\left(\frac{M_*}{{\rm M}_\odot}\right)^{-1}\left(\frac{R_{\rm p}}{10\,R_{\rm S}}\right)^4\left(\frac{1-e}{10^{-5}}\right)^{-1/2}$$

- Relaxation is key
  - Brings stars to capture orbits ☺
  - Can kick stars onto plunge orbits ☹
  - Brings stellar BHs to the center ☺
- EMRBs in the MC code: rough and ready... Freitag 2001, 2003
  - Star swallowed when $t_{\rm GW} < t_{\rm rlx,peri}$. Should look out for "premature" plunge (Hopman & Alexander 05)
  - $t_{\rm rlx,peri}$ estimated from last encounter (not orbit-$\oint$)
  - Approximate treatment of small-scale orbit diffusion
  - Inspiral and GW emission computed off-line from e and a at capture. MC simulation yield list of capture events (M_i,e_i,a_i)

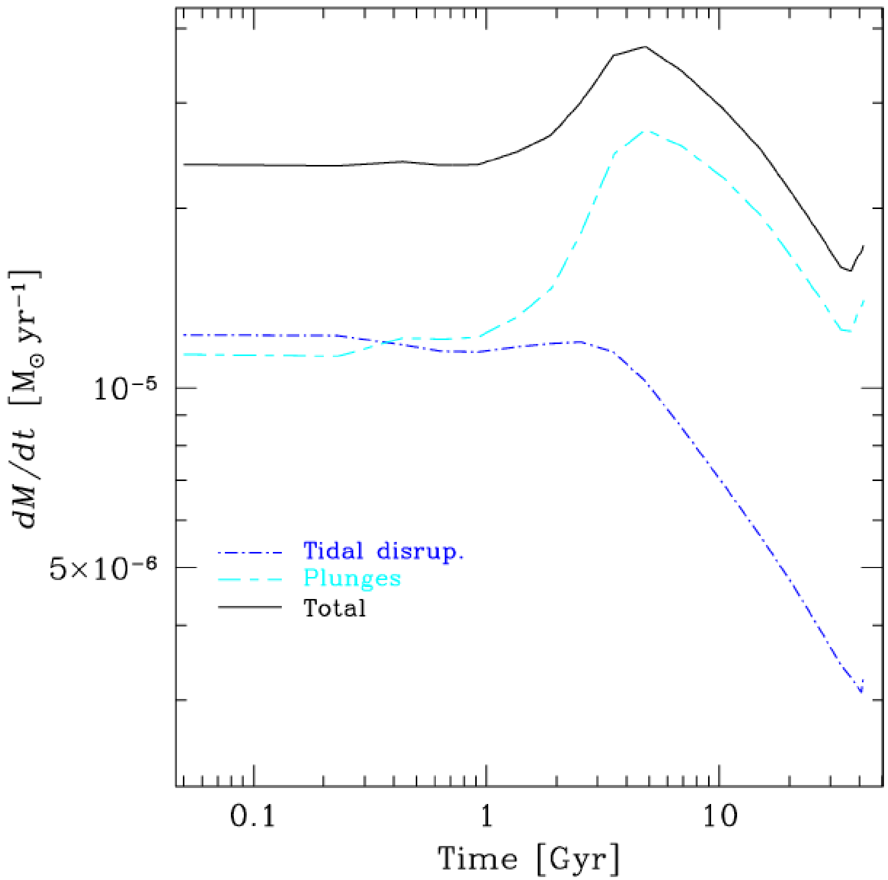
EMR Binary inspiral in (e,a) plane



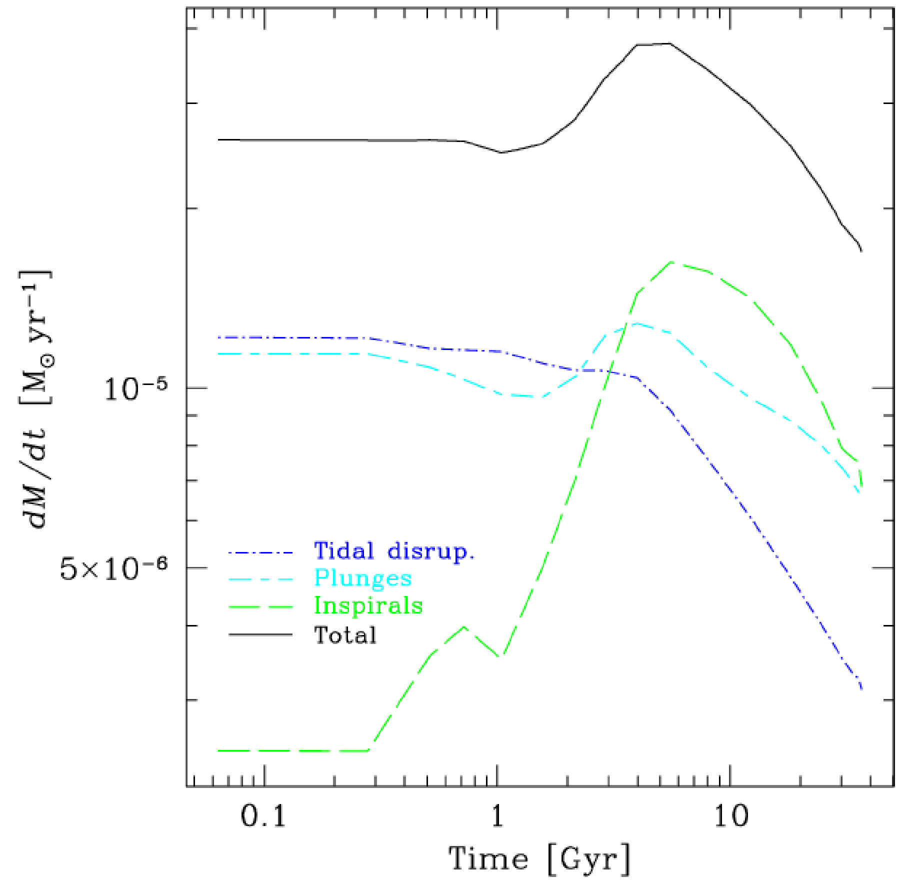
For possible contribution of “binary splitting”, see Miller et al. 2005

Rates of MBH-star mergers

If EMR inspirals are not allowed



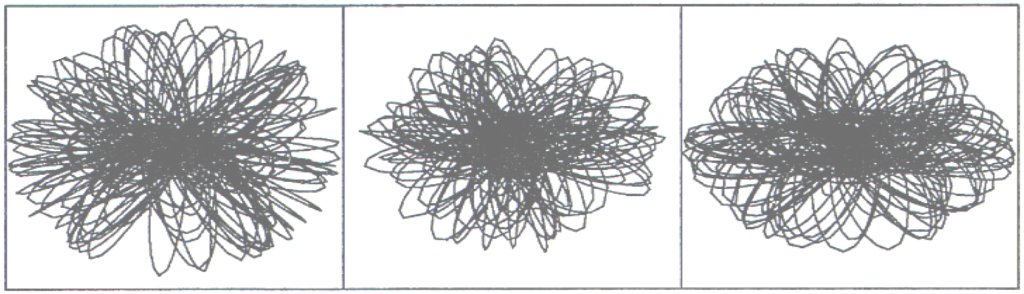
If EMR inspirals are allowed (naive treatment)



Key question: fraction of mergers in “LISA friendly” inspiral regime?

EMRB inspirals: Problems to address

- Very rare events: very high resolution required
- Relaxation plays a role on time scales << t_{rlx}
- LISA detection rates dominated by stellar BHs
  - Mass segregation is key
  - Role of natal kicks?
  - Role of large-angle scatterings (ejections from cusp)
- Effects of non-sphericity? (centrophilic orbits)

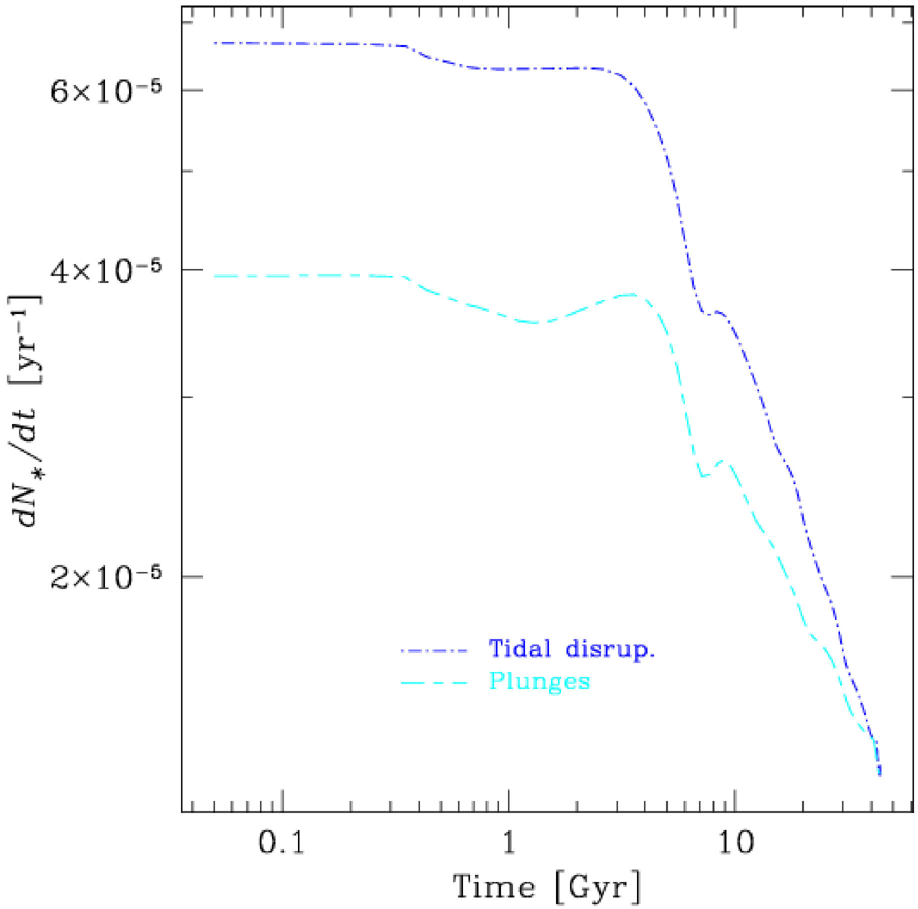


Poon & Merritt 01

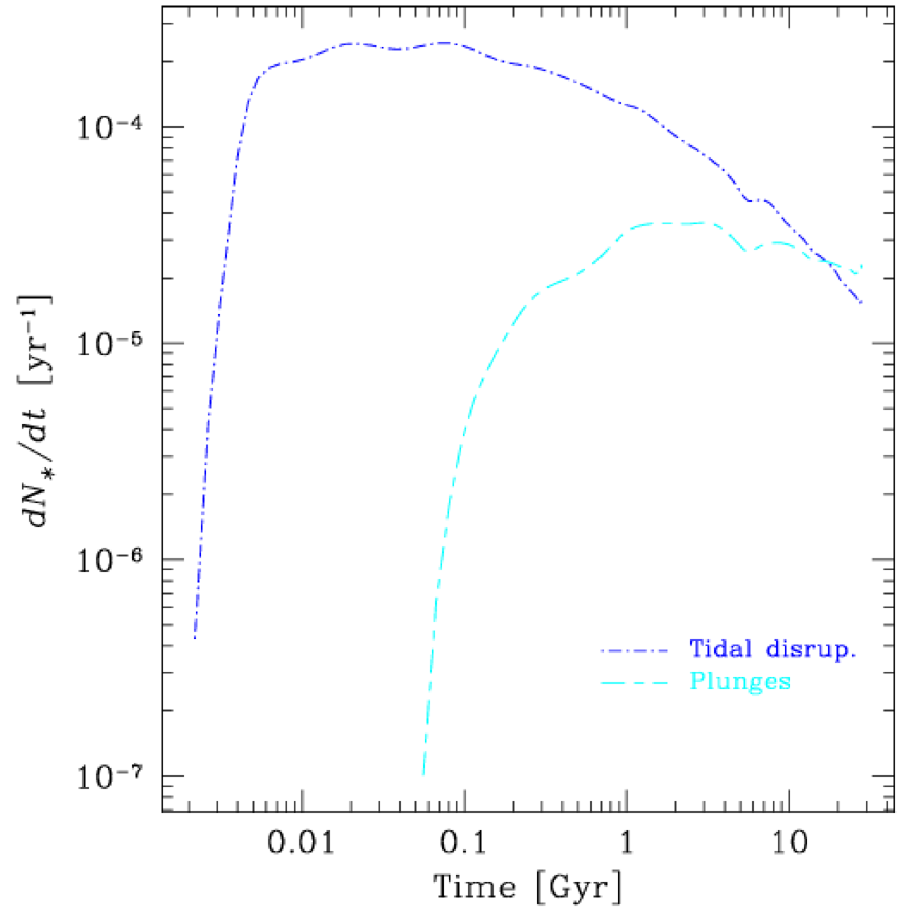
- Star interaction/formation with/in accretion disk

Event rates

Large initial MBH, no stellar evolution



Small initial MBH, stellar evolution



Approx. same rates at 10 Gyr

How does the MC work?

- Initialization
  - Realization of cluster with N particles according to DF $F(E)$ ↪ E_i, J_i, R_i
  - Attribution of masses M_i according to IMF

- Main loop (modifies 2 particles per step)

1) Selection of pair of neighboring particles $P_{\rm selec} \propto \delta t(R)^{-1}$
2) Test for collisions: rand() < $P_{\rm coll}$; modify $M_{1,2}$ & $V_{1,2}$ if needed
3) Relaxation simulated by "Super-encounter"

$$\theta_{\rm SE} = \frac{\pi}{2}\sqrt{\frac{\delta t}{t_{\rm rlx}}}$$

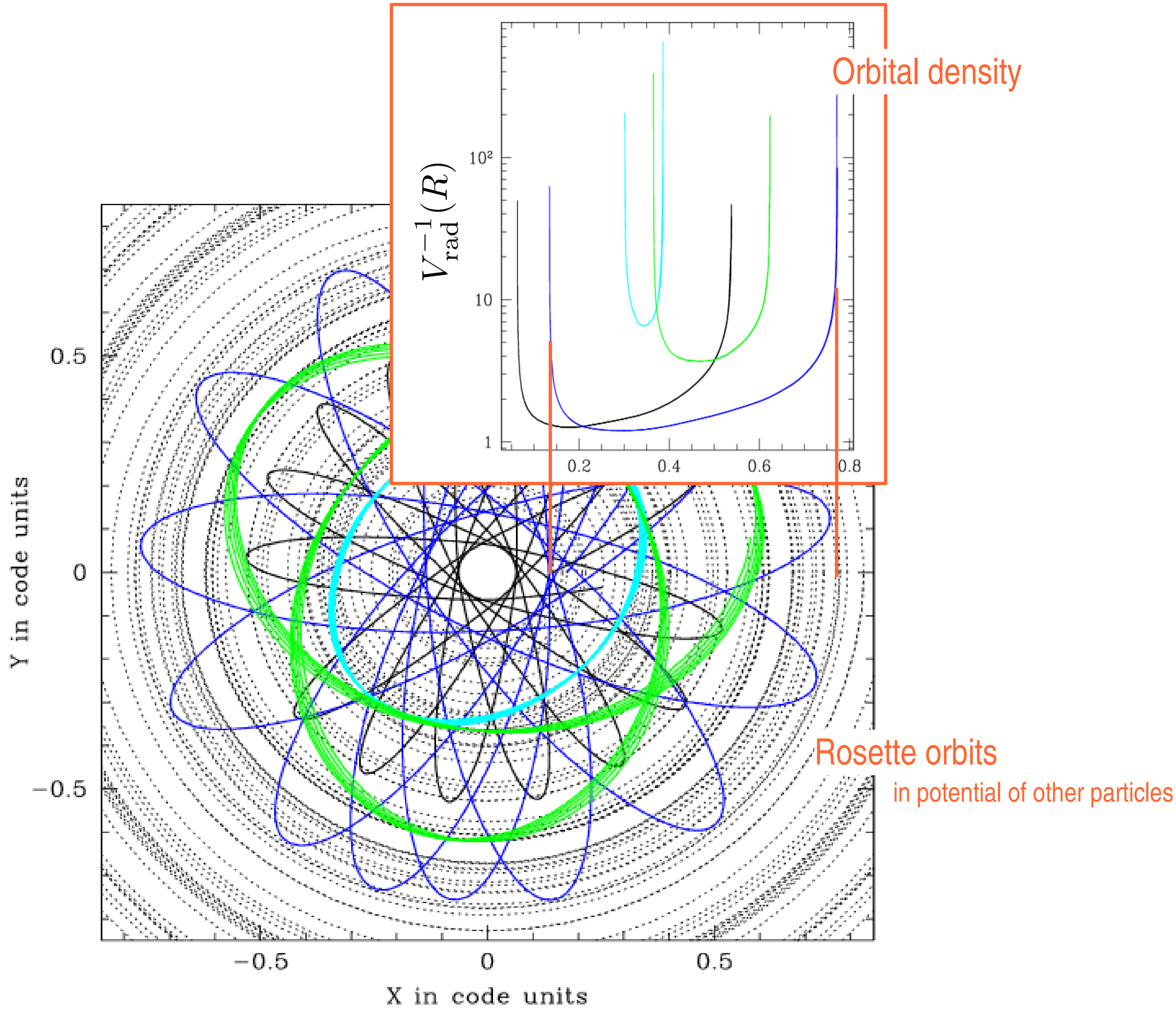
4) New orbital parameters $E_{1,2}$ & $J_{1,2}$ computed
5) For each particle, new position R_i picked at random on (E_i,J_i)-orbit
   Cluster's potential updated

$$\frac{dP}{dR}(R) \propto \frac{1}{V_{\rm r}(R)}$$

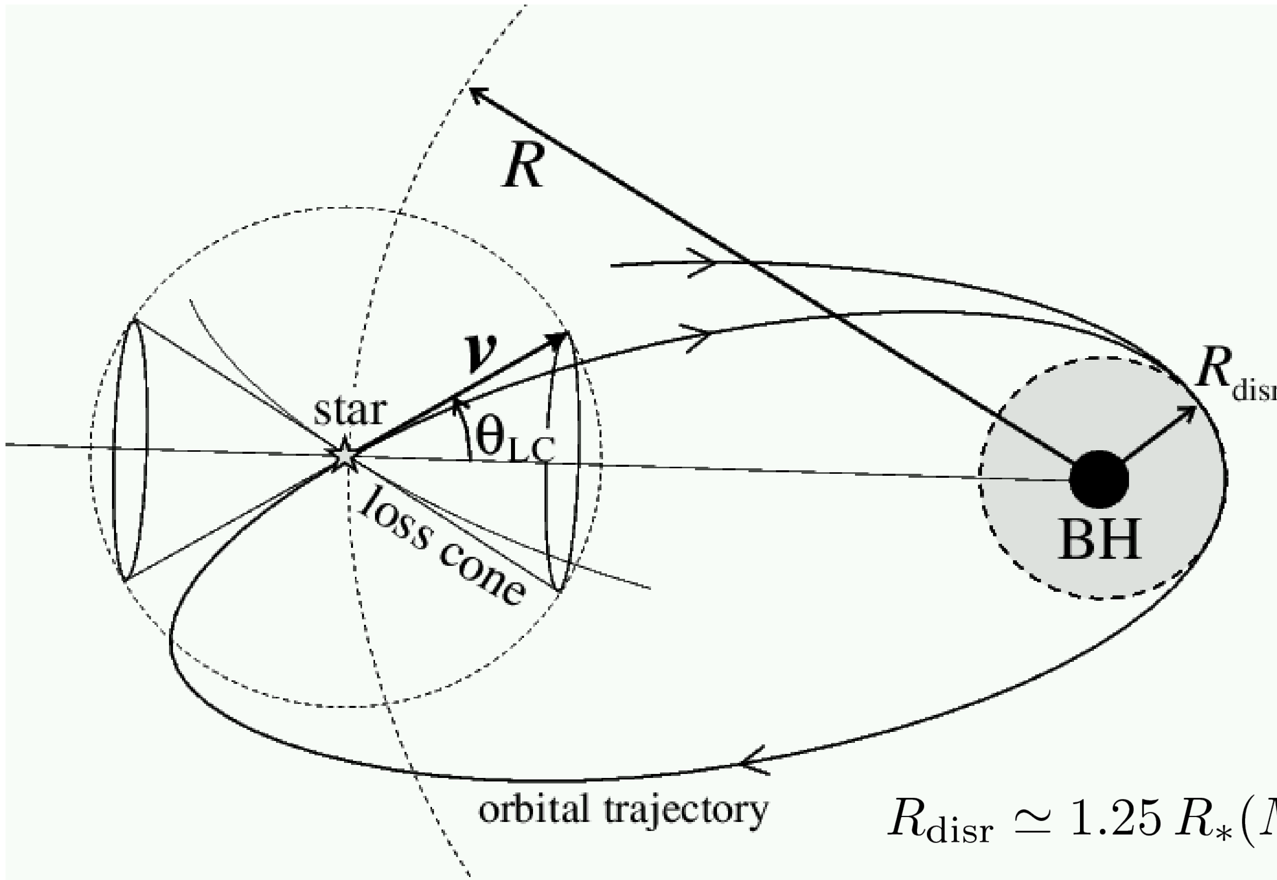
Go back to 1

And add many complications!...

Selection of particle position in MC code



Loss Cone



$$R_{\rm disr} \simeq 1.25\, R_*(M_{\rm BH}/M_*)^{1/3}$$

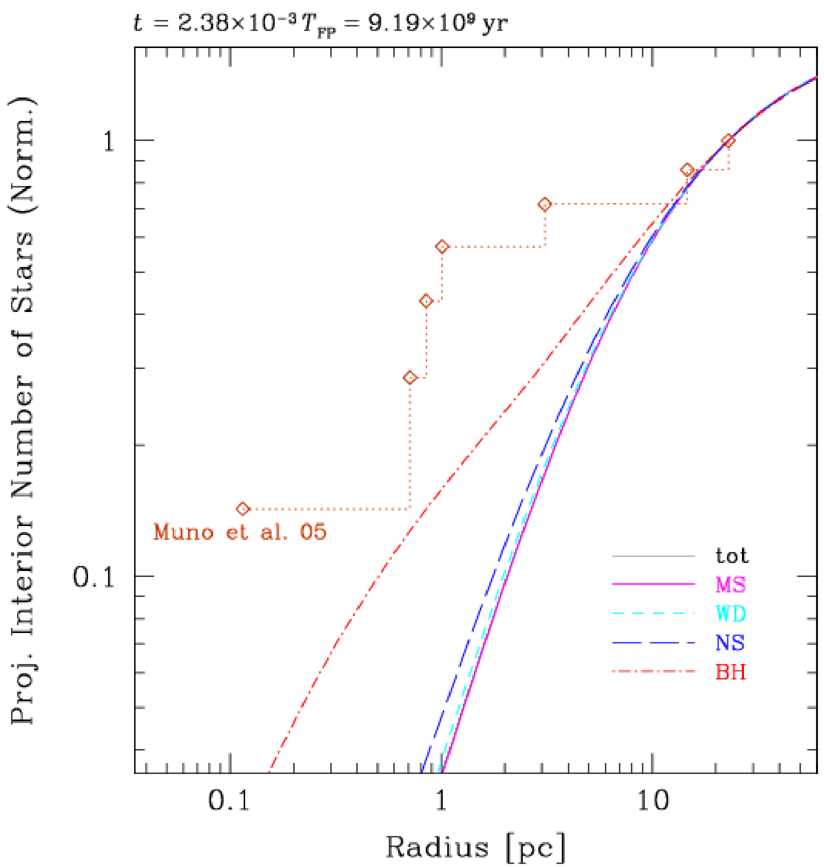
Loss cone apperture: $J < J_{\rm LC} \simeq \sqrt{2GM_{\rm BH}R_{\rm disr}}$

$$\theta_{\rm LC} \simeq \frac{J_{\rm LC}}{Rv} \approx \sqrt{\frac{R_{\rm disr}}{R}}$$

Evidence of segregation around SgrA* ?

- 7 transient X-ray sources within 25 pc (Muno et al. 2005)
  - 4/7 within 1 pc of projected distance
  - Probably LMXBs with NS or BH accretor

Comparison of cumulative numbers with MC simulation



Central concentration through passive segregation of BHs not excluded (!) but...

Sources probably formed through 3-body effects.
Need to take binary dynamics into account.

$$\left.\frac{dn}{dt}\right|_{\rm exchange} \propto n_{\rm bin}\, n_{\rm CO}\, \sigma\, \Sigma$$

Collisions in galactic nuclei

SgrA* model

Collisions treated thanks to table of SPH simulations (Freitag & Benz 2005)