astro8405

An Introduction to the
Cosmic Microwave Background

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astro8405: The Cosmic Microwave Background

Aktionen ▾

This course intends to give you a modern and up-to-date introduction to the science and experimental techniques relating to the Cosmic Microwave Background. No prior knowledge of cosmology is necessary, your prerequisite are a basic understanding of electrodynamics and thermal physics and some familiarity with Python programming.

Lecture 13:

Sunyaev–Zeldovich Effect
(Part I)

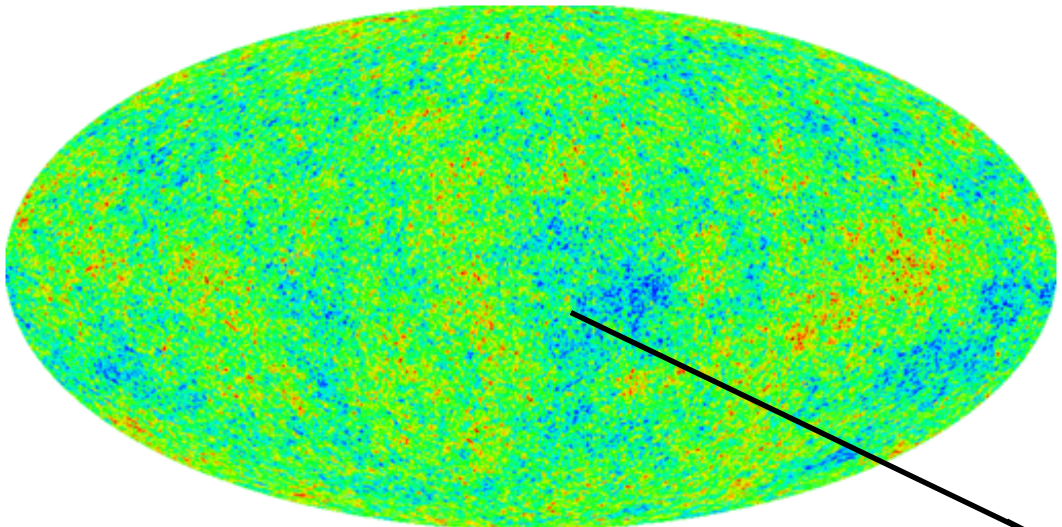
SECONDARY temperature anisotropies

From the 1995 review by Max Tegmark

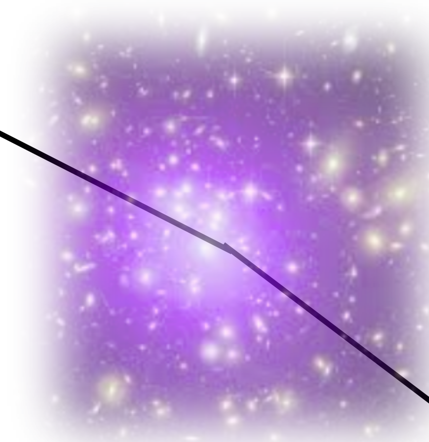
Table 1. Sources of temperature fluctuations.

PRIMARY	Gravity	
	Doppler	
	Density fluctuations	
	Damping	
	Defects	Strings
		Textures
SECONDARY	Gravity	Early ISW
		Late ISW
		Rees-Sciama
		Lensing
	Local reionization	Thermal SZ
		Kinematic SZ
	Global reionization	Suppression
		New Doppler
		Vishniac
"TERTIARY"	Extragalactic	Radio point sources
		IR point sources
(foregrounds & headaches)	Galactic	Dust
		Free-free
		Synchrotron
	Local	Solar system
		Atmosphere
		Noise, *etc.*

CMB photons on their way to us



Last scattering surface

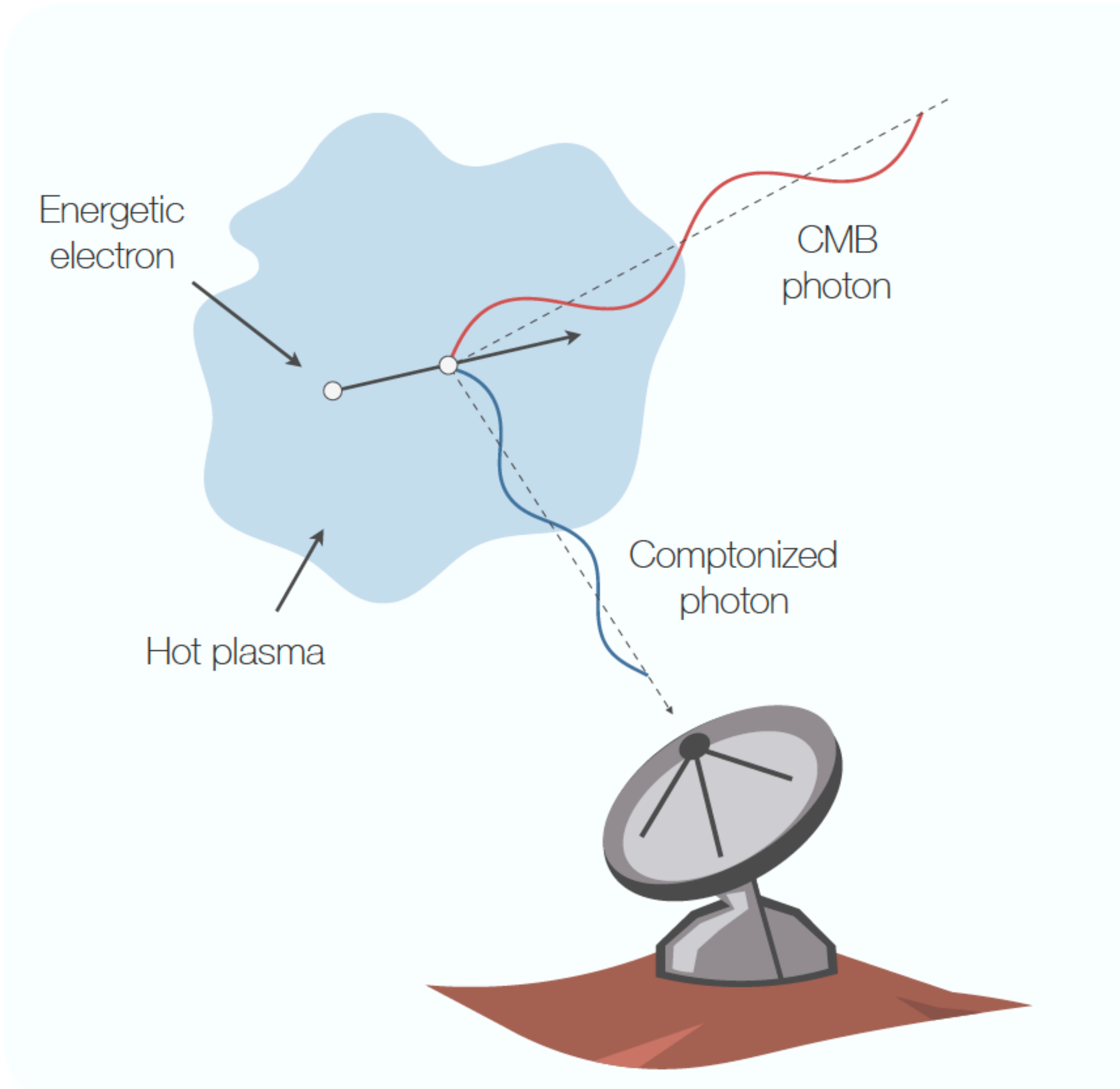


Observer

Two things can happen to the CMB photons:

1. ***they are deflected by gravitational potentials***
2. ***they get scattered off electrons and atoms***

Scattering of CMB by free electrons



Mroczkowski, Nagai, Basu, Chluba et al. (2019)

Scattering of an isotropic radiation (as in CMB) by stationary electrons produces no net effect, because as many photons are lost from the line of sight as are gained.

However, if the electrons are in motion with respect to the CMB rest frame, they will impart some of their kinetic energy to the photons, which will lead to a spectral distortion. This is the **Sunyaev-Zeldovich (SZ) effect**. This is nothing but the ***Compton-y*** distortion in the single-scattering limit (i.e. $y << 1$).

The spectral distortion shape will depend on the velocity distribution of the electrons, e.g., thermal electrons will have Maxwell-Boltzmann velocity distribution, which leads to the unique spectrum of the ***thermal*** SZ effect. Electrons with nonthermal velocities or bulk velocities will leave their own distinct spectral signatures.

Recap: Compton-y distortion

Compton-y distortion is created when scattering between electrons and photons are inefficient in causing an energy exchange. This is typically the case when the electron and photon temperatures are vastly different, so that electrons practically don't change energy after scattering. The energy exchange is parametrized by the Compton y-parameter, and we typically have $y \ll 1$.

The Kompaneets equation (right) is the general equation describing the Comptonization problem, which can be solved analytically for the limiting case of a small y-parameter ($\Delta\tau \ll 1$):

$$\frac{\partial n}{\partial \tau} \equiv \frac{\theta_e}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial}{\partial x}n + \frac{T_\gamma}{T_e}\,n(1+n)\right],$$

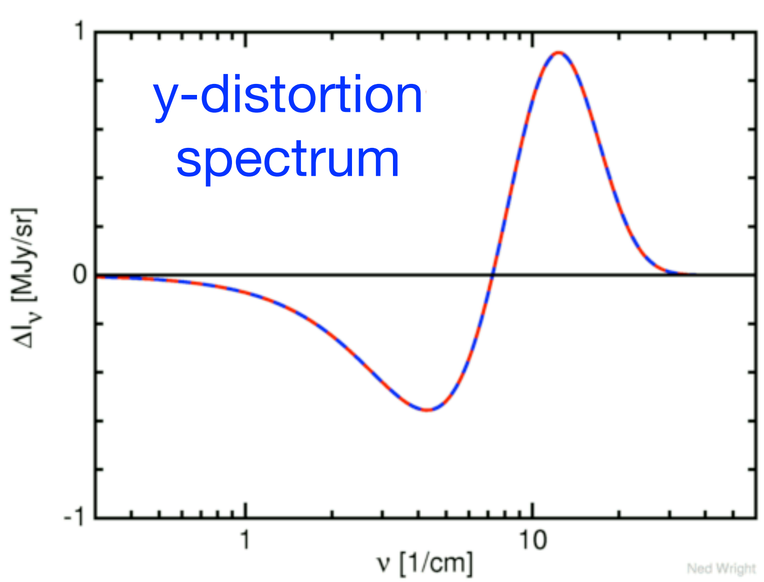
$$\Delta n \approx \frac{\Delta\tau\,\theta_e}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial}{\partial x}n_{bb} + \frac{T_\gamma}{T_e}\,n_{bb}(1+n_{bb})\right] \approx \frac{\Delta\tau\,(\theta_\gamma-\theta_e)}{x^2}\frac{\partial}{\partial x}x^4 n_{bb}(1+n_{bb})$$

$$\approx \Delta\tau\,(\theta_\gamma-\theta_e)\left[4xn_{bb}(1+n_{bb}) - x^2 n_{bb}(1+n_{bb})(1+2n_{bb})\right]$$

$$\approx \Delta\tau\,(\theta_e-\theta_\gamma)\,G(x)\left[x\,\frac{e^x+1}{e^x-1}-4\right] \equiv \Delta\tau\,(\theta_e-\theta_\gamma)\,Y_{SZ}(x),$$

$$G(x) \equiv xe^x/(e^x-1)^2$$

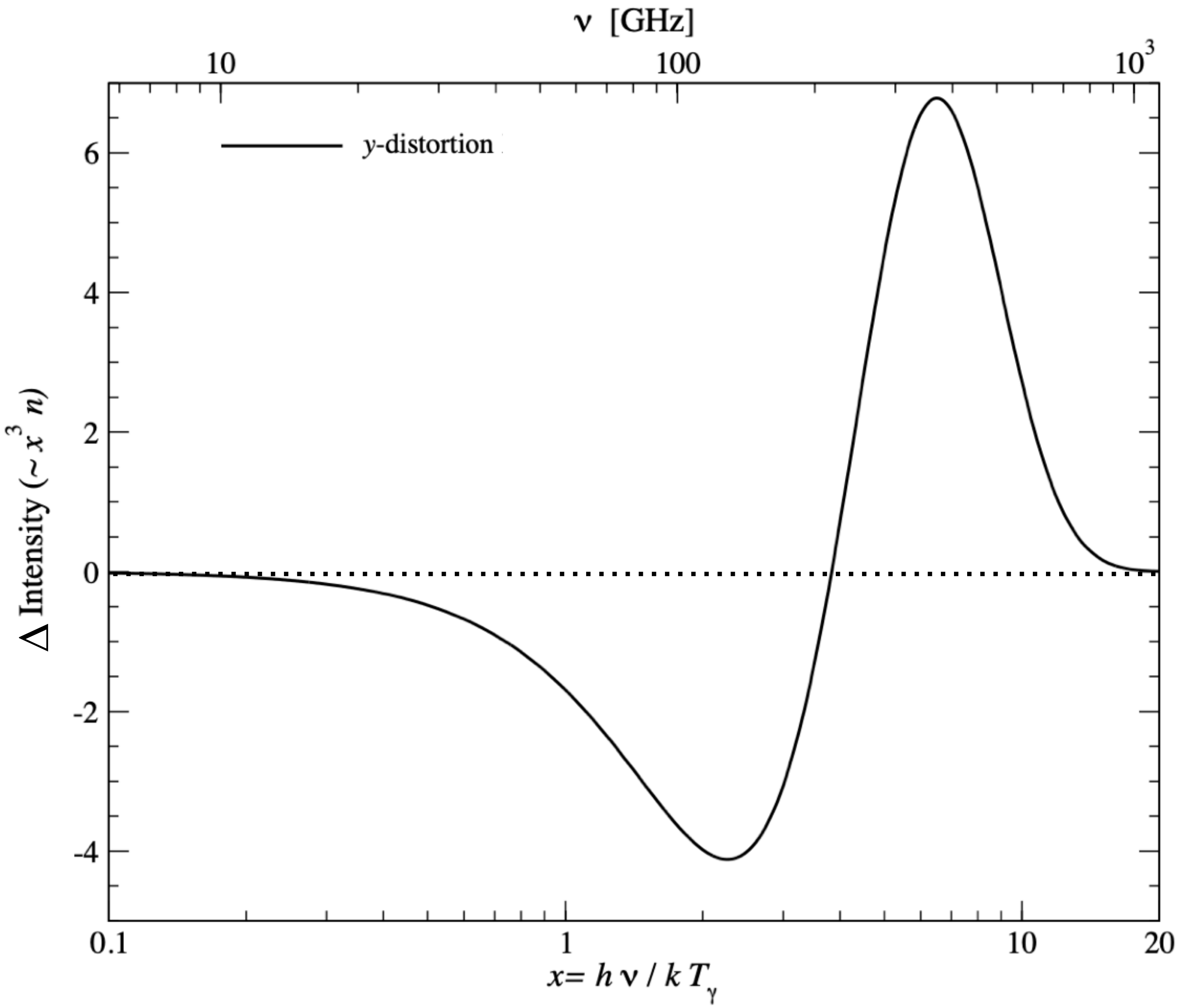
$$Y_{SZ}(x) = G(x)\left[x\,\frac{e^x+1}{e^x-1}-4\right] \approx \begin{cases} -\frac{2}{x} & \text{for} \quad x \ll 1 \\ x(x-4)e^{-x} & \text{for} \quad x \gg 1. \end{cases}$$



Recap: y–distortion & the SZ effect

The Compton y-parameter depends on the number of scatterings (dependence on optical depth, τ) and the net energy transfer per scattering, $\Delta\nu/\nu \simeq 4(\theta_e - \theta_\gamma)$,

$$y = \int_0^\tau \frac{k(T_e - T_\gamma)}{m_e c^2}\, d\tau' = \int_0^t \frac{k(T_e - T_\gamma)}{m_e c^2} \sigma_T N_e c\, dt',$$



In the local universe we have $T_\gamma << T_e$ such that the Compton y-parameter is simply proportional to the line of sight integral of the electron pressure ($y << 1$):

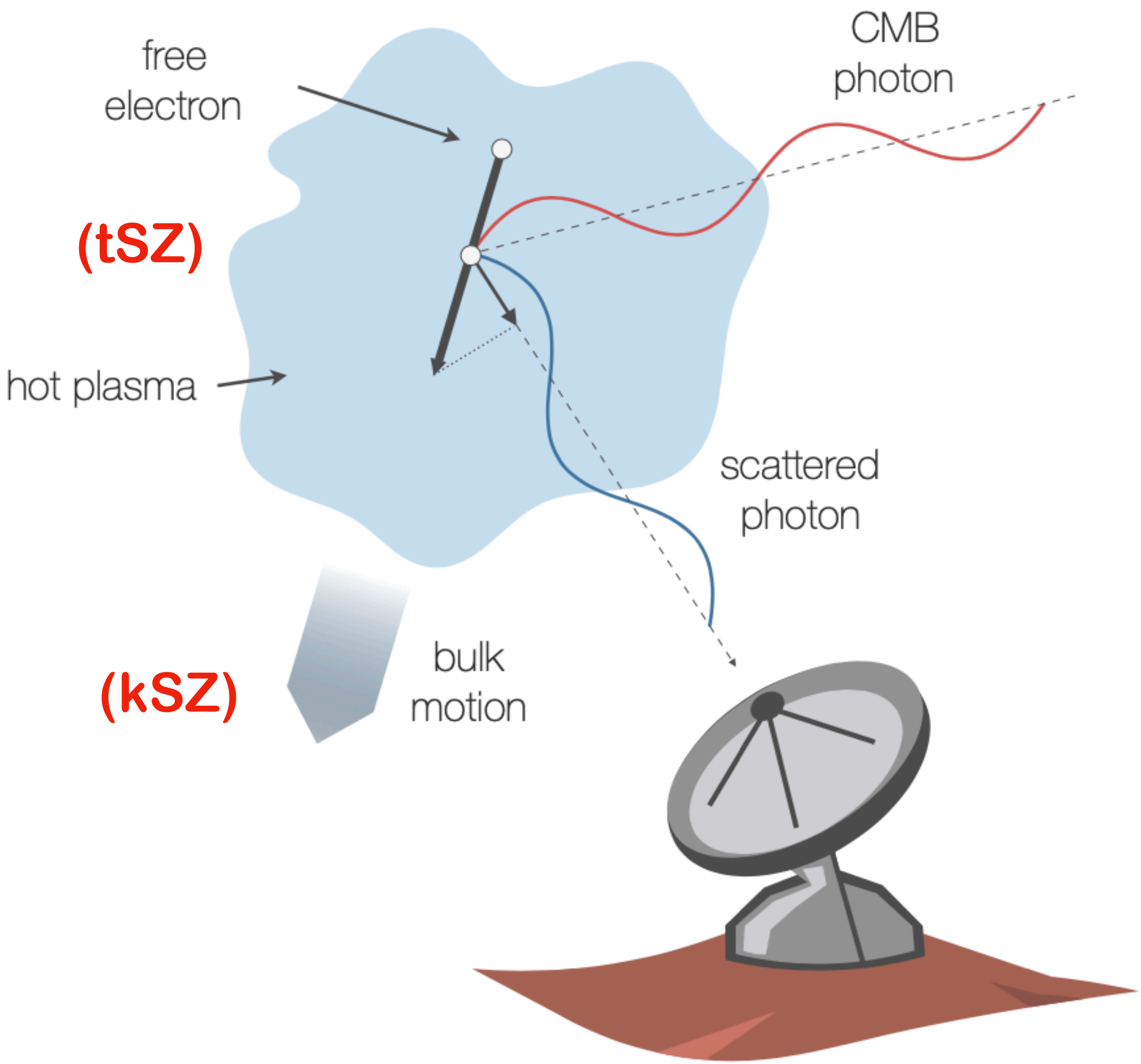
$$y = \int_0^\tau \frac{kT_e}{m_e c^2}\, d\tau' \approx \theta_e \tau$$

This is the thermal Sunyaev-Zeldovich effect, first studied by Sunyaev & Zeldovich (1968), for the scattering of CMB photons by thermal electrons inside galaxy clusters.

$$\boxed{\frac{\Delta T_{\mathrm{CMB}}}{T_{\mathrm{CMB}}} \approx y\left(x\,\frac{e^x+1}{e^x-1} - 4\right) = y\, f(x).}$$

$$\Delta I_\nu \approx I_0\, y \frac{x^4 e^x}{(e^x-1)^2}\left(x\,\frac{e^x+1}{e^x-1} - 4\right) \equiv I_0\, y\, g(x)$$

Single IC scattering of CMB photons



Mroczkowski, Nagai, Basu, Chluba et al. (2019)

Single IC scattering of CMB photons

The physics of the SZ effect is very simple. Moving electrons transfer some of their kinetic energy to the low-energy CMB photons via the Doppler effect. This is the well-known formula for the energy ratio of photons after Compton scattering, when $h\nu \ll \gamma m_e c^2$

$$\frac{\nu'}{\nu} = \frac{1-\beta\mu}{1-\beta\mu' + \frac{h\nu}{\gamma m_e c^2}(1-\mu_{sc})} \approx \frac{1-\beta\mu}{1-\beta\mu'}.$$

Here $\beta = v/c$ is the speed of the scattering electron with Lorentz factor $\gamma = 1/\sqrt{1-\beta^2}$ in units of the speed of light, c; m_e is the electron mass; h is the Planck constant; μ and μ' are respectively the direction cosines of the incoming and scattered photon with respect to the incoming electron; and μ_{sc} is the corresponding direction cosine between the incoming and scattered photons.

In single-scattering events, with electron-speeds drawn from an isotropic velocity distribution, there is no net effect in the first order, as the gains and losses average out to leading order, leaving **second and higher order terms.** The average energy gained by a CMB photon in each scattering is determined by $\Delta\nu/\nu = (4/3)\,\beta^2 = 4k_BT_e / m_ec^2$.

$$\frac{\nu'}{\nu} \approx \frac{1-\beta\mu}{1-\beta\mu'} \overset{\beta\ll 1}{\approx} 1 - \beta(\mu-\mu') - \beta^2(\mu-\mu')\mu' + O(\beta^3).$$

$$\boxed{\langle\beta^2\rangle = 3kT_e/m_ec^2}$$

(for Maxwell-Boltzmann)

The original papers

THE INTERACTION OF MATTER AND RADIATION IN A HOT-MODEL UNIVERSE*

YA. B. ZELDOVICH and R. A. SUNYAEV
Institute of Applied Mathematics, U.S.S.R. Academy of Sciences, Moscow, U.S.S.R.

Published July 1969 in Astrophys. & Space Science

$$\frac{\partial n}{\partial y} = x^{-2} \frac{\partial}{\partial x} x^4 \cdot \frac{\partial n}{\partial x}$$

$$y = -\int_{t_0}^{t} \frac{\kappa T_e}{m_e c^2} n_e \sigma_0 c \mathrm{d}t = \int_0^{\tau} \frac{\kappa T_e}{m_e c^2} \mathrm{d}\tau \,,$$

$$\frac{\Delta n}{n_0} = \frac{\Delta J}{J_0} = xy \frac{e^x}{e^x - 1} \left\{ \frac{x}{\tanh(x/2)} - 4 \right\}.$$

The original papers

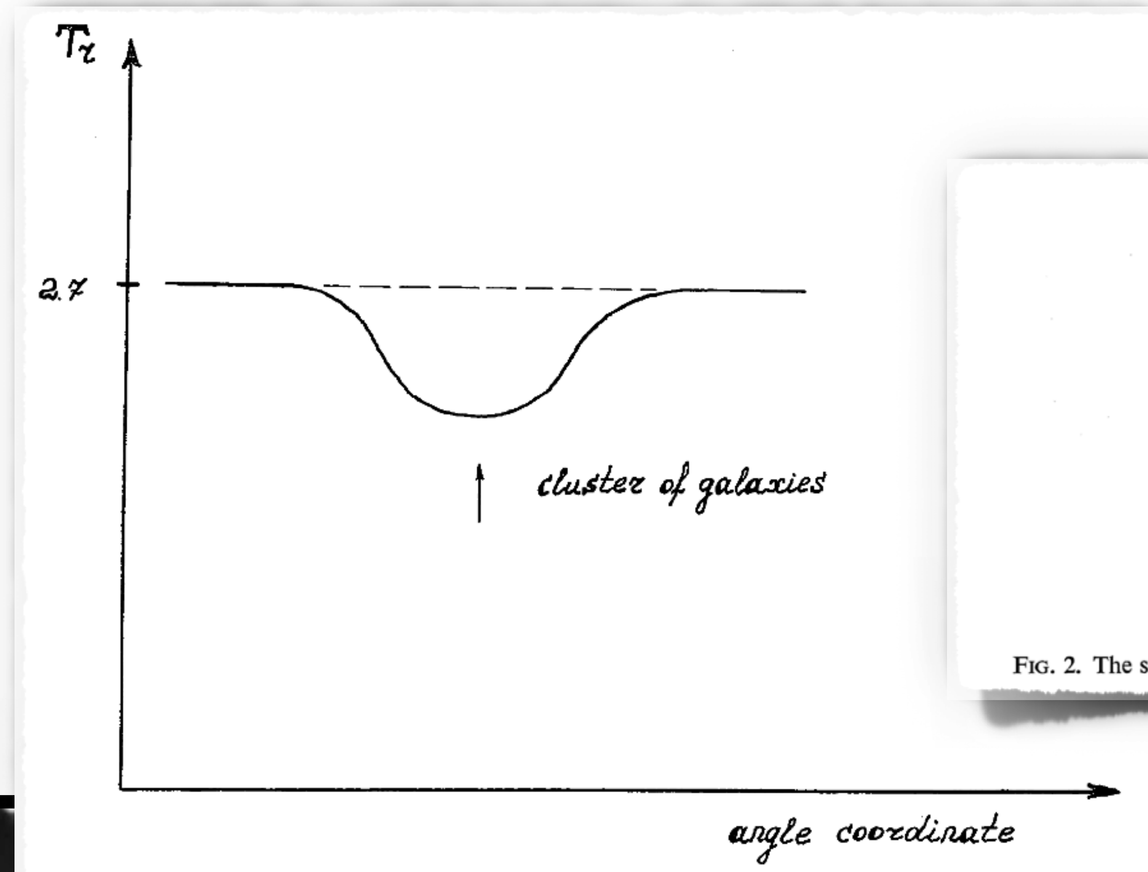


FIG. 1. The "hole" in the microwave background.

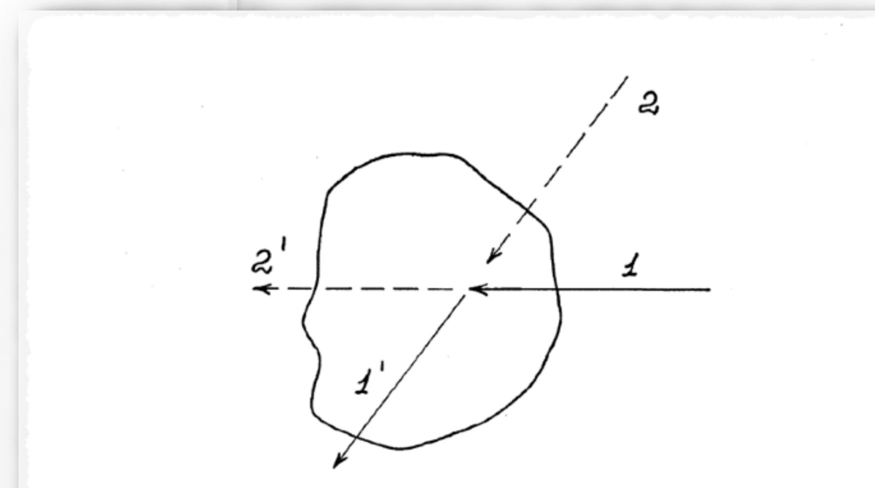


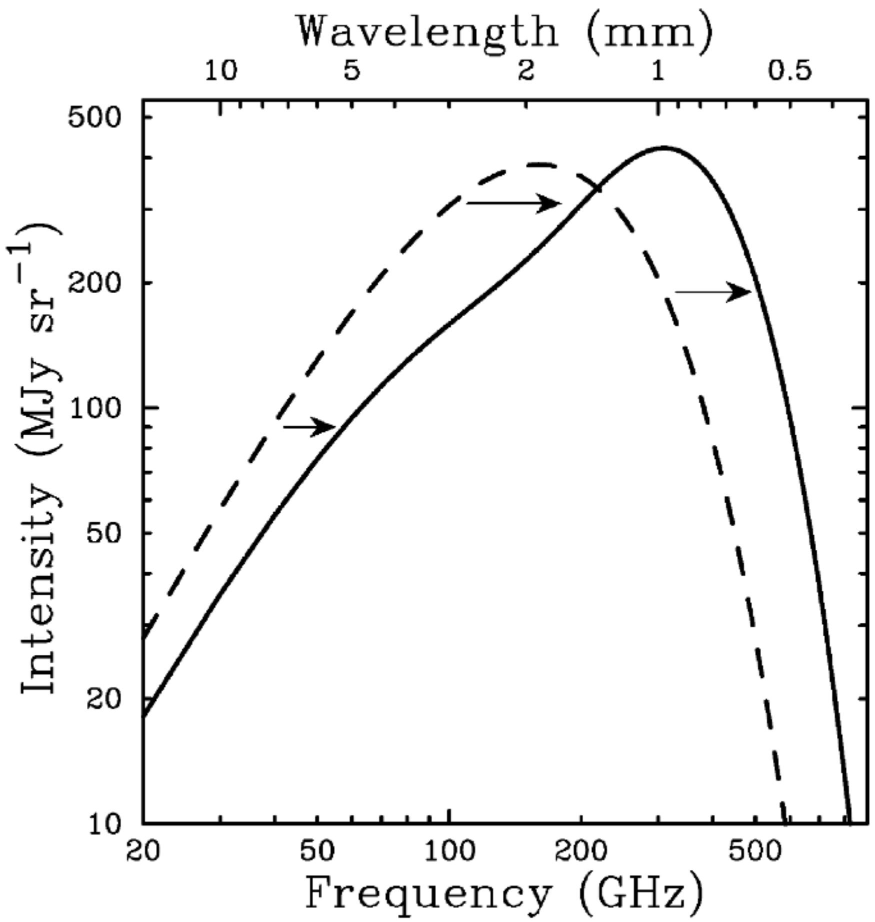
FIG. 2. The scattering of isotropic radiation field by the cloud of electrons.

The Observation of Relic Radiation as a Test of the Nature of X-Ray Radiation from the Clusters of Galaxies

R. A. Sunyaev & Ya. B. Zeldovich

Comments on Astrophysics & Space Physics, 1972

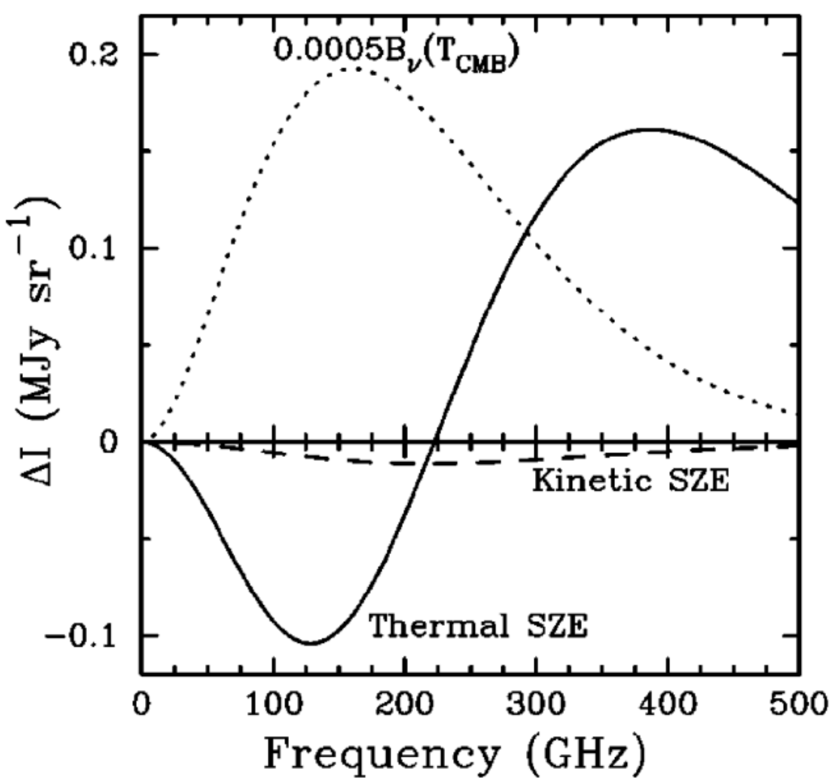
The thermal SZ effect

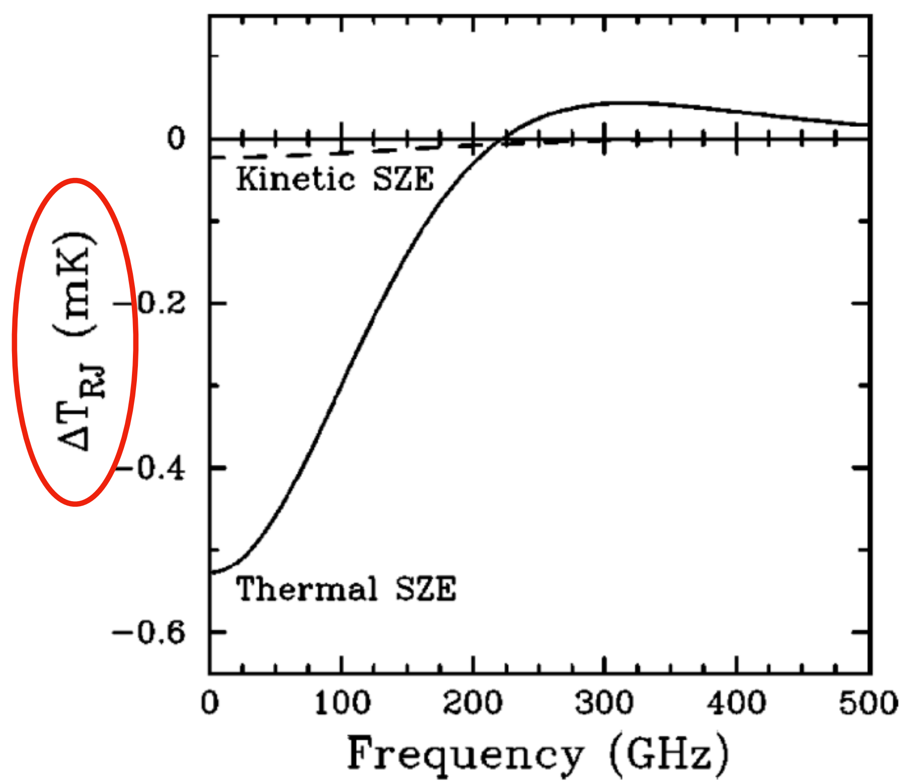


This is the effect caused by thermal (Maxwell-Boltzmann) motion of the electrons, leading to the β^2 effect. The following two well-known expression describe this “non-relativistic” SZ effect, since for a full relativistic description (see later for rSZ effect), the Maxwell-Jüttner distribution needs to be used.

$$\frac{\Delta T_{\rm CMB}}{T_{\rm CMB}} \approx y\left(x\,\frac{e^x+1}{e^x-1}-4\right) = y\,f(x).$$

$$\Delta I_\nu \approx I_0\,y\,\frac{x^4 e^x}{(e^x-1)^2}\left(x\,\frac{e^x+1}{e^x-1}-4\right) \equiv I_0\,y\,g(x)$$





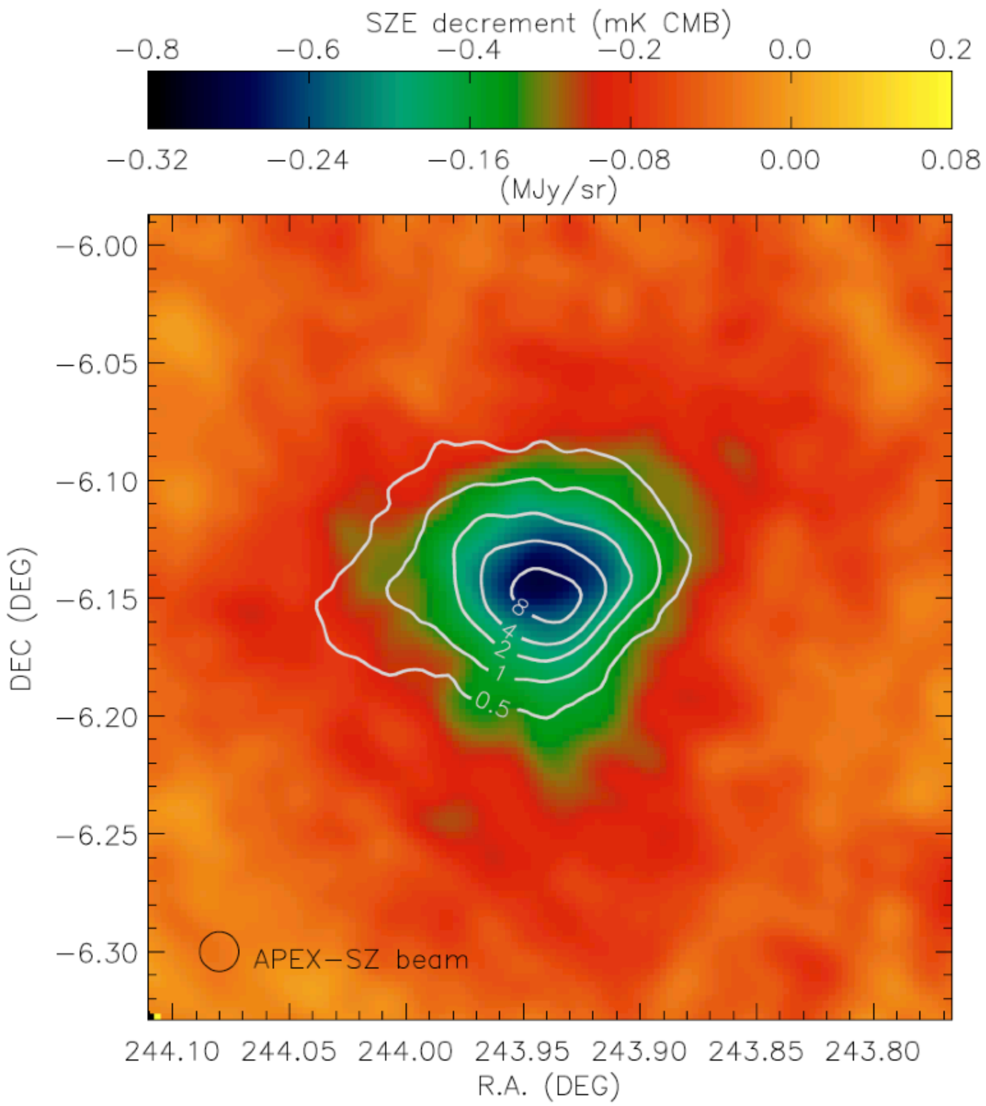
$x \equiv \frac{h\nu}{k_B T_{CMB}}$ is the dimensionless frequency

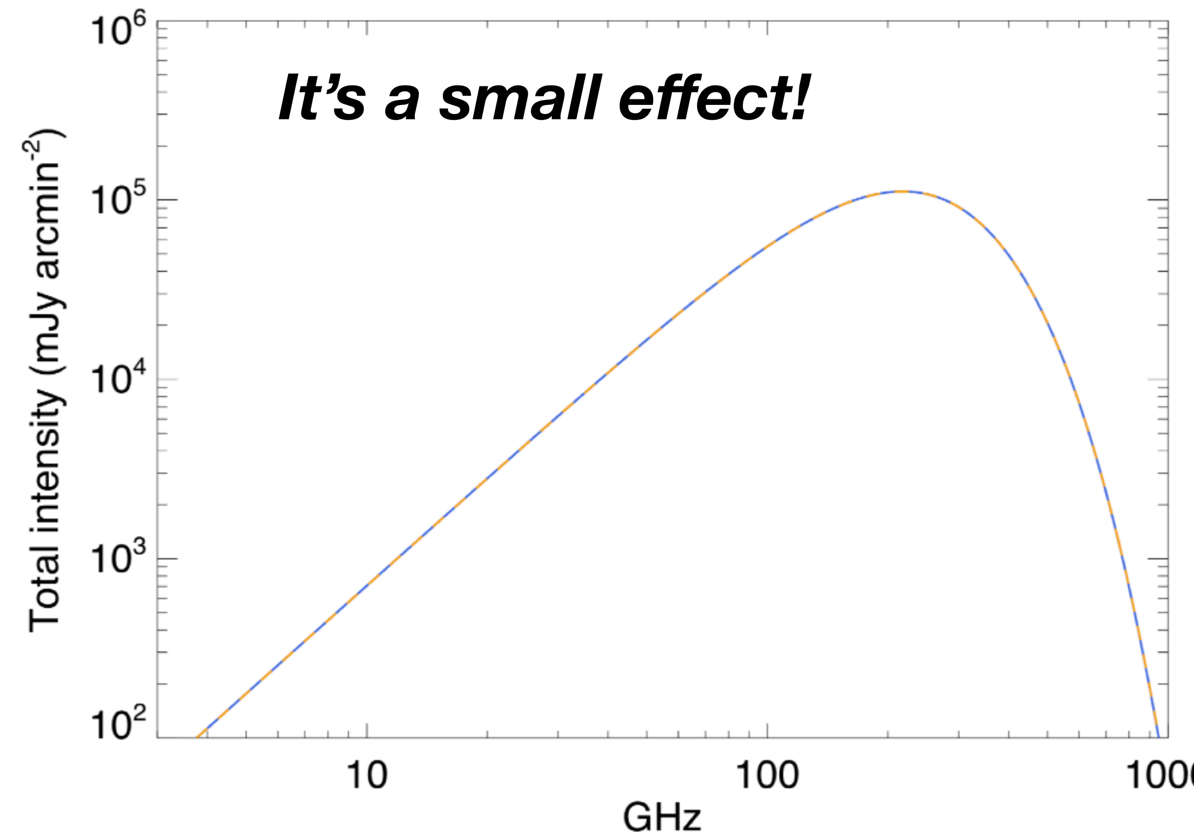
Compton-y parameter is the *amplitude* of the SZ effect signal:

$$y \equiv \int \frac{k_{\rm B}T_{\rm e}}{m_{\rm e}c^2}\,{\rm d}\tau_{\rm e} = \int \frac{k_{\rm B}T_{\rm e}}{m_{\rm e}c^2}\,n_{\rm e}\sigma_{\rm T}{\rm d}l$$

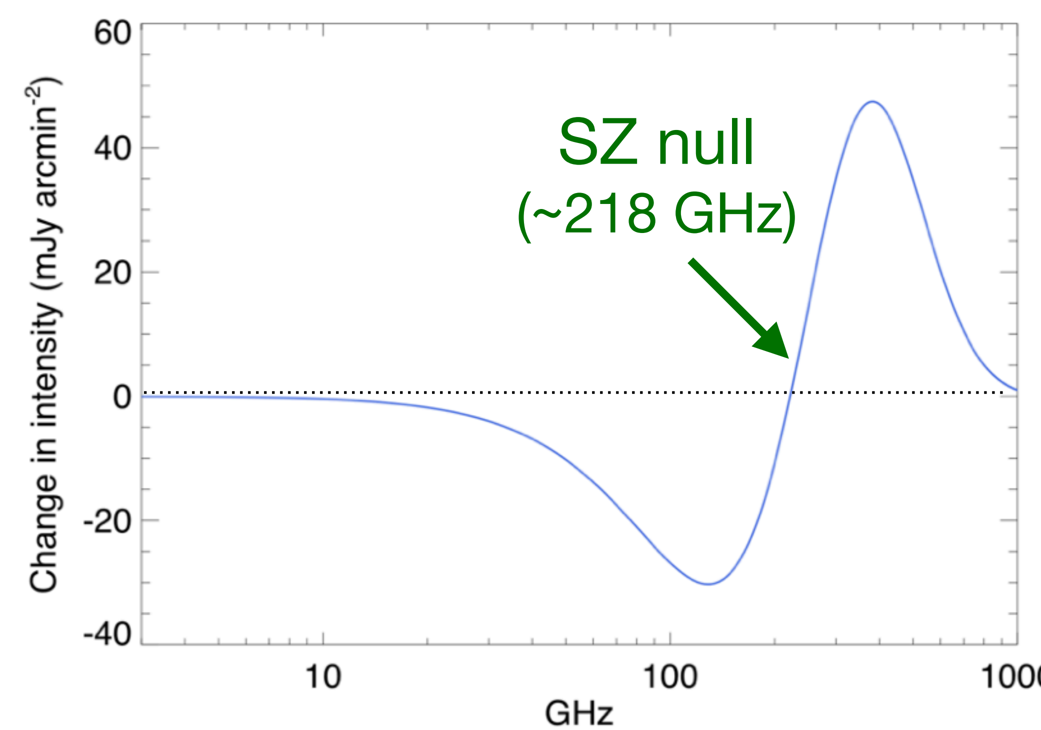
Figures from review by Carlstrom, Holder and Reese (2002)

The thermal SZ effect





Actual SZ measurement (APEX-SZ 150 GHz) for the galaxy cluster Abell 2163



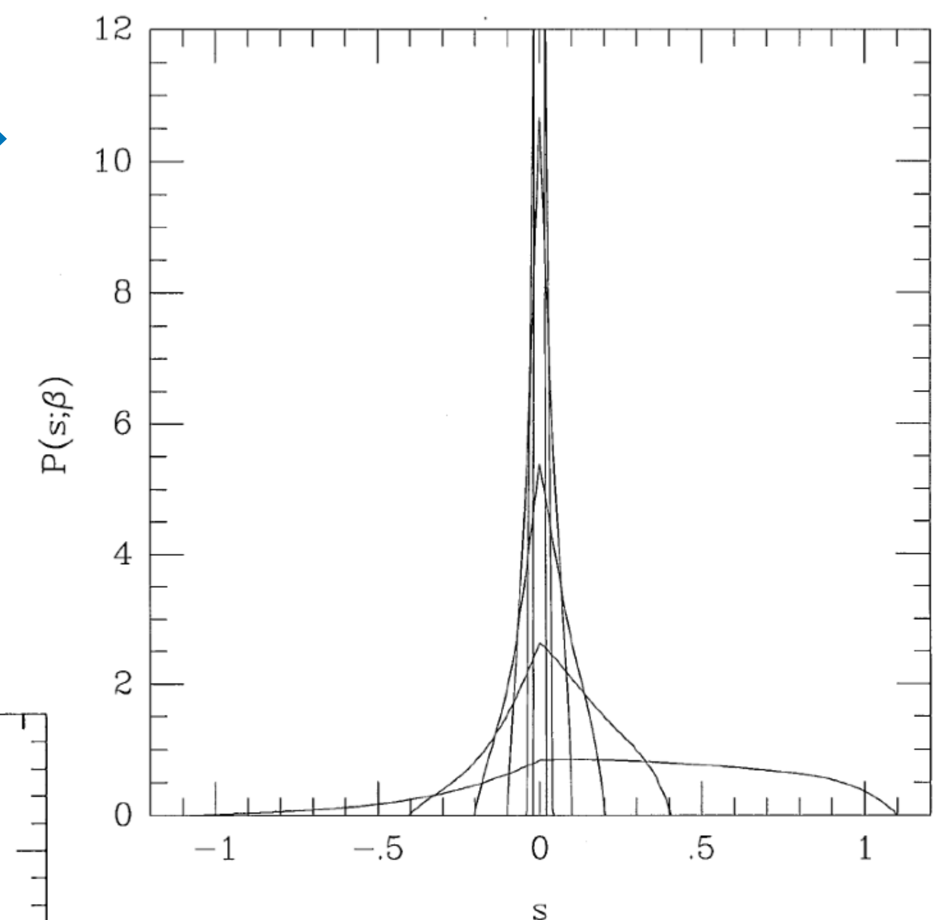
In this example, $y_0 = 3.4\text{x}10^{-4}$

$y << 1 \;\Rightarrow\; \Delta I / I_{CMB} = g(\nu)\, y << 1$

$g(\nu)$ is the intensity spectrum

Scattering kernel for the tSZ effect

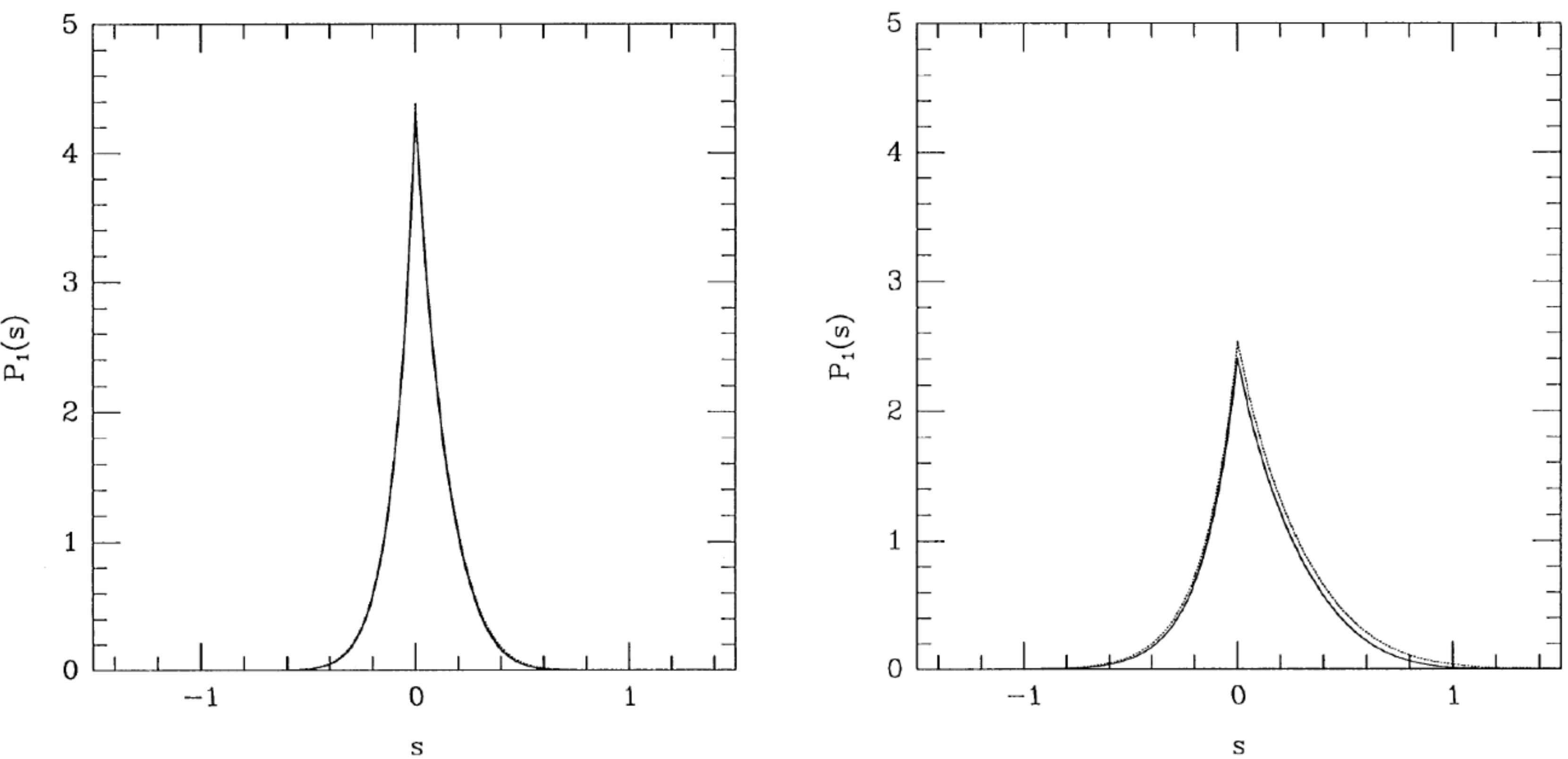
The **scattering probability function** P(s;β) from a single electron with velocity β (β = 0.01, 0.02, 0.05, 0.10, 0.20, and 0.50)



s = logarithmic frequency shift due to scattering

$$s = \log(\nu''/\nu)$$

This function needs to be integrated over the velocity distribution of electrons.

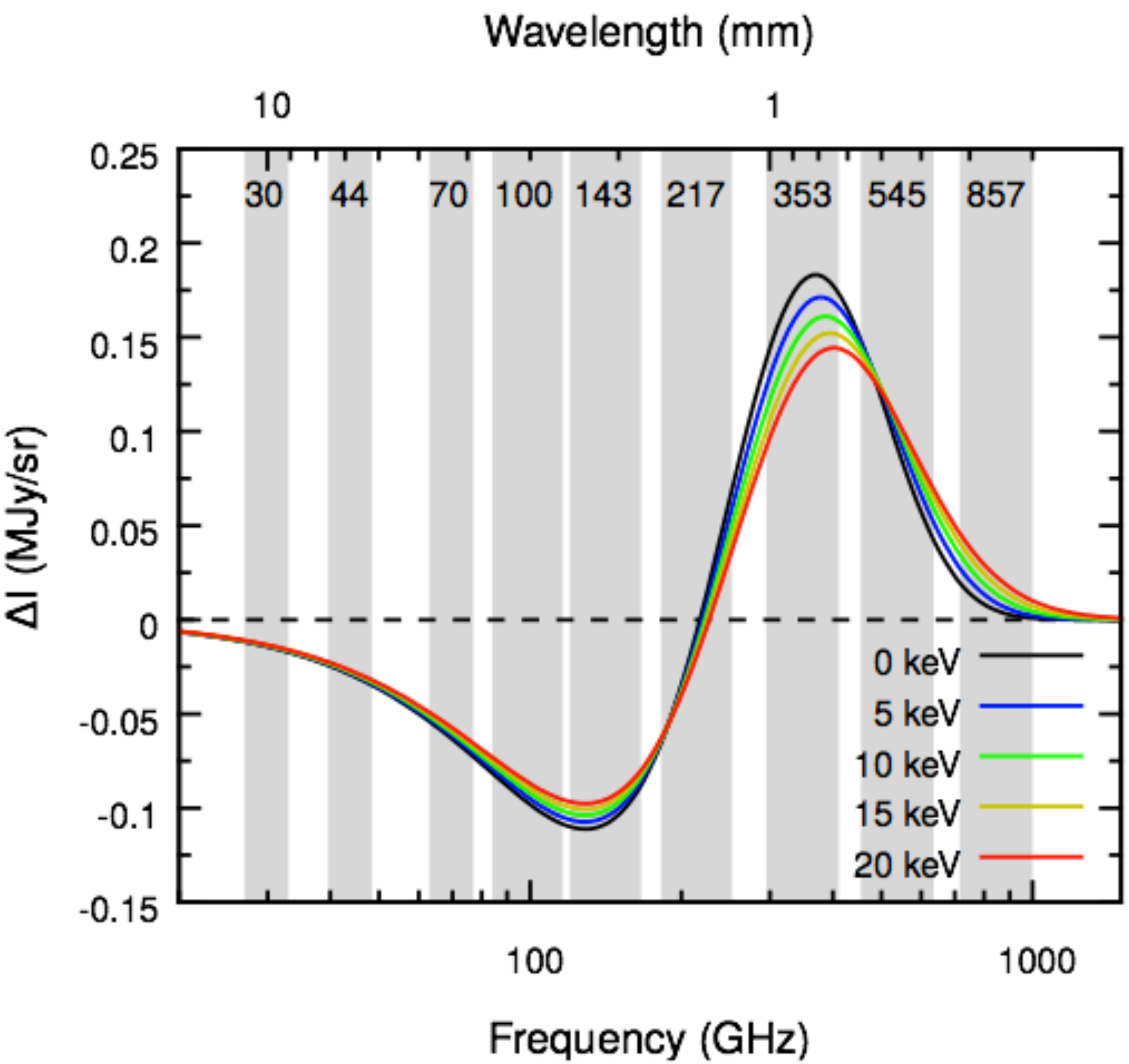


Scattering kernel for **5 keV and 15 keV thermal plasma** (taken from Birkinshaw 1999)

It can be seen that the distribution of scattered photon frequencies is asymmetric, with a stronger upscattering tail than a downscattering tail.

This is the origin of the SZ effect!

Full spectrum of the tSZ effect



$$\frac{\Delta I_{\rm SZ}}{I_0} = h(x)\left[\underbrace{f(x, T_{\rm e})\, y}_{\rm tSZ} \underbrace{- \tau_{\rm e}\left(\frac{v_{\rm pec}}{c}\right)}_{\rm kSZ}\right]$$

The y-parameter is the line-of-sight integral of pressure:

$$y = \frac{\sigma_{\rm T}}{m_{\rm e}c^2}\int_{\rm l.o.s.} n_{\rm e}k_{\rm B}T_{\rm e}\,{\rm d}l,$$

The following definitions are used:

$$x \equiv h\nu/(k_{\rm B}T_{\rm CMB})$$

$$I_0 = 2(k_{\rm B}T_{\rm CMB})^3/(hc)^2,$$

$$h(x) = x^4\exp(x)/(\exp(x)-1)^2$$

and

$$f(x, T_{\rm e}) = \left(x\frac{\exp(x)+1}{\exp(x)-1} - 4\right)(1+\delta_{\rm SZE}(x, T_{\rm e}))$$

Relativistic correction to the tSZ effect

For very high energy electrons, the Komaneets approximation breaks down (scattering can no longer be considered elastic). This is often the case for hot galaxy clusters, where average temperature can exceed 10 keV, so there are enough relativistic ($\gamma \gtrsim 100$) electrons in the thermal tail (sometimes called the Maxwell-Jüttner distribution).

$$f_{MB}(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}. \quad \Longrightarrow \quad f_{MJ}(\beta)d\beta = \frac{\beta^2\gamma^5 e^{\left(-\frac{m_0c^2}{kT}\gamma\right)}}{Z_{MJ}}d\beta\,, \quad Z_{MJ} = \int_0^1 \beta^2\gamma^5 e^{\left(-\frac{m_0c^2}{kT}\gamma\right)}d\beta.$$

This spectral departure from the classical non-relativistic calculation is termed as the **relativistic SZ, or rSZ effect**. Just like measuring the X-ray Bremsstrahlung spectrum in the high-energy part, this is an effective tool to measure cluster temperatures directly.

$$\left\langle\frac{\Delta\nu}{\nu}\right\rangle \approx 4\Theta_e + 10\Theta_e^2 + \frac{15}{2}\Theta_e^3 - \frac{15}{2}\Theta_e^4 + O(\Theta_e^5),$$

$$\left\langle\left(\frac{\Delta\nu}{\nu}\right)^2\right\rangle \approx 2\Theta_e + 47\Theta_e^2 + \frac{1023}{4}\Theta_e^3 + \frac{2505}{4}\Theta_e^4 + O(\Theta_e^5),$$

$$\Theta_e = kT_e/m_ec^2$$

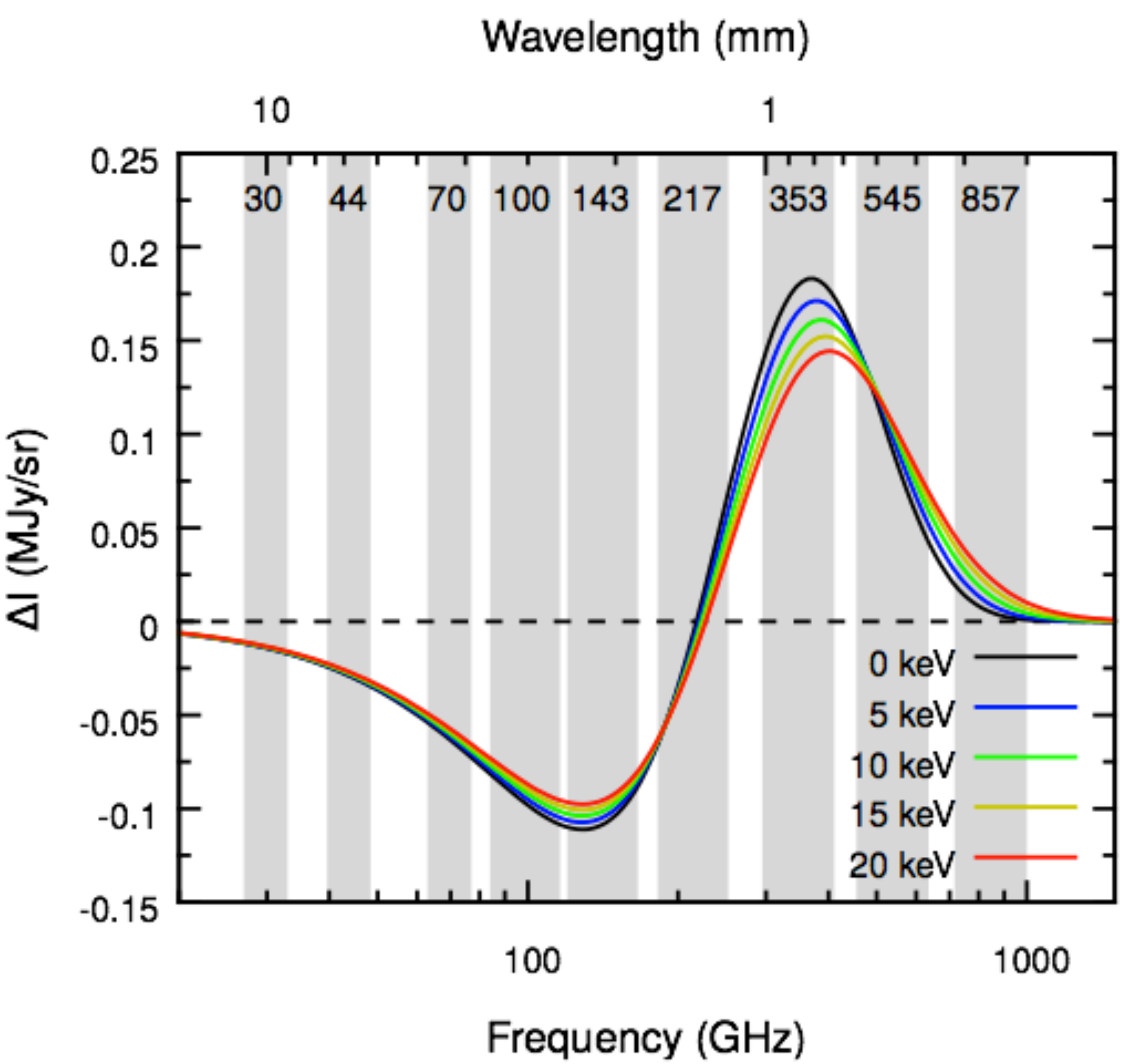
At higher temperatures (roughly kT_e> 5 keV) it is no longer accurate to assume $\Delta\nu/\nu$ << 1 and higher order moments of the scattering kernel becomes important.

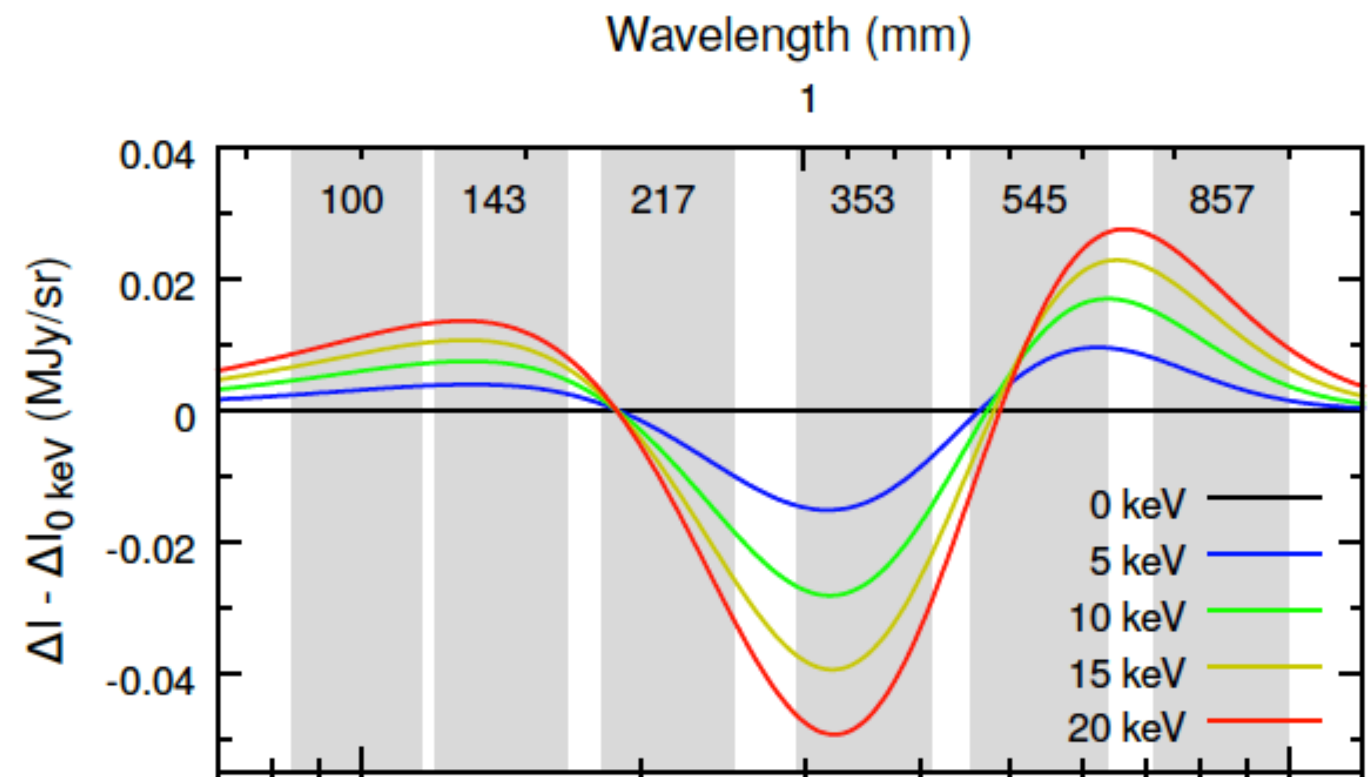
Relativistic SZ (or rSZ) effect

$$\frac{\delta n(\nu)}{n(\nu)} = \tau \frac{x e^x}{e^x-1}\left\{\frac{V}{c}\mu + \frac{kT_e}{m_e c^2}(-4+F) + \left(\frac{V}{c}\right)^2\left(-1-\mu^2+\frac{3+11\mu^2}{20}F\right) + \frac{V}{c}\frac{kT_e}{m_e c^2}\mu\left[10-\frac{47}{5}F+\frac{7}{10}(2F^2+G^2)\right] + \left(\frac{kT_e}{m_e c^2}\right)^2\left[-10+\frac{47}{2}F-\frac{42}{5}F^2+\frac{7}{10}F^3+\frac{7}{5}G^2(-3+F)\right]\right\}.$$

Sazonov & Sunyaev (1998)

$F = x \coth(x/2)$, and $G = x/\sinh(x/2)$.





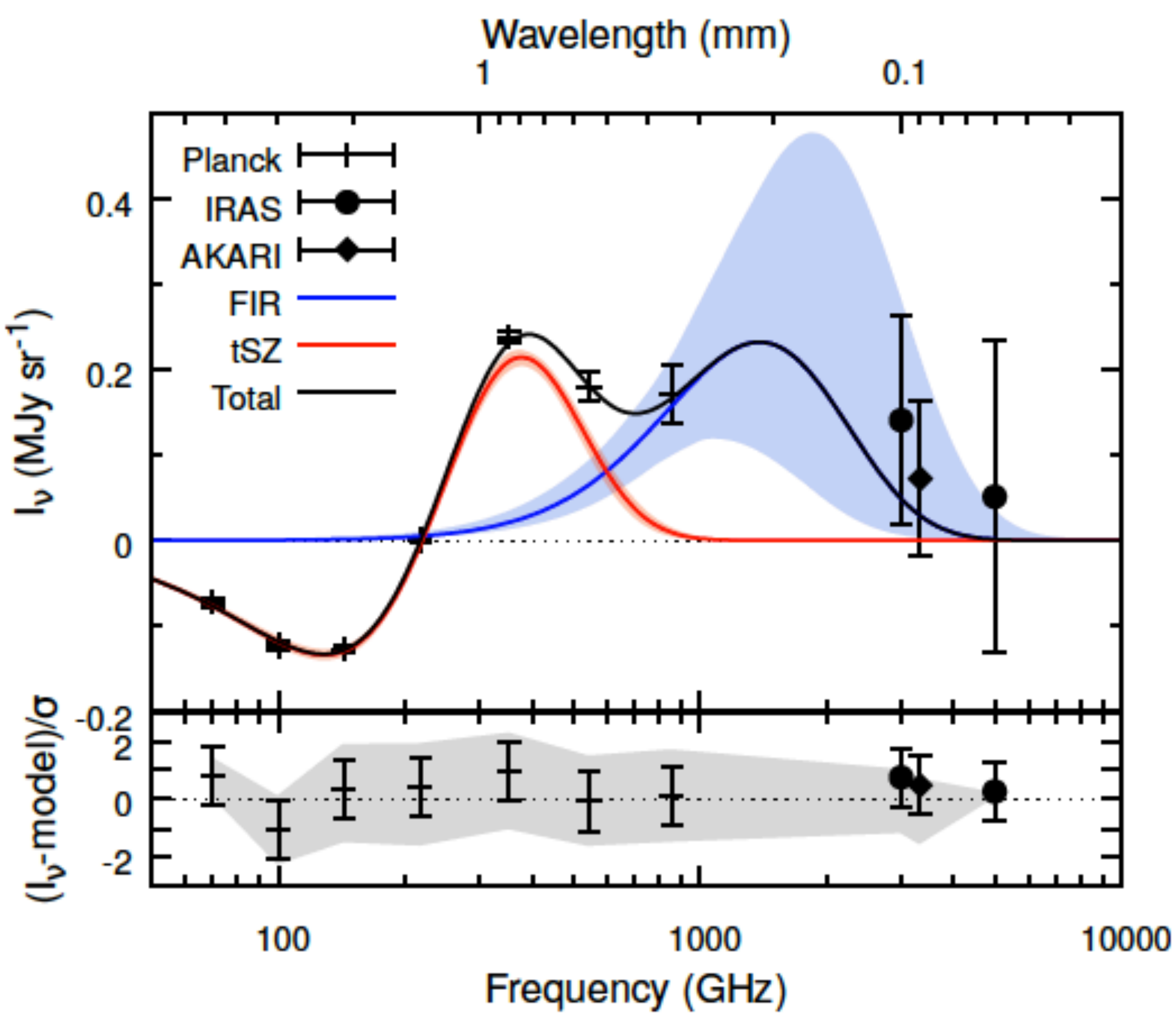
(difference from 0 keV)

There are widely used analytic expressions to compute the rSZ effect (e.g. by Itoh et al. 1998 or Nozawa et al. 1998), but it is best to use numerical packages like SZpack (Chluba et al. 2012) for better accuracy.

rSZ effect measurements

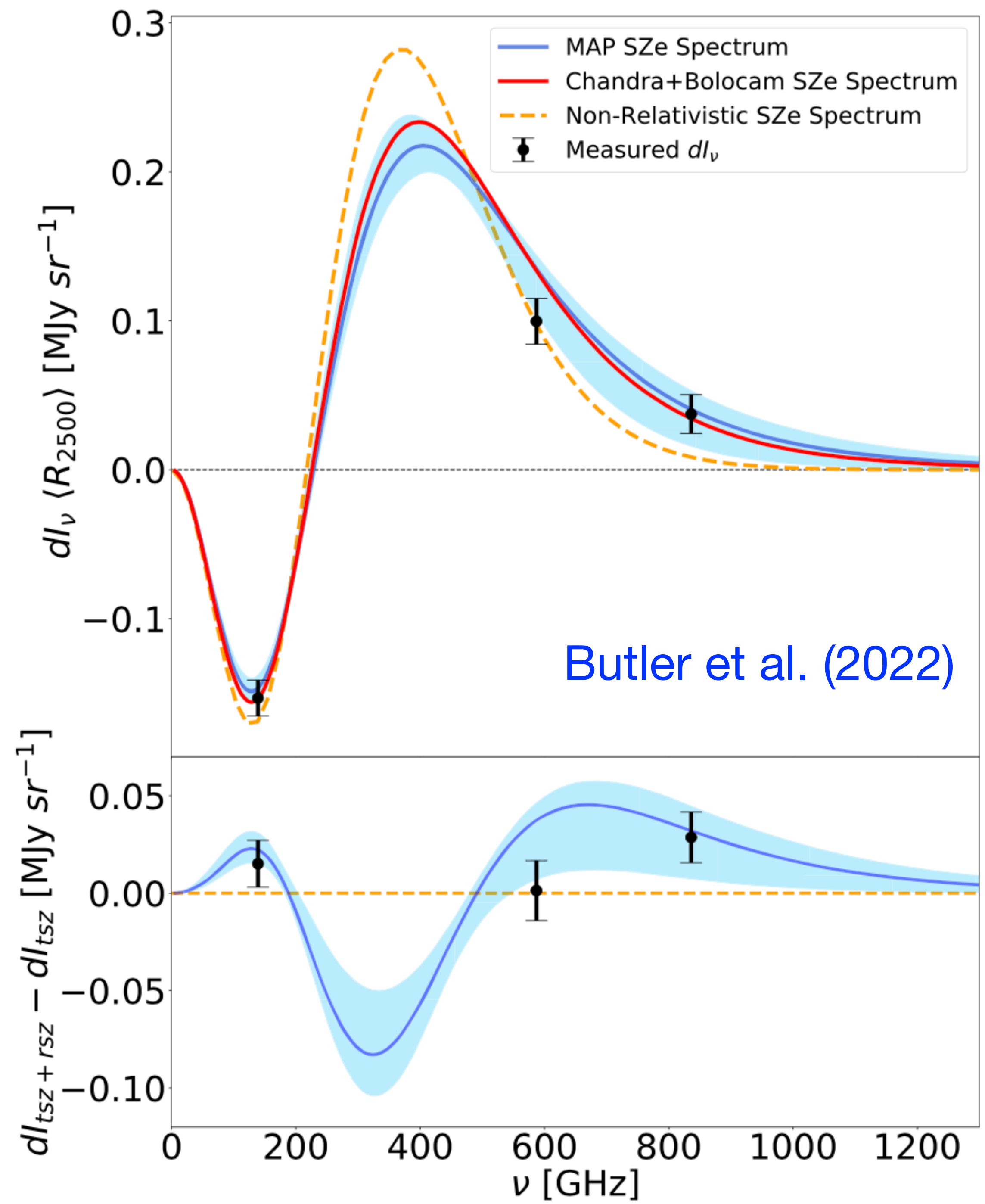
Current best measurement with *Planck* data, stacking ~700 clusters

$$\boxed{k_B \langle T_{SZ} \rangle = 4.4^{+2.1}_{-2.0}\ \mathrm{keV}}$$



Erler, Basu et al. (2018)

Measurement in a single, massive cluster RXC J1347

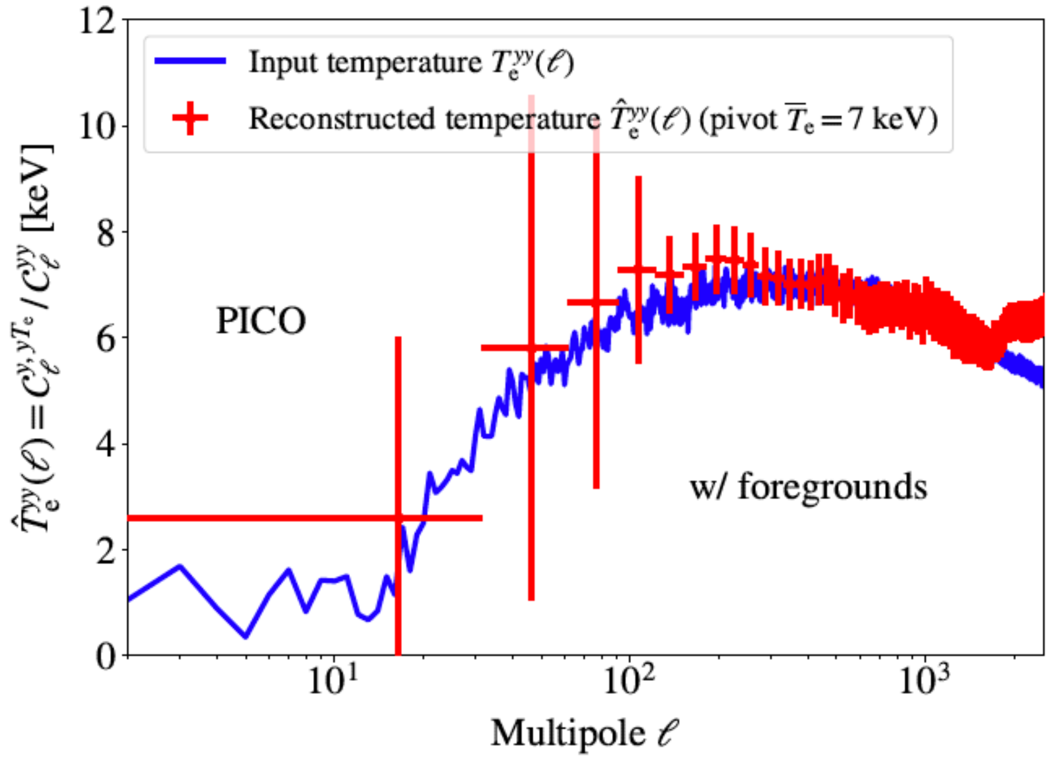


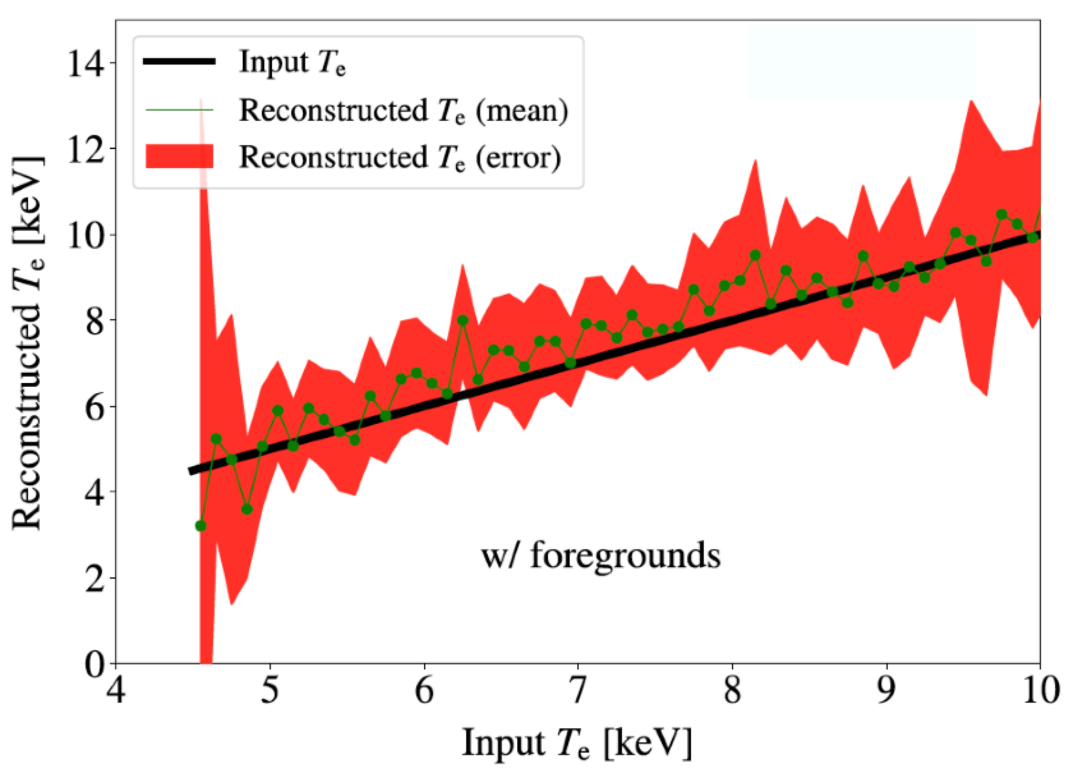
$\langle T_{sz} \rangle_{2500} = 12.3$ keV with a 68% credible interval

$5.8 < \langle T_{sz} \rangle_{2500} < 20.5$ keV

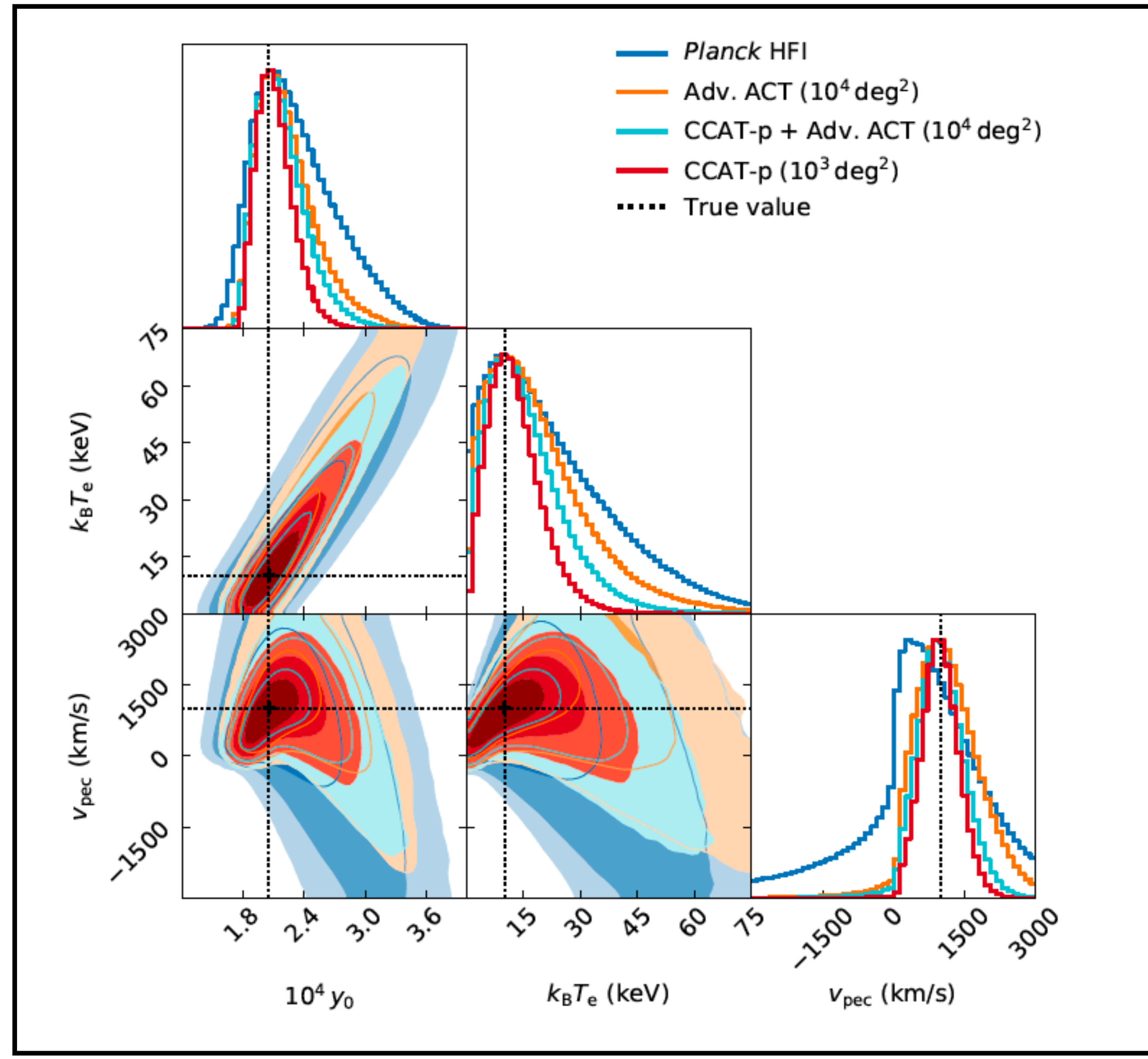
rSZ measurement forecasts

Forecast for the Coma cluster (top) and astacked sample of clusters (bottom) with the proposed PICO satellite (Remazeilles et al. (2020)





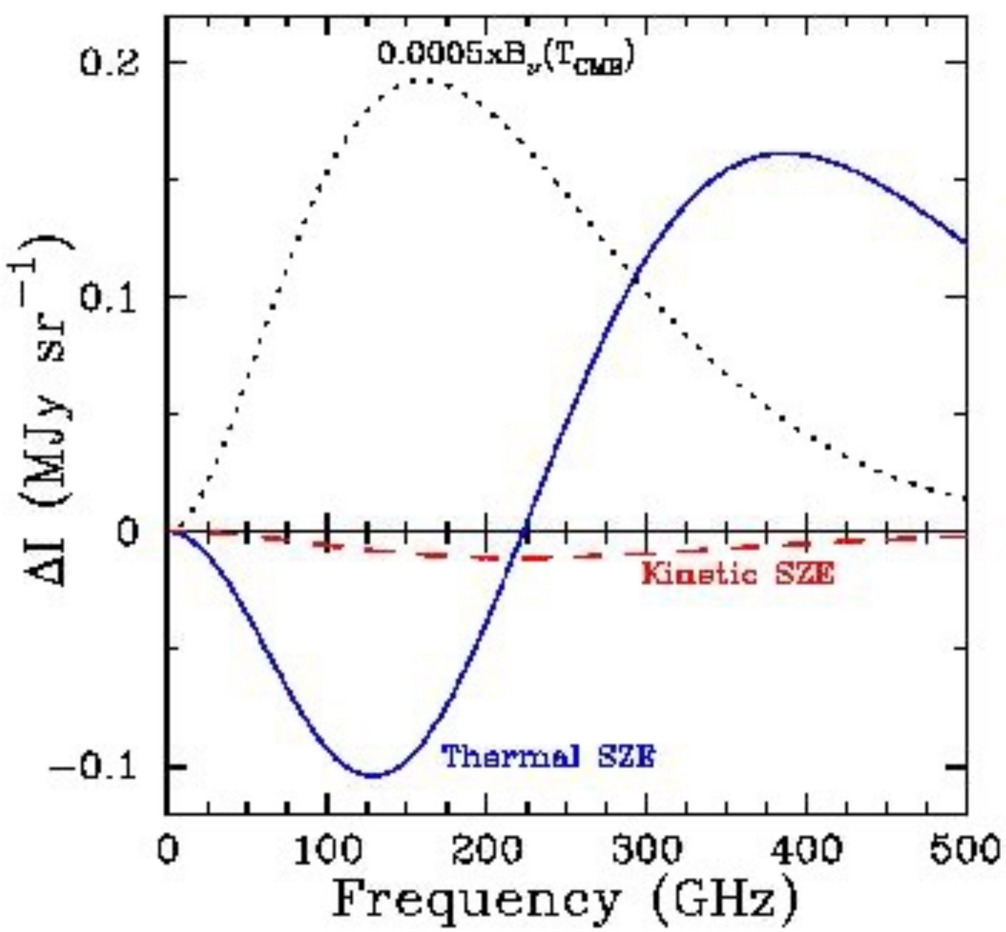
Predicted measurement with CCAT-prime (for a single, massive cluster)



Jens Erler Ph.D. Thesis

A

Different types of SZ effect: kSZ, pSZ



tSZ

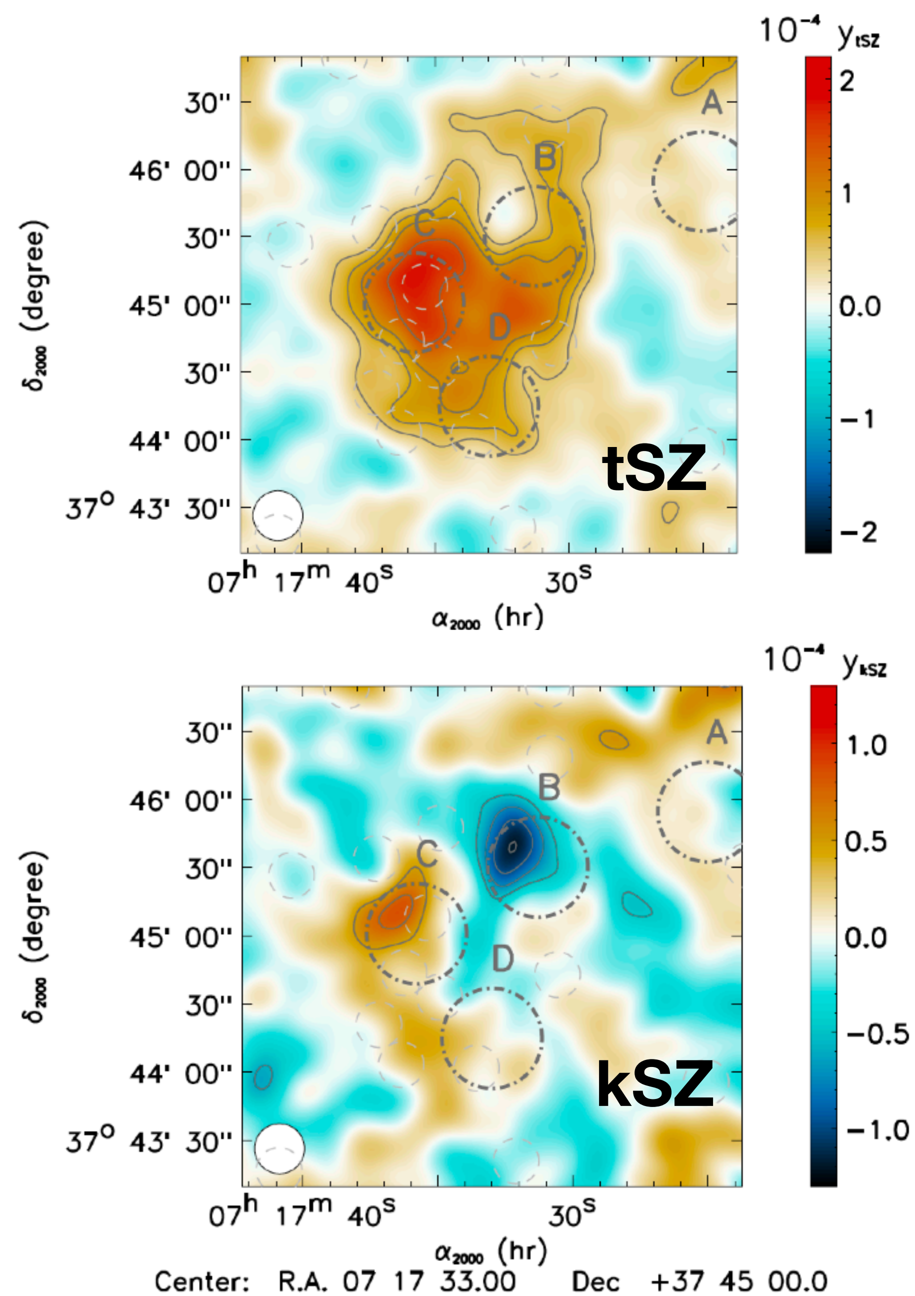
kSZ

rSZ

pSZ

ntSZ

Below is the *first measurement* of the kSZ effect from internal gas motions in a cluster (MACS J0717.5; Mroczkowski et al. 2012, Adam et al. 2017)



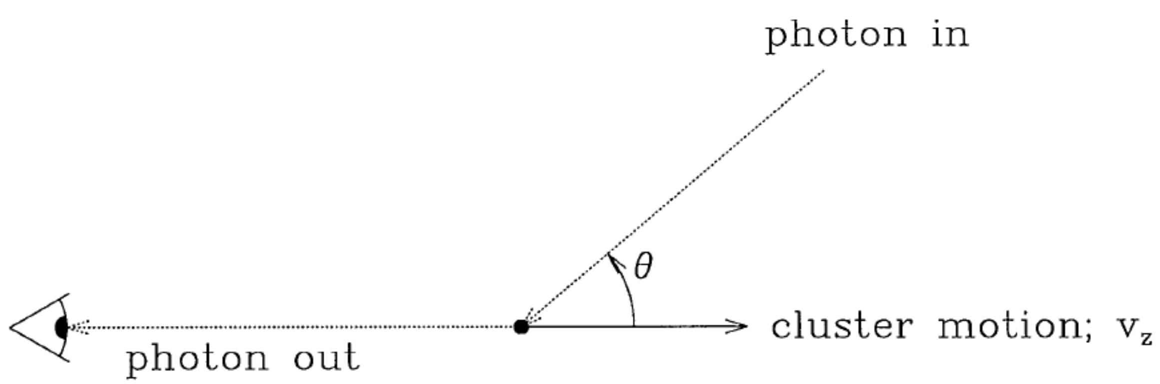
The kinematic SZ (kSZ) effect is caused by the motion of the clusters (i.e. the scattering electrons) as a whole, or from its internal bulk motion.

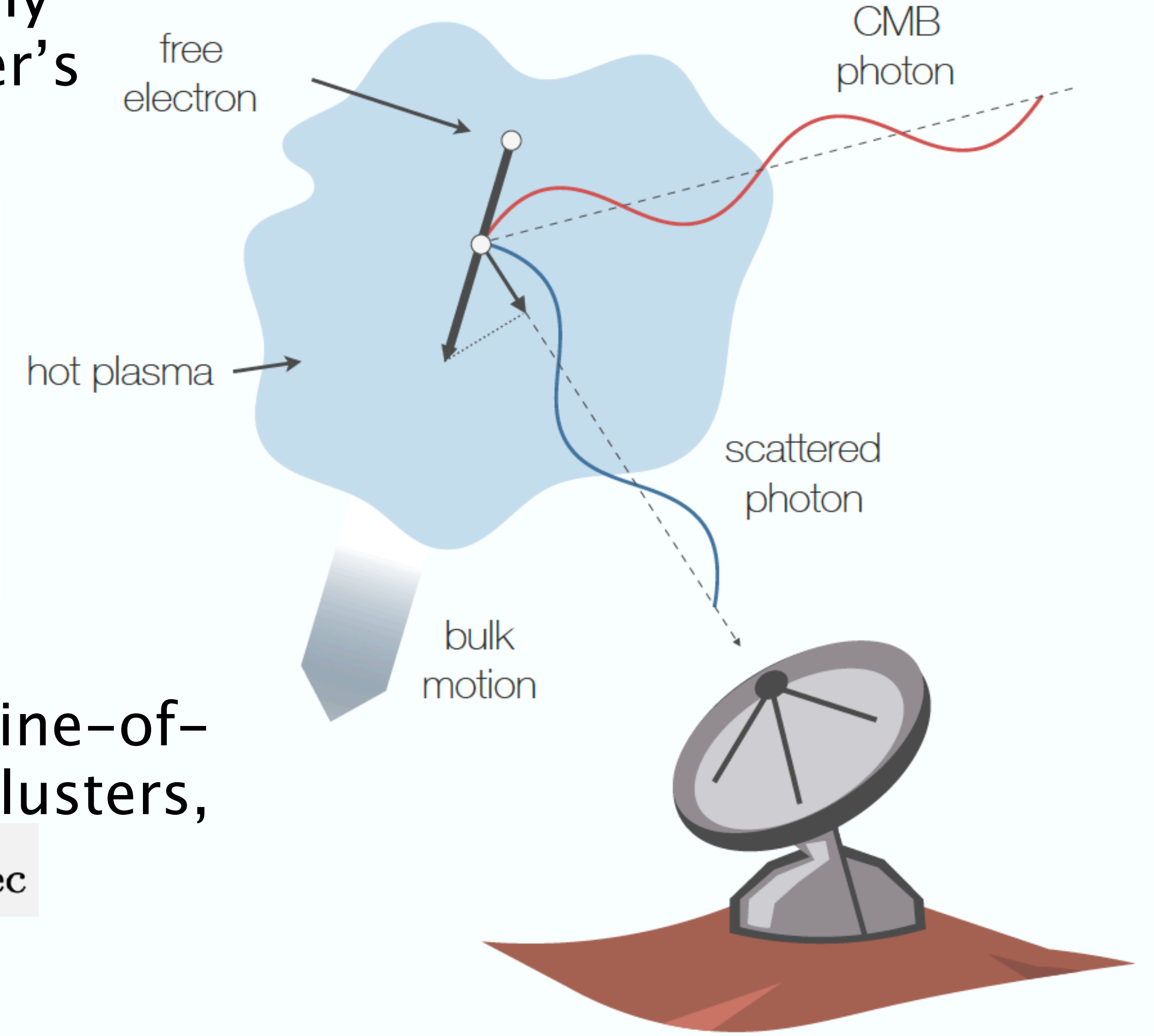
A polarized SZ (pSZ) effect can arise from scattering of the quadrupole radiation in the cluster frame, both primordial and due to cluster's transverse motion (this is much smaller).

Nonthermal SZ (ntSZ) is the effect caused by nonthermal distribution of electrons, mostly power-law electrons.

Kinematic SZ effect

Caused by simple Doppler shift of photon energy. In the electron's rest frame there is a CMB dipole, which the IC scattering partially isotropize. Then, transferred to the observer's frame, there is a net anisotropic signal.





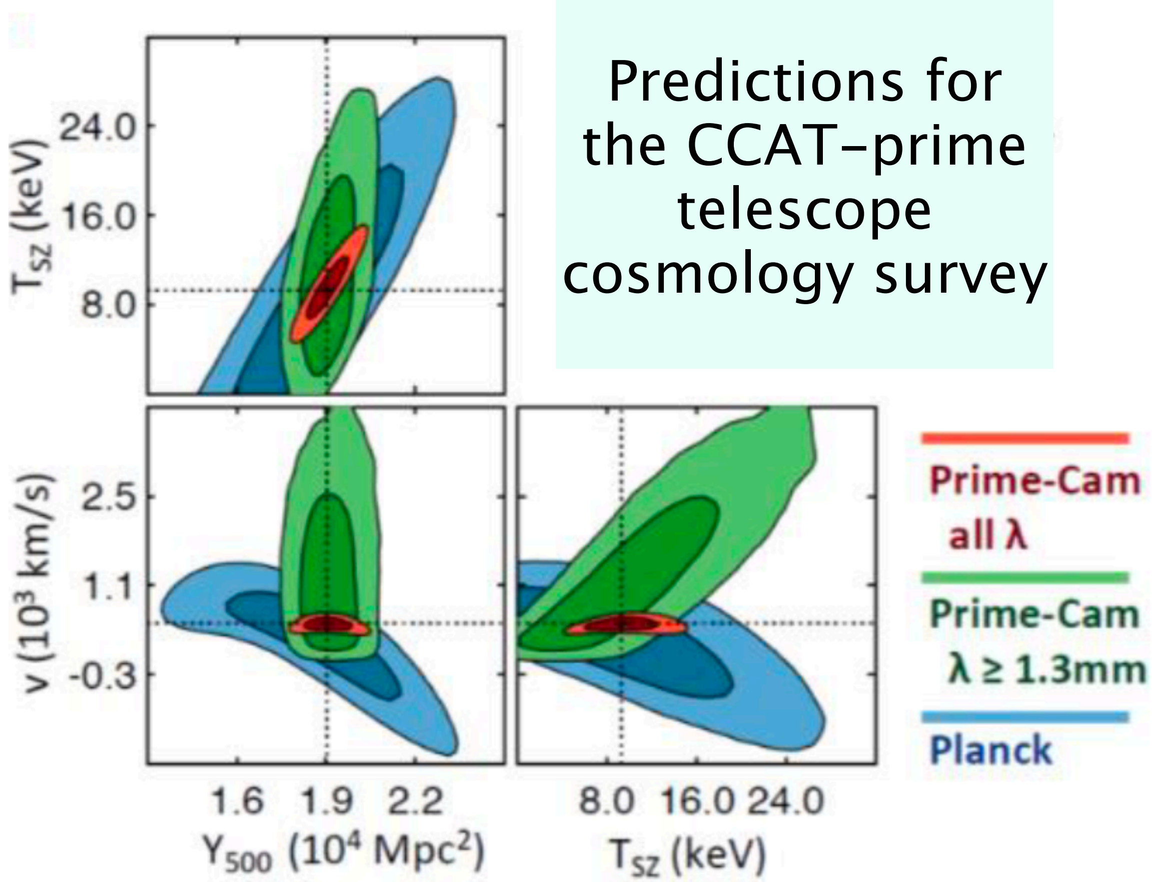
The signal is simply proportional to the line-of-sight component of the velocity of the clusters, multiplied by its optical depth, $\Delta T \propto \tau_e \, v_{\rm pec}$

$$\boxed{\frac{\Delta T_{SZE}}{T_{CMB}} = -\tau_e \left(\frac{v_{pec}}{c}\right)_{||}}$$

Importance of measuring the kSZ effect

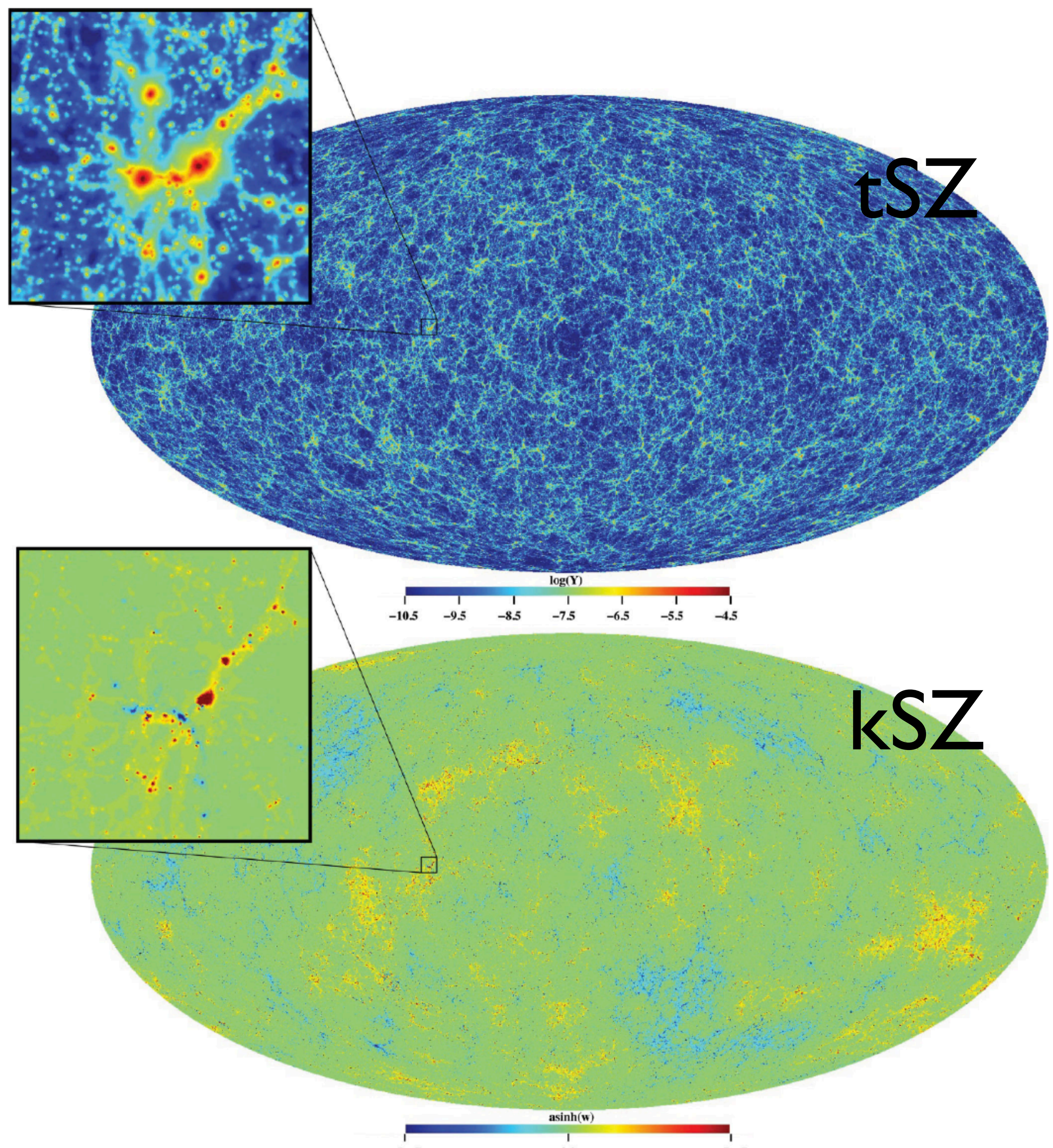
kSZ effect is one of the most promising tool to map the cosmic velocity field in the linear regime. This has huge potential for cosmology, since the amplitude of the velocity field is directly proportional to the growth rate of structure and the matter density.

$$\vec{v}(\vec{k}) = i\frac{d\ln D}{d\ln a}\frac{aH\delta(\vec{k})\vec{k}}{k^2}$$



Predictions for the CCAT-prime telescope cosmology survey

(Simulations by K. Dolag)



Cosmology recap: velocity and overdensity

The continuity equation relates the divergence of the peculiar velocity to the time rate of change of the total density perturbations:

$$\nabla \cdot \mathbf{v} = -a \frac{\partial \delta}{\partial t},$$

The density perturbations are commonly expressed in terms of the linear growth rate:

$$f(a) = \frac{a}{D_+} \frac{\mathrm{d}D_+}{\mathrm{d}a} = \frac{\mathrm{d} \log D_+}{\mathrm{d} \log a},$$

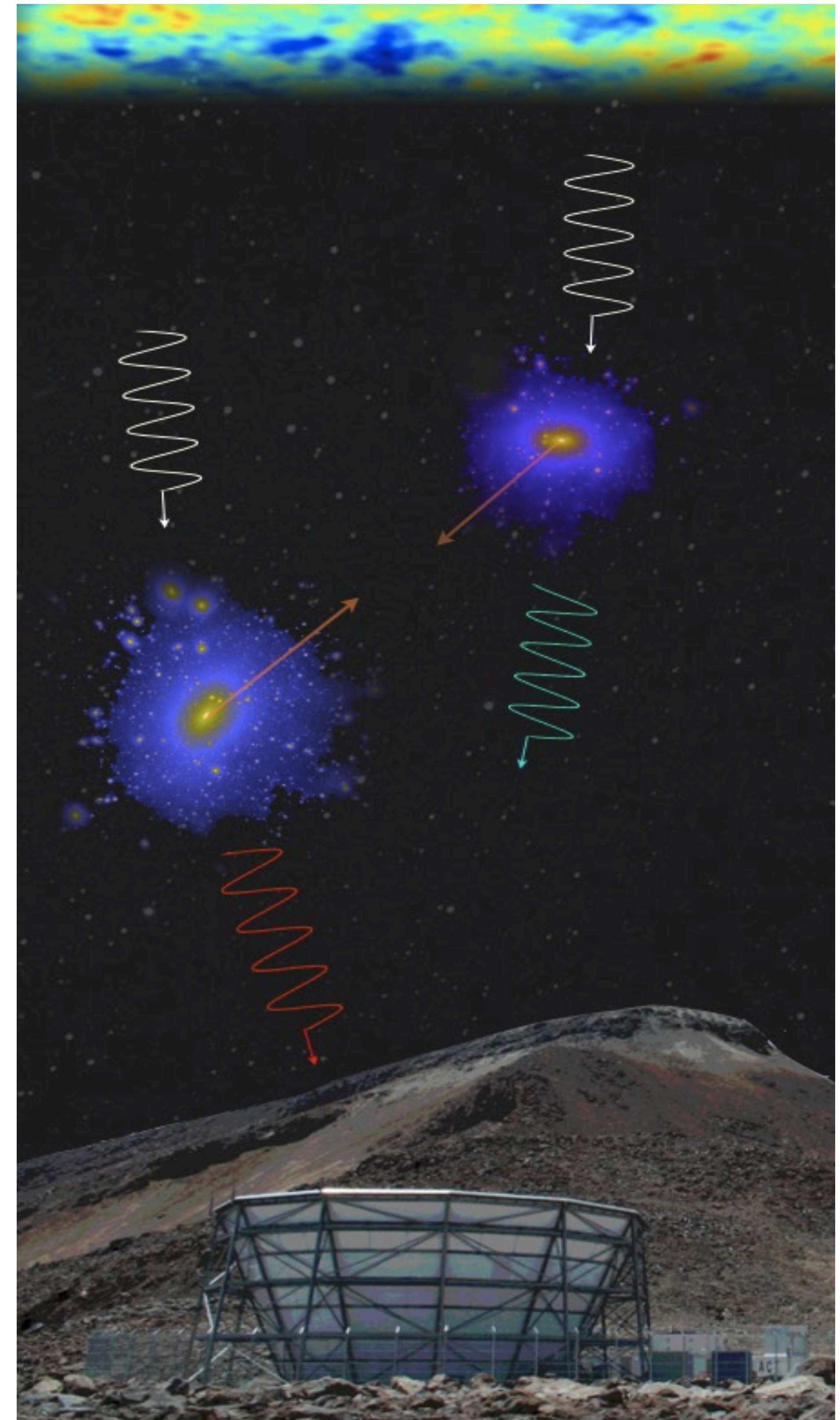
The below approximation for growth rate is valid only for ΛCDM cosmology, for other cosmologies it would be different.

$$\nabla \cdot \mathbf{v} = -f(a) \dot{a} \delta \simeq -\Omega_m^{0.545}(a) a H(a) \delta,$$

In the Fourier domain the derivatives are replaced by multiplications:

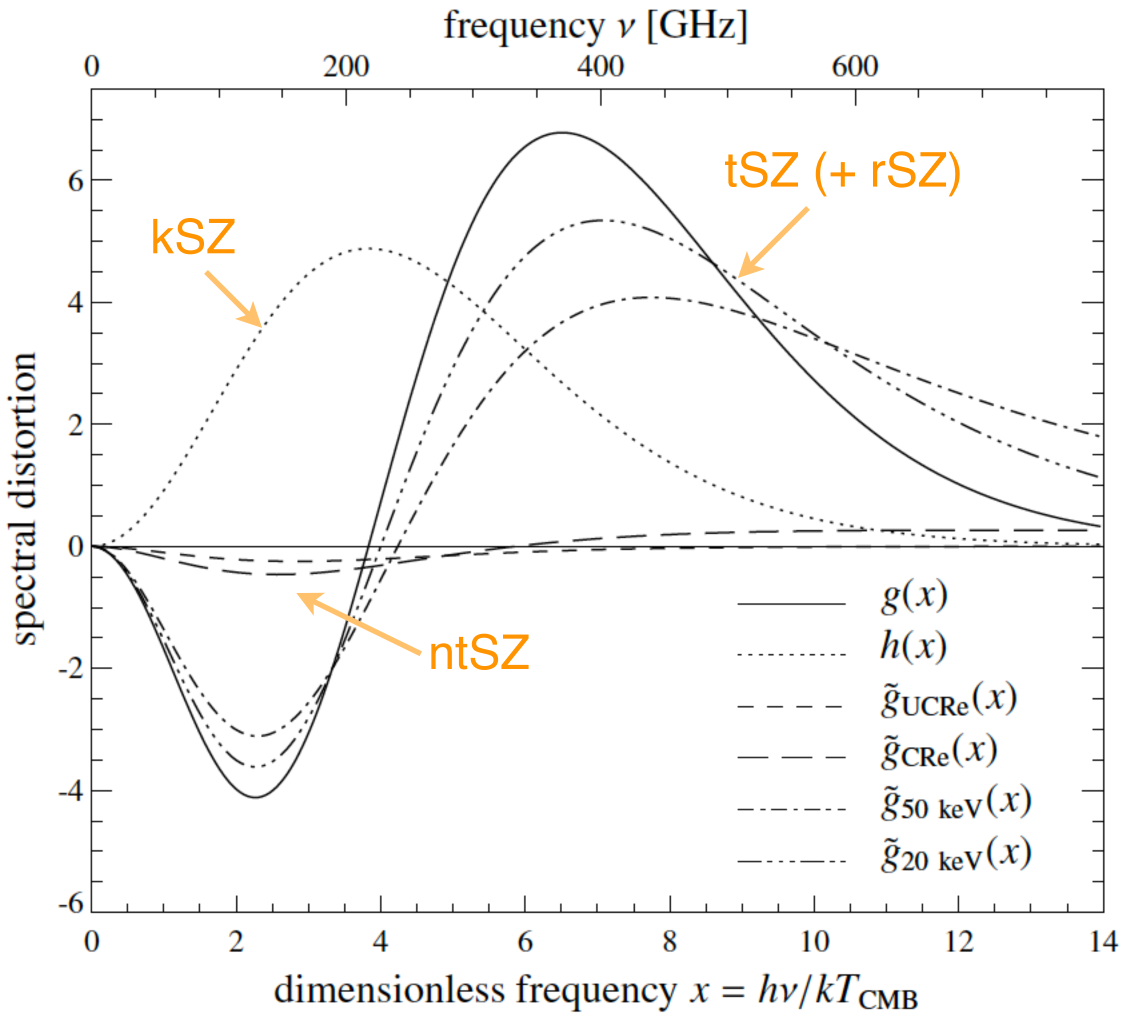
$$\vec{v}(\vec{k}) = i \frac{d \ln D}{d \ln a} \frac{a H \delta(\vec{k}) \vec{k}}{k^2}.$$

But beware, **kSZ is actually measuring the momentum**, i.e., the product of mass and velocity. So we need a prior knowledge of the baryon distribution (i.e. optical depth) in clusters first. Alternatively, one can use prior velocity measurements to gain insight on the baryonic content in clusters via kSZ.

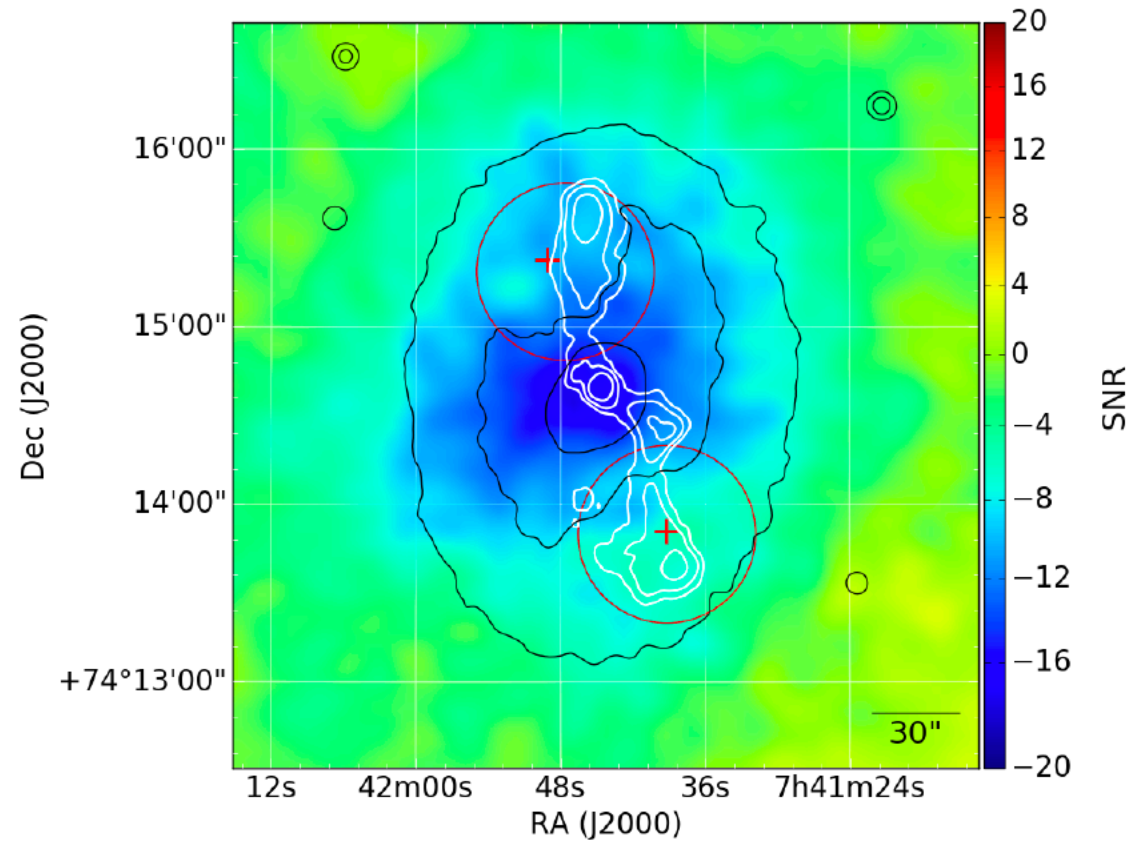


Credit: Hand et al. ACT collaboration

Another types of SZ effect: non-thermal



Pfrommer et al. (2005)



Abdulla et al. (2018)

arXiv:1806.05050

tSZ

kSZ

rSZ

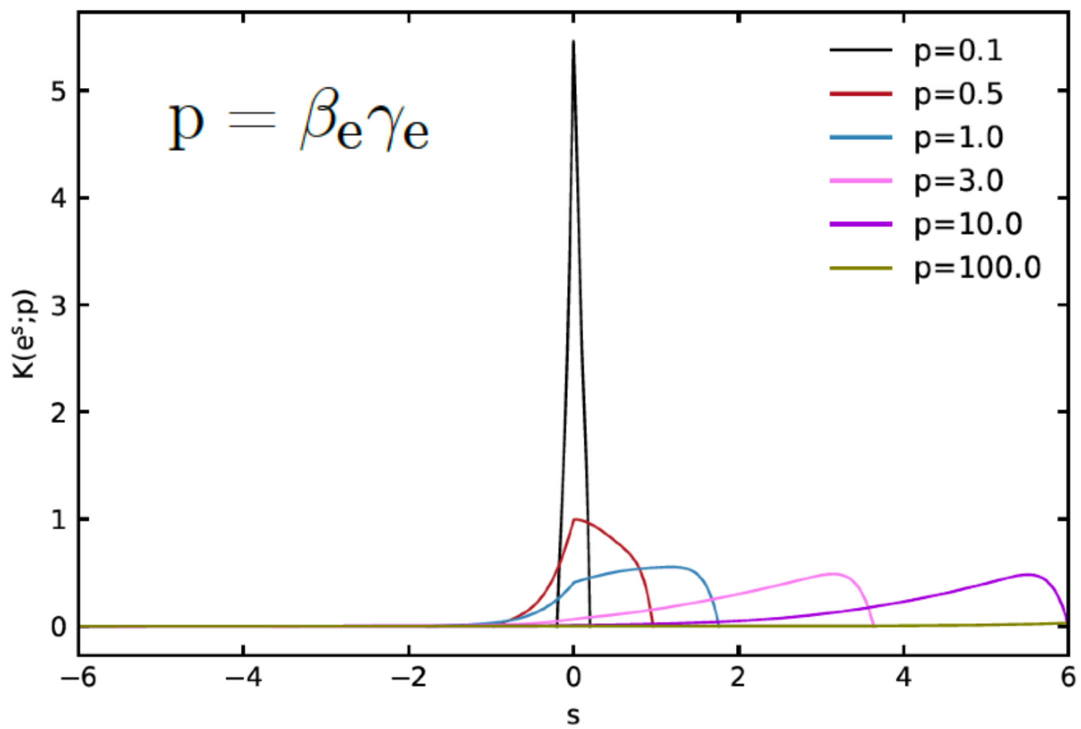
pSZ

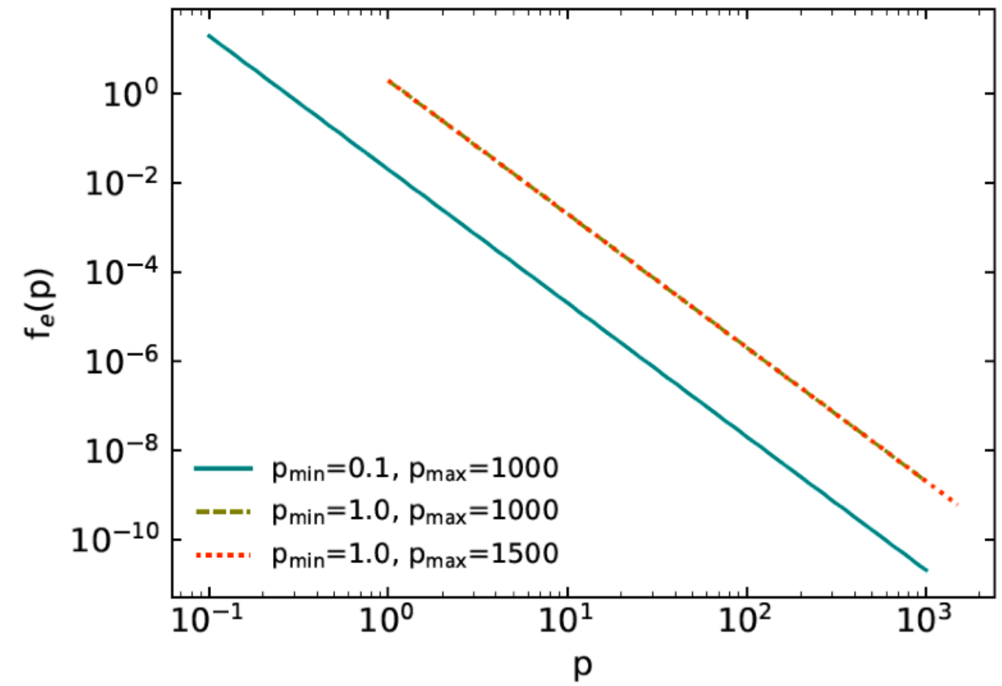
ntSZ

Non-thermal SZ is the spectral distortion from ultra-high energy electrons with power-law energy distribution (i.e. cosmic ray electrons). A very recent observation has provided strong evidence for this signal inside AGN bubbles on galaxy clusters.

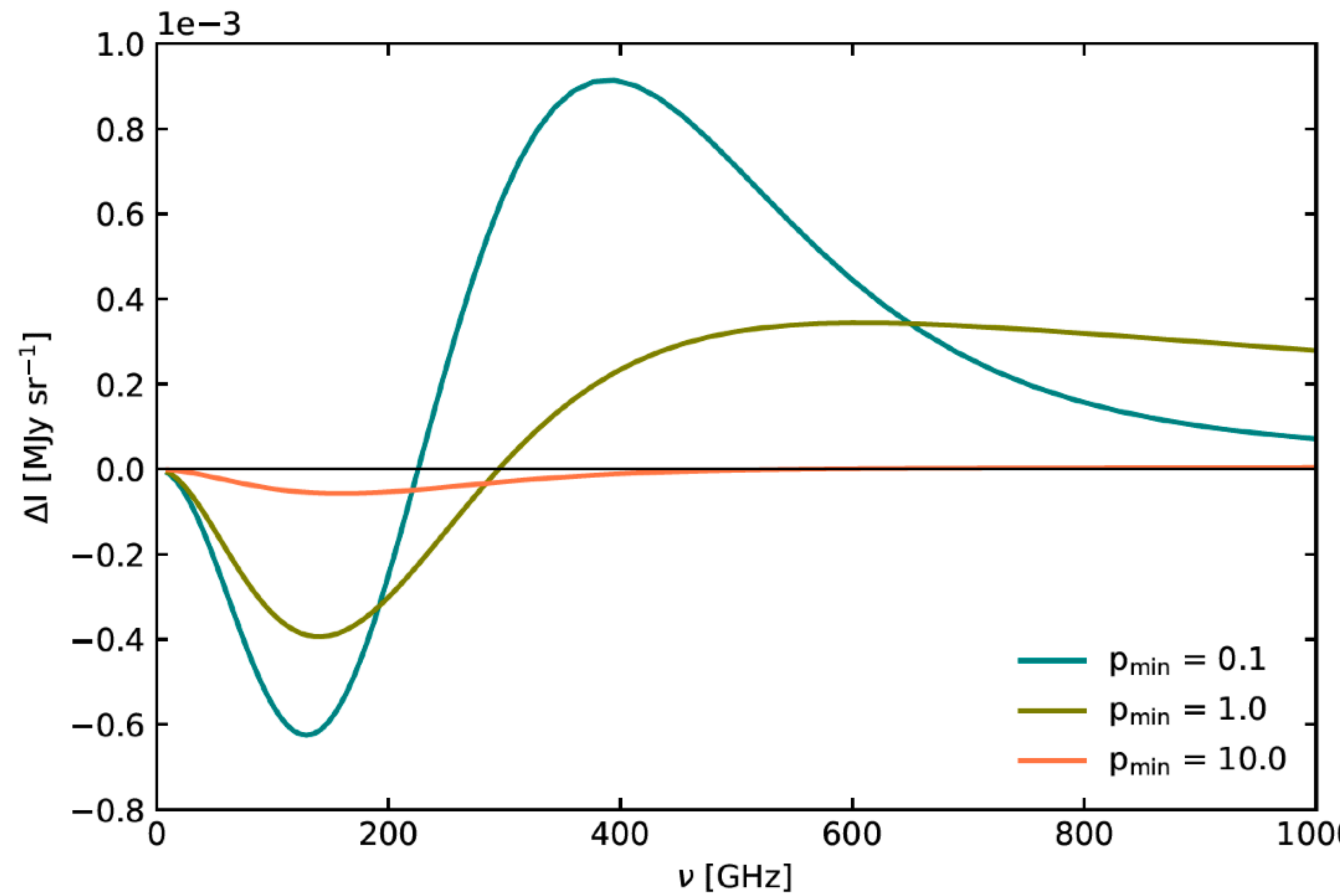
Nonthermal SZ (ntSZ) effect

$p = p_{\mathrm{phys}}/m_e c$ is the dimensionless electron momentum





Figures from master's thesis by Vyoma Muralidhara (2019)

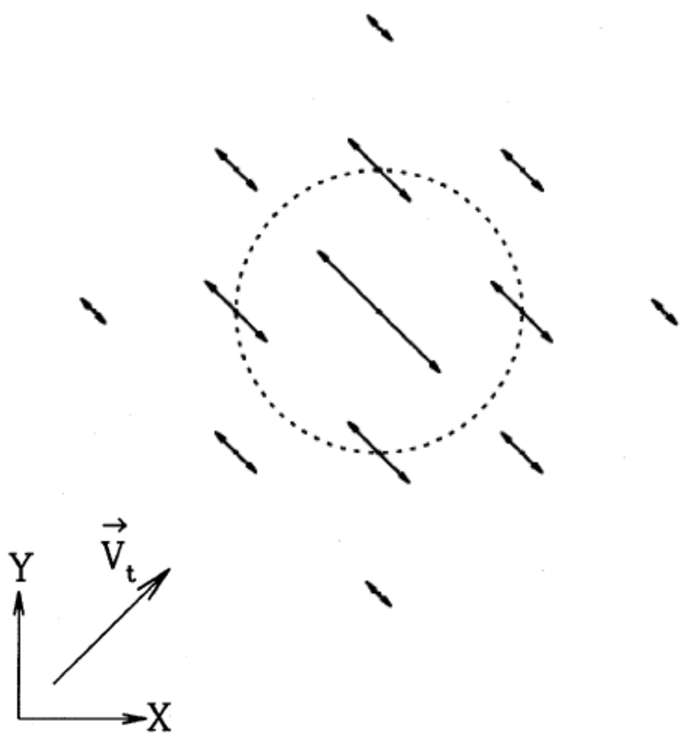


We are on the verge of detecting the ntSZ signal from galaxy clusters, which can be a game-changer in the study of cluster magnetic fields.

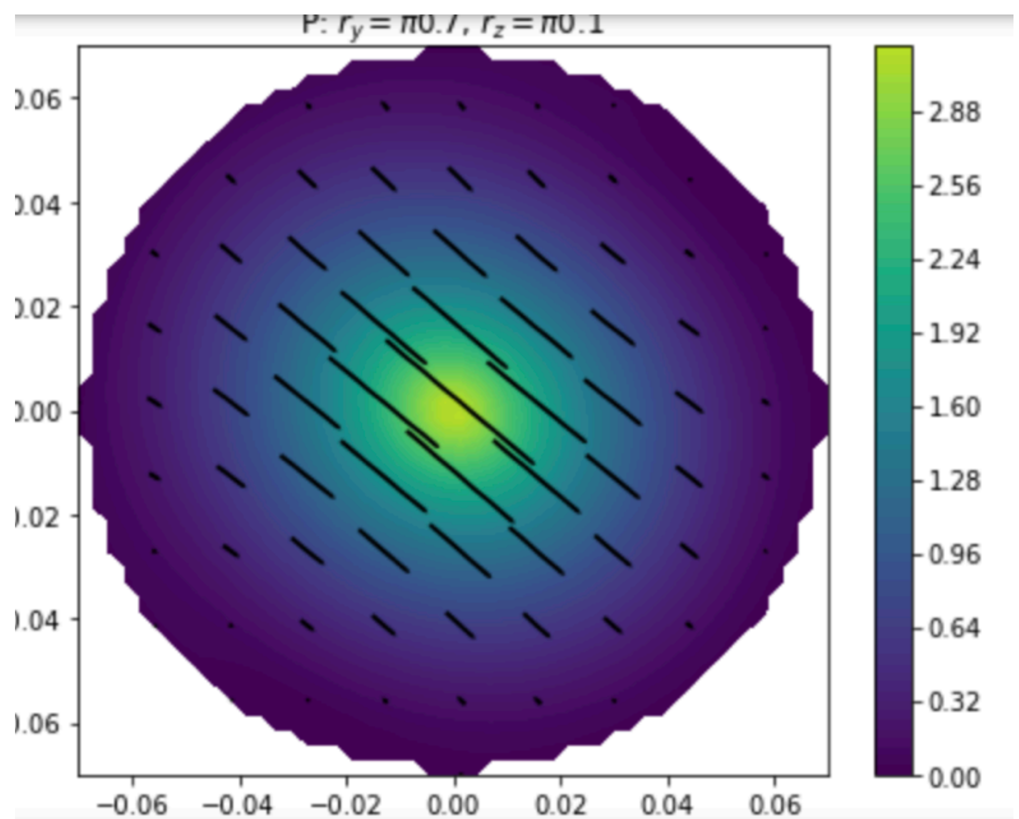
Polarized SZ (pSZ) effect

Recall that polarization is caused by a **quadrupole temperature (intensity) anosotropy**.

CMB itself has an intrinsic quadrupole moment. Also transverse motion of galaxy clusters will create a quadrupole moment from relativistic aberration. A second-order effect can be created also from the anisotropic distribution of electrons within the cluster, via second scattering.



Polarization angles from transverse motion (Sazonov & Sunyaev 1999)



Bachelor's thesis of Nikolas Pässler

Polarized SZ (pSZ) effect

There are many possible sources of polarized SZ signal, among which the ones introduced by the intrinsic quadrupole and bulk transverse motions are the most prominent (and cosmologically interesting). The hope will be that we can separate all these from their different spectral dependence or via stacking techniques.

Effect causing polarization	Fiducial level P_0	Scaling $\alpha(\tau, \beta_t, ...)$	Spectral shape $\varphi(x)$
CMB quadrupole	10^{-8}	$\propto \frac{Q_{\rm rms}}{T_{\rm CMB}}\,\tau$	$\frac{xe^x}{e^x-1}$
Bulk transverse motion	10^{-8}	$\propto \beta_{\rm t}^2\,\tau$	$\frac{e^x(e^x+1)}{2(e^x-1)^2}x^2$
Second scatterings (τ^2)	10^{-8}	$\propto \frac{kT_{\rm e}}{m_{\rm e}c^2}\,\tau^2$	$\frac{xe^x}{e^x-1}\left(x\frac{e^x+1}{e^x-1}-4\right)$
Bulk transverse anisotropy	10^{-8}	$\propto \left\langle\beta_{\rm t}^2\right\rangle\,\tau$	$\frac{e^x(e^x+1)}{2(e^x-1)^2}x^2$
Pressure anisotropy	10^{-8}	$\propto \frac{\Delta T_{\rm e}}{T_{\rm e}}\frac{kT_{\rm e}}{m_{\rm e}c^2}\,\tau$	$\frac{e^x(e^x+1)}{2(e^x-1)^2}x^2$
Moving lens	10^{-9}	$\propto \beta_{\rm t}\Delta\theta\,\tau$	$\frac{xe^x}{e^x-1}$
Cluster rotation	10^{-10}	$\propto \beta_{\rm r}^2\,\tau$	$\frac{e^x(e^x+1)}{2(e^x-1)^2}x^2$
CMB fluctuations	10^{-8}	$\propto \frac{\sqrt{D_\ell^{EE}}}{T_{\rm CMB}}$	$\frac{xe^x}{e^x-1}$

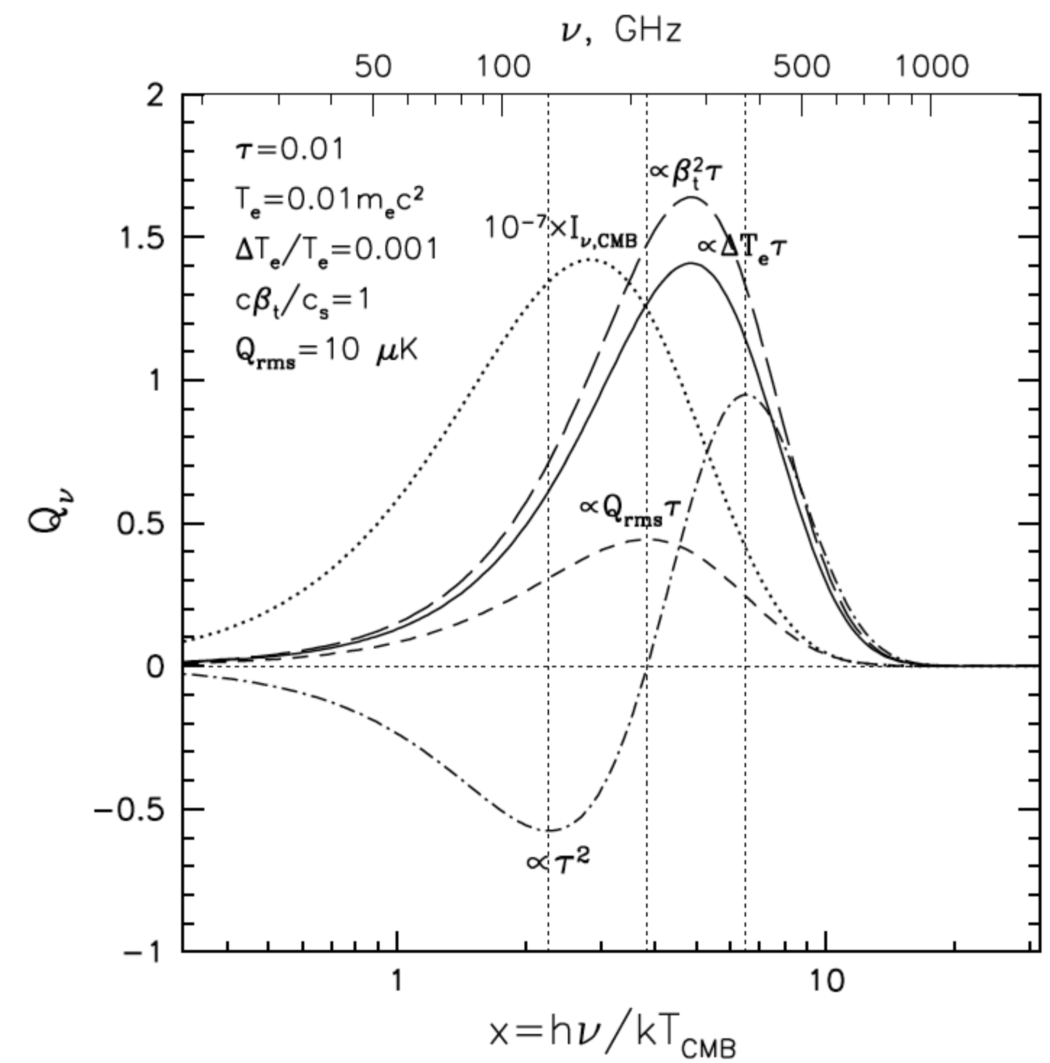
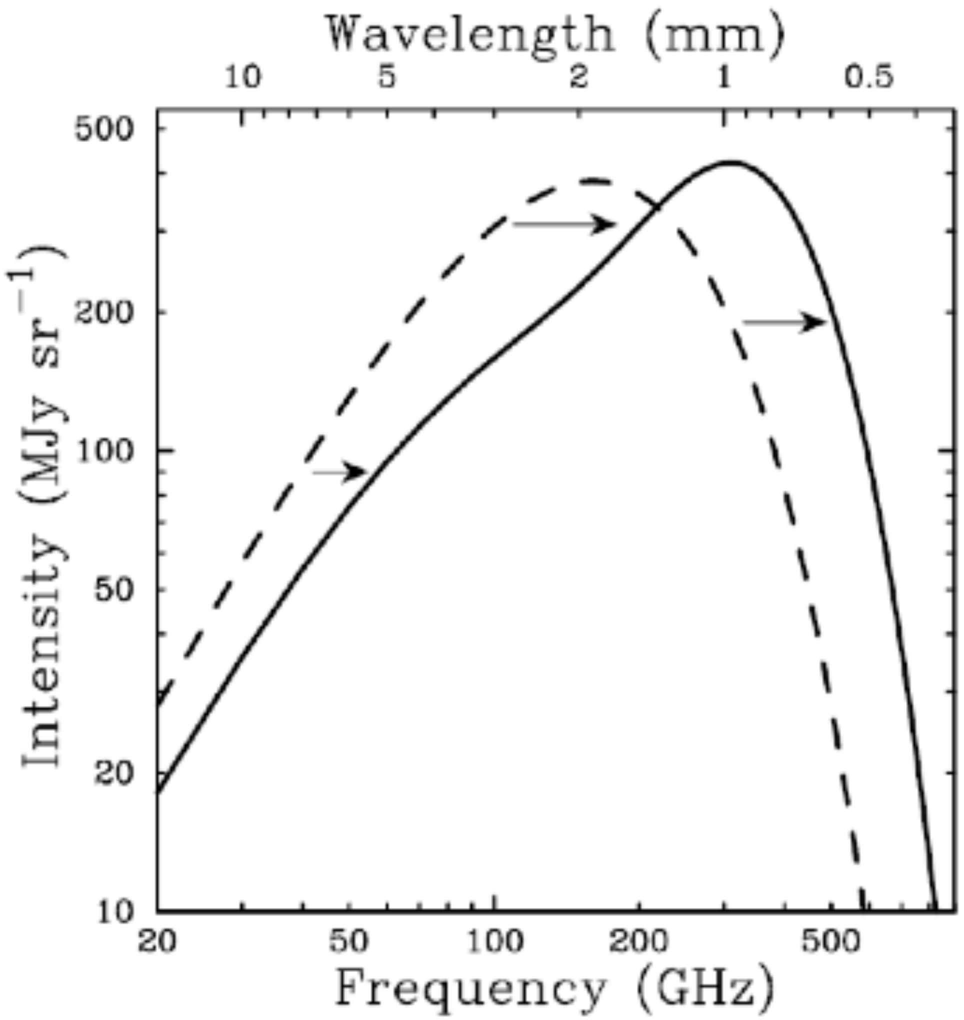
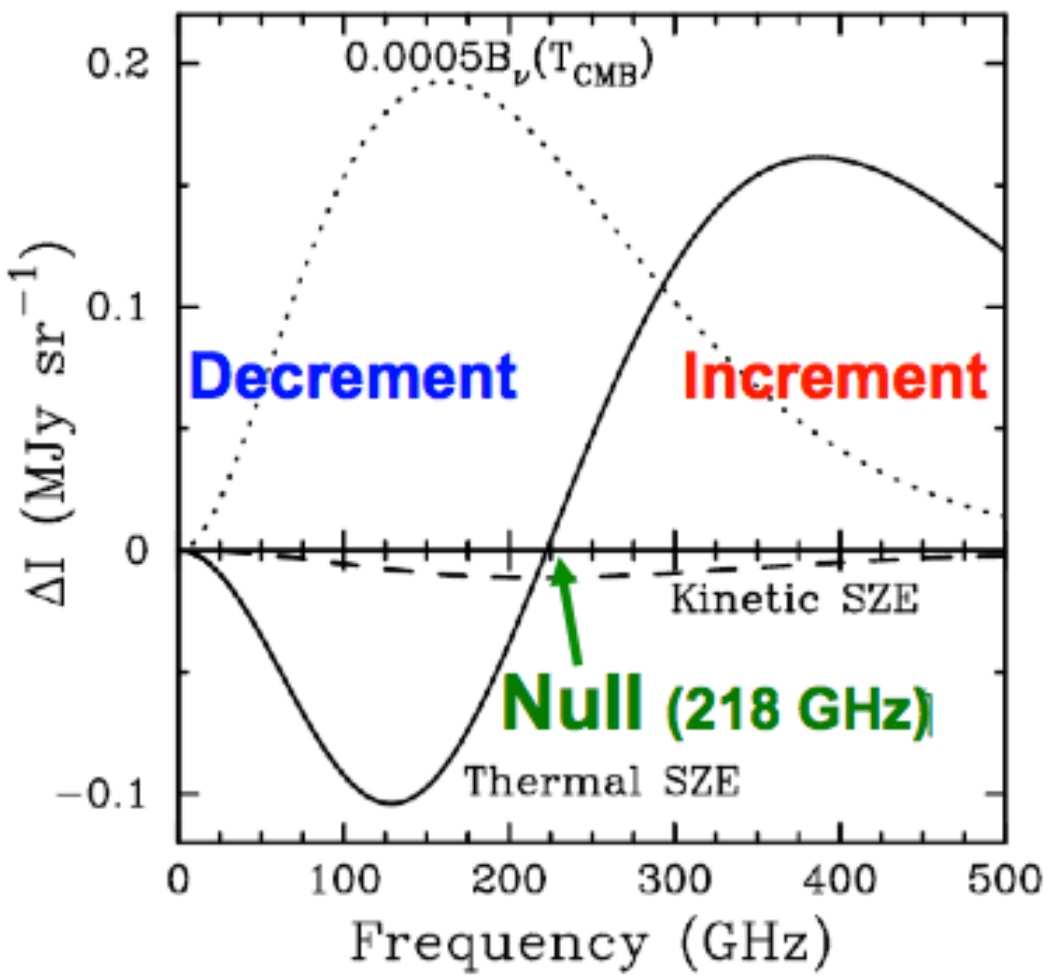


Table and figure from Voyage 2050 science paper, which were adapted from Khabibullin et al. (2018).

Thermal SZ effect summary

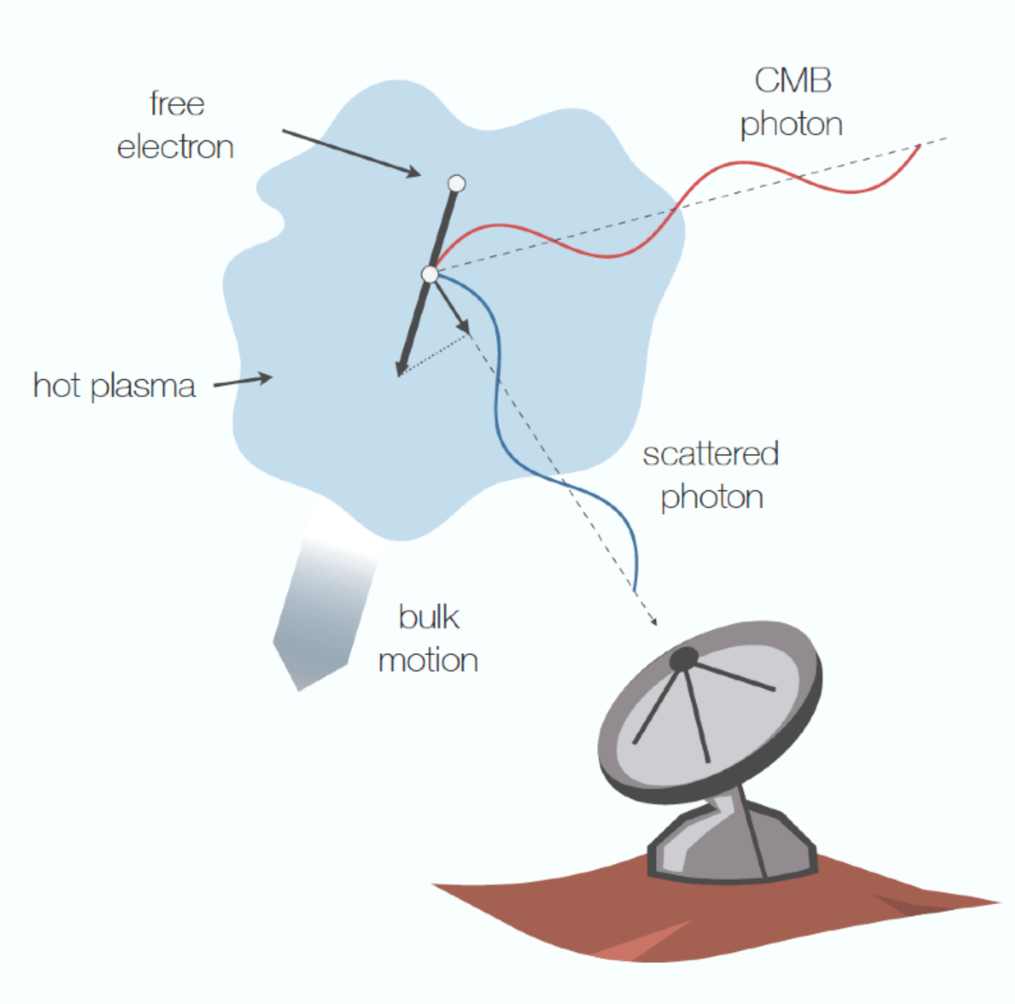


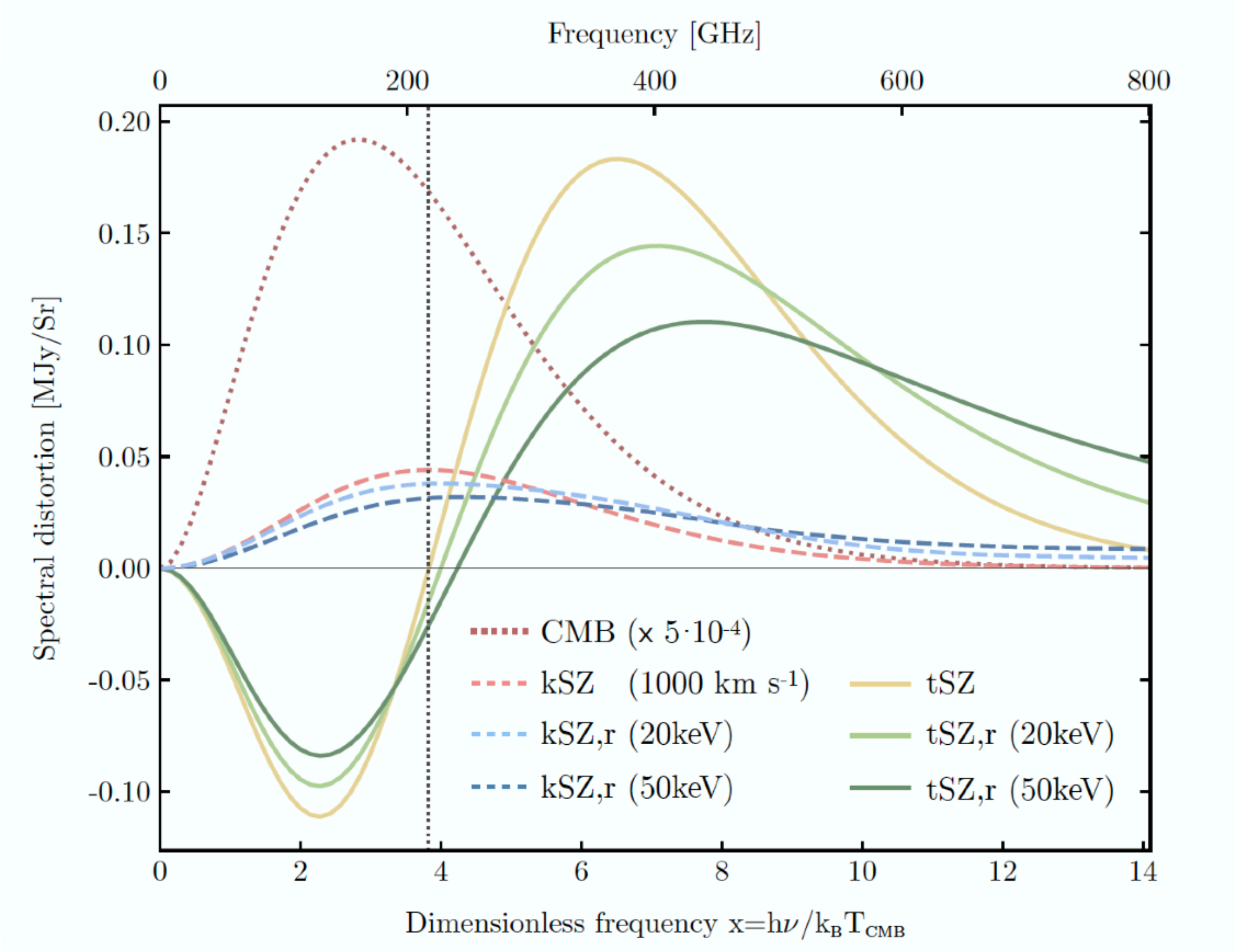


$$y \equiv \int \frac{k_B T_e}{m_e c^2}\, d\tau_e = \int \frac{k_B T_e}{m_e c^2}\, n_e \sigma_T dl = \frac{\sigma_T}{m_e c^2} \int P_e\, dl.$$

$$\Delta I_\nu \approx I_0\, y \frac{x^4 e^x}{(e^x - 1)^2} \left(x \frac{e^x + 1}{e^x - 1} - 4 \right) \equiv I_0\, y\, g(x) \qquad \frac{\Delta T_{CMB}}{T_{CMB}} \approx y \left(x \frac{e^x + 1}{e^x - 1} - 4 \right) = y\, f(x).$$

Kinematic SZ effect summary





$$\frac{\Delta T_{\rm CMB}}{T_{\rm CMB}} \approx -\int \sigma_{\rm T} n_{\rm e}\, \boldsymbol{n} \cdot \boldsymbol{\beta}_{\rm p}\, {\rm d}l = -\int \boldsymbol{n} \cdot \boldsymbol{\beta}_{\rm p}\, {\rm d}\tau_{\rm e} \equiv -y_{\rm kSZ}$$

$$\Delta I_\nu \approx -I_0 \frac{x^4 {\rm e}^x}{({\rm e}^x - 1)^2}\, y_{\rm kSZ}$$

Relativistic corrections (2.order) summary

kSZ **tSZ** **relativistic kSZ (rkSZ)**

$$\delta I_\nu = \tau \frac{2(kT_{CMB})^2}{hc^2} \frac{x^4 e^x}{(e^x-1)^2} \left\{ \boxed{\frac{V}{c}\mu} + \boxed{\frac{kT_e}{m_e c^2}(-4+F)} + \boxed{\left(\frac{V}{c}\right)^2 \left(-1-\mu^2+\frac{3+11\mu^2}{20}F\right)} \right.$$

$$+ \boxed{\frac{V}{c}\frac{kT_e}{m_e c^2}\mu\left[10-\frac{47}{5}F+\frac{7}{10}(2F^2+G^2)\right]}$$

thermal-kinematic SZ (tkSZ)

$$\left. + \boxed{\left(\frac{kT_e}{m_e c^2}\right)^2 \left[-10+\frac{47}{2}F-\frac{42}{5}F^2+\frac{7}{10}F^3+\frac{7}{5}G^2(-3+F)\right]} \right\}$$

relativistic tSZ (rtSZ / rSZ)

From Sazonov & Sunyaev (1998)

where $F = x \coth(x/2)$, and $G = x/\sinh(x/2)$.

For a massive cluster with hot plasma (~ 5 keV) and moving with very high peculiar velocity (~ 300 km/s):

$$\left(\frac{kT_e}{m_e c^2}\right) \sim 10^{-2}$$

$$\left(\frac{v_{\rm pec}}{c}\right) \sim 10^{-3}$$

All these different components of the SZ effect have different spectral dependences (i.e. different combination of *F* and *G*), so it is possible to separate them using any ILC-like method.

Relativistic corrections (2.order) summary

kSZ tSZ relativistic kSZ (rkSZ)

$$\delta I_\nu = \tau \frac{2(kT_{CMB})^2}{hc^2}\frac{x^4 e^x}{(e^x-1)^2}\left\{\frac{V}{c}\mu + \frac{kT_e}{m_e c^2}(-4+F) + \left(\frac{V}{c}\right)^2\left(-1-\mu^2+\frac{3+11\mu^2}{20}F\right)\right.$$
$$+\frac{V}{c}\frac{kT_e}{m_e c^2}\mu\left[10-\frac{47}{5}F+\frac{7}{10}(2F^2+G^2)\right]$$
$$\left.+\left(\frac{kT_e}{m_e c^2}\right)^2\left[-10+\frac{47}{2}F-\frac{42}{5}F^2+\frac{7}{10}F^3+\frac{7}{5}G^2(-3+F)\right]\right\}$$

thermal-kinematic SZ (tkSZ)

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All these different components of the SZ effect have different spectral dependences (i.e. different combination of *F* and *G*), so it is possible to separate them using any ILC-like method.

The tSZ and kSZ are now both well established observables, and the rtSZ is marginally detected. There is forecast for detecting the tkSZ in the next 10 years. The rkSZ is still a long shot..

Questions?

Feel free to email me or ask questions in our eCampus Forum