astro8405

An Introduction to the

Cosmic Microwave Background

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eCampus | Lernplattform der Universität Bonn

astro8405: The Cosmic Microwave Background

Aktionen

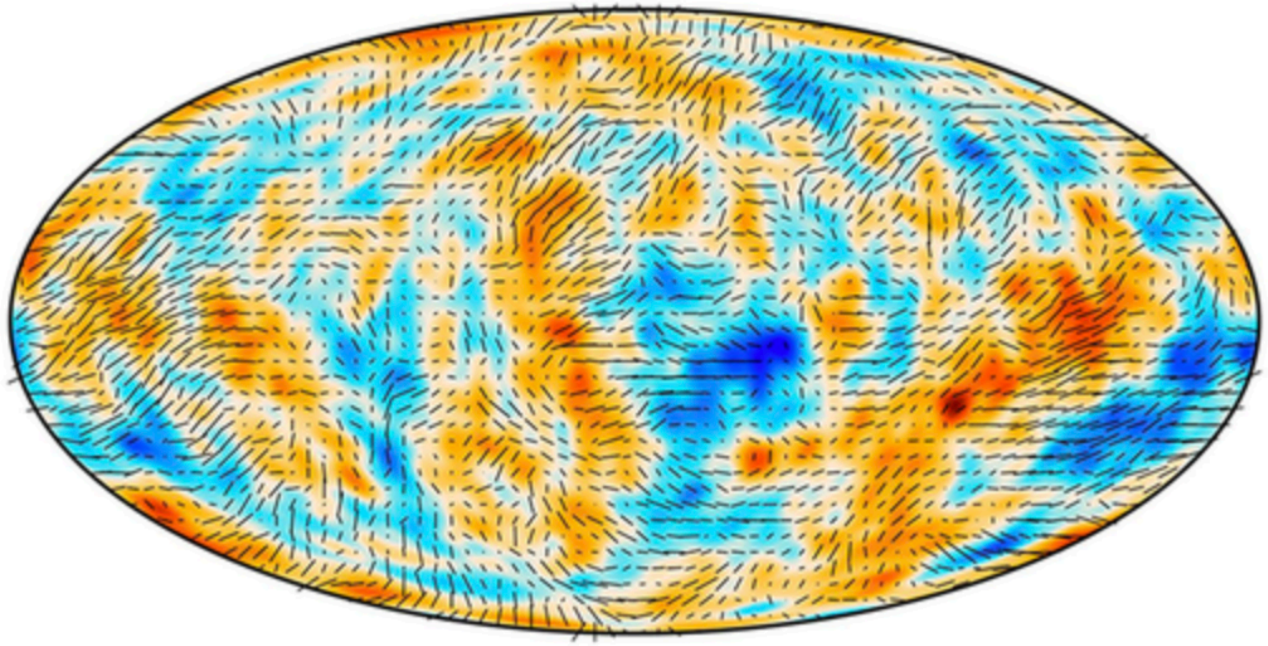
This course intends to give you a modern and up-to-date introduction to the science and experimental techniques relating to the Cosmic Microwave Background. No prior knowledge of cosmology is necessary, your prerequisite are a basic understanding of electrodynamics and thermal physics and some familiarity with Python programming.

Lecture 8:

Polarization Anisotropies:

B-modes and the primordial gravitational waves

Recap: E- and B-modes of polarization



CMB has weak linear polarization (Thomson scattering does not create circular polarization). Polarization anisotropies are described by a vector field, whereas temperature anisotropies are represented by a scalar field.

The rotational transformation of Stokes Q and U parameters lead us to define two coordinate-free variables, E and B, using the properties of spin-weighted spherical harmonics.

$$T(\hat{\boldsymbol{n}}) = \sum_{lm} a_{T,lm} Y_{lm}(\hat{\boldsymbol{n}})$$

$$(Q+iU)(\hat{\boldsymbol{n}}) = \sum_{lm} a_{2,lm}\ {}_2Y_{lm}(\hat{\boldsymbol{n}})$$

$$(Q-iU)(\hat{\boldsymbol{n}}) = \sum_{lm} a_{-2,lm}\ {}_{-2}Y_{lm}(\hat{\boldsymbol{n}}).$$

$$\tilde{E}(\hat{\boldsymbol{n}}) = \sum_{lm} \left[\frac{(l+2)!}{(l-2)!}\right]^{1/2} a_{E,lm} Y_{lm}(\hat{\boldsymbol{n}})$$

$$\tilde{B}(\hat{\boldsymbol{n}}) = \sum_{lm} \left[\frac{(l+2)!}{(l-2)!}\right]^{1/2} a_{B,lm} Y_{lm}(\hat{\boldsymbol{n}})$$

$$a_{E,lm} = -(a_{2,lm} + a_{-2,lm})/2$$

$$a_{B,lm} = i(a_{2,lm} - a_{-2,lm})/2.$$

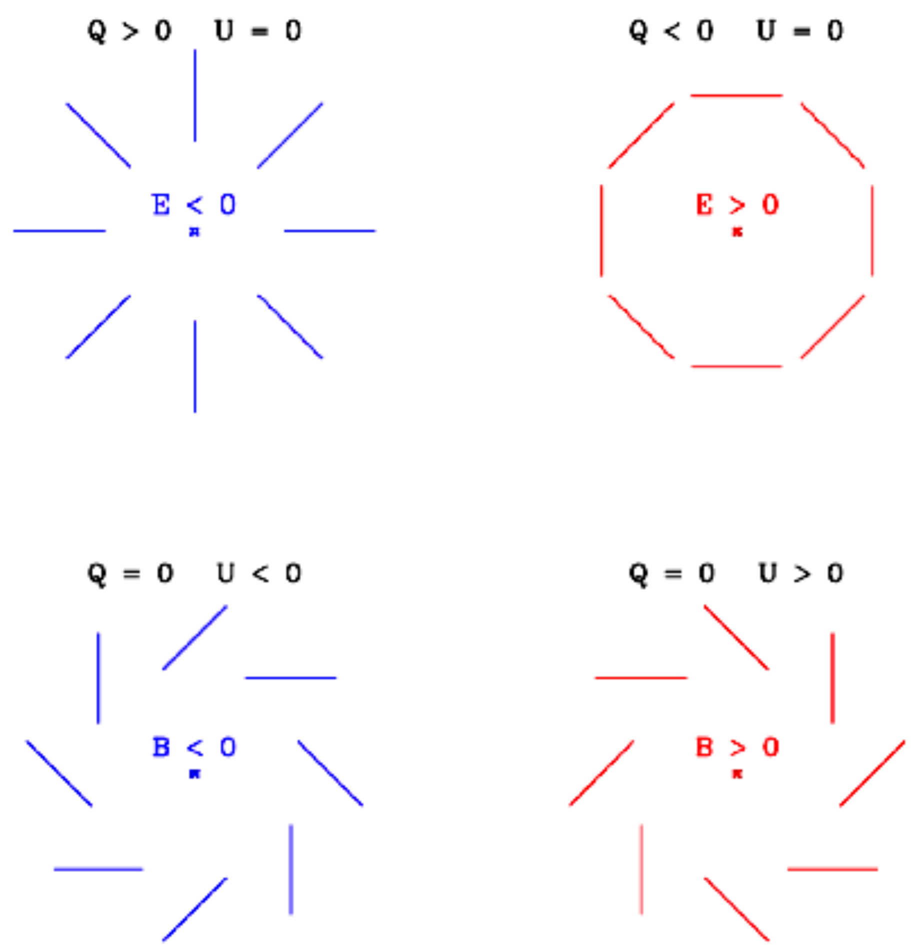
Recap: E and B mode characteristics

Two flavours of CMB polarization:

E mode → even parity → scalar perturbations
B mode → odd parity → tensor perturbations

E-mode

B-mode



- We break down the polarization field into two components which we call E and B modes. This is the spin-2 analog of the gradient/curl decomposition of a vector field.
- E modes are generated by density (scalar) perturbations via Thomson scattering.
- Additional vector modes are created by vortical motion of the matter at recombination – this is small
- B modes are generated by gravity waves (tensor perturbations) at last scattering, or by gravitational lensing (which transforms E modes into B modes along the line of sight to us) later on.

What causes the CMB quadrupole?

Two things:

"Normal" CDM: Density perturbations at $z \approx 1100$ lead to velocities that create local quadrupoles seen by scattering electrons.

=> E-mode polarization ("parity-even")

Gravity waves: create local quadrupoles seen by the scattering electrons.

=> B-mode polarization ("parity-odd")

The problem of understanding the polarization pattern of the CMB thus reduces to understanding the quadrupole temperature fluctuations at the *moment of last scattering*.

From gravity waves to B-mode polarization

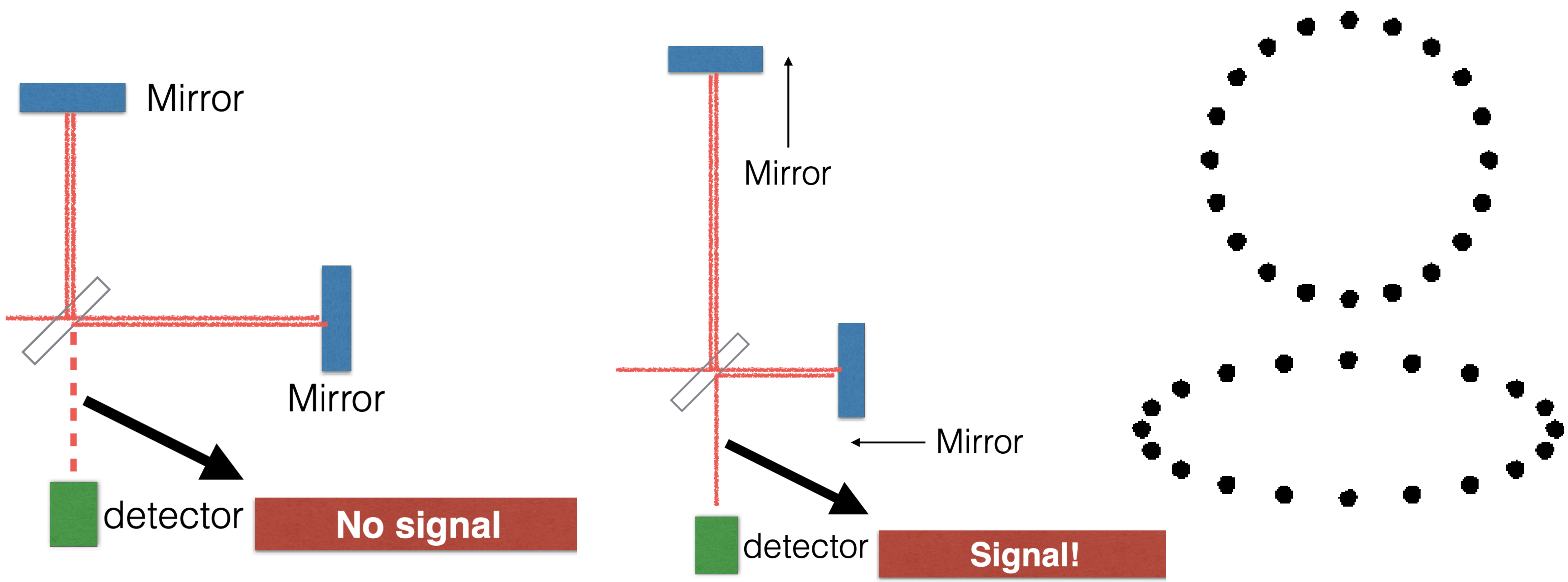
Arguably the cleanest prediction of inflation is a spectrum of primordial gravitational waves. These are tensor perturbations to the spatial metric,

$$ds^2 = a^2 \sum_{i=1}^{3} \sum_{j=1}^{3} (\delta_{ij} + h_{ij}) dx^i dx^j$$

"metric perturbation"

-> CURVED SPACE!

A plane gravitational wave causes a "quadrupolar stretching" of the space (tensor mode perturbations, as opposed to scalar modes from density).



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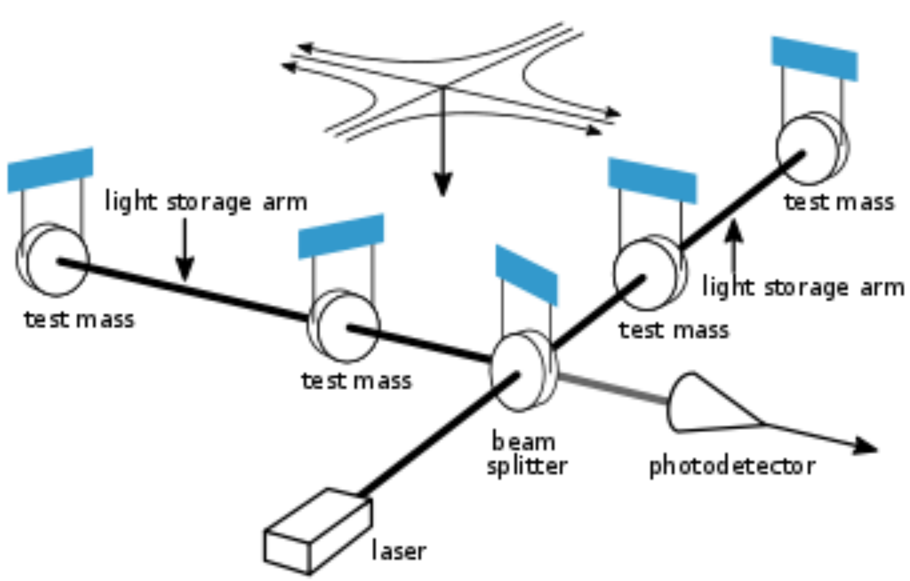
A plane gravitational wave causes a "quadrupolar stretching" of the space (tensor mode perturbations, as opposed to scalar modes from density).

This changes a circle of test particles into an ellipse, and the radiation acquires a *m=2* quadrupole pattern → primordial B-mode signal



Gravitational waves detected by LIGO → wavelength thousands of kilometers

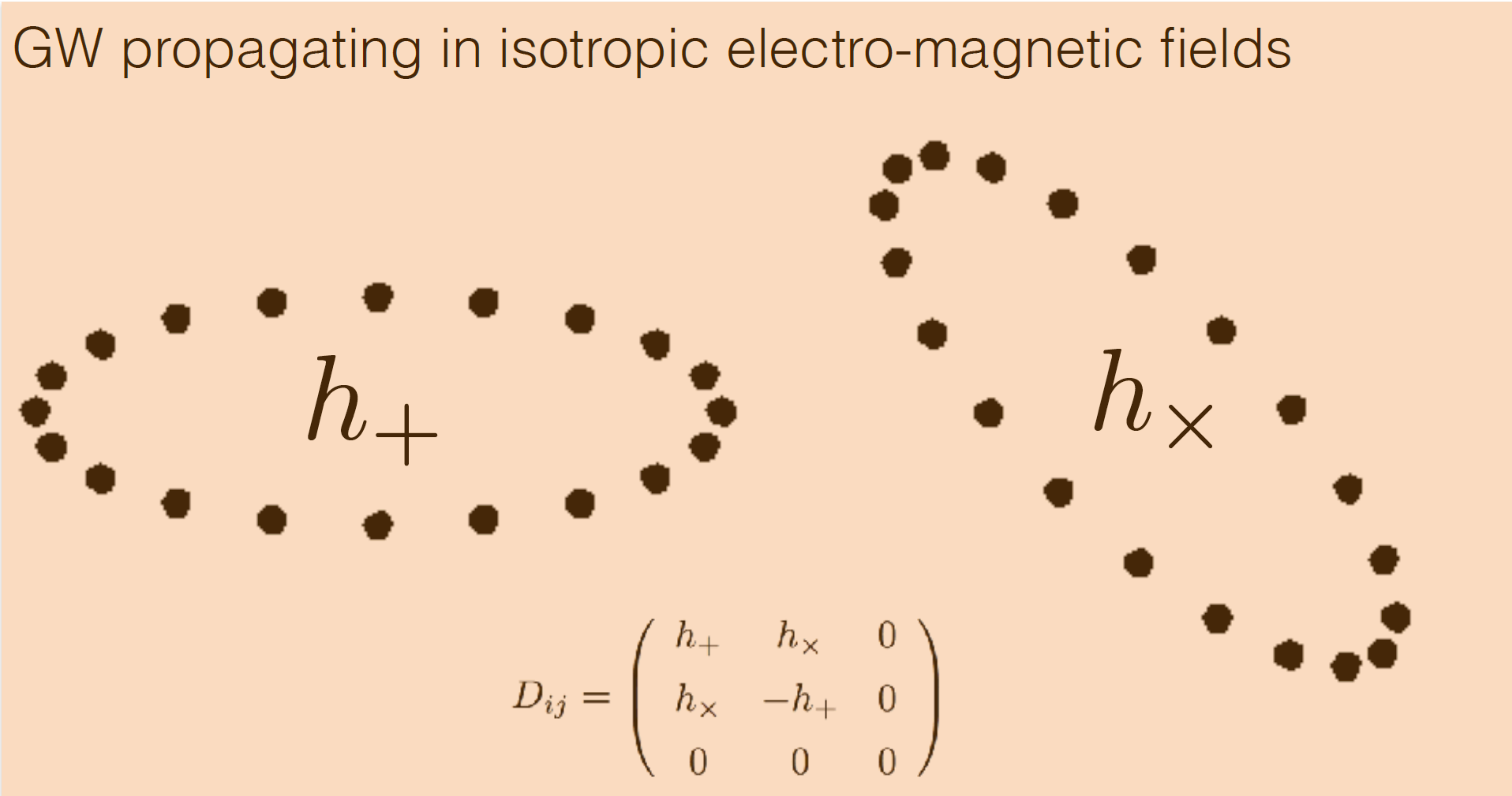
Primordial gravitational waves produced by inflation → wavelength billions of light-years!

From GW to CMB polarization

Isotropic electro-magnetic fields

Slide courtesy: E. Komatsu

From GW to CMB polarization



D_{ij} **: Tensor metric perturbation [=gravitational waves]**

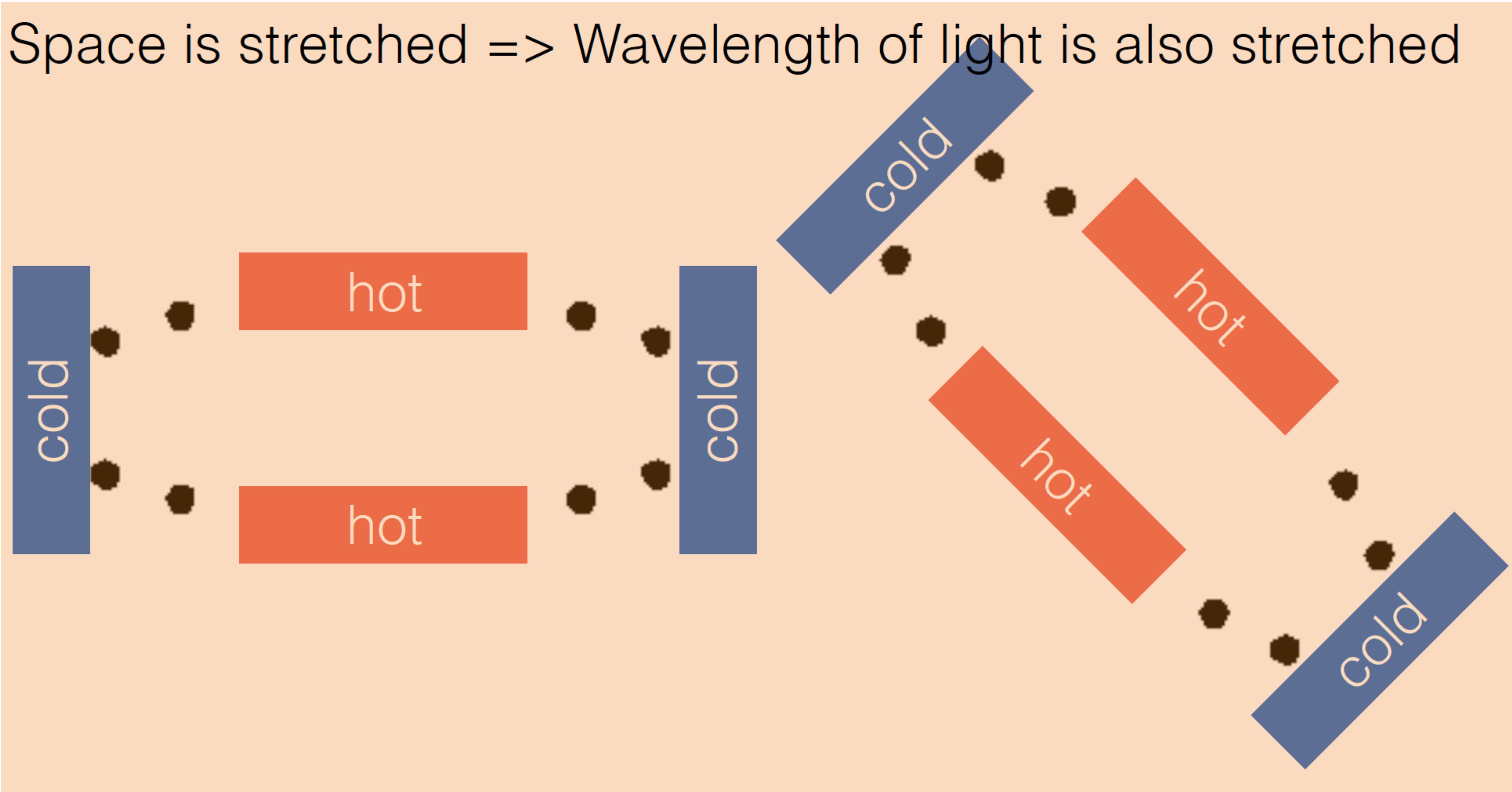
Slide courtesy: E. Komatsu

$$ds^2 = a^2 \sum_{i=1}^{3} \sum_{j=1}^{3} (\delta_{ij} + h_{ij}) dx^i dx^j$$

"metric perturbation"

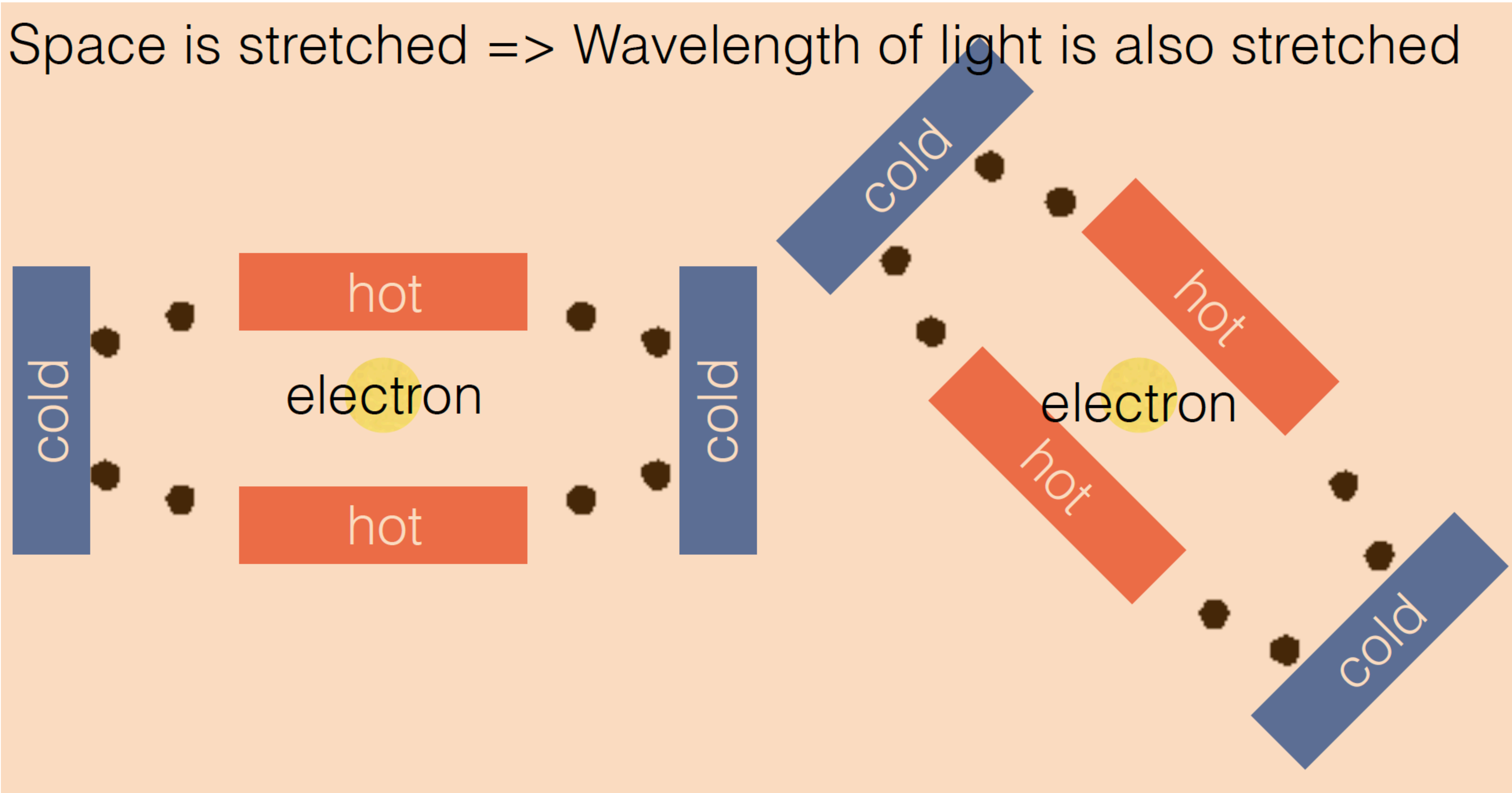
(D_{ij} and h_{ij} are the same thing, just different notations!)

From GW to CMB polarization



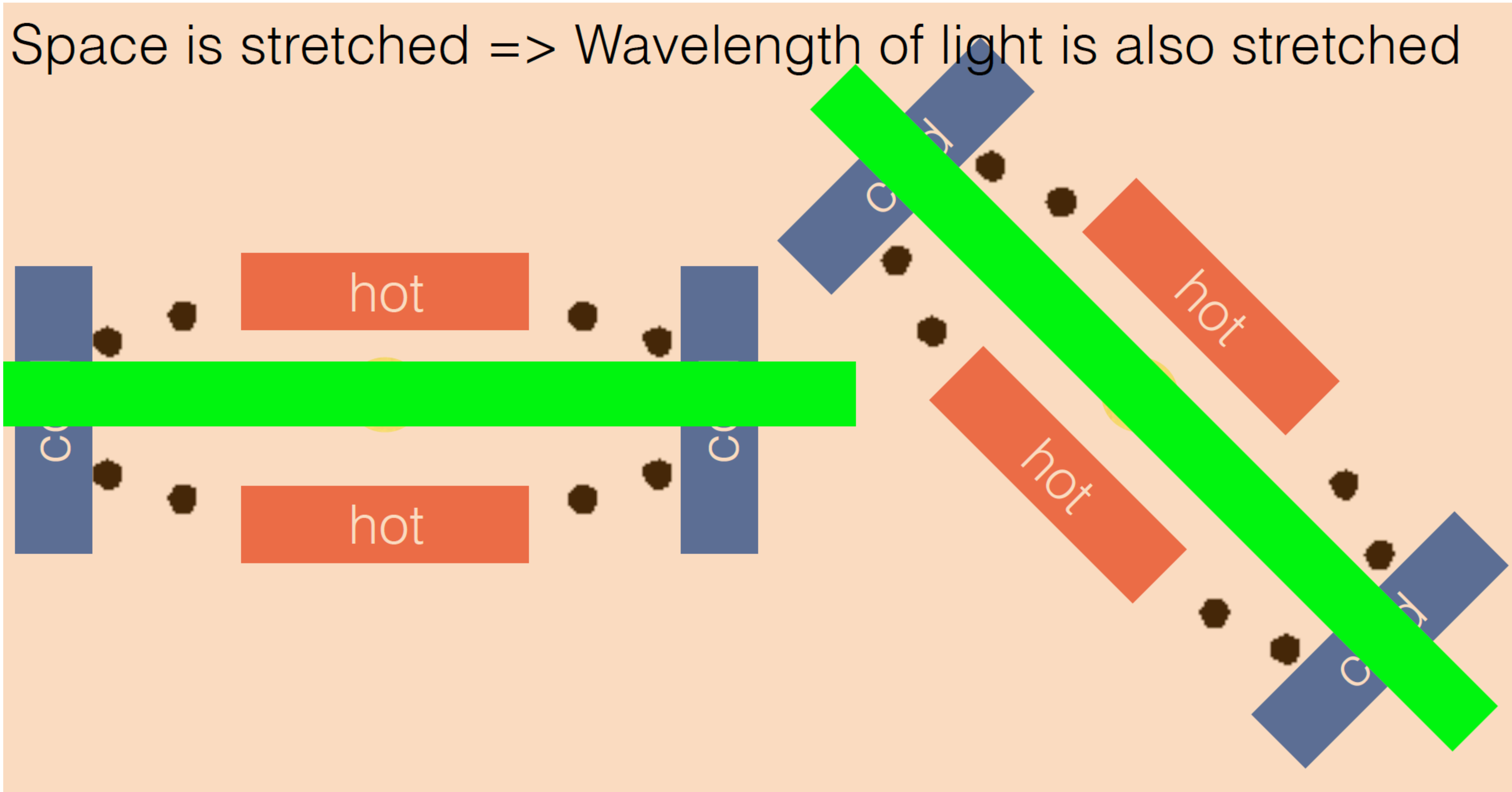
Slide courtesy: E. Komatsu

From GW to CMB polarization



Slide courtesy: E. Komatsu

From GW to CMB polarization



Slide courtesy: E. Komatsu

From GW to CMB polarization

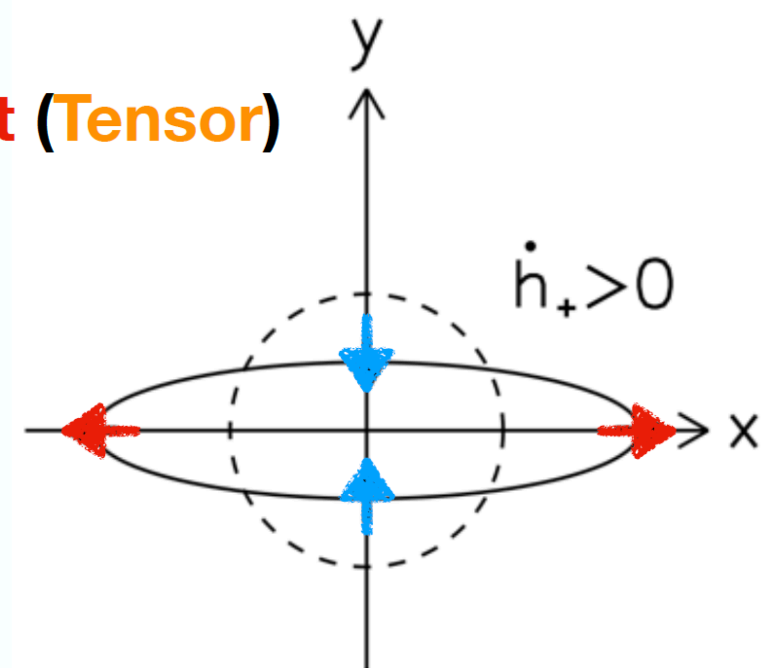
Spatial curvature metric

Gravitational potential

$$\frac{1}{p}\frac{dp}{dt} = -\frac{\dot{a}}{a} + \dot{\Psi} - \frac{1}{a}\sum_i \frac{\partial \Phi}{\partial x^i}\gamma^i - \frac{1}{2}\sum_{ij}\dot{D}_{ij}\gamma^i\gamma^j$$

- **Gravitational blue/redshift (Tensor)**

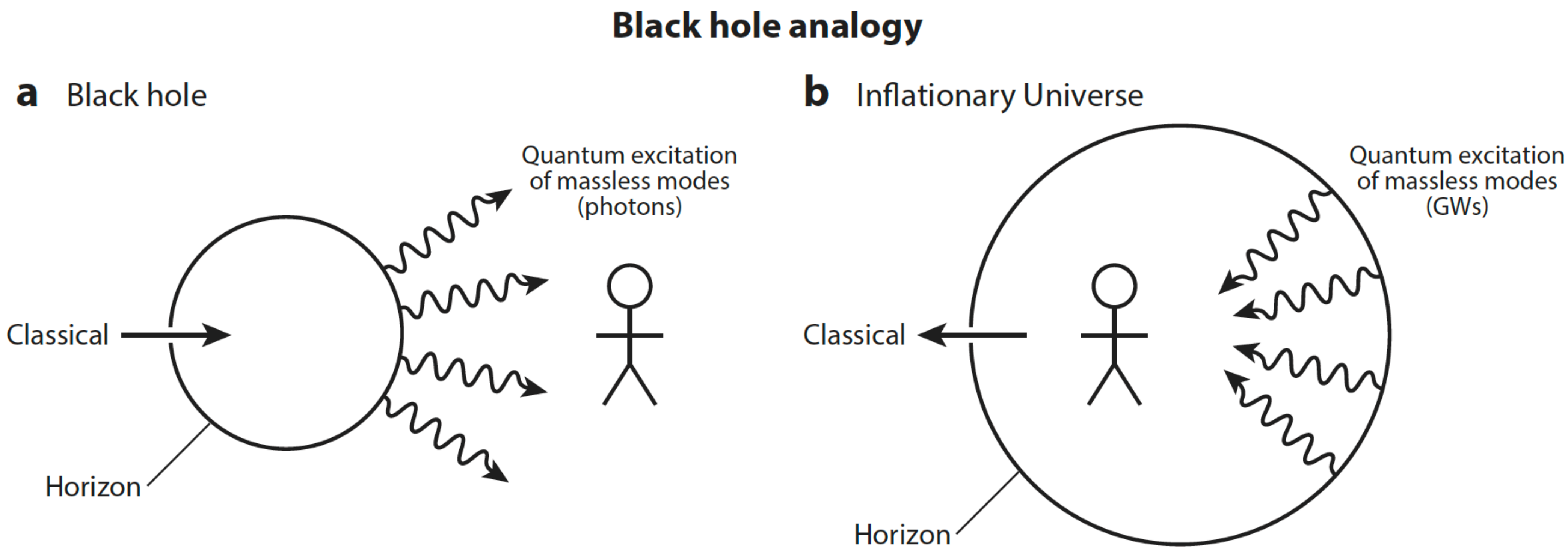
$$D_{ij} = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



The change in photon energy (or its wavelength) is proportional to the time derivative of the metric perturbation, i.e., the difference in the metric perturbation at different epochs. As a result of inflation, the gravitational waves are very large (many times the horizon scale), hence the difference of metric perturbations between two different cosmic times is also large, resulting in the quadrupole temperature anisotropy from gravitational waves.

Why inflation would produce GW?

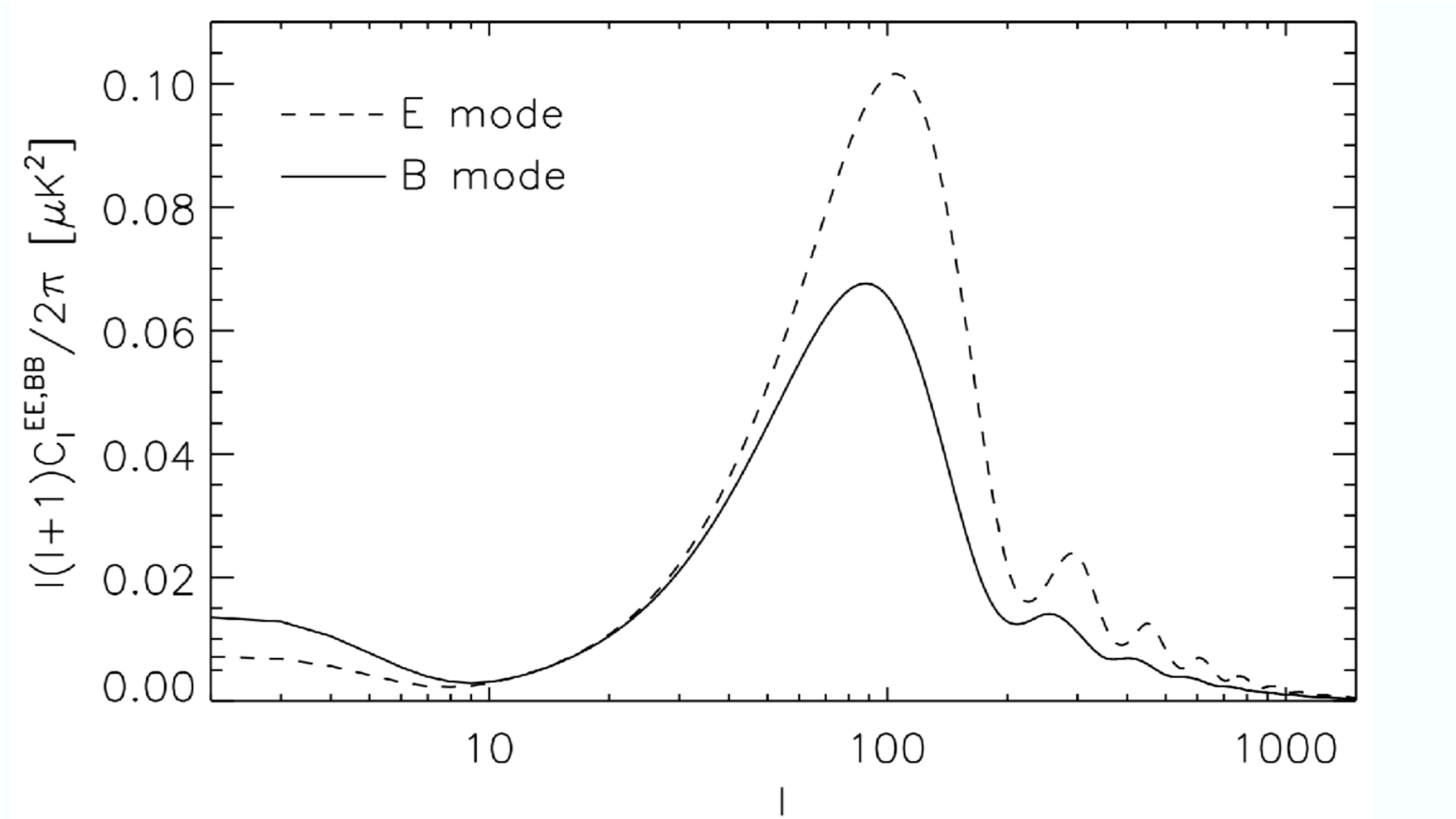
(From Kamionkowski & Kovetz, ARAA 2016)



HEURISTIC UNDERSTANDING OF INFLATIONARY GRAVITATIONAL WAVES

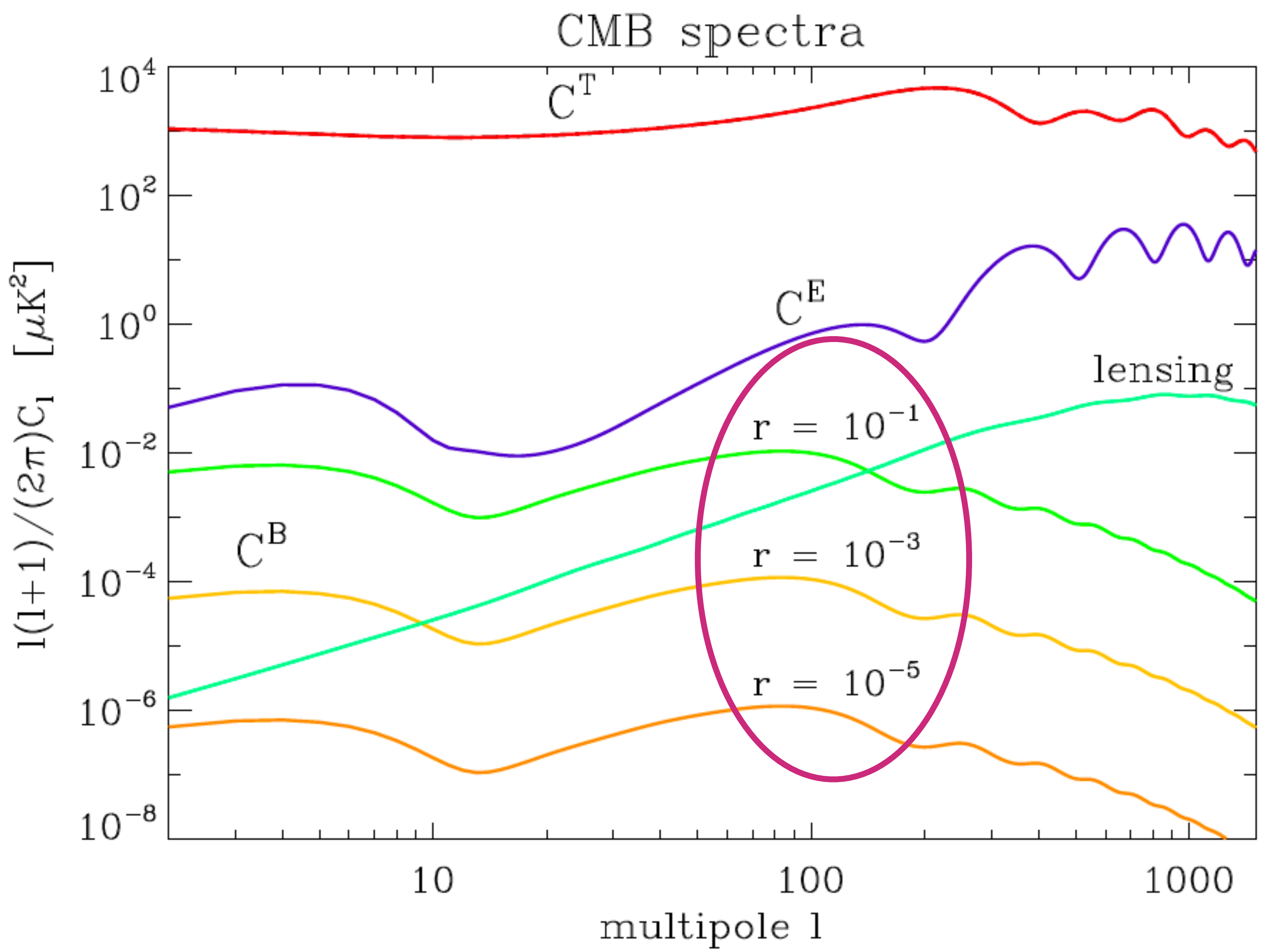
Here we present in simple heuristic terms the origin of IGWs. Consider first a black hole. As shown in **Figure 2**, it has an event horizon, a spherical surface beyond which, according to (classical) general relativity, objects and information disappear without a trace. Hawking showed, however, that when quantum mechanics is taken into consideration, the horizon glows—it emits electromagnetic radiation (Hawking 1975). Hawking's argument also applies, however, to any radiation field with massless quanta, and so the black hole also radiates GWs. In an FRW Universe with an accelerated expansion, there is also a horizon, a spherical surface beyond which (according to general relativity) objects and information disappear. This time, though, the observer sees this spherical surface from the inside, rather than the outside. Just as was the case with the black hole horizon, this horizon also radiates GWs, according to quantum mechanics. These GWs are produced throughout inflation, and the expansion rate and thus horizon temperature are nearly constant during inflation. These GWs thus remain, after inflation, as a primordial-GW background with a nearly scale-invariant spectrum.

Gravity waves also create E-modes



We should note that the tensor perturbations, caused by gravitational waves, also generate E-mode polarization (as well as temperature anisotropies). In fact, E- and B-modes are created almost equally (see figure above). However, the E-modes created by density (scalar) perturbations at the last scattering are much stronger, so one can neglect the tensor-origin of E-modes.

Recap on inflation and the meaning of the tensor-to-scalar ratio, *r*



Tensor–to–scalar ratio

1. Scalar power spectrum

The scaler perturbations are Gaussian, so all information about them is contained in the two-point correlation function:

$$\langle \mathcal{R}(\mathbf{k})\mathcal{R}^*(\mathbf{k}')\rangle = \frac{P(k)}{(2\pi)^3}\delta(\mathbf{k}-\mathbf{k}'),$$

The mean square value of the initial perturbation amplitude is

$$\langle \mathcal{R}^2(\mathbf{x})\rangle = \langle \int e^{i\mathbf{kx}} R(\mathbf{k}) d^3k \int e^{-i\mathbf{k}'\mathbf{x}} R^*(\mathbf{k}') d^3k' \rangle = \int d^3k \frac{P(k)}{(2\pi)^3} = \int_0^\infty \frac{dk}{k}\mathcal{P}(k),$$

Where $\mathcal{P}(k) = k^3 P(k)/(2\pi^2)$ is also called the power spectrum, and is approximated as follows:

$$\mathcal{P}_s(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$$

In 1960's, Zel'dovich and Harrison independently predicted the flat spectrum of perturbations (n_s=1). But we now know that the spectrum is *slightly red.* The WMAP5 values for a fixed k_* = 500 Mpc^{-1} are:

$$A_s = (2.46 \pm 0.09)\cdot 10^{-9},$$
$$n_s = 0.960 \pm 0.014.$$

Tensor-to-scalar ratio

2. Tensor power spectrum & power ratio, *r*

Actually, the derivation of approximately flat power spectrum does not depend on whether we deal with scalar or tensor fields. **Inflation also generates tensor perturbations** (transverse traceless perturbations of spatial metric h_{ij}, i.e. gravitational waves).

We have the same picture for tensor perturbations: primordial perturbations are Gaussian random field with almost flat power spectrum. In this case we have

$$\mathcal{P}_T(k) = A_T \left(\frac{k}{k_*}\right)^{n_T} .$$

It is convenient to introduce the parameter $r = \mathcal{P}_T/\mathcal{P}_s$ which measures the ratio of tensor to scalar perturbations.

For some early inflation theories with power-law potentials, prediction was $r \sim 0.1 - 0.3$

➜ **these are now practically ruled out by Planck data**

3. Inflation and the spectral index, n_s

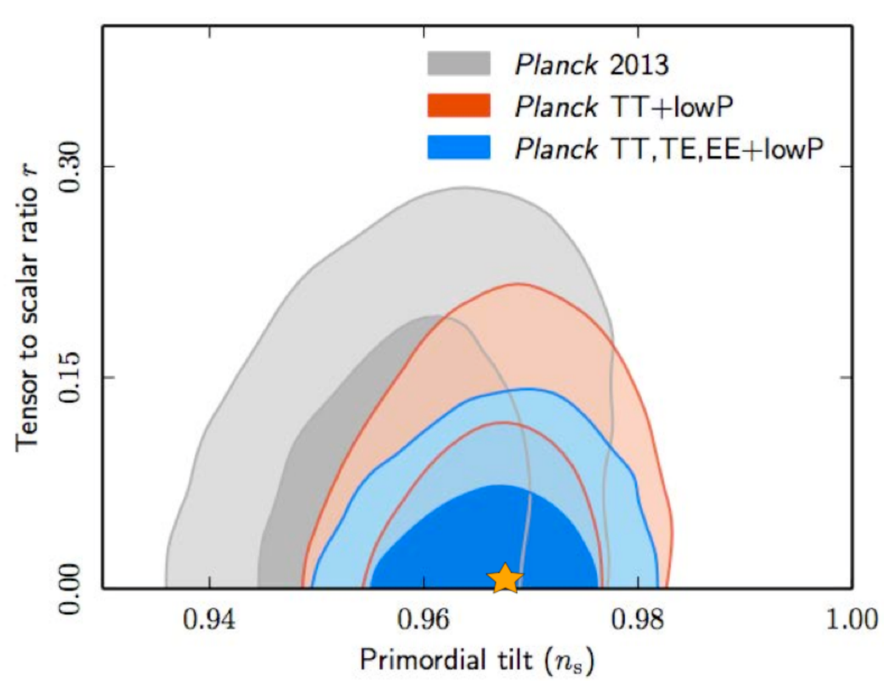
Inflation occurs if the universe is filled with a scalar field φ, which has non-vanishing scalar potential V(φ). The homogeneous field φ then satisfies the equation

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{dV}{d\varphi}. \qquad a(t) \propto \exp\left(\int H dt\right), \quad H \approx \text{const.}$$

For a relatively flat potential (dV/dφ small), the acceleration term can be neglected. The Friedmann equation in this case is $H^2 = 8\pi/3G\, V(\varphi)$. So if φ varies slowly, then V(φ) and thus H also varies slowly, and the parameters of inflation are almost time independent *(slow-roll inflation)*.

Yet, the parameters are not *exactly* time-independent (inflation has to end!), so the predicted value of the spectral tilt (n_s – 1) is small but non-zero. It can be positive or negative, depending on the scalar potential V(φ). In particular, it is negative for the simplest power-law potentials like

$$V(\varphi) = \frac{m^2}{2}\varphi^2 \quad \text{or} \quad V(\varphi) = \frac{\lambda}{4}\varphi^4.$$



For the case of slow-roll inflation,

$$n_s - 1 = -3M_{\rm pl}^2\left(\frac{V'}{V}\right)^2 + 2M_{\rm pl}^2\frac{V''}{V}$$

with $' \equiv \frac{\partial}{\partial \phi}$.

Tensor-to-scalar ratio

4. Possible shapes of inflation potential

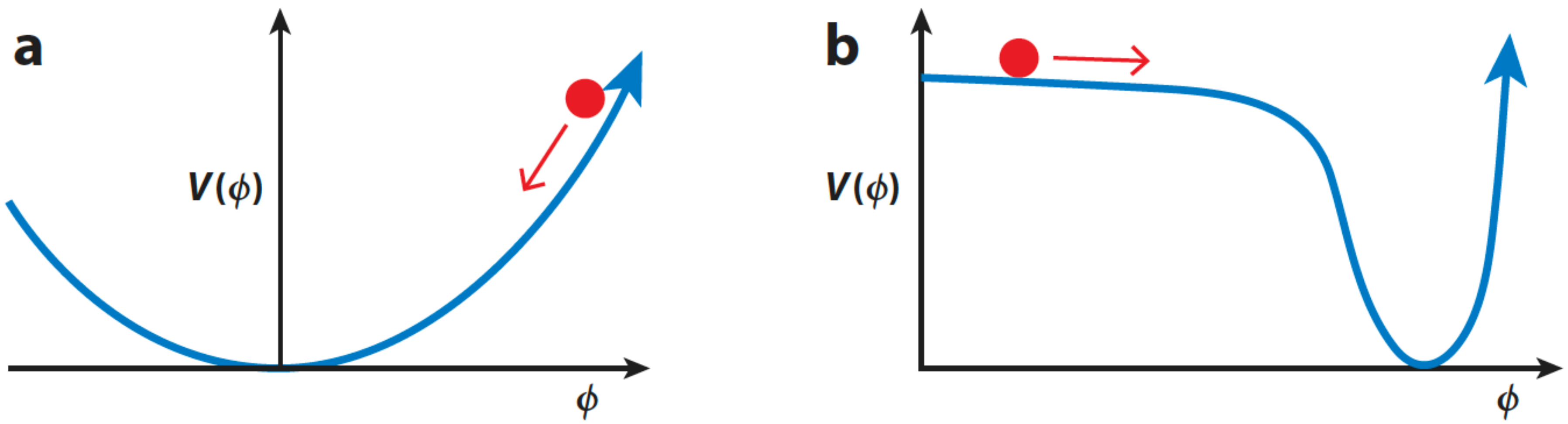


Figure 1

Inflation postulates that at some point in the early history of the Universe, the cosmic energy density was dominated by the vacuum energy associated with the displacement of some scalar field ϕ (the inflaton) from the minimum of its potential. Shown here for illustration are two toy models for the inflaton potential: Panel *a* shows a quadratic potential, and panel *b* shows a hilltop potential. Adapted from Kamionkowski & Kosowsky (1999) with permission.

$$\epsilon = 3\frac{\dot{\phi}^2/2}{V + \dot{\phi}^2/2} \simeq \frac{{M_{\rm Pl}}^2}{2}\left(\frac{V'}{V}\right)^2$$

The number of *e-folding* during inflation is determined by the slope of the potential, characterized by the parameter $\epsilon \equiv -\dot{H}/H^2 < 1$

$$N(t) \equiv \ln\frac{a(t_{\rm end})}{a(t)} = \int_t^{t_{\rm end}} H\ {\rm d}t = -\frac{1}{2m_{\rm Pl}^2}\int_{\phi_t}^{\phi_{\rm end}} \frac{H}{H'}\,{\rm d}\phi = \int_{\phi_{\rm end}}^{\phi_t} \frac{{\rm d}\phi}{M_{\rm Pl}}\frac{1}{\sqrt{2\epsilon(\phi)}}.$$

Tensor–to–scalar ratio

5. Predictions for single-field, slow-roll inflation

Slow–Roll inflation requires the acceleration term for the potential is zero, and the shape of the potential is parametrized by the “slow–roll parameters”

$$\ddot{\varphi} + 3H\dot{\varphi} = -V'(\varphi)$$

$$H^2 = \frac{V(\varphi)}{3M_{\rm Pl}^2}$$

$$3H\dot{\varphi} = -V'(\varphi)$$

$$\varepsilon(\varphi) \equiv \frac{1}{2}M_{\rm Pl}^2\left(\frac{V'}{V}\right)^2$$

$$\eta(\varphi) \equiv M_{\rm Pl}^2\frac{V''}{V}$$

For all physical, slow-roll inflation models, ϵ, η << 1

For scalar perturbations:

$$\Delta_{\mathcal{R}}^2(k) \equiv A_s\left(\frac{k}{k_\star}\right)^{n_s-1}$$

$$n_s - 1 = -2\varepsilon - \eta$$

For tensor perturbations:

$$\Delta_t^2(k) \equiv A_t\left(\frac{k}{k_\star}\right)^{n_t}$$

$$r \equiv \frac{\Delta_t^2}{\Delta_{\mathcal{R}}^2} = 16\epsilon$$

$$n_t = \frac{d\ln\Delta_t^2(k)}{d\ln k} = -2\epsilon$$

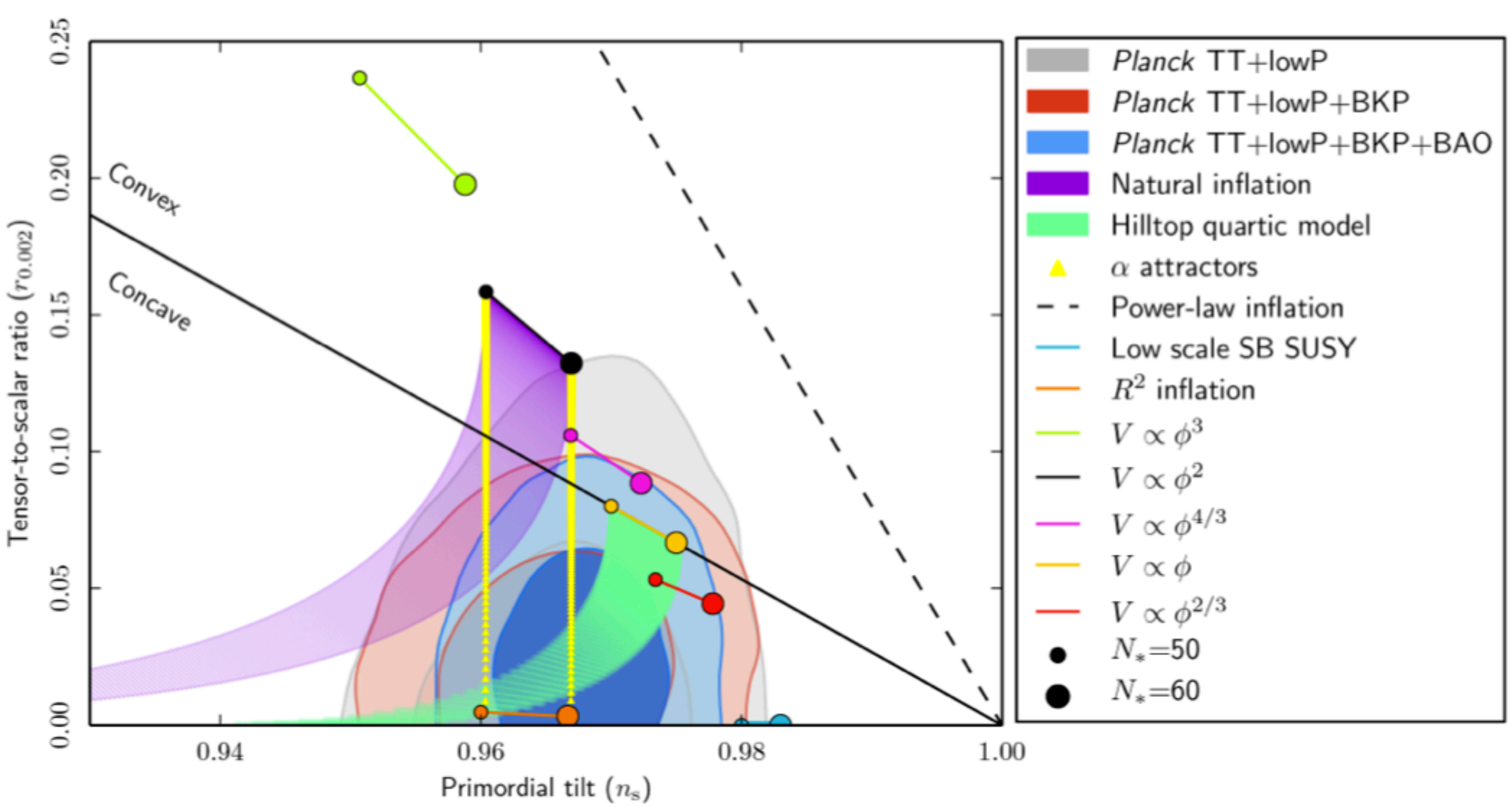
$$r = 16\varepsilon$$

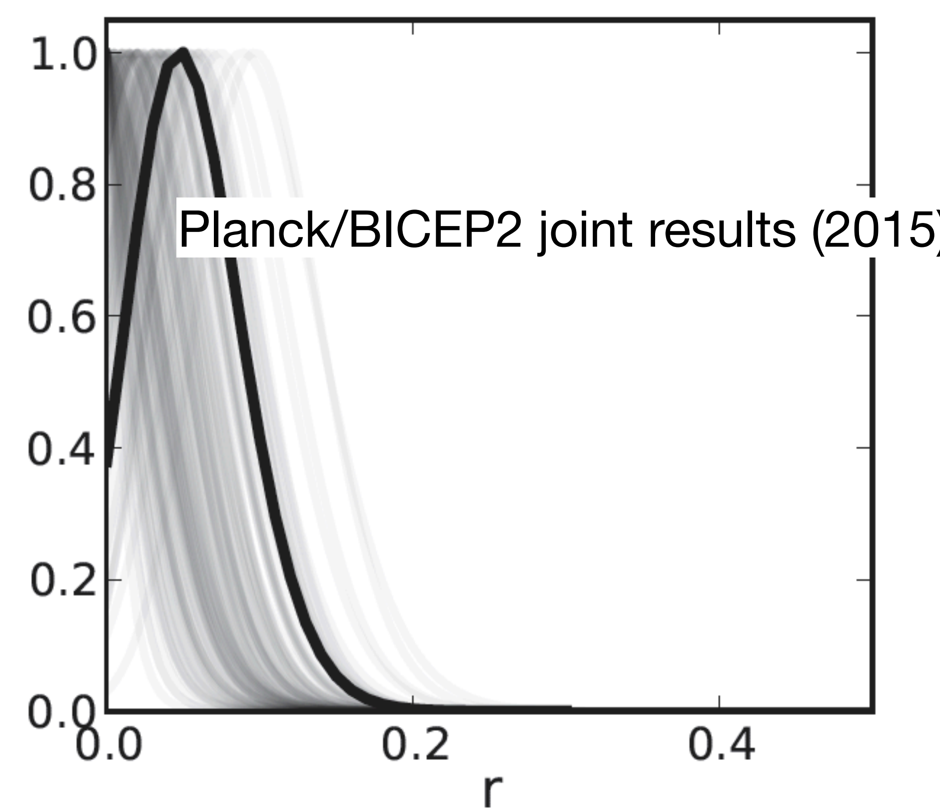
$$n_t = -2\varepsilon$$

The tensor amplitude is therefore a direct measure of the expansion rate H during inflation. Knowing this, one can completely determine the energetics of inflation!

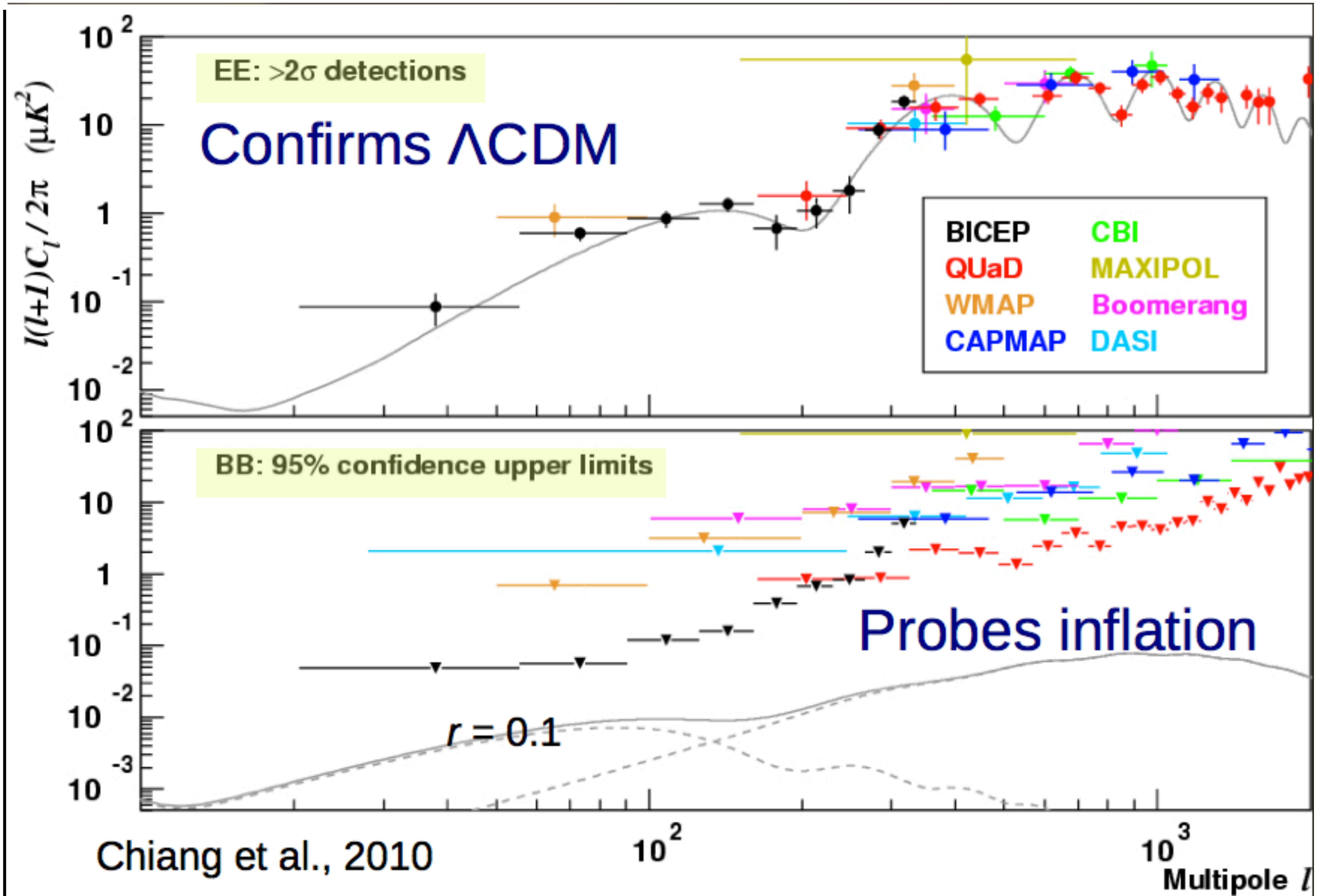
- Single-field slow-roll inflation looks remarkably good:
  - **Super-horizon fluctuation**
  - **Adiabaticity**
  - **Gaussianity**
  - **$n_s<1$**

But we want a direct confirmation of inflation and probe its energy scale: ***Gravitational waves!***





Measurements of E, B power from ground



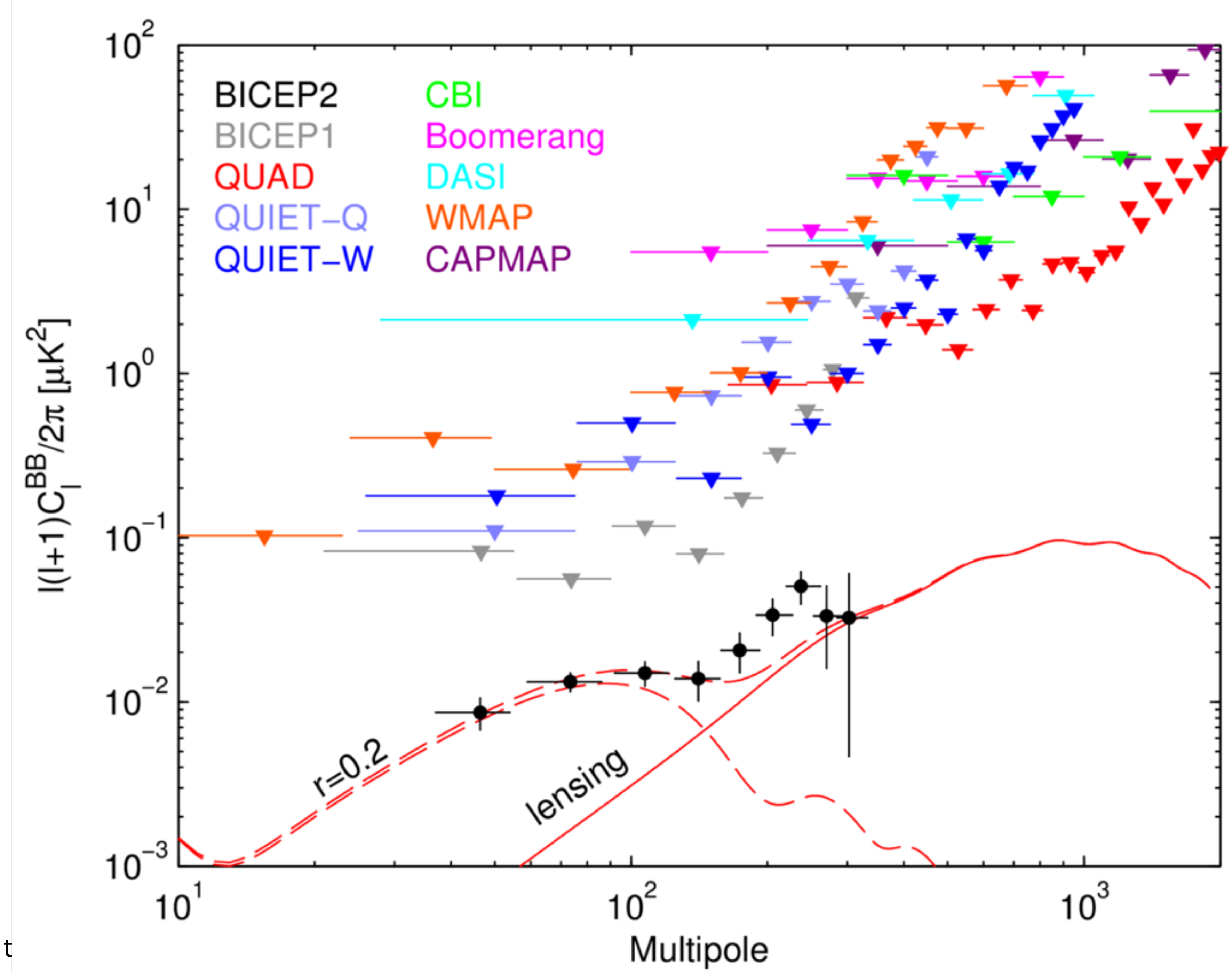
BICEP2 result in 2014

arXiv.org > astro-ph > arXiv:1403.3985

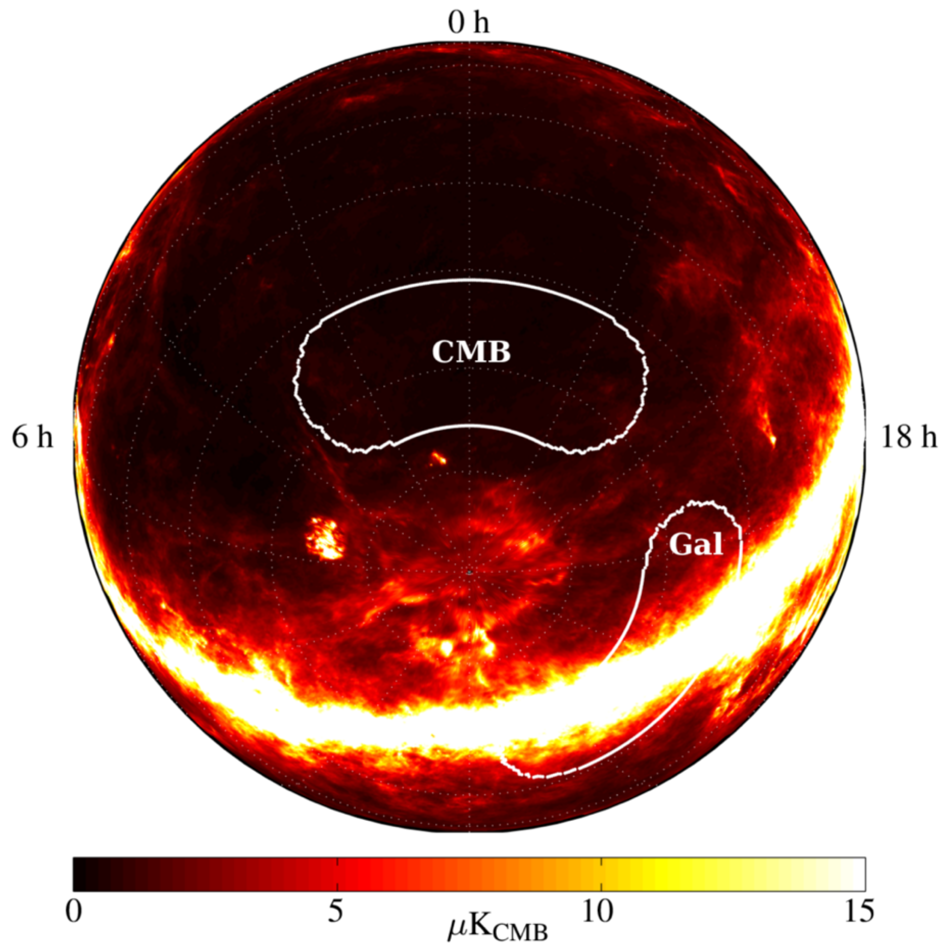
Astrophysics > Cosmology and Nongalactic Astrophysics

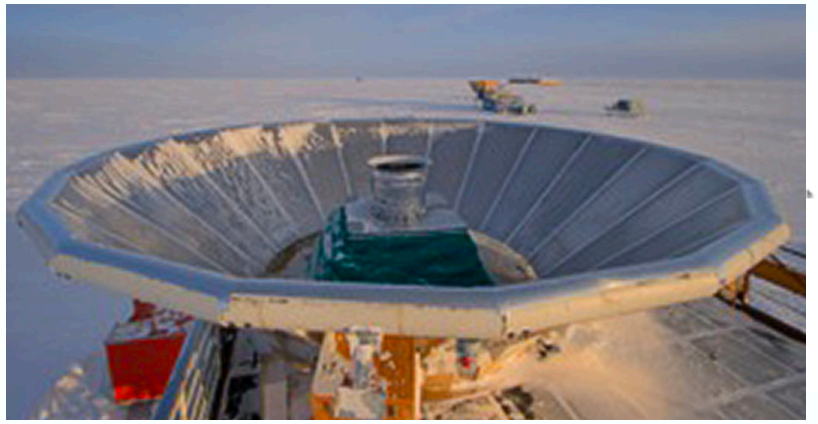
[Submitted on 17 Mar 2014 (v1), last revised 23 Jun 2014 (this version, v3)]

BICEP2 I: Detection Of B-mode Polarization at Degree Angular Scales

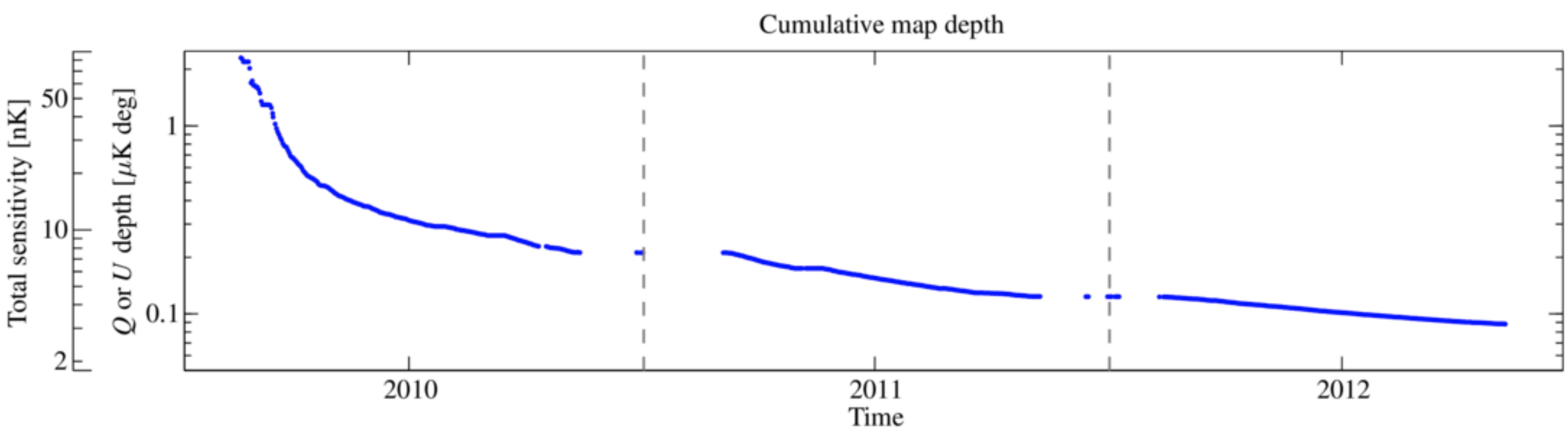


BICEP2 observation of the CMB polarization

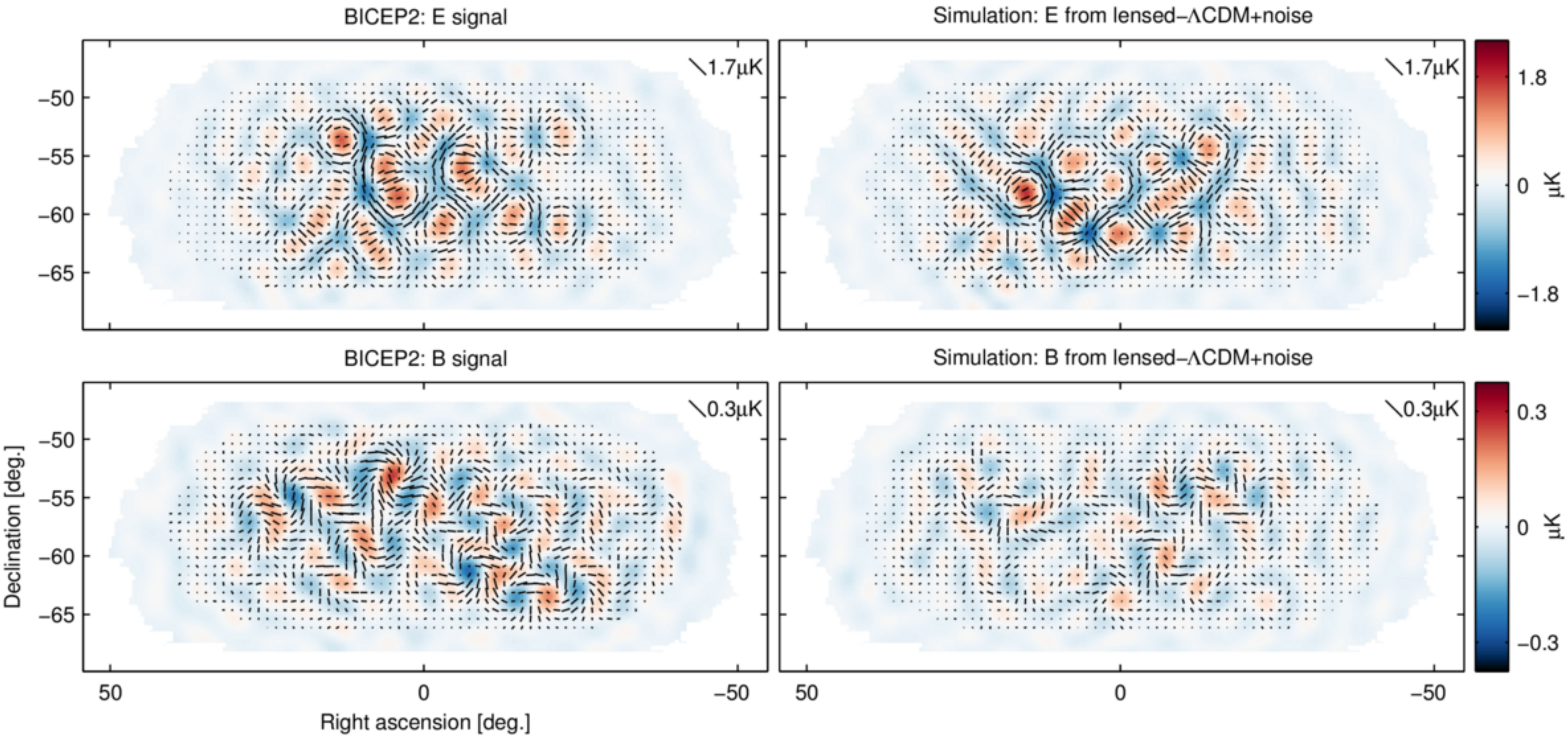




- Small telescope at South Pole
- 512 bolometers at 150 GHz
- Observed 380 square degrees for three years (2010 - 2012)
- Previous BICEP1 at 100 and 150 GHz (2006-2008)
- Current: Keck Array = 5 x BICEP2 at 150 GHz (2011 - 2013) and additional detectors at 100 and 220 GHz (2014 onwards)



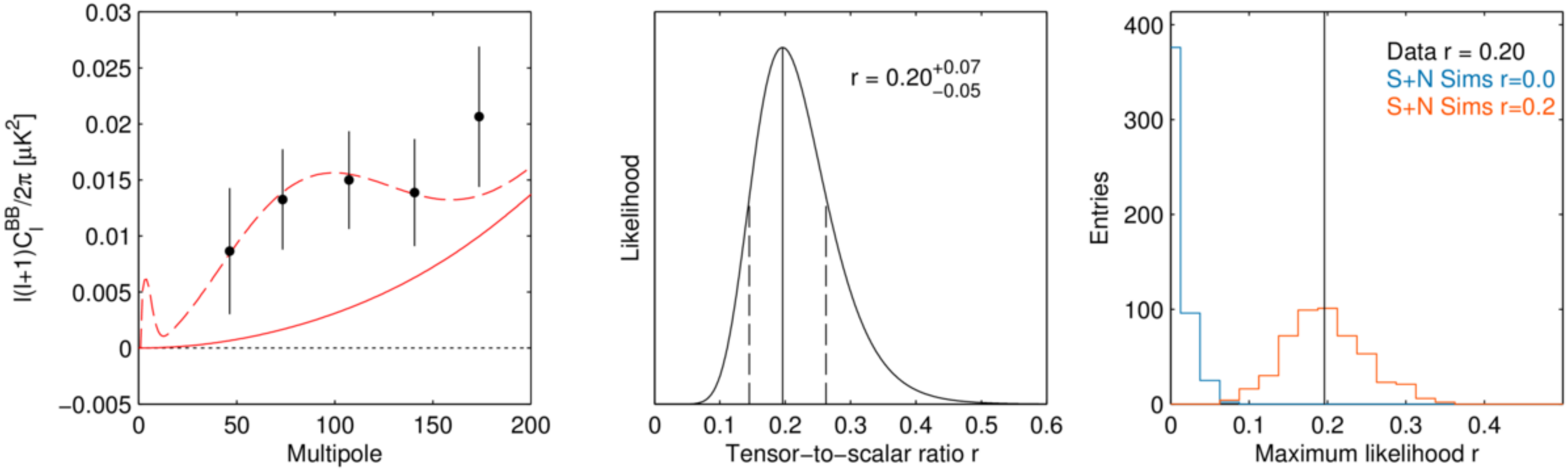
BICEP2 E- and B-mode CMB maps

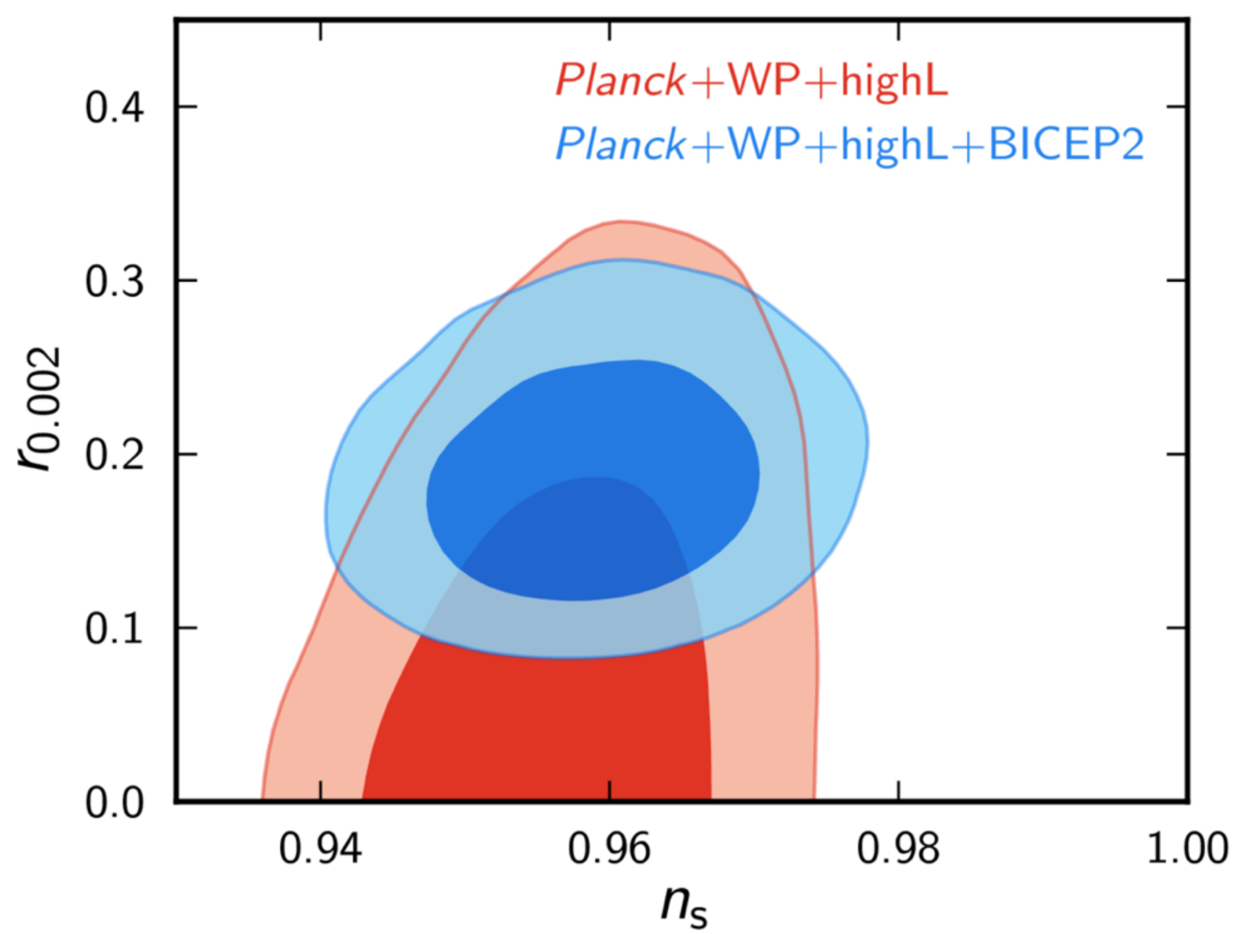


Detection of B-Mode Polarization at Degree Angular Scales by BICEP2

P. A. R. Ade *et al.* (BICEP2 Collaboration)
Phys. Rev. Lett. **112**, 241101 – Published 19 June 2014

BICEP2 scaler-to-tensor ratio



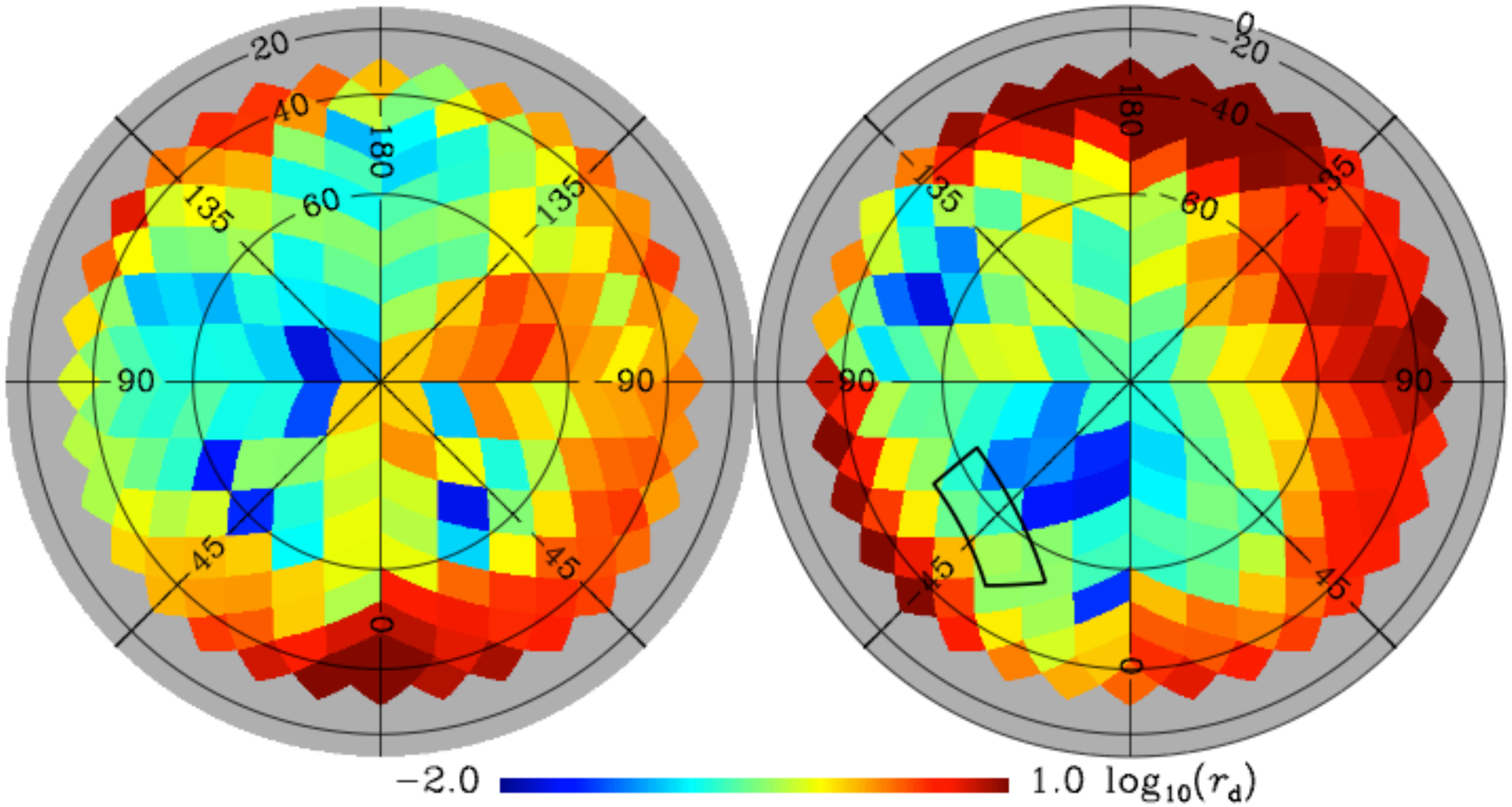


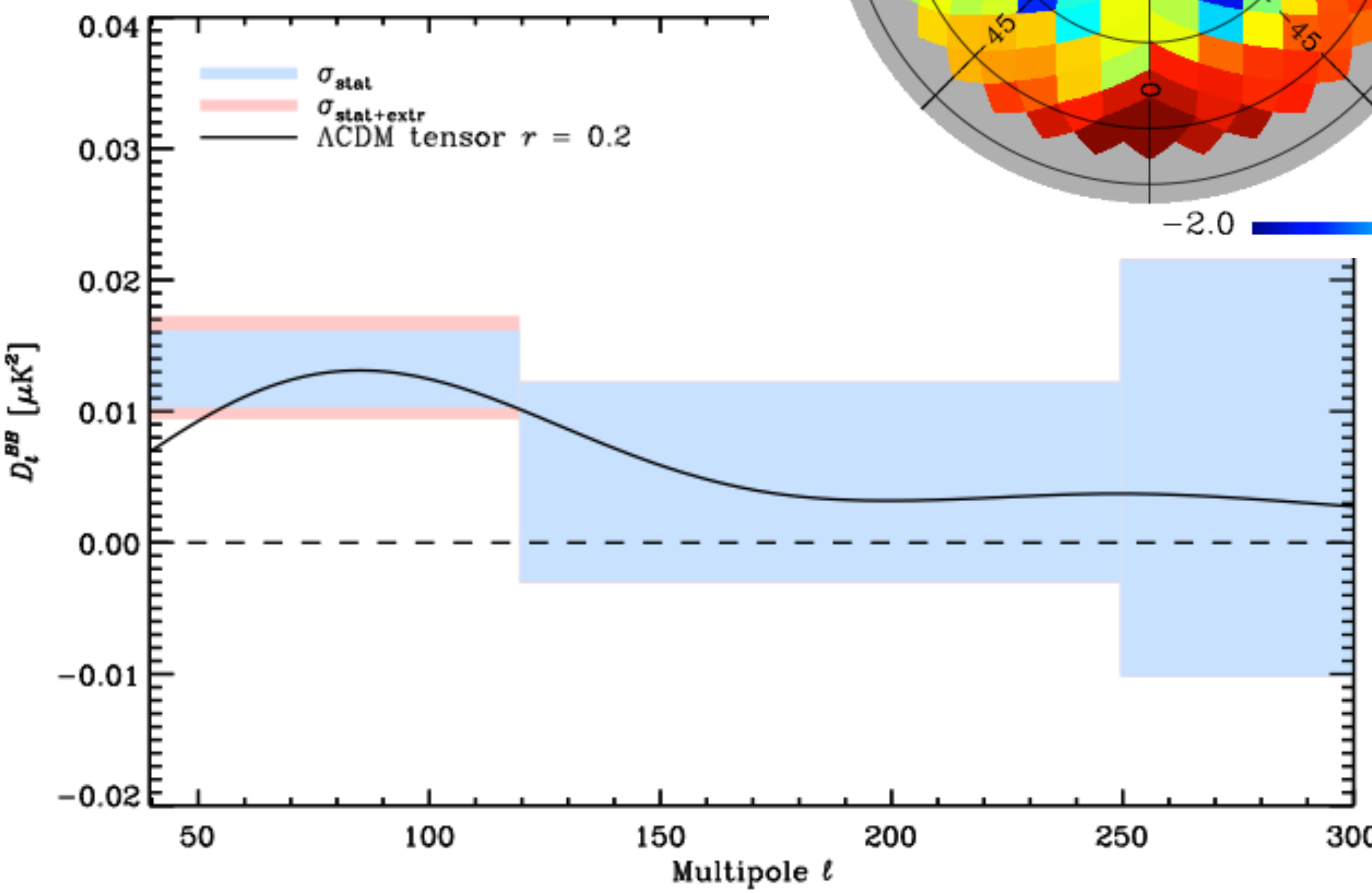
Comparison with Planck 2013 result → a clear tension!

Then… proved wrong by *Planck*!

Planck Intermediate Results XXX (2016)

Dust polarization at high galactic latitudes



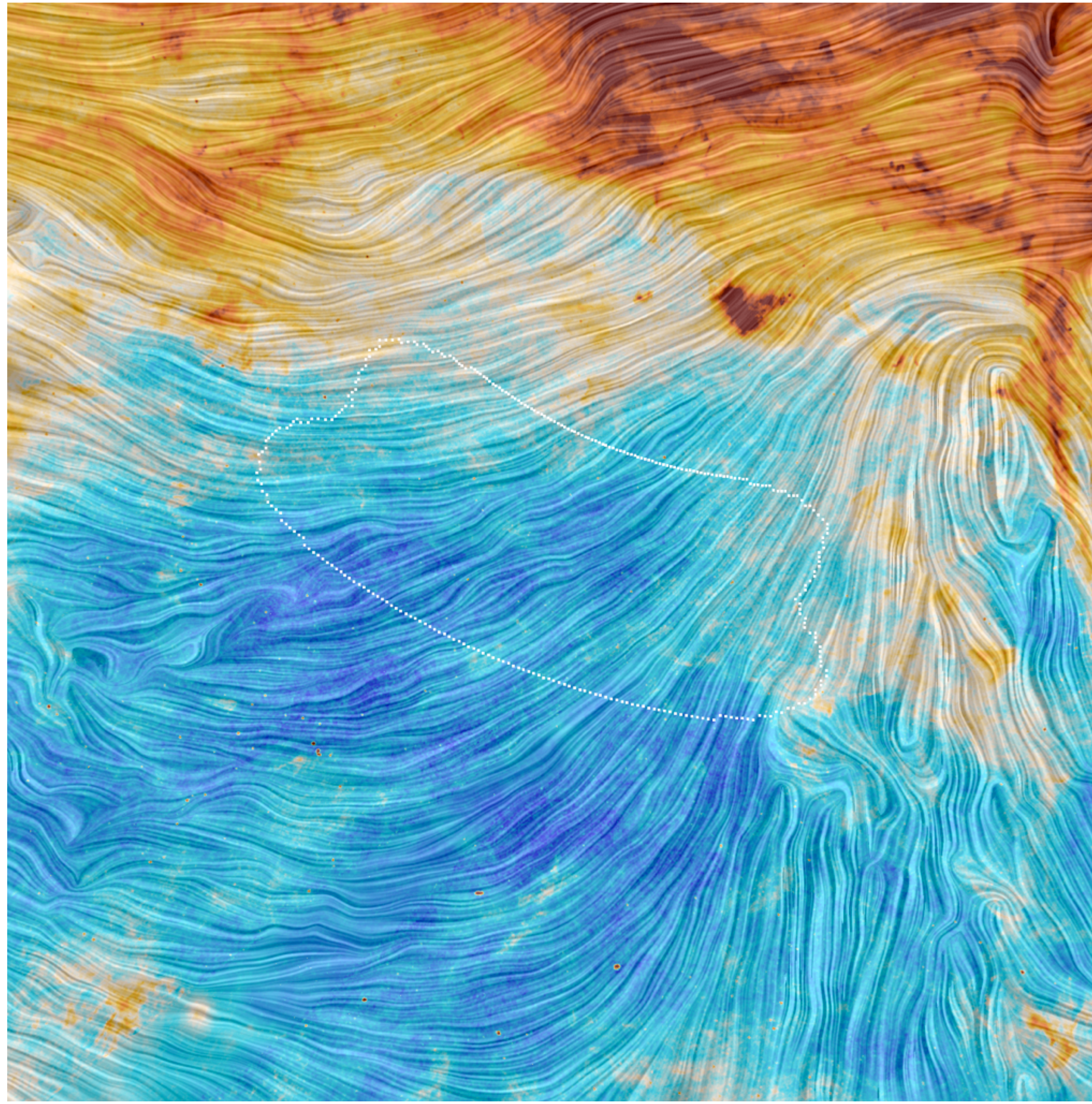


BICEP-2 severely underestimated the dust polarization power at low multipoles

Planck's view of the BICEP2 field

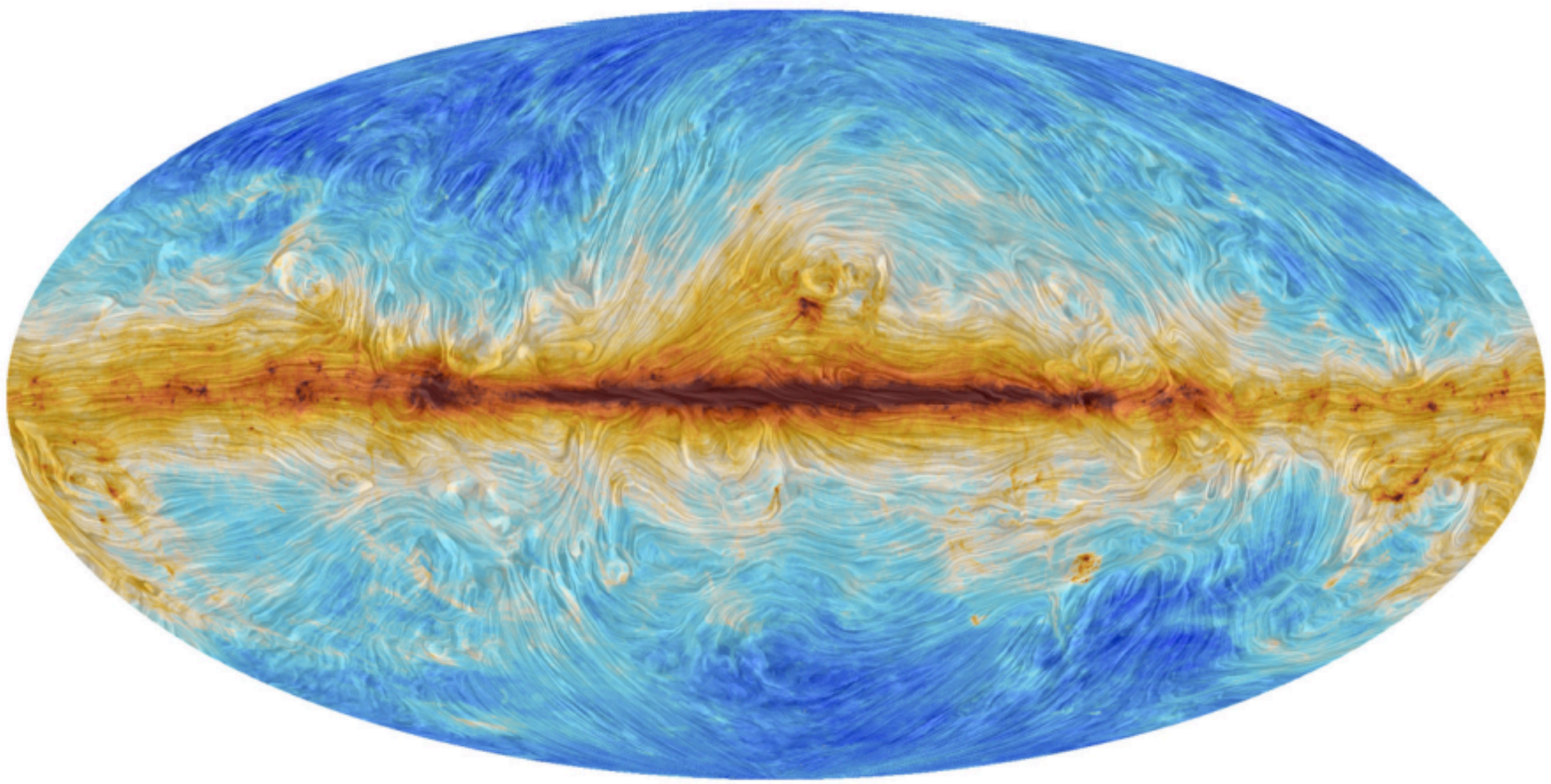
colors → dust intensity

"engravings" → magnetic field

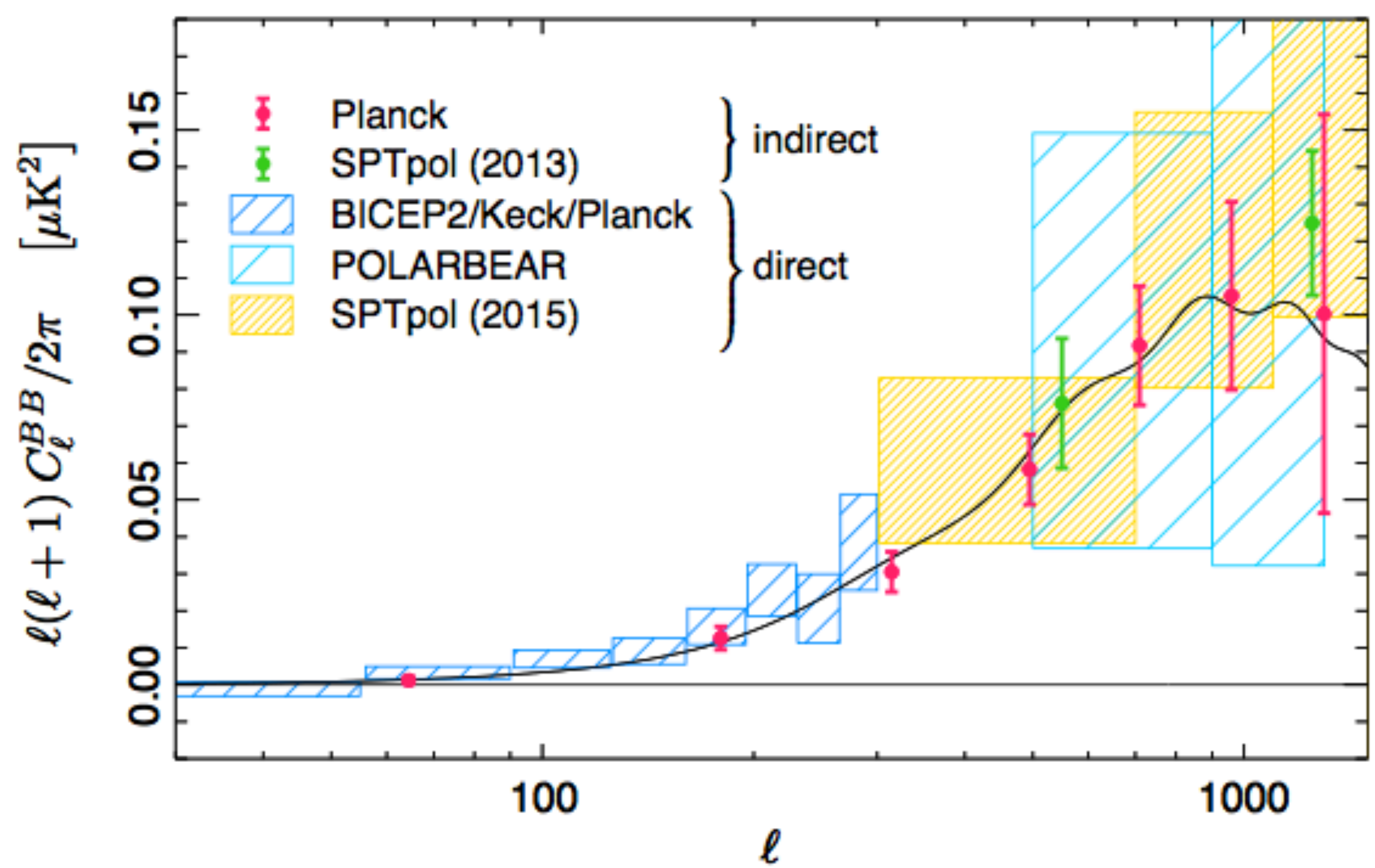


Credit: ESA/Planck collaboration

Planck results on the B-mode



Dust polarization map
(Planck collaboration 2015)



Lensing B-mode
(Planck collaboration 2015)

STARSTRUCK

Discovery of Big Bang's Gravitational Waves Goes Bust, Due to Dust

A leaked analysis suggests that findings of gravitational waves from the big bang were illusory.

nature

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nature > news > article

Published: 30 January 2015

Gravitational waves discovery now officially dead

Ron Cowen

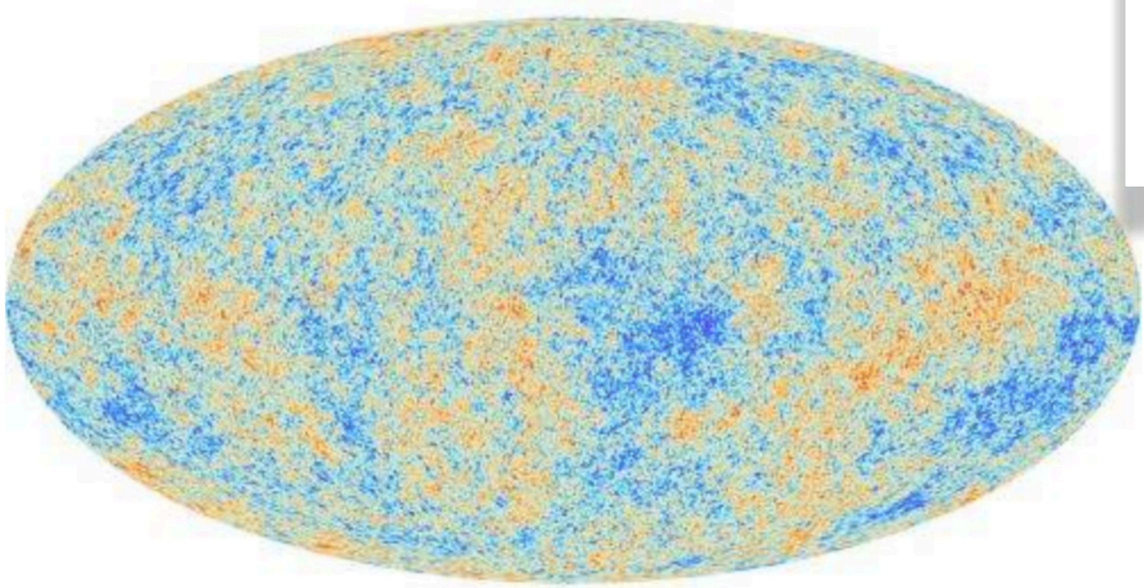
Nature (2015) | Cite this article

775 Accesses | 4 Citations | 677 Altmetric | Metrics

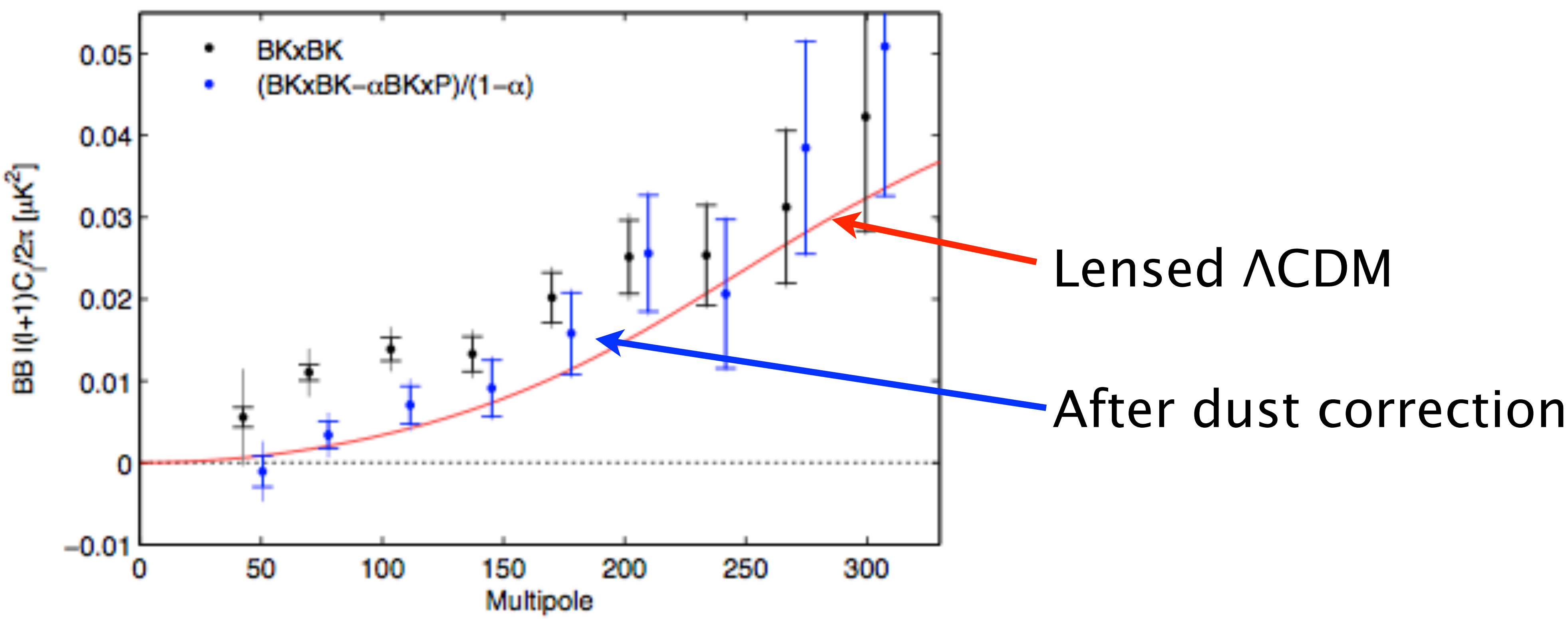
Combined data from South Pole experiment BICEP2 and Planck probe point to Galactic dust as confounding signal.

Cosmic inflation: Dust finally settles on BICEP2 results

by Amina Khan, Los Angeles Times

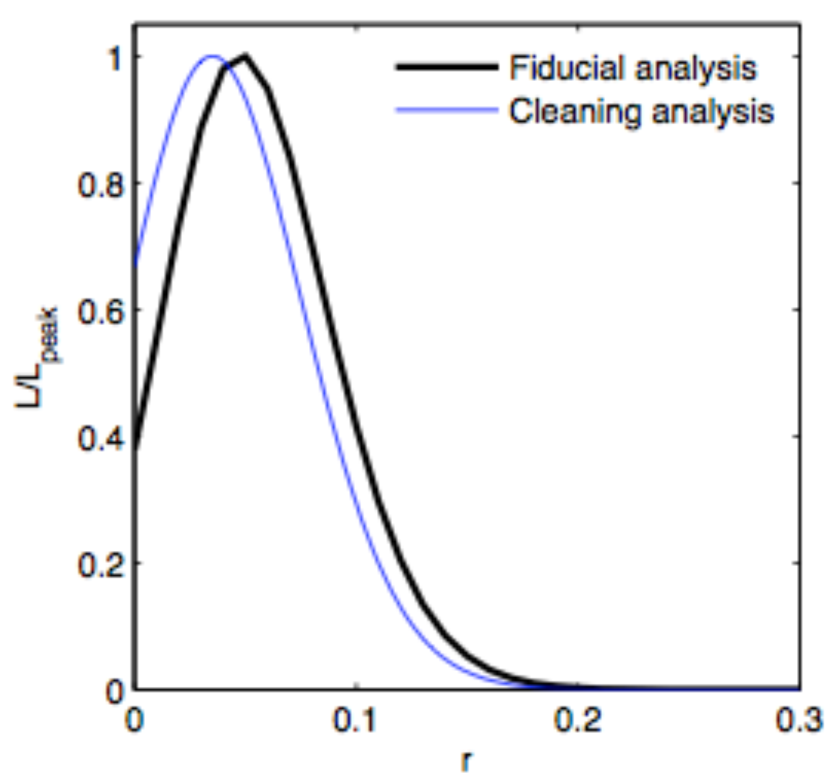


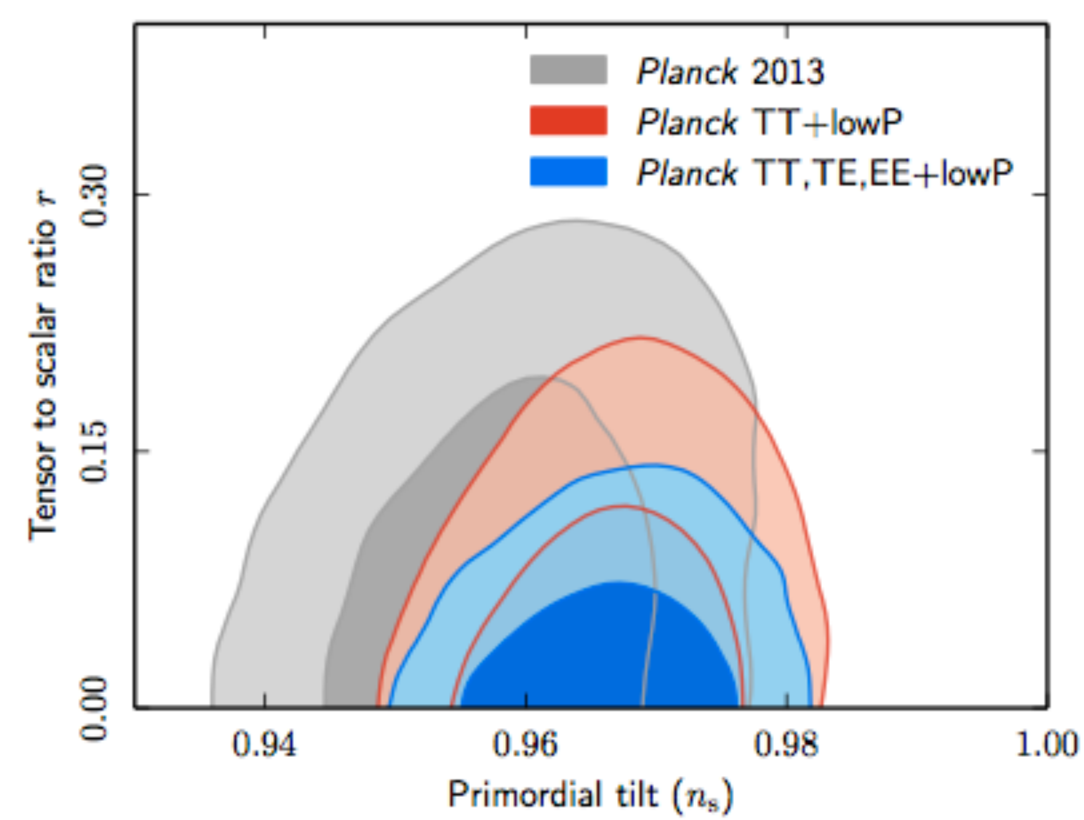
Planck 2015 + BICEP



BICEP2 + Planck joint analysis (2015)

95% upper limit: r < 0.08

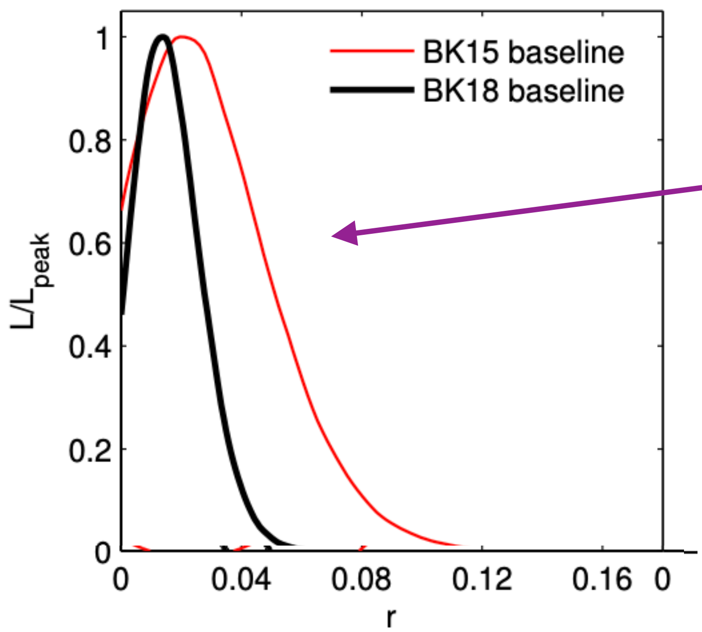




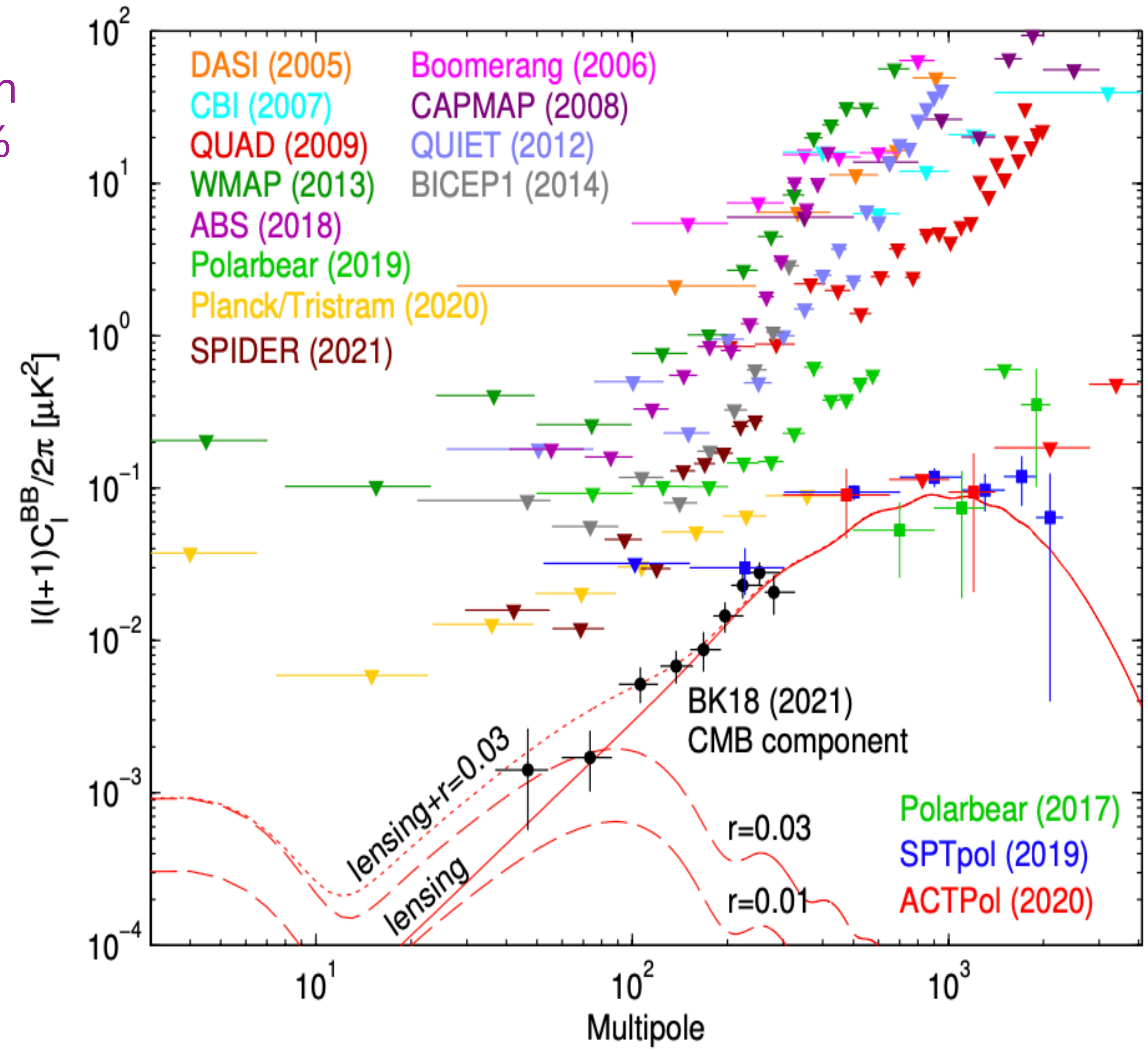
Latest constraints (late 2021)

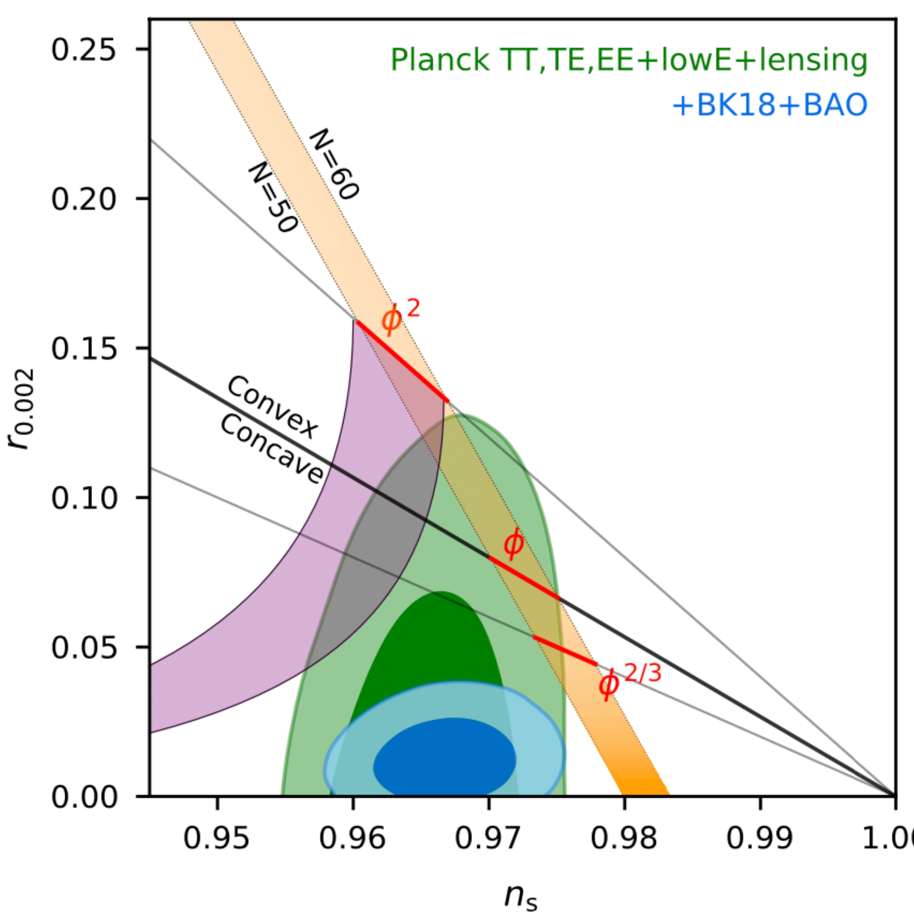
BICEP / *Keck* XIII: Improved Constraints on Primordial Gravitational Waves using *Planck*, WMAP, and BICEP/*Keck* Observations through the 2018 Observing Season

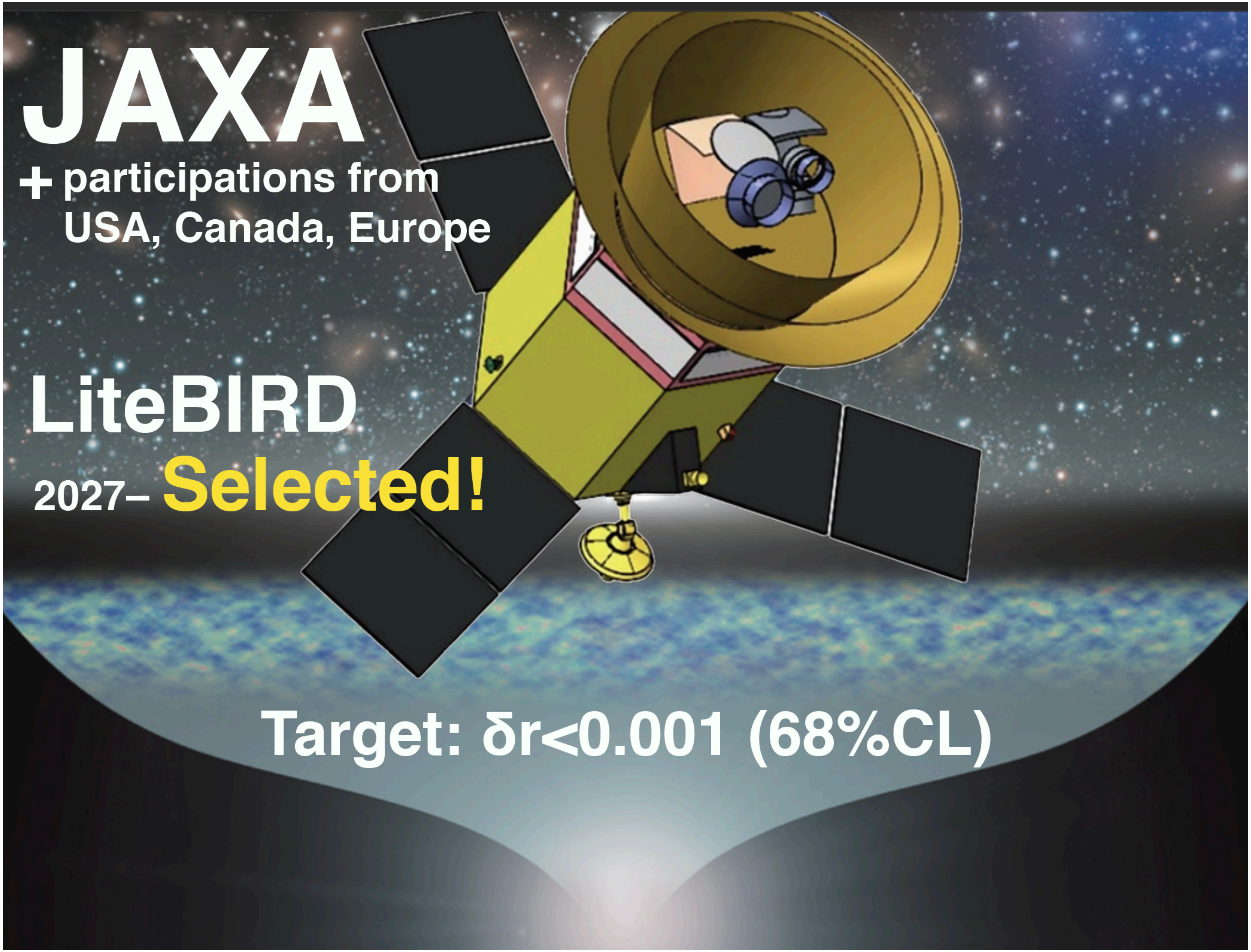
astro-ph > arXiv:2110.00483

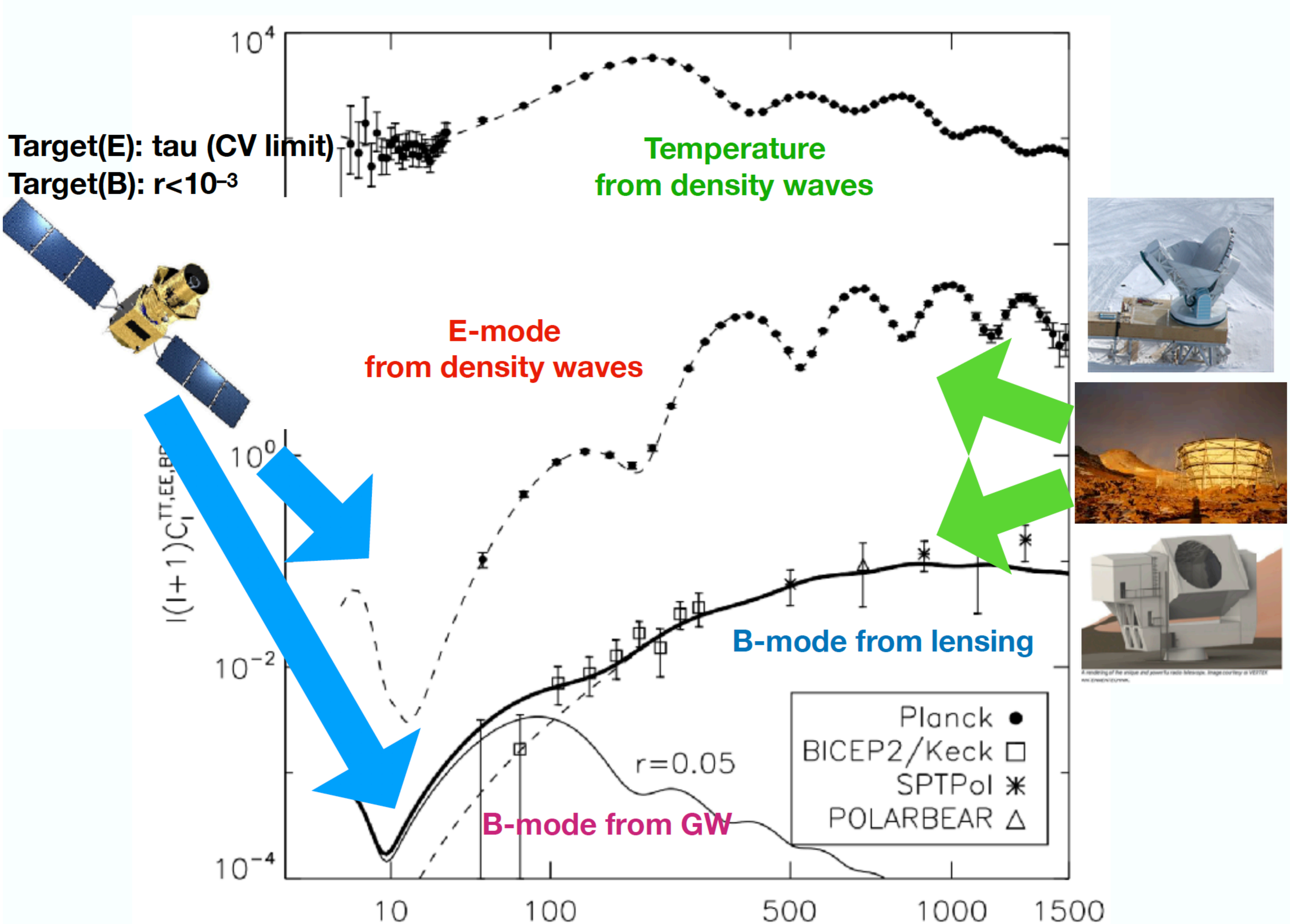


Improved constraints, but still non-detection (**r < 0.036** at 95% confidence)






JAXA
+ participations from
USA, Canada, Europe
LiteBIRD
2027– Selected!
Target: δr<0.001 (68%CL)



Slide courtesy: E. Komatsu


South Pole Telescope "3G"
SIMONS
OBSERVATORY
CMB-S4
Next Generation CMB Experiment
BICEP/Keck Array
CLASS

SO will start observing very soon..

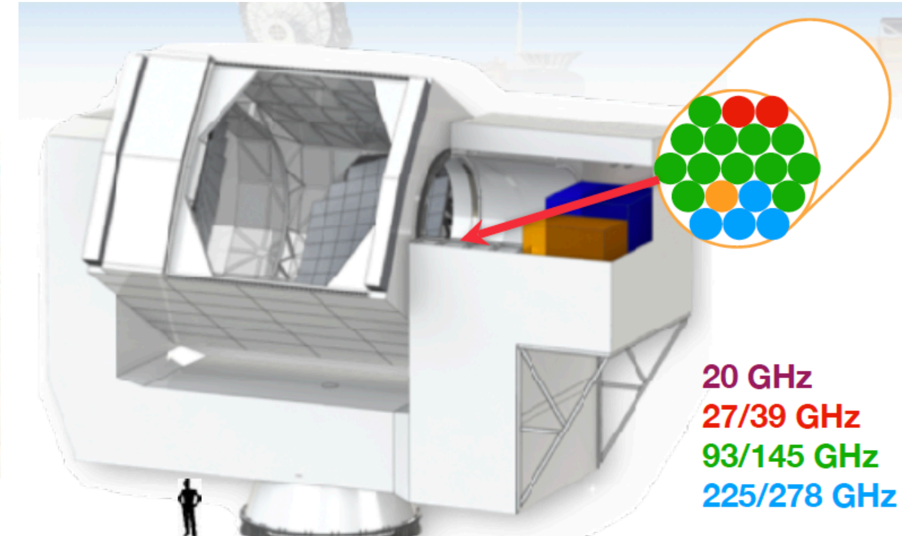
March 08, 2017

The Eternal Sky: A Short Film about Building the Simons Observatory (Part 1)

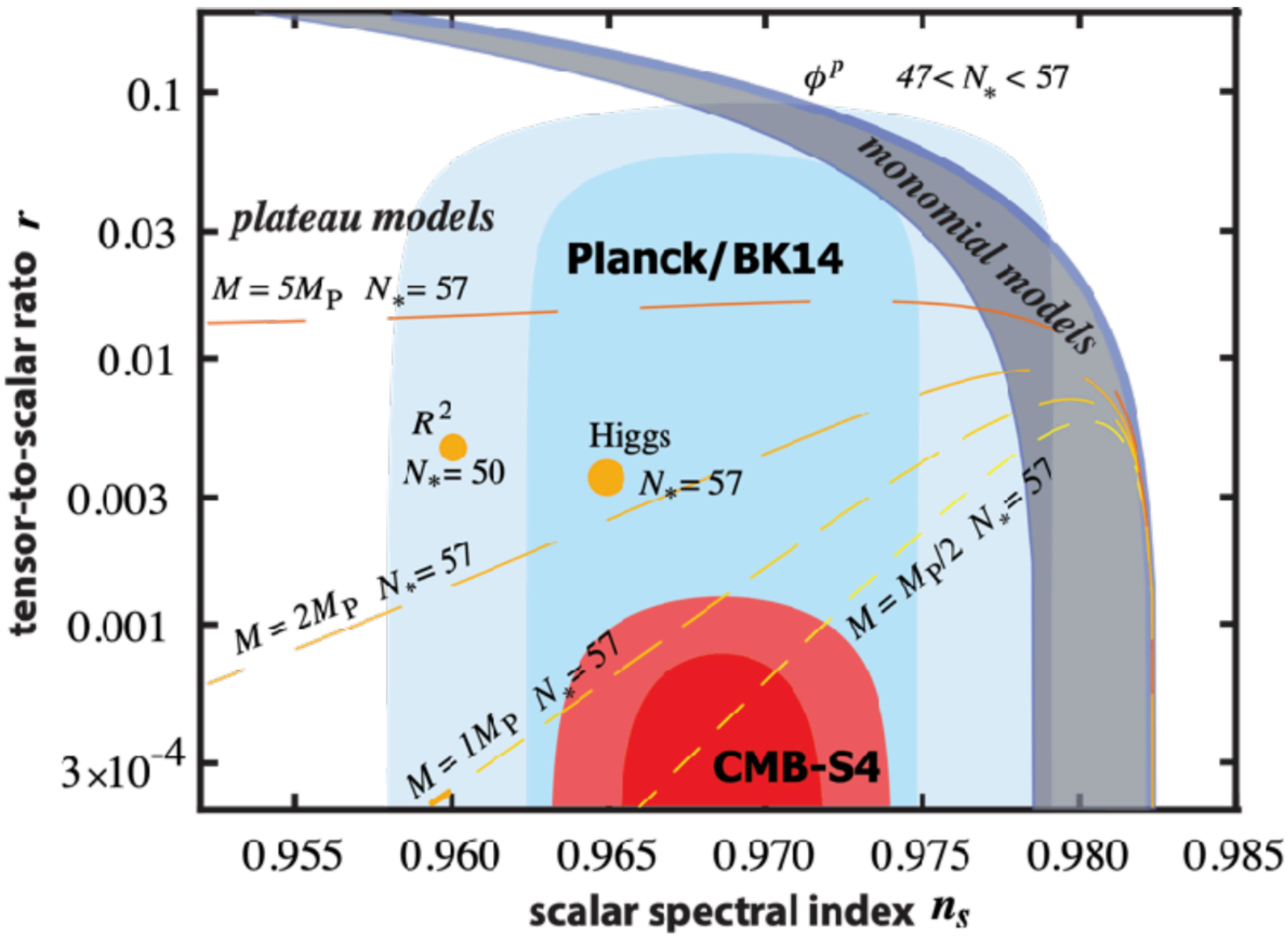
October 03, 2017

The Eternal Sky, Part 2: The Hunt for Inflation

CMB-S4 endorsed by Astro2020



Large Aperture Telescope (LAT)



Detection of ***r*** would give the energy scale of inflation, provide evidence for the quantization of gravity, and fundamental insights into physics and cosmology.

All inflation models that naturally explain the observed deviation from scale invariance and that also have a characteristic scale equal to or larger than the gravitational mass scale predict $r > 10^{-3}$. A well-motivated sub-class within this set of models is detectable by CMB-S4 at 5σ.

CMB-S4 sensitivity of $\sigma(r) < 0.0005$ and ensures that a non-detection of r will rule out the leading inflationary models, and motivate alternate models for the origin of the universe.

CMB-S4 upper limit goal $r < 10^{-3}$ at 95% C.L. (SPO and SO goals $\sim 10^{-2}$)

Questions?

Feel free to email me or ask questions in our eCampus Forum