astro8405

An Introduction to the Cosmic Microwave Background

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astro8405: The Cosmic Microwave Background

Aktionen

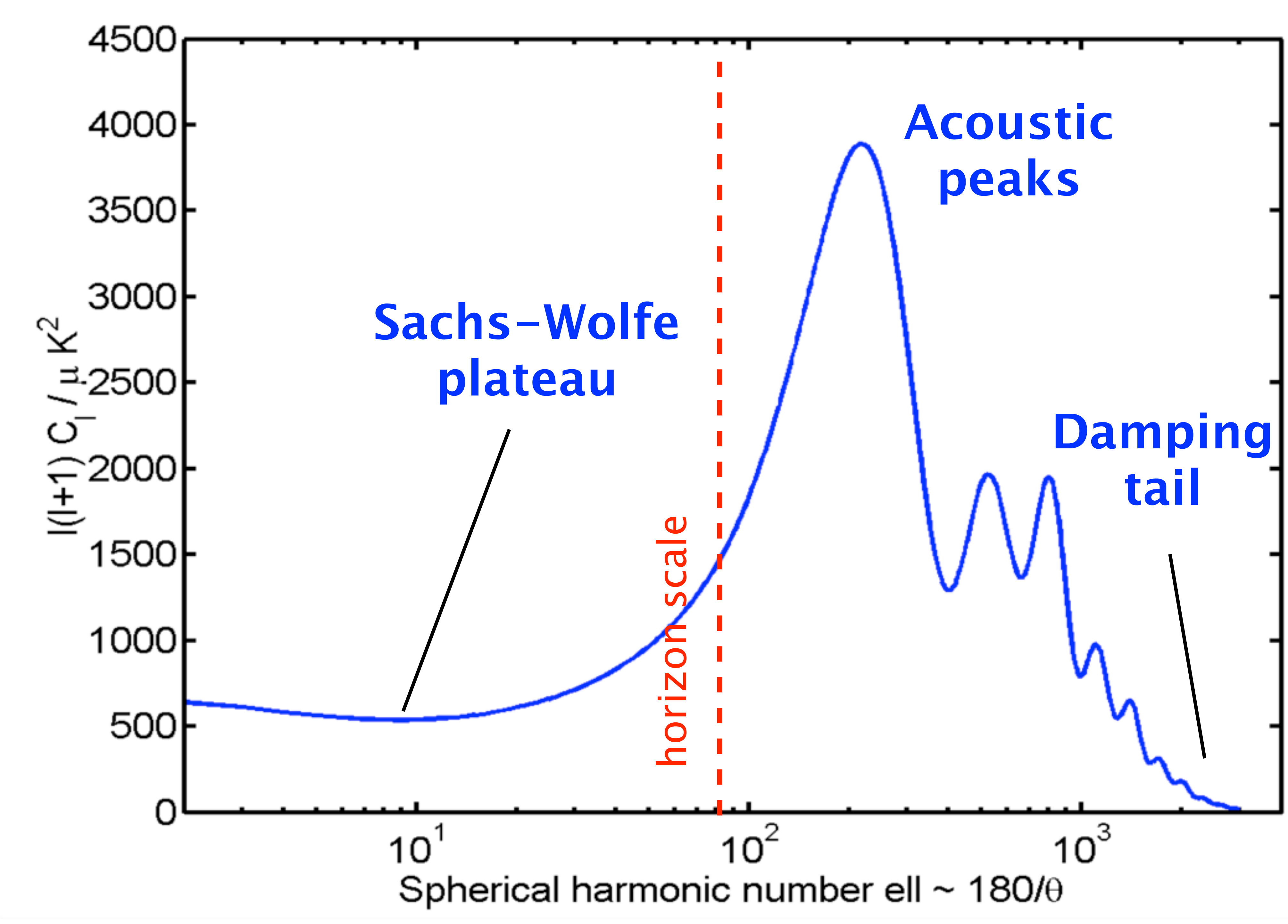
This course intends to give you a modern and up-to-date introduction to the science and experimental techniques relating to the Cosmic Microwave Background. No prior knowledge of cosmology is necessary, your prerequisite are a basic understanding of electrodynamics and thermal physics and some familiarity with Python programming.

Lecture 6:

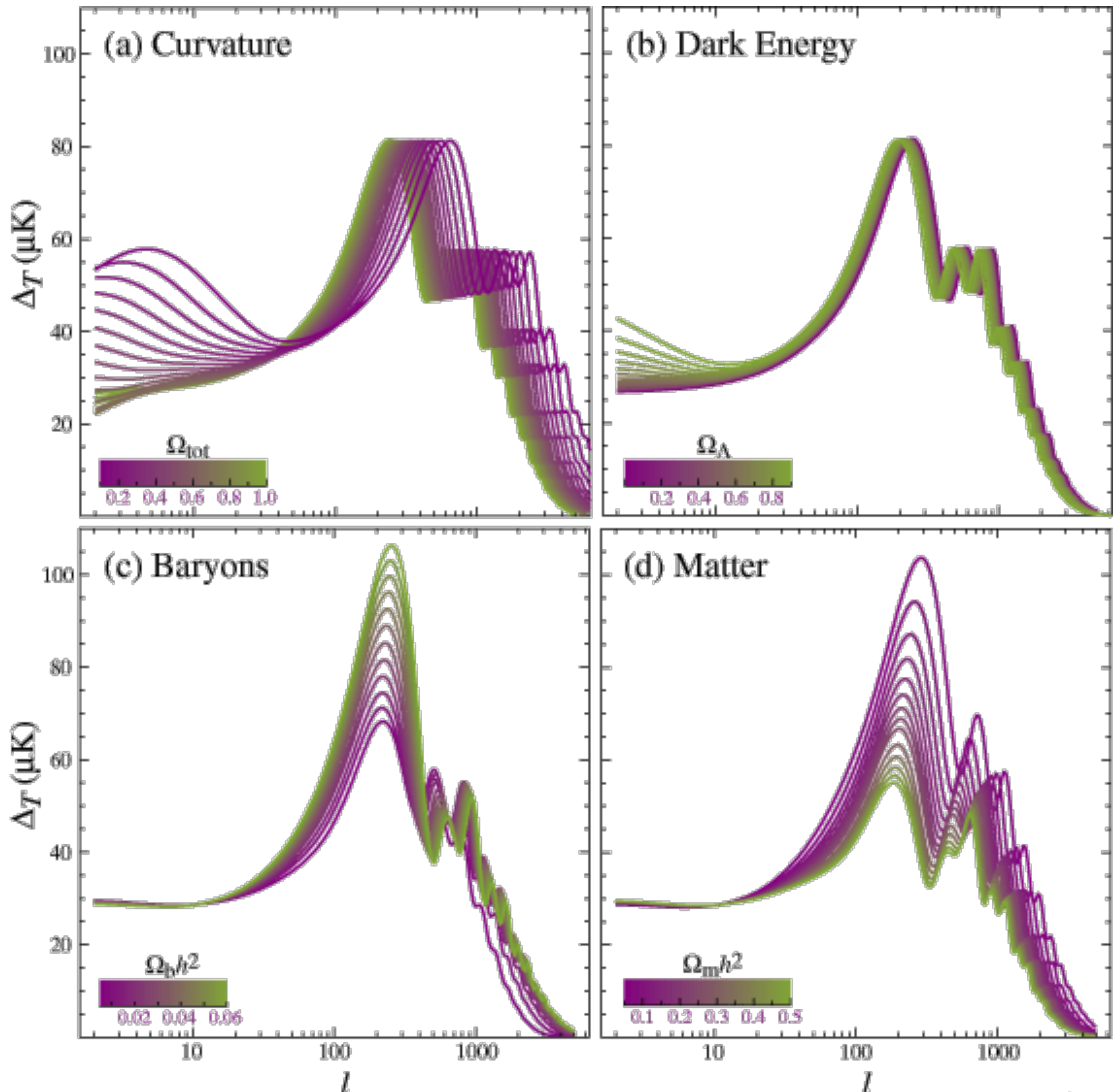
Doing cosmology with the temperature anisotropy power spectrum

TT power spectrum: Primary anisotropies

Summary of the three main effects:



Which way the peaks move?



Credit: Wayne Hu

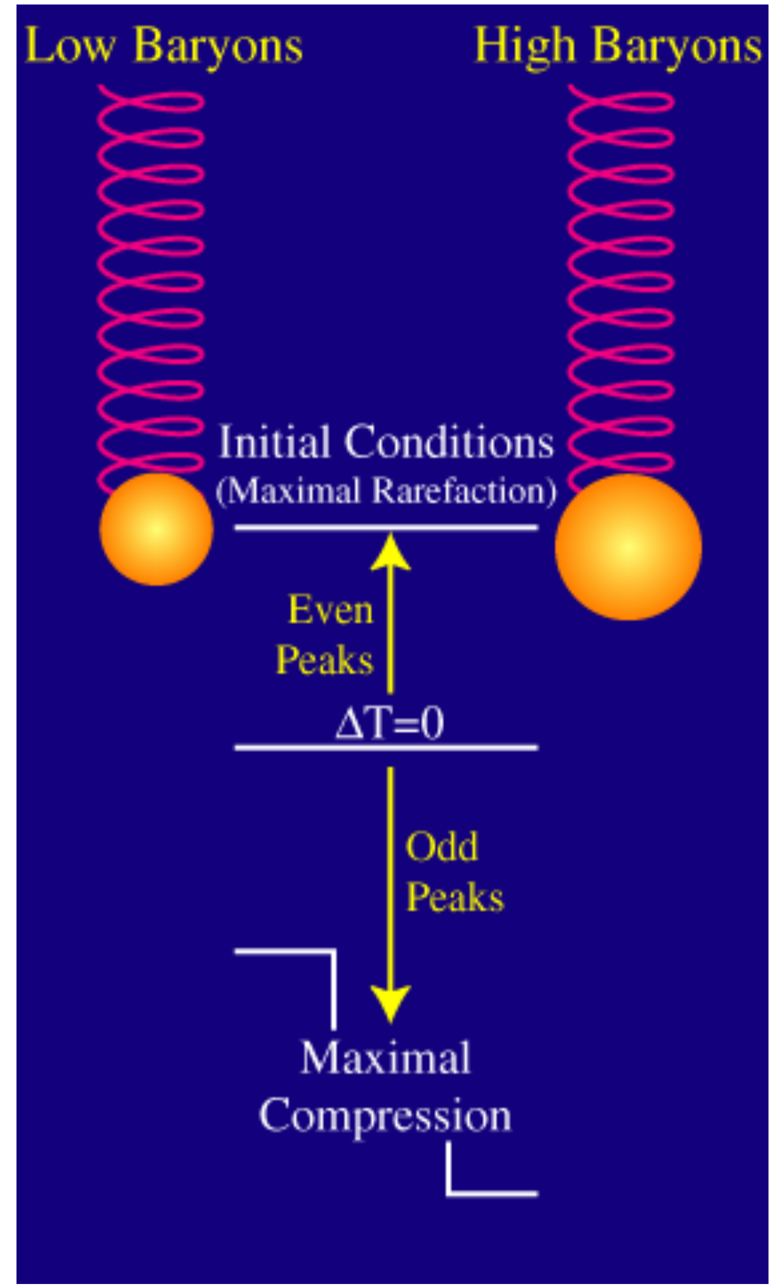
Baryon loading

The presence of more baryons **add inertia**, and increases the amplitude of the oscillations (baryons drag the fluid into potential wells).

Perturbations are then compressed more before radiation pressure can revert the motion.

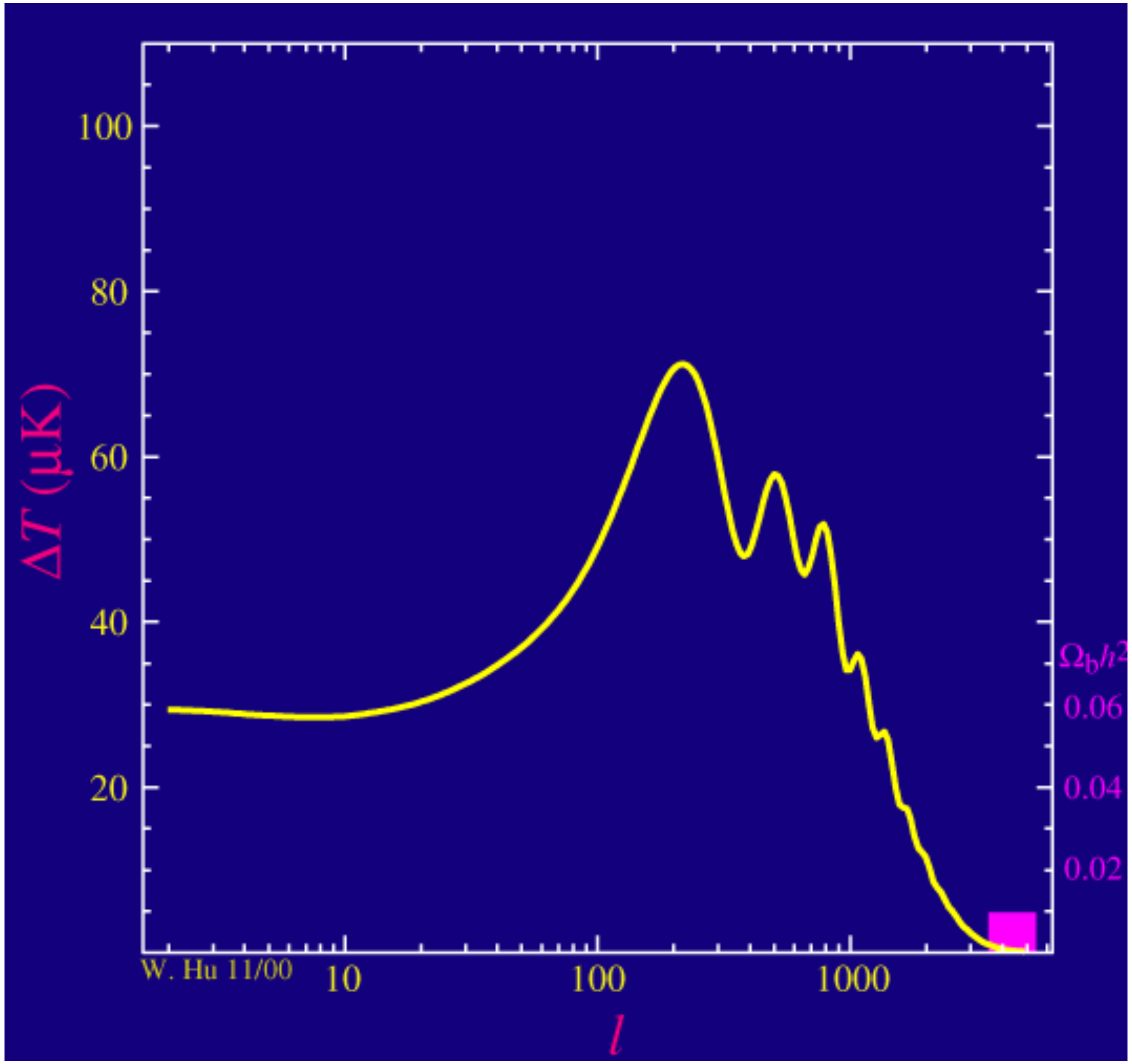
This causes a **breaking of symmetry in the oscillations**, enhancing only the compressional phase (i.e., every odd-numbered peak: 1st, 3rd,.. etc).

This can be used to measure the abundance of cosmic baryons.



Credit: Wayne Hu

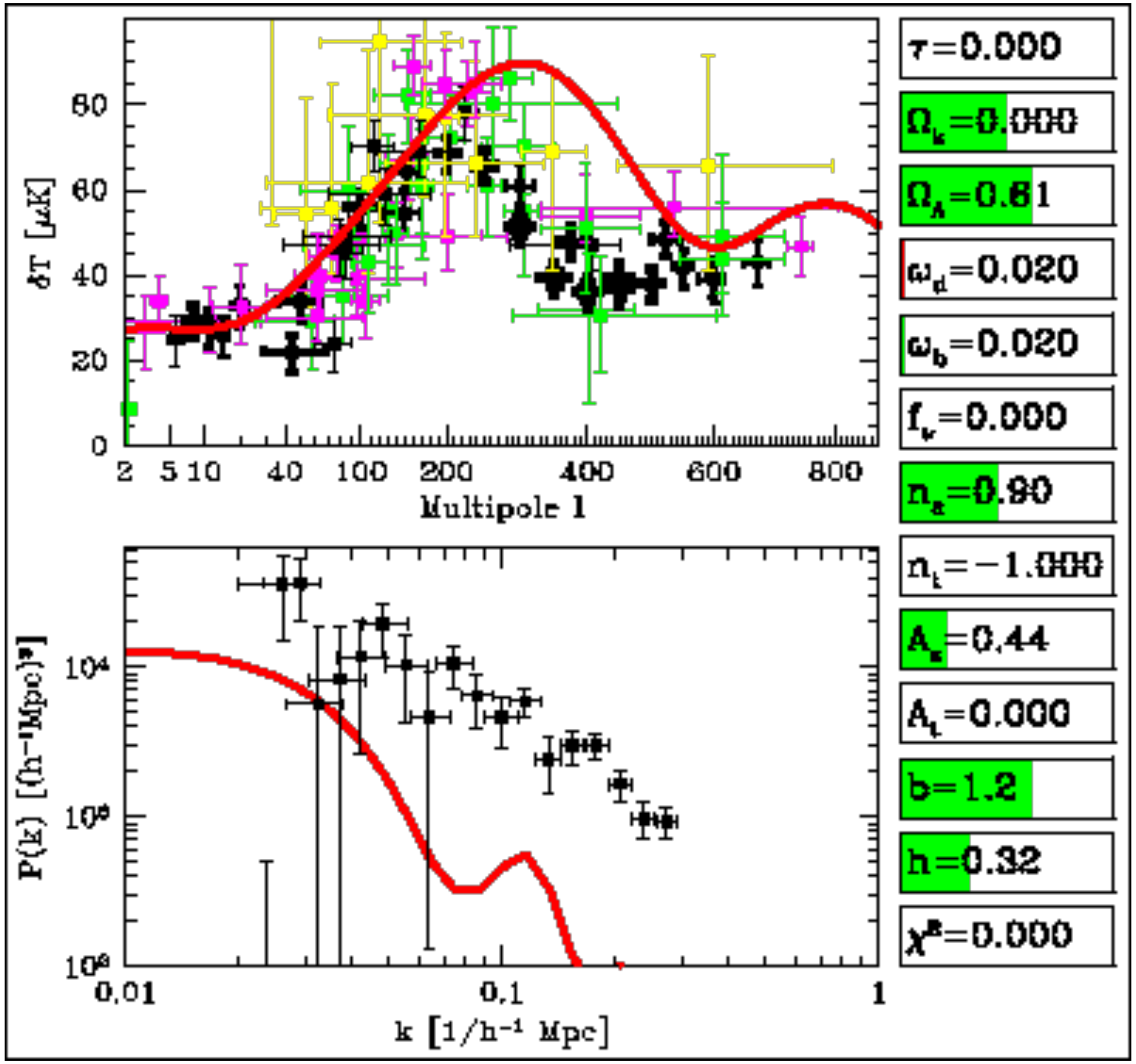
Baryons in the power spectrum



Credit: Wayne Hu

Power spectrum shows baryons enhance every odd-numbered peak, which helps to distinguish baryons from cold dark matter.
(Baryons also change the damping scale at the tail)

Dark matter in the power spectrum



Credit: Max Tegmark

As the Dark Matter density *decreases*, while keeping other parameters fixed, two things happen:

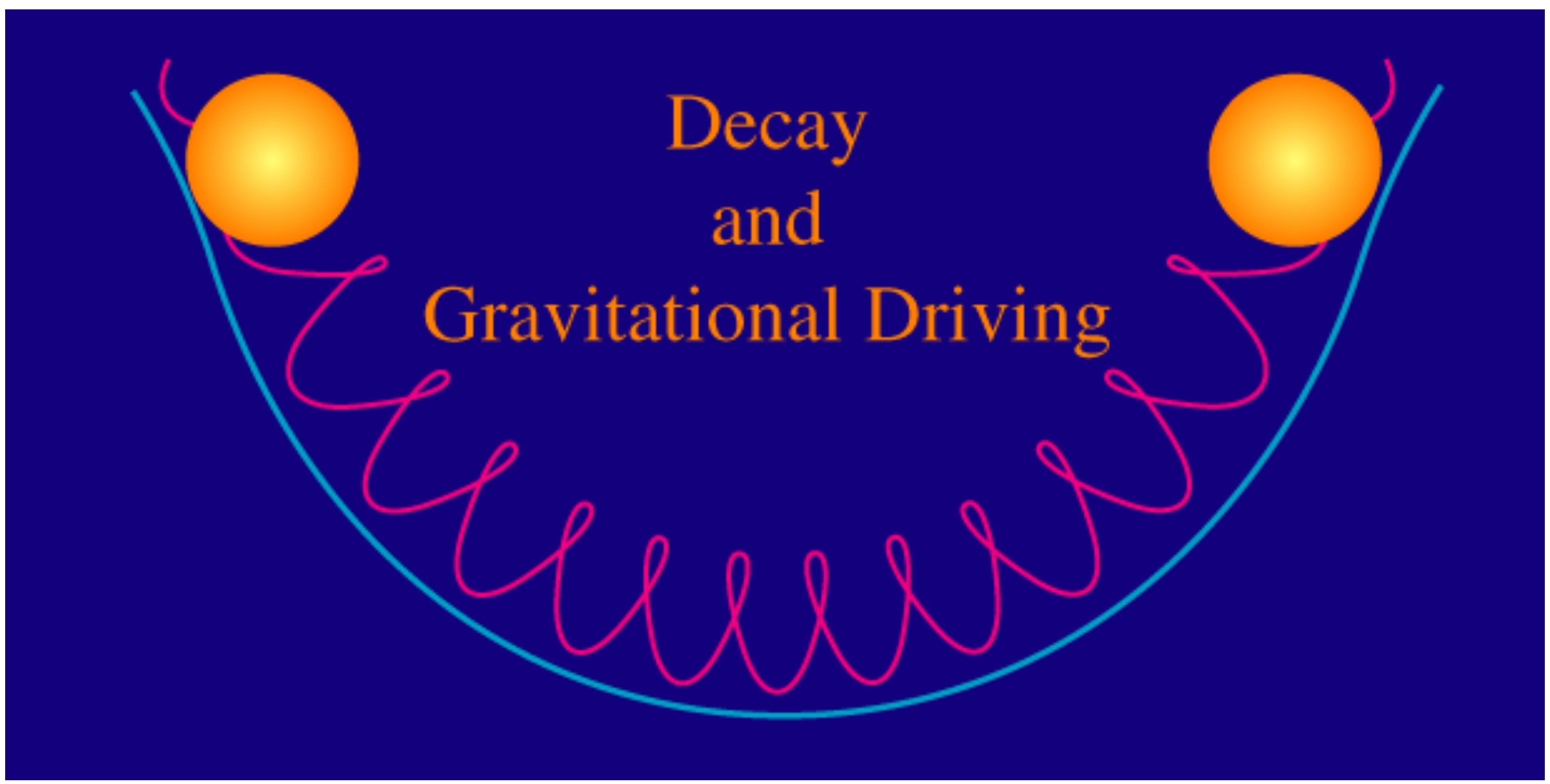
(1) The baryon fraction increases, creating an effect similar to raising the baryon density.

(2) The matter-to-radiation ratio changes, making the contribution of radiation in the total matter-energy density of the universe more significant. The enhanced radiation pressure causes the gravitation potential to be less pronounced, a phenomenon known as **radiation driving**. This in turn drives the oscillations stronger by eliminating the force that otherwise would oppose it.

Radiation driving force

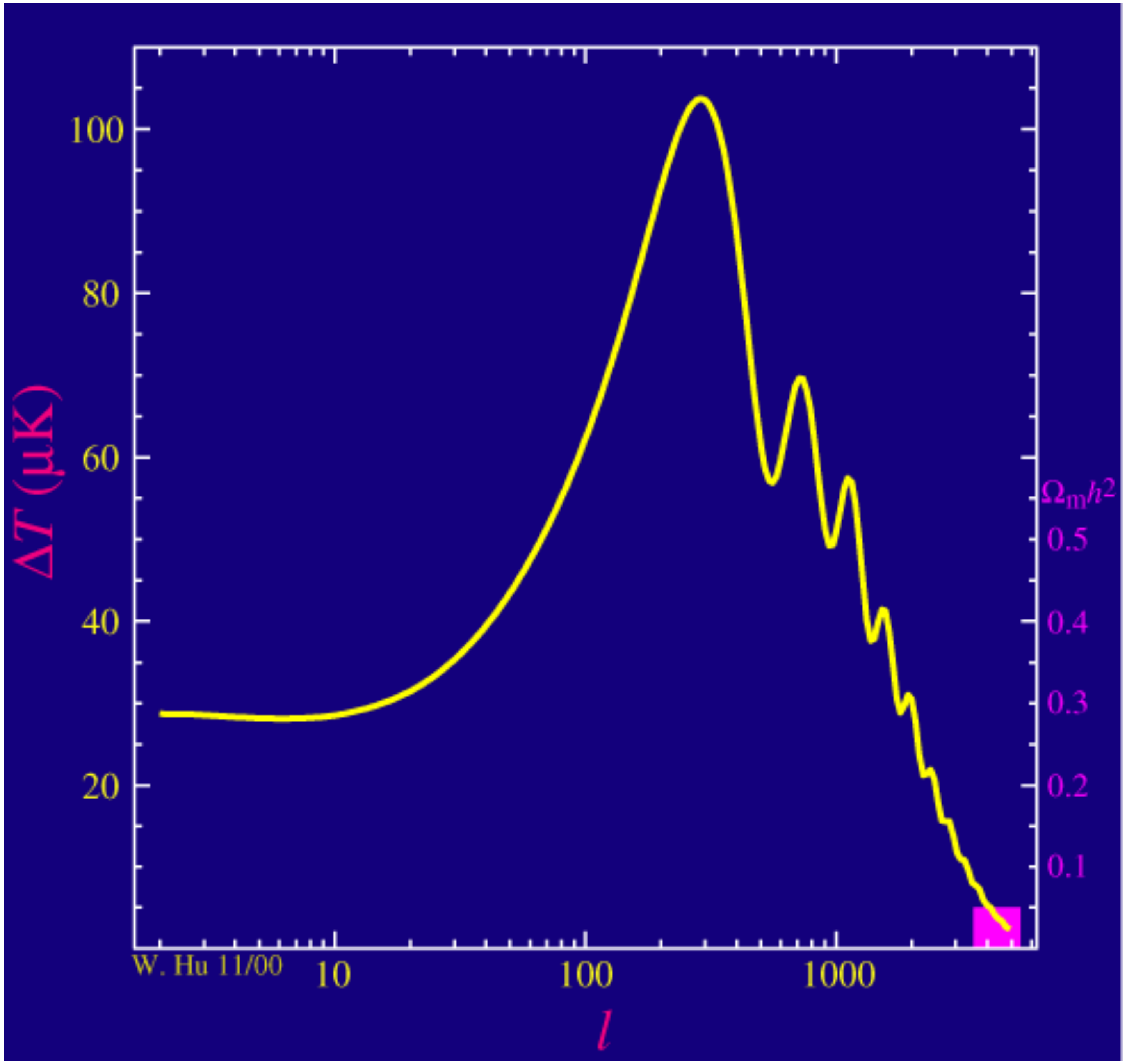
Decreasing the matter density correspondingly increases the contribution of radiation in determining the gravitational potentials, and brings the epoch of matter-radiation equality close to the epoch of recombination.

When the radiation (CMB photons) energy density starts to dominate over the matter energy density, the gravitational potential in which the photon-baryon fluid oscillates can not be taken as a constant. The potential decays to drive the amplitude of the oscillation up.



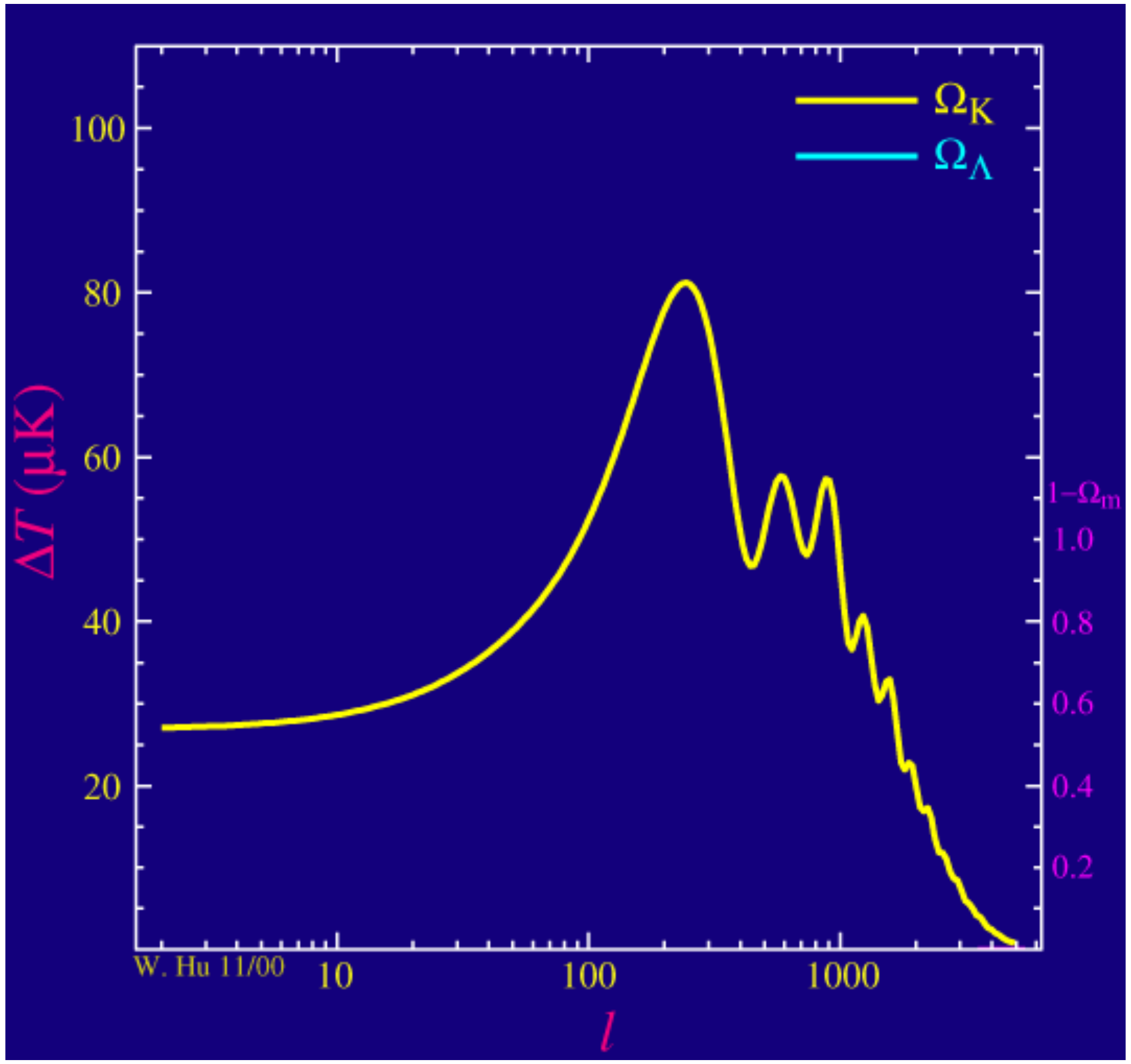
Credit: Wayne Hu (see "Radiation Driving Force")

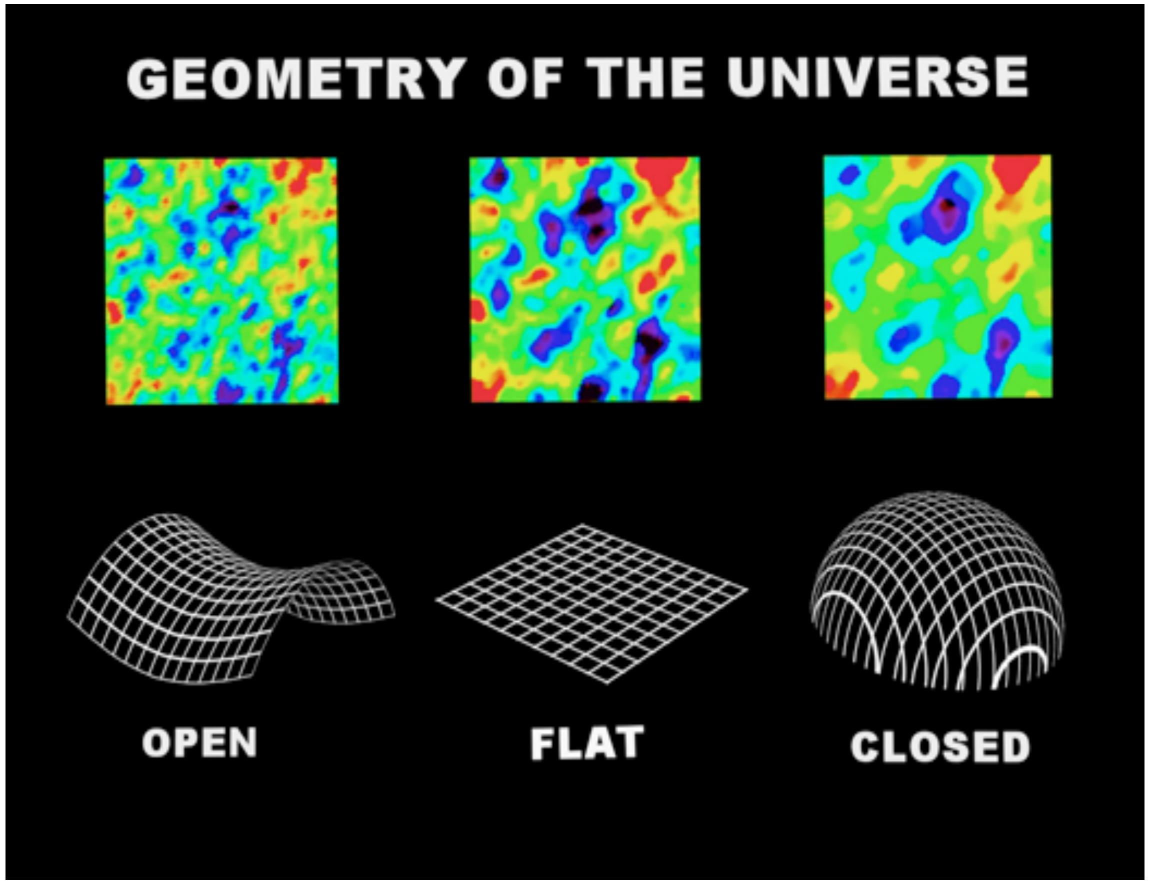
Dark matter in the power spectrum



Credit: Wayne Hu

Effect of curvature





Credit: Wayne Hu

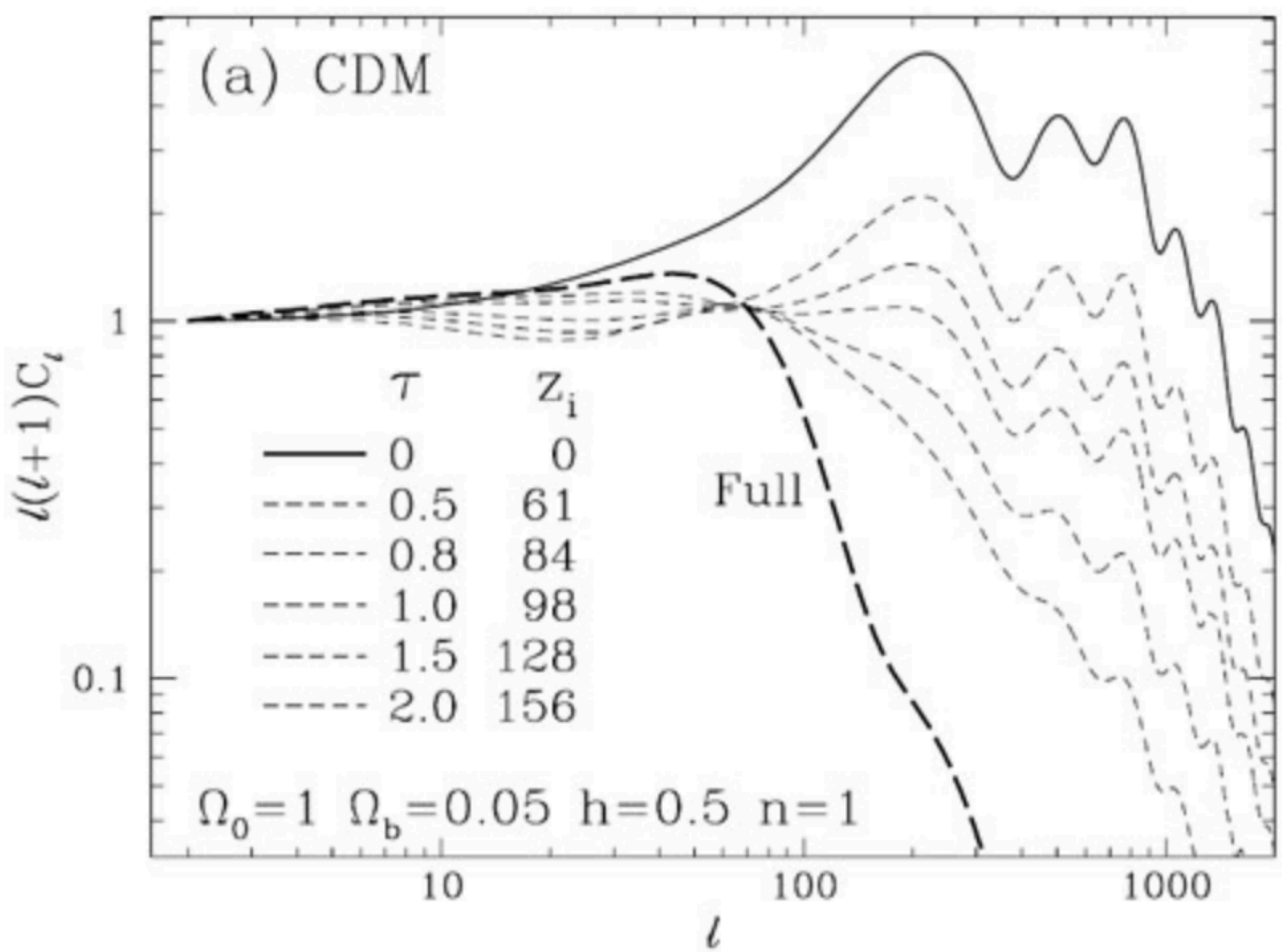
Ω_k does not change the amplitude of the power spectrum, rather it shifts the peaks sideways. This follows from the conversion of the physical scales (on the LSS) to angular scales (that we observe), which depends on the geometry.

Curvature (cosmological constant, Ω_Λ) also causes the ISW effect on large scales, by altering the growth of structures in the path of CMB photons.

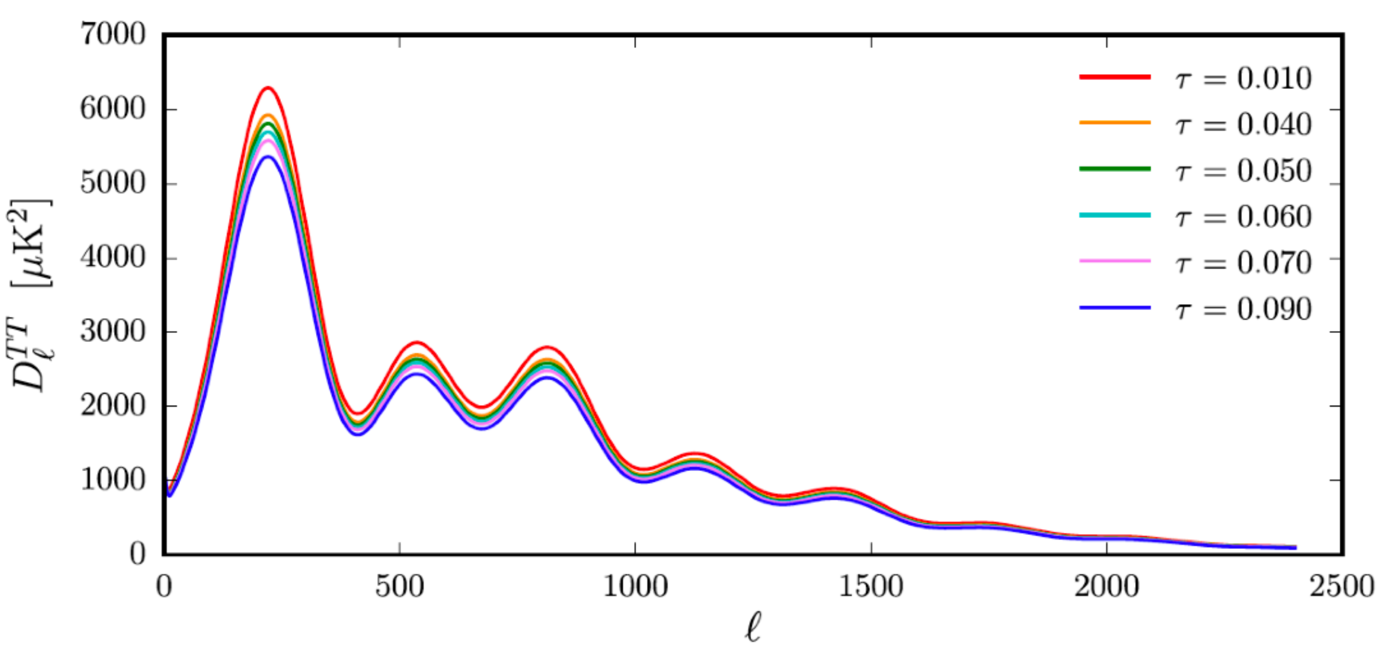
Effect of optical depth (reionization)

The main impact of cosmic reionization at redshift z ~ 6-10 is to dampen the temperature anisotropies, and create new E-mode anisotropies at very large angles.

τ degenerate with other cosmological parameters, especially A_s (the amplitude of the density perturbations) and related parameters n_s, σ_8, and foregrounds.



$$C_\ell^{TT} \propto A_s e^{-2\tau}$$

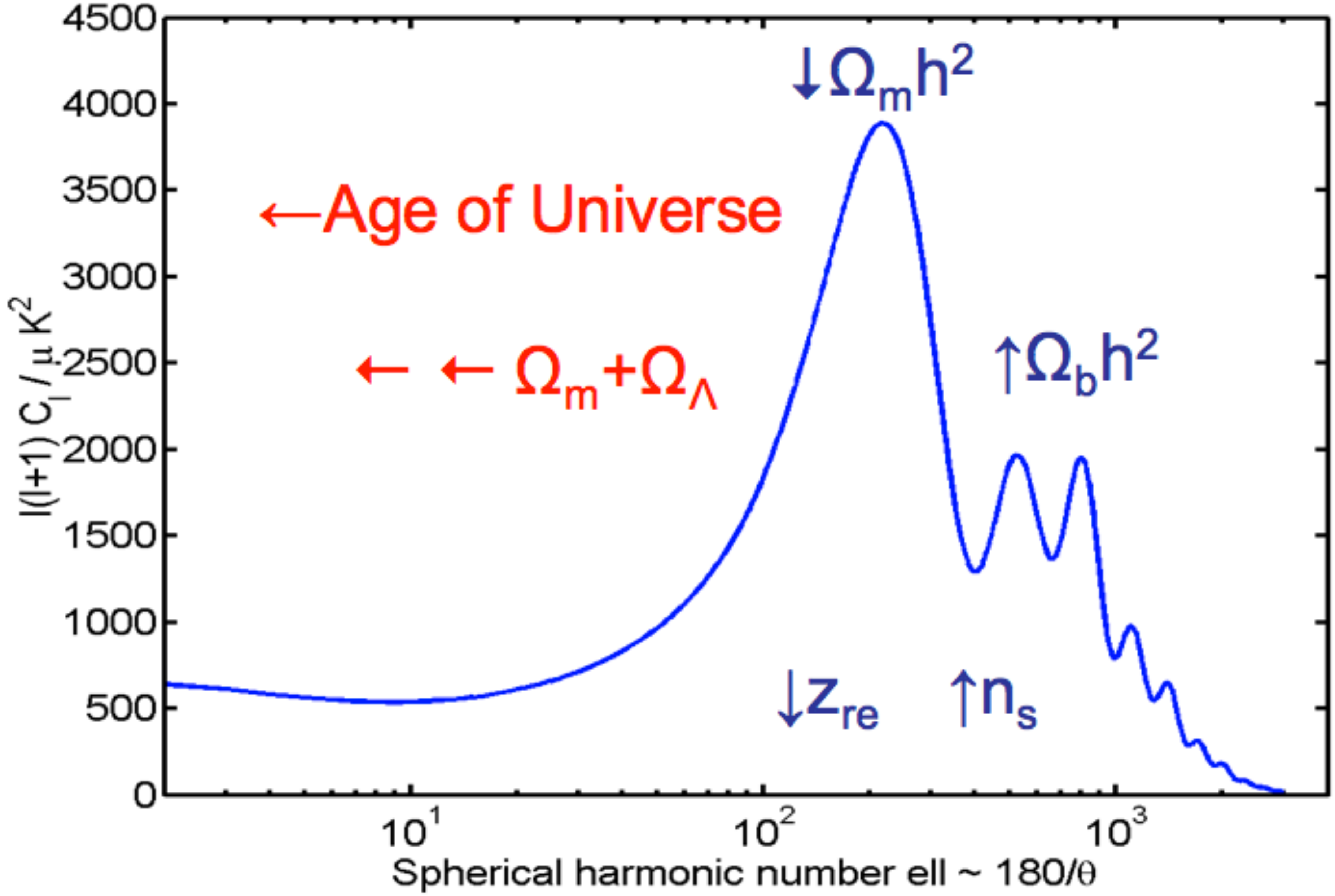


Current constraint from Planck (2018):

τ = 0.056 ± 0.007

$$\tau(z) = \int_{t(z)}^{t_0} n_e \sigma_T c dt'$$

CMB parameter cheat sheet



Only six parameters for the CMB

The TT power spectrum is adequately described by six independent parameters:

- Ω_0 total density parameter ⟷ used interchangeably with Ω_Λ
- A amplitude of primordial scalar perturbations (at some pivot scale k_p)
- n spectral index of primordial scalar perturbations
- τ optical depth due to reionization (discussed in Sec. 12.9.6)
- $\omega_b \equiv \Omega_b h^2$ "physical" baryon density parameter
- $\omega_m \equiv \Omega_m h^2$ "physical" matter density parameter

Adding extra parameters like neutrino mass or DE equation of state does not improve the goodness of the fit significantly, so those are taken at their fiducial values.

Other cosmological parameters, for example H_0, are derived from these six:

$$\Omega_0 = \Omega_m + \Omega_\Lambda \quad \Rightarrow \quad \Omega_m = \Omega_0 - \Omega_\Lambda$$

$$h = \sqrt{\frac{\omega_m}{\Omega_m}} = \sqrt{\frac{\omega_m}{\Omega_0 - \Omega_\Lambda}}$$

$$\Omega_b = \frac{\omega_b}{h^2} = \frac{\omega_b}{\omega_m}(\Omega_0 - \Omega_\Lambda)$$

Parameter constraints from *Planck*

All columns assume the ΛCDM cosmology with a power-law initial spectrum, no tensors, **spatial flatness**, cosmological constant as dark energy, and sum of neutrino masses 0.06 eV.

Planck CMB results 2013

		Planck+WP +highL	*Planck*+WP +highL+BAO	*WMAP*9+eCMB +BAO
6-parameter combination to fit the TT data	$\Omega_b h^2$	0.02207 ± 0.00027	0.02214 ± 0.00024	0.02211 ± 0.00034
	$\Omega_c h^2$	0.1198 ± 0.0026	0.1187 ± 0.0017	0.1162 ± 0.0020
	$100\,\theta_{MC}$	1.0413 ± 0.0006	1.0415 ± 0.0006	–
	n_s	0.958 ± 0.007	0.961 ± 0.005	0.958 ± 0.008
	τ	$0.091^{+0.013}_{-0.014}$	0.092 ± 0.013	$0.079^{+0.011}_{-0.012}$
	$\ln(10^{10}\Delta^2_{\mathcal{R}})$	3.090 ± 0.025	3.091 ± 0.025	3.212 ± 0.029
Derived from the above	h	0.673 ± 0.012	0.678 ± 0.008	0.688 ± 0.008
	σ_8	0.828 ± 0.012	0.826 ± 0.012	$0.822^{+0.013}_{-0.014}$
	Ω_m	$0.315^{+0.016}_{-0.017}$	0.308 ± 0.010	0.293 ± 0.010
	Ω_Λ	$0.685^{+0.017}_{-0.016}$	0.692 ± 0.010	0.707 ± 0.010

$100\,\theta_{MC}$ is 100x the "acoustic scale", $\theta = r/D_{LS}$, which defines the position of the peaks, hence total curvature

Planck 2013 cosmological results

Planck collaboration (2013)

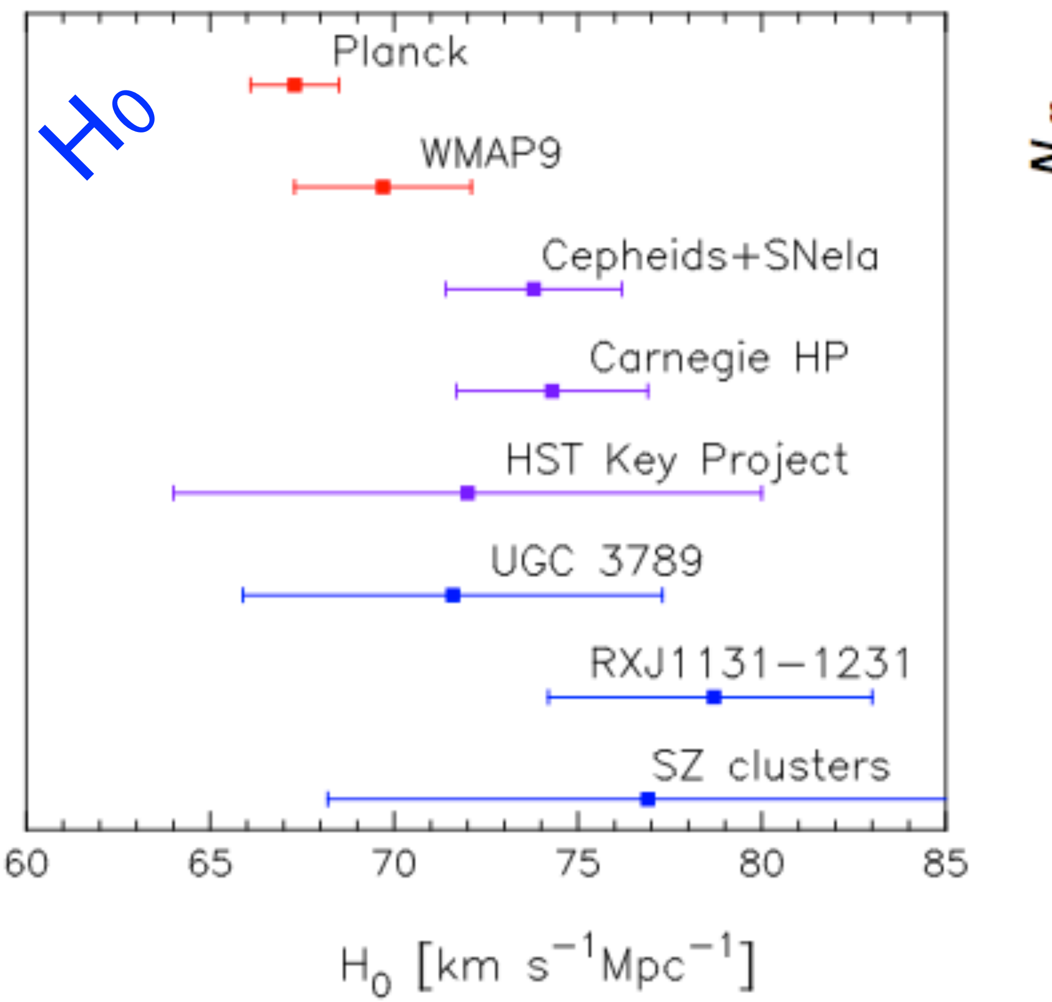
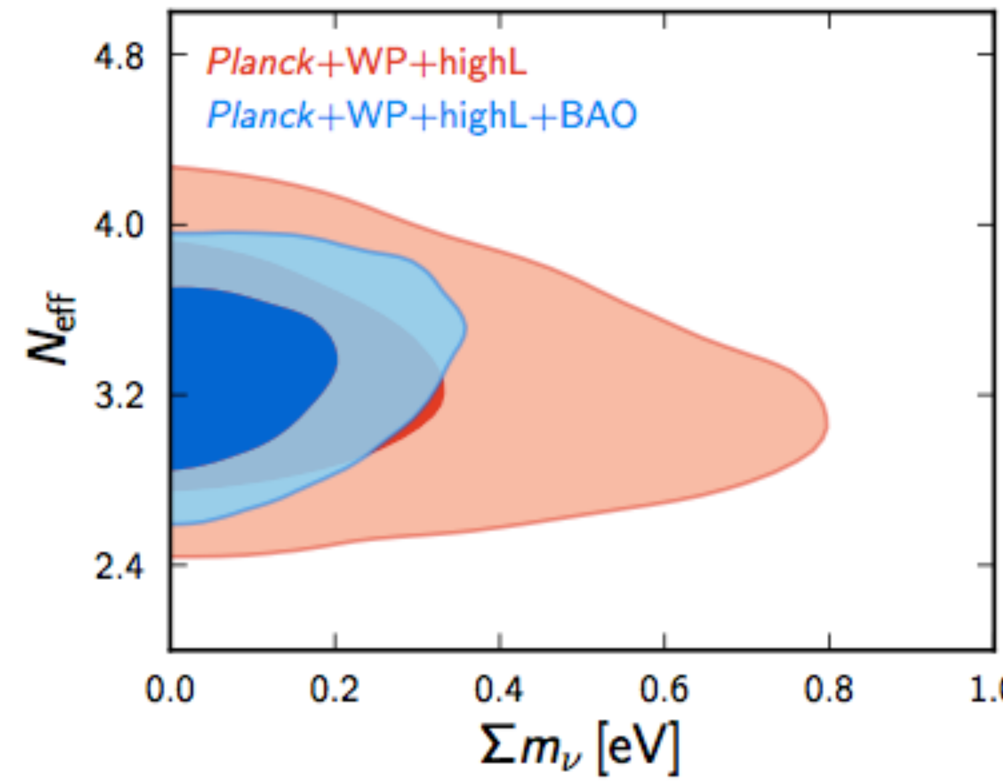


Fig. 16. Comparison of H_0 measurements, with estimates of $\pm 1\,\sigma$ errors, from a number of techniques (see text for details). These are compared with the spatially-flat ΛCDM model constraints from *Planck* and *WMAP*-9.

Neutrino mass



$$\frac{\rho_\nu}{\rho_\gamma} = \frac{7}{8} N_{\rm eff}\left(\frac{4}{11}\right)^{4/3}$$

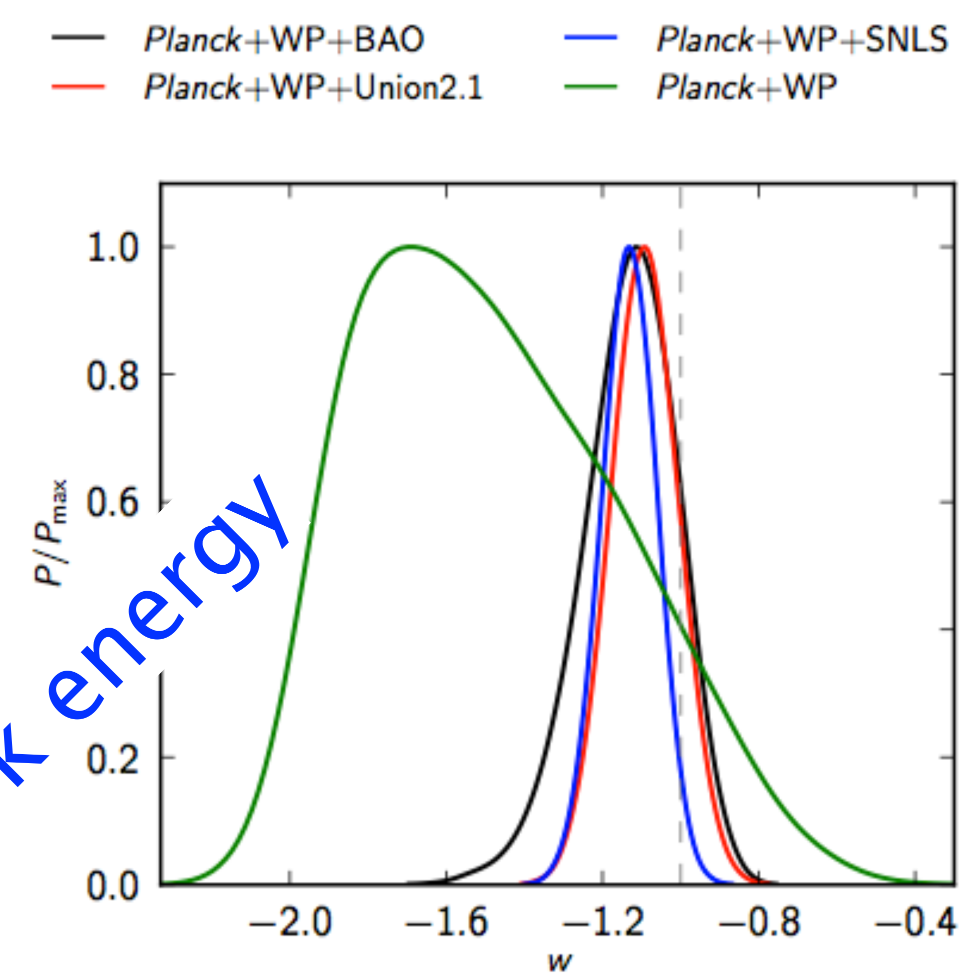
$$\Omega_\nu = \frac{\rho_\nu^0}{\rho_{\rm crit}^0} = \frac{\sum m_\nu}{93.14 h^2\,{\rm eV}}$$

$w(a) \equiv \rho_{DE}/\rho_{DE}$

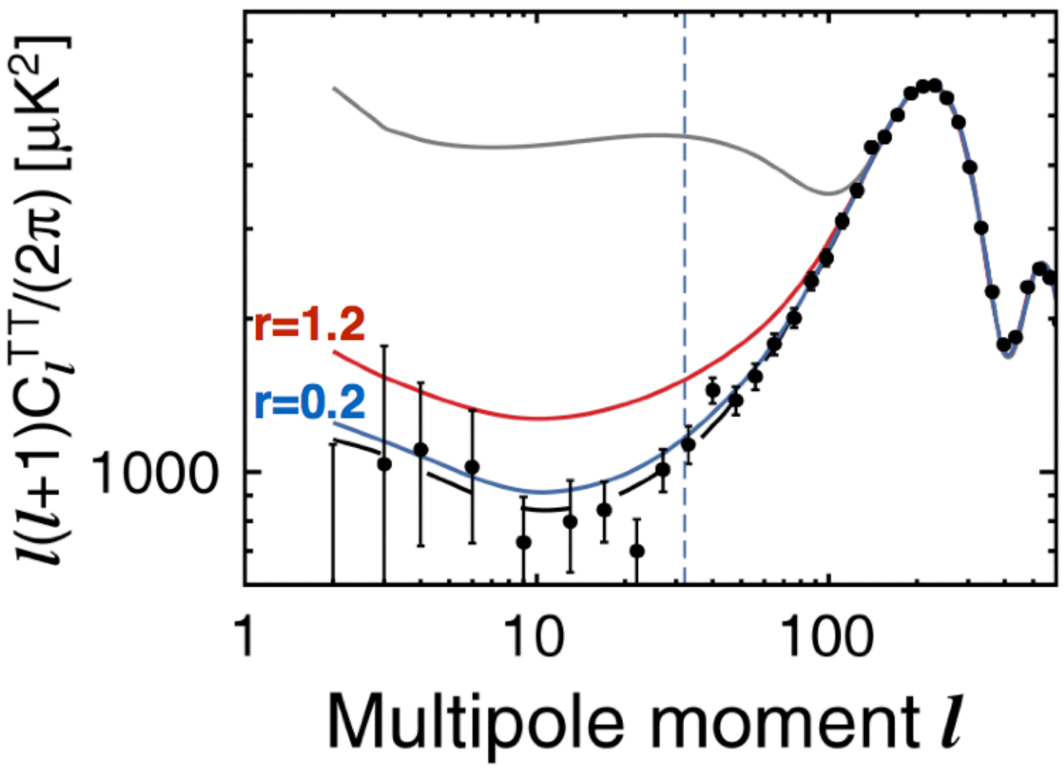
$\rho_{DE} \sim a^{-3(1+w)}$

w = -1 is cosmological constant

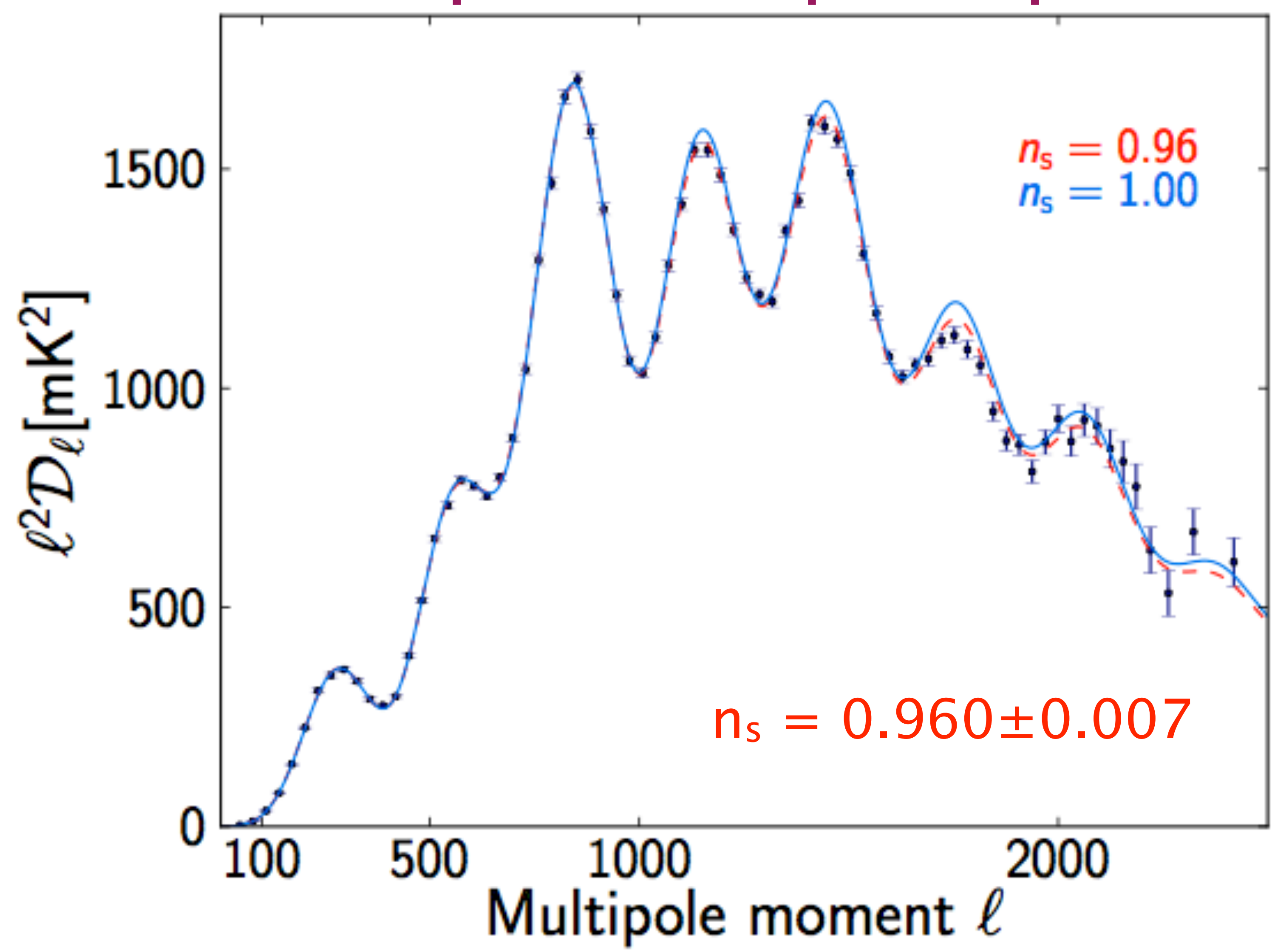
Dark energy

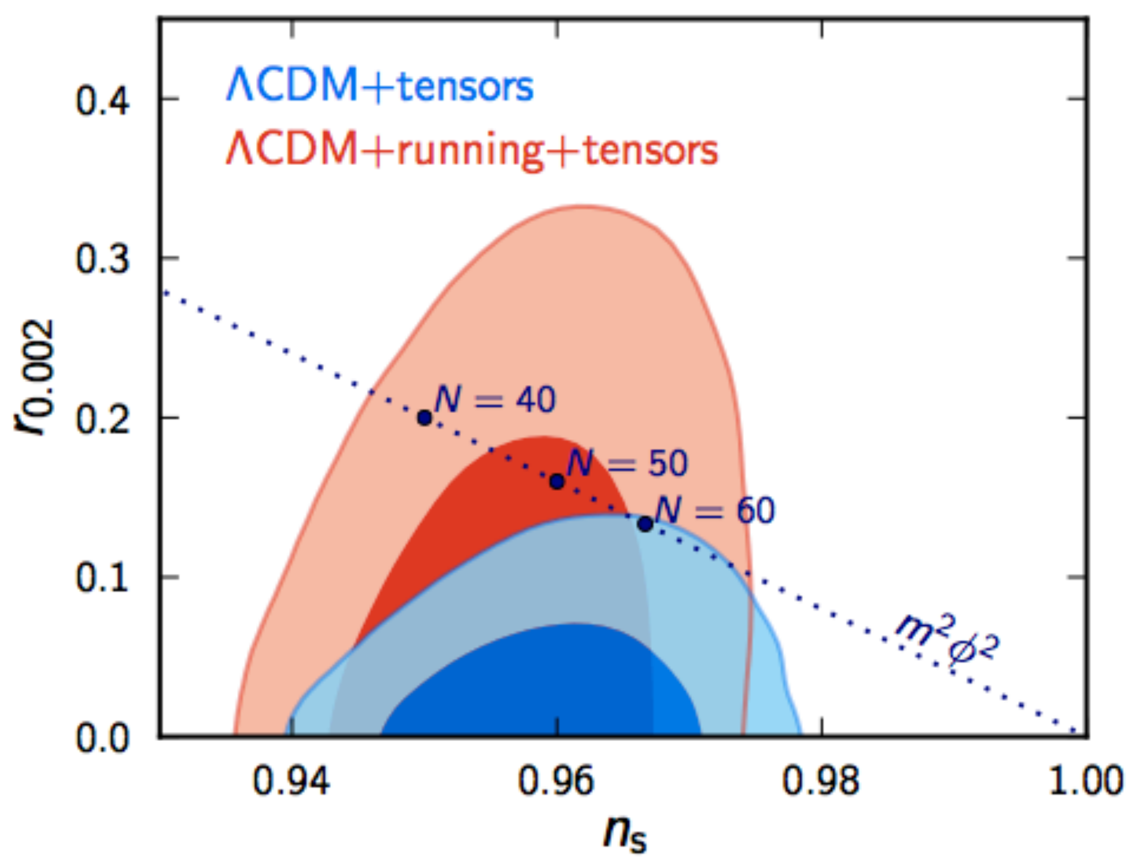


Planck 2013 cosmological results



EE polarization power spectrum





Inflation

$$\mathcal{P}_s(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$$

$$r = \mathcal{P}_T / \mathcal{P}_s$$

$$\mathcal{P}_T(k) = A_T \left(\frac{k}{k_*}\right)^{n_T} .$$

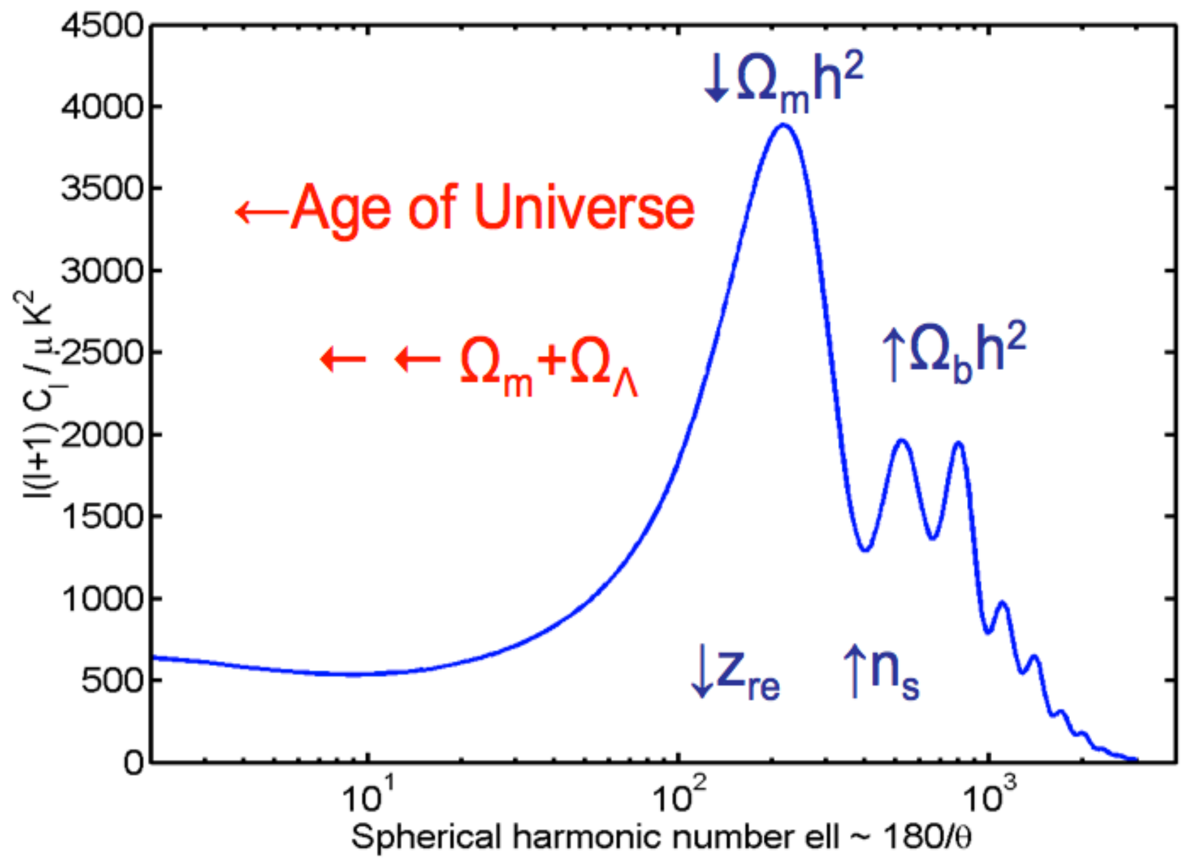
H_0 from CMB measurements

The Hubble parameter is obtained from CMB data by assuming spatial flatness (or some other constraint on the total matter-energy content of the universe).

$$\Omega_0 = \Omega_m + \Omega_\Lambda \quad \Rightarrow \quad \Omega_m = \Omega_0 - \Omega_\Lambda$$

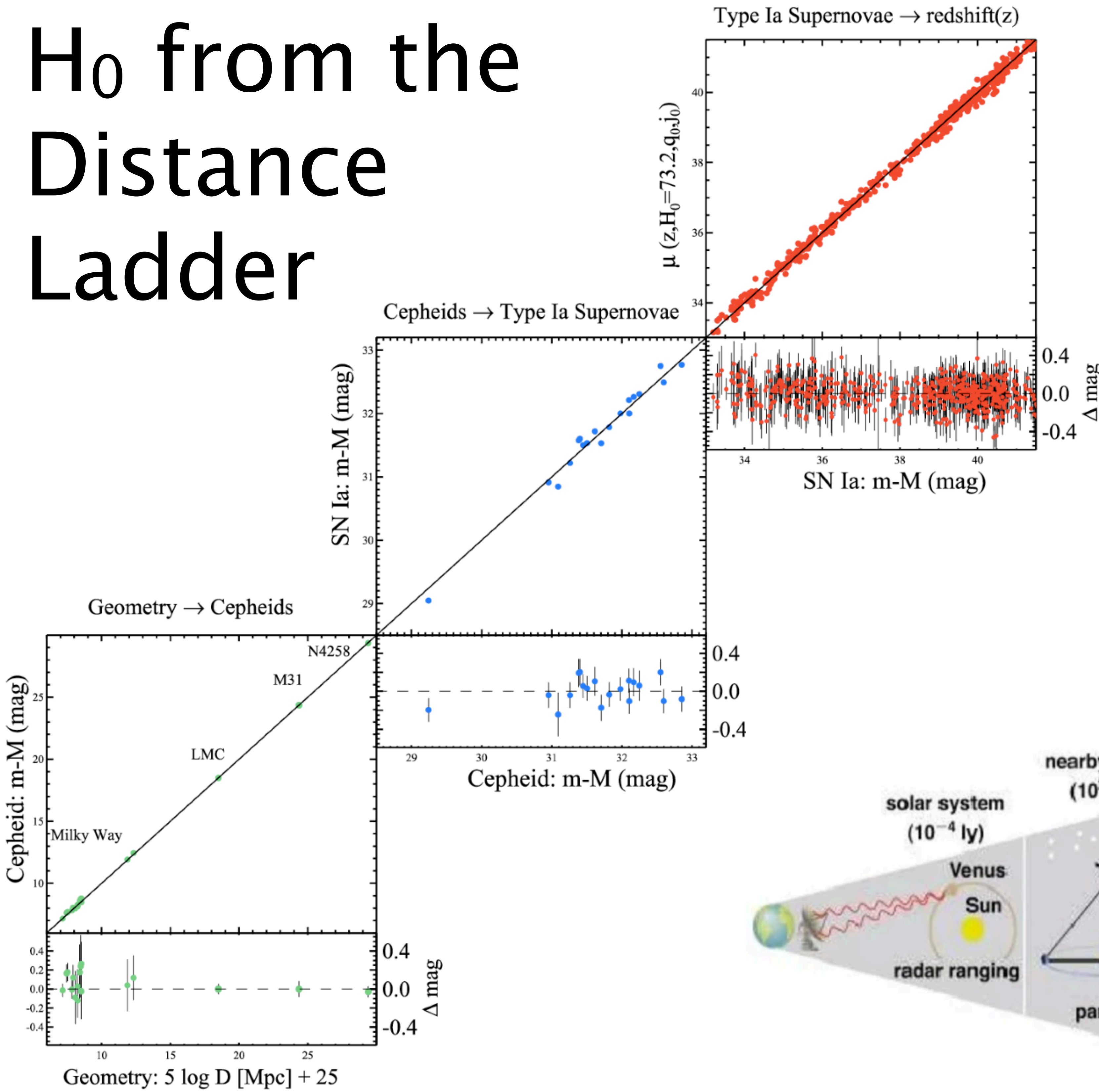
$$h = \sqrt{\frac{\omega_m}{\Omega_m}} = \sqrt{\frac{\omega_m}{\Omega_0 - \Omega_\Lambda}}$$

Since CMB measurement is (primarily) a snapshot of the universe at z=1100, it only provides an estimate of H(z) at that epoch. To connect it to the present-epoch H_0 value, one needs a cosmological model, which is the flat-ΛCDM that comes from fitting CMB data.

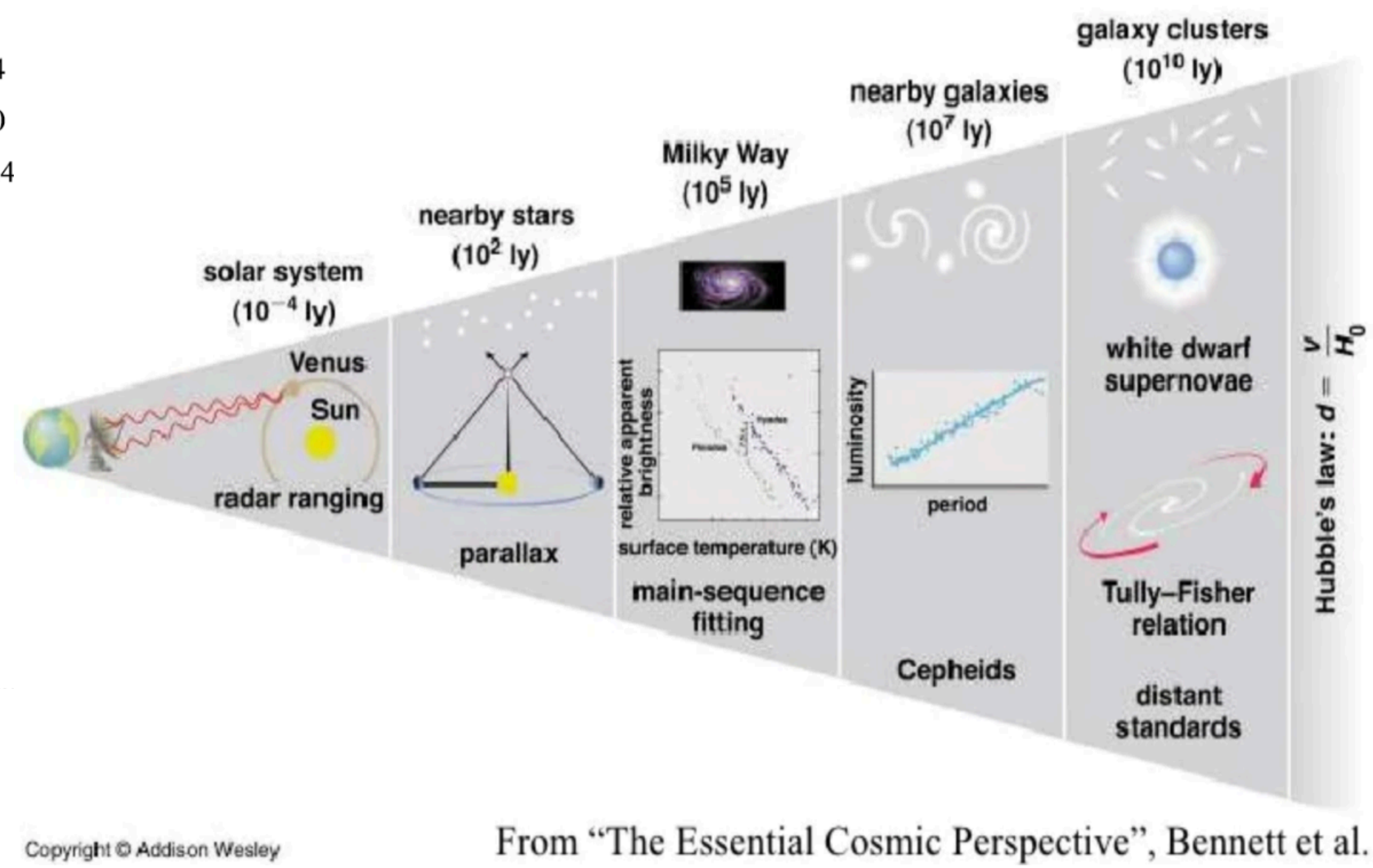


It is possible to partially break the degeneracy with the flat-universe assumption using CMB lensing data (to be discussed in January). But otherwise, CMB measurements leave very little wiggle room to deviate from the flat-universe ΛCDM model (in combination with the BBN result).

H_0 from the Distance Ladder



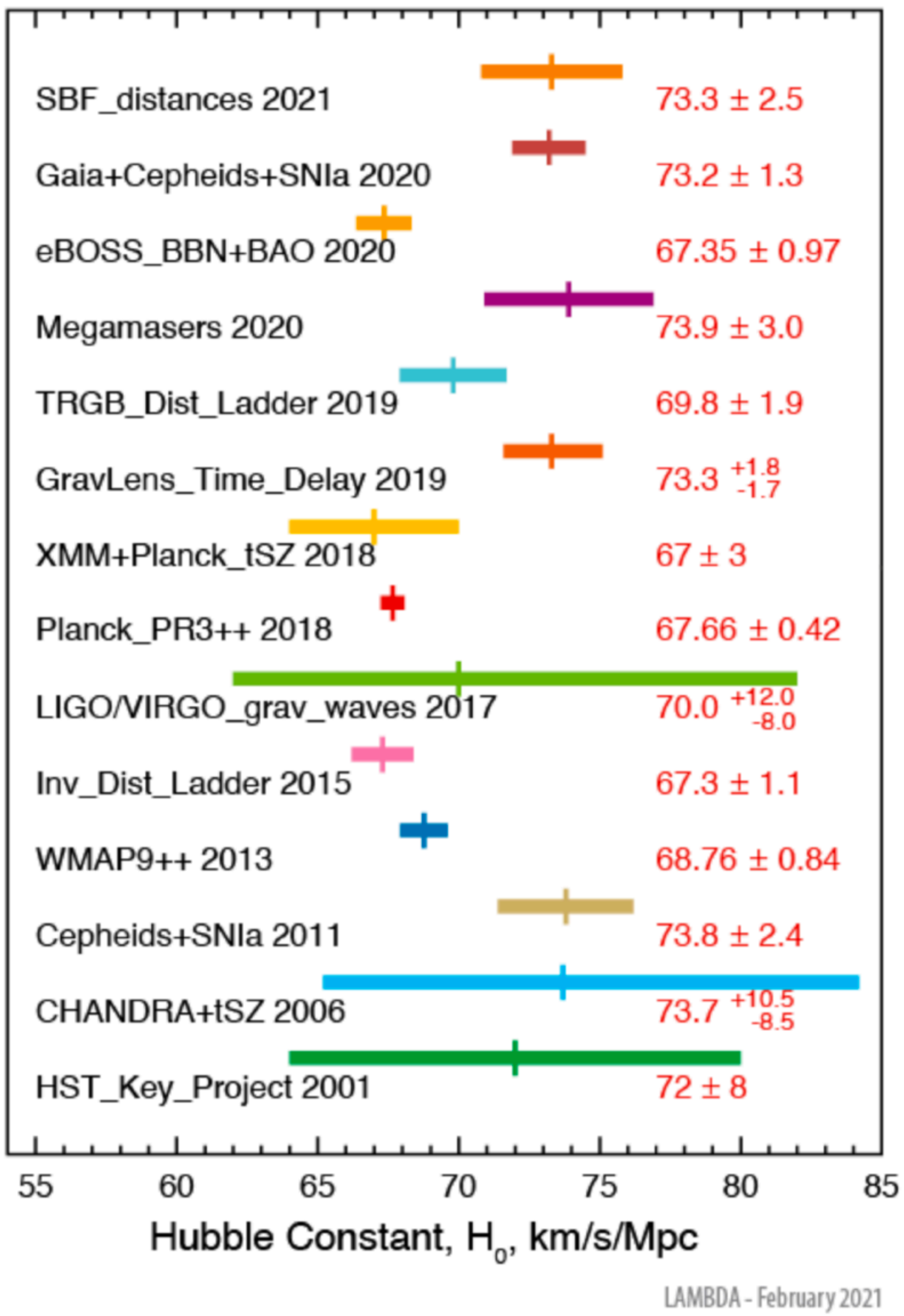
Traditional 'distance ladder' method builds on the reliability of direct measurements close-by and the extension of those measurements outward through the use of standard candles such as Cepheid variables and Type Ia supernovae.



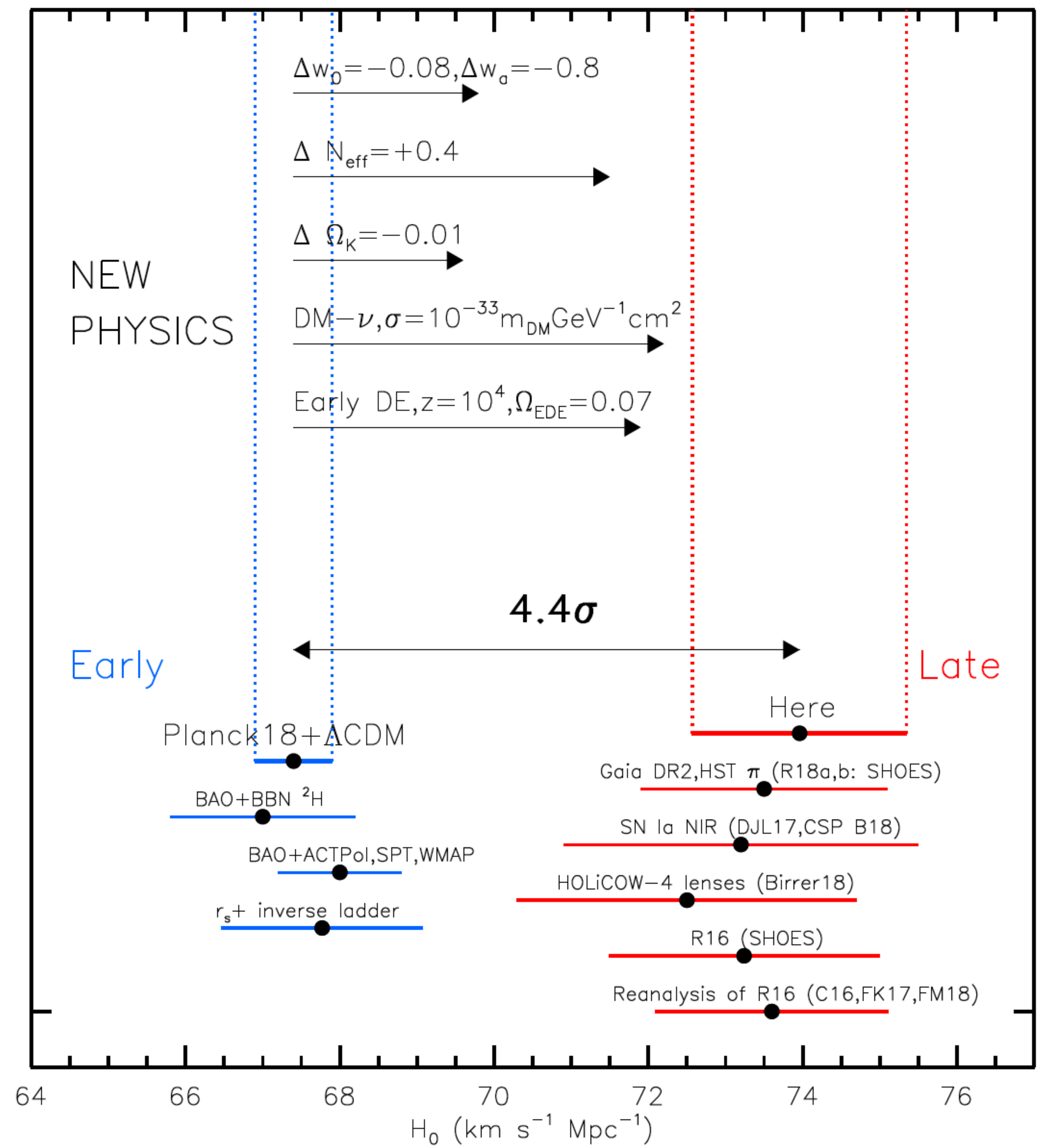
(Fig. From Riess et al. 2016)

From "The Essential Cosmic Perspective", Bennett et al.

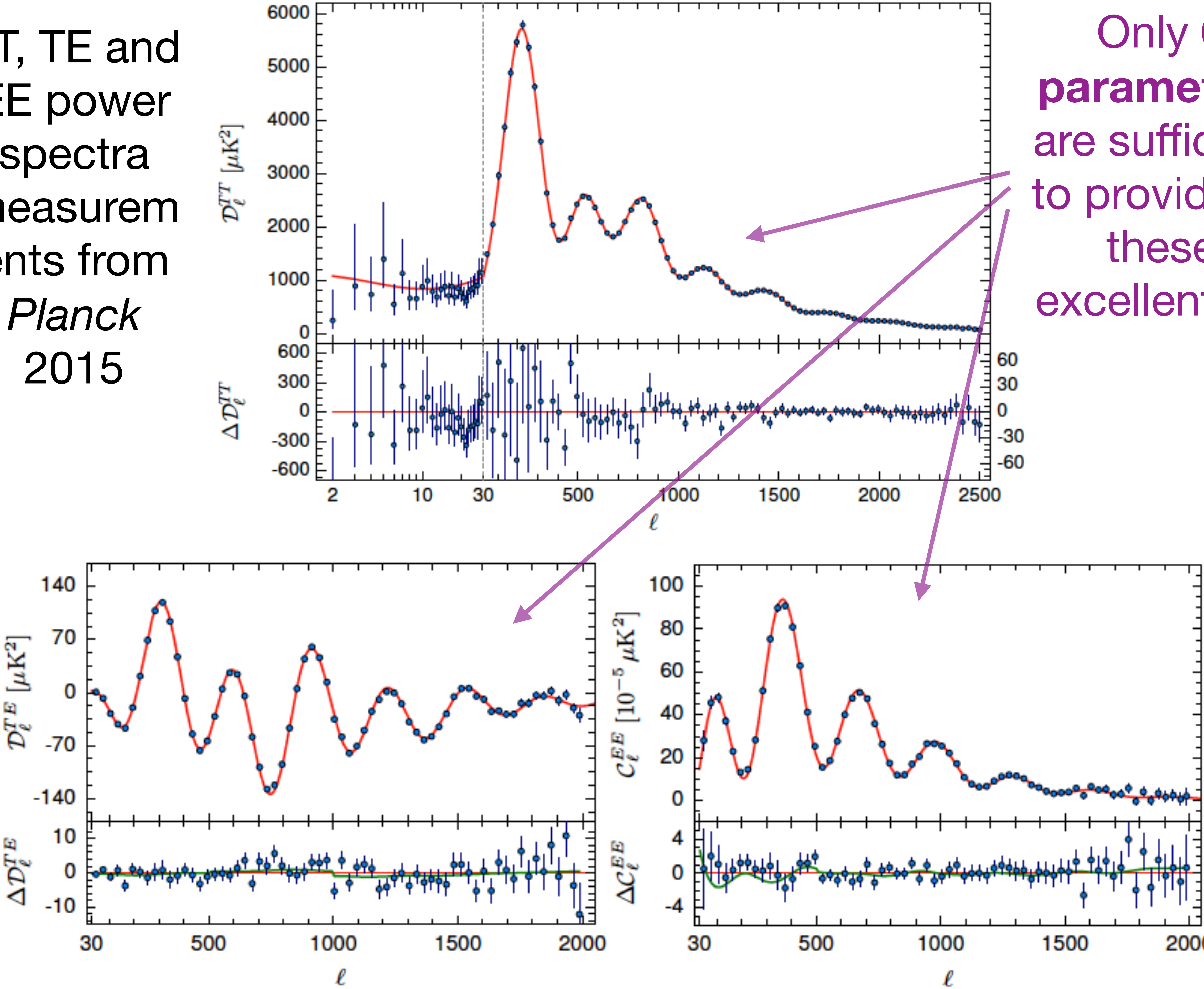
The "Hubble tension"



(Riess et al. 2019, ApJ)



TT, TE and EE power spectra measurements from *Planck* 2015



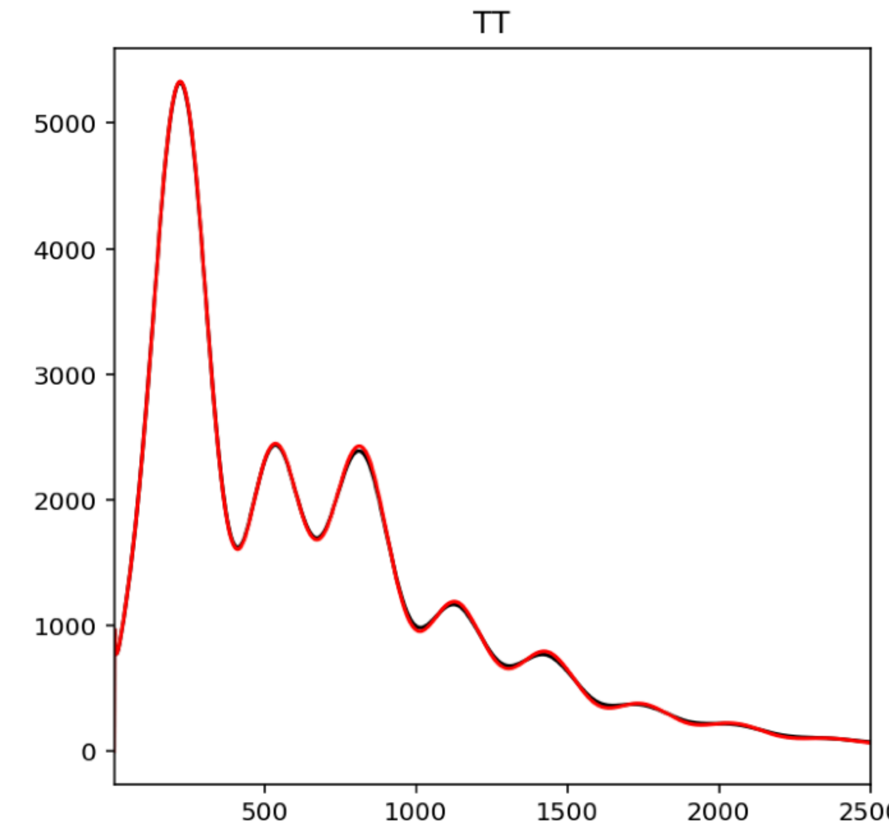
Only **6 parameters** are sufficient to provide all these excellent fits

How to generate random CMB skies?

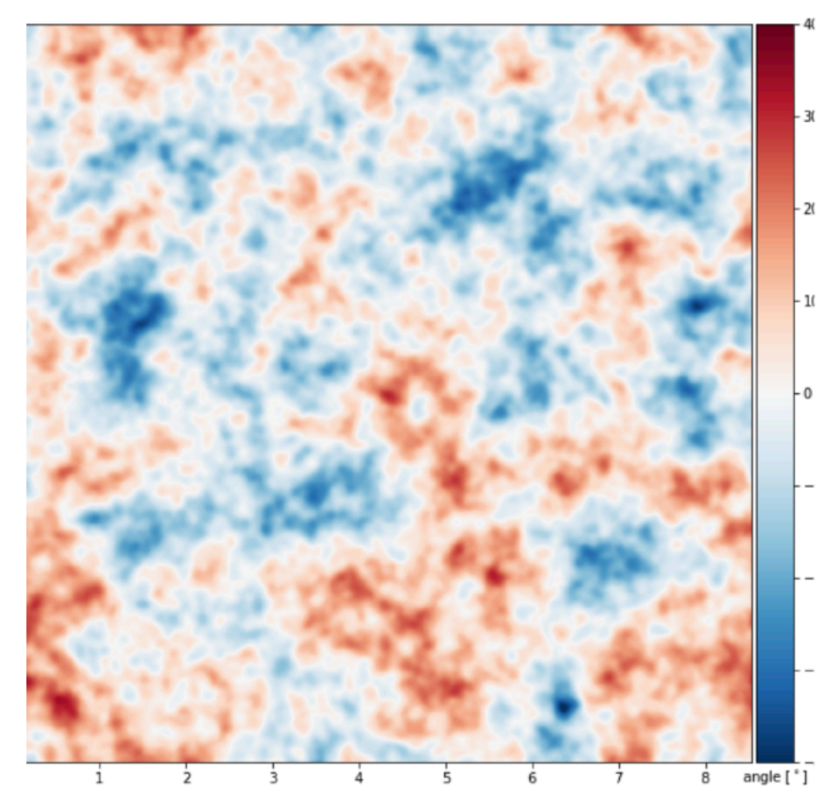
- Start with an artificially generated power spectrum with your choice of a cosmological model

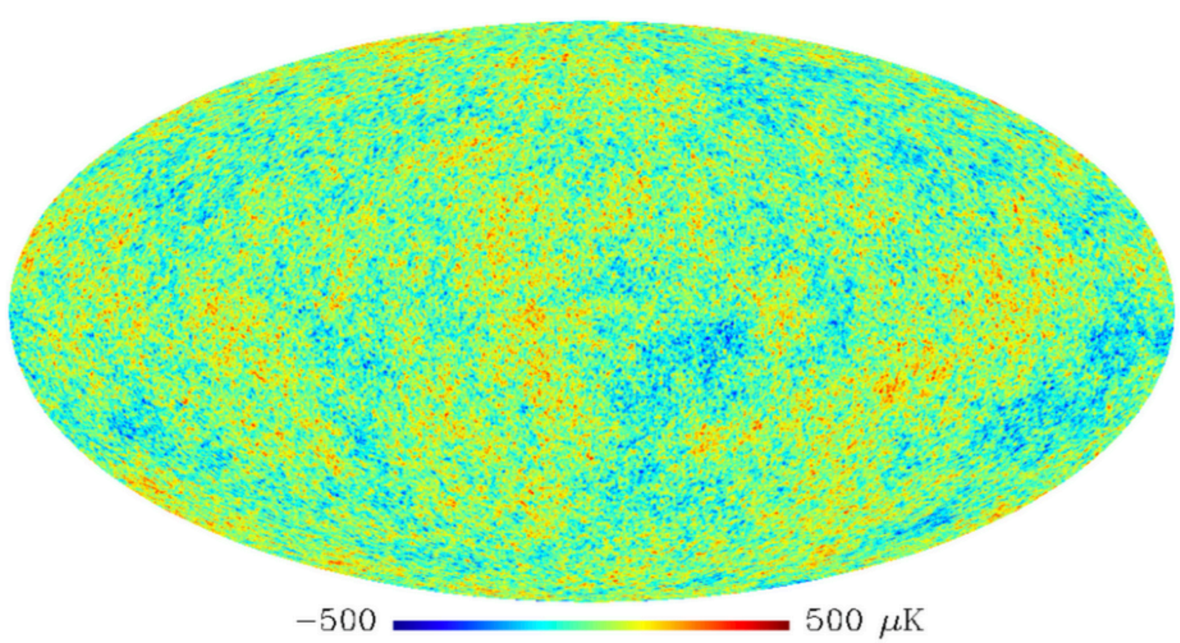
- Compute a randomly-generated a_{lm} value (m is the random part) whose amplitudes are consistent with the C_l-s.

- For flat-sky patches, you have simply used the inverse Fourier-transform of a 2D P(k). For full-sky realizations, use spherical harmonics.

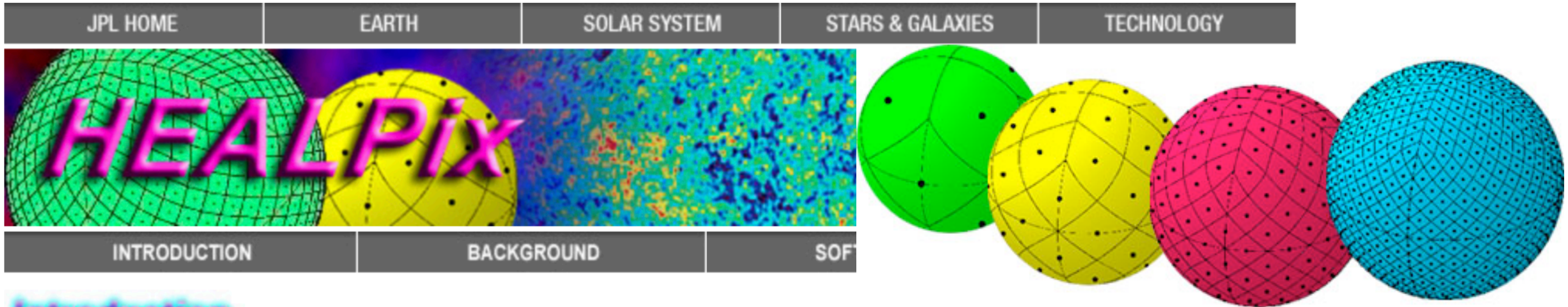


$$c_\ell = \langle |a_{\ell m}| \rangle^2 \qquad \frac{\Delta T}{T}(\theta,\phi) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_m^\ell(\theta,\phi)$$





HealPix (or HealPy) for full sky



HEALPix is an acronym for **H**ierarchical **E**qual **A**rea iso**L**atitude **Pix**elization of a sphere. As suggested in the name, this pixelization produces a subdivision of a spherical surface in which each pixel covers the same surface area as every other pixel. The figure below shows the partitioning of a sphere at progressively higher resolutions, from left to right. The green sphere represents the lowest resolution possible with the HEALPix base partitioning of the sphere surface into 12 equal sized pixels. The yellow sphere has a HEALPix grid of 48 pixels, the red sphere has 192 pixels, and the blue sphere has a grid of 768 pixels (~7.3 degree resolution).

healpy.sphtfunc.synfast

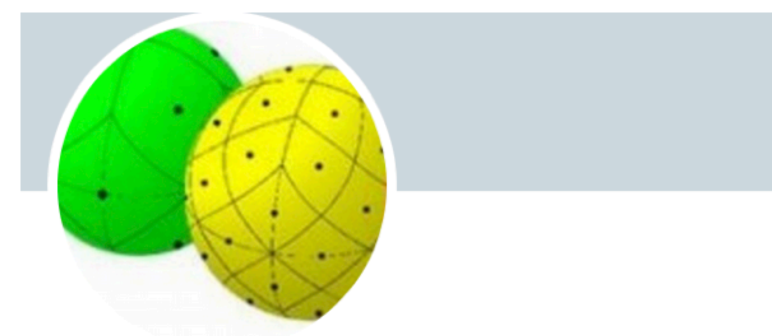
`healpy.sphtfunc.synfast`(*cls, nside, lmax=None, mmax=None, alm=False, pol=True, pixwin=False, fwhm=0.0, sigma=None, new=False, verbose=True*)

Create a map(s) from cl(s).

healpy.sphtfunc.anafast

`healpy.sphtfunc.anafast`(*map1, map2=None, nspec=None, lmax=None, mmax=None, iter=3, alm=False, pol=True, use_weights=False, datapath=None, gal_cut=0, use_pixel_weights=False*)

Computes the power spectrum of a Healpix map, or the cross-spectrum between two maps if *map2* is given. No removal of monopole or dipole is performed. The input maps must be in ring-ordering. Spherical harmonics transforms in HEALPix are always on the full sky, if the map is masked, those pixels are set to 0. It is recommended to remove monopole from the map before running *anafast* to reduce boundary effects.

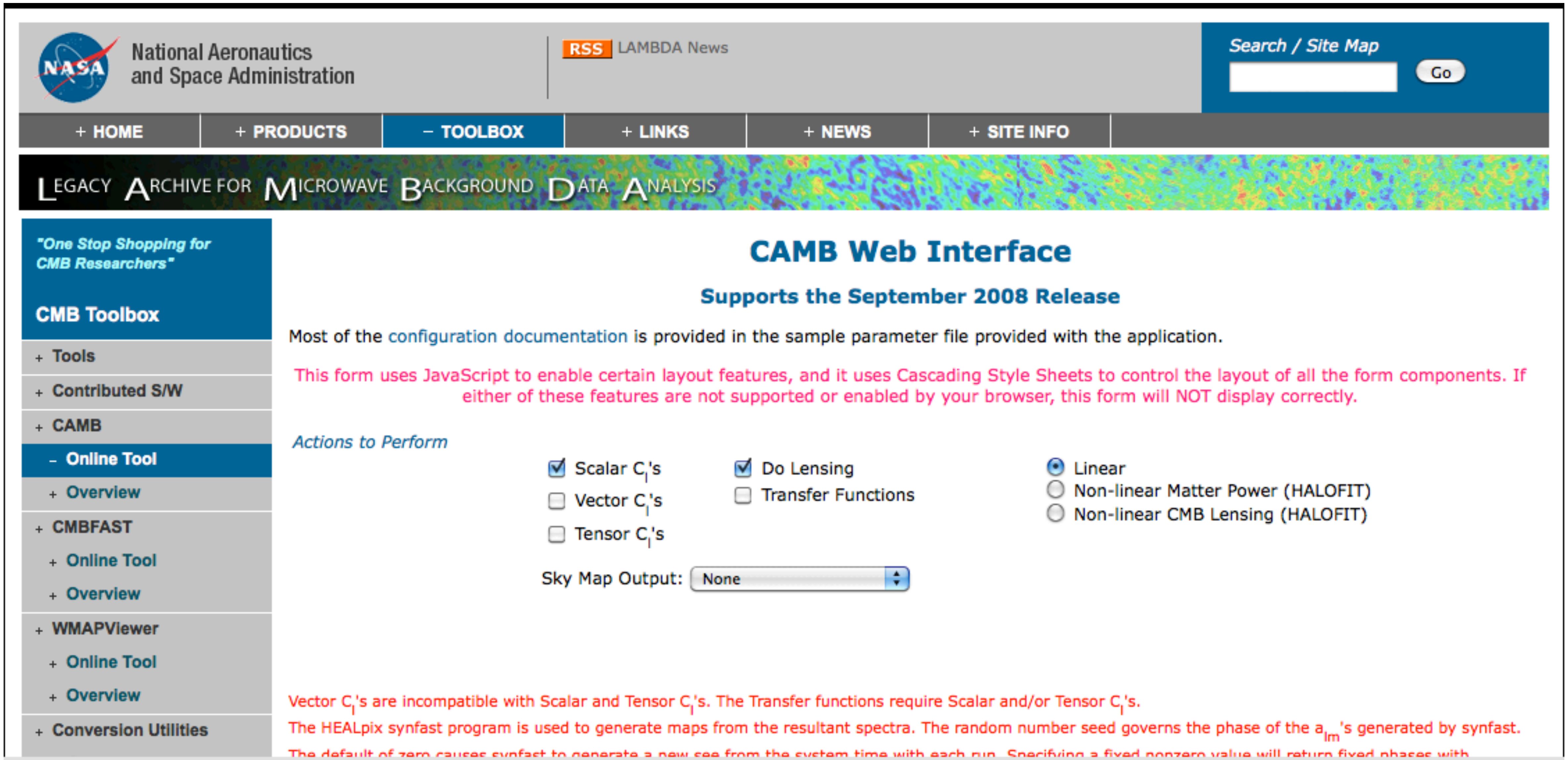


healpy
@healpy

HEALPix maps tools for Python

github.com/healpy   Joined April 2012

Online C_l calculators



CMB Toolbox: https://lambda.gsfc.nasa.gov/toolbox/

CAMB website: http://camb.info/

CAMB Python page: http://camb.readthedocs.io/en/latest/

(try the online example notebook!)

Calculation of the C_l–s (codes like CMBFast & CAMB)

Boltzmann transport equation describes the evolution of the photon distribution function

$$\delta f_T(\hat{\mathbf{n}}, \mathbf{x}, \eta) = \left(T\frac{\partial f}{\partial T}\right)_{\rm CMB}\frac{\Delta T}{T}$$

Collisional part describes the scattering of the photons with electrons

Gravitational part describes the motion of the photons in the perturbed background

$$\frac{\partial}{\partial\eta}\frac{\Delta T}{T}(\hat{\mathbf{n}}, \mathbf{x}, \eta) = \text{Coll.} + \text{Grav.}$$

Differential form in Fourier space

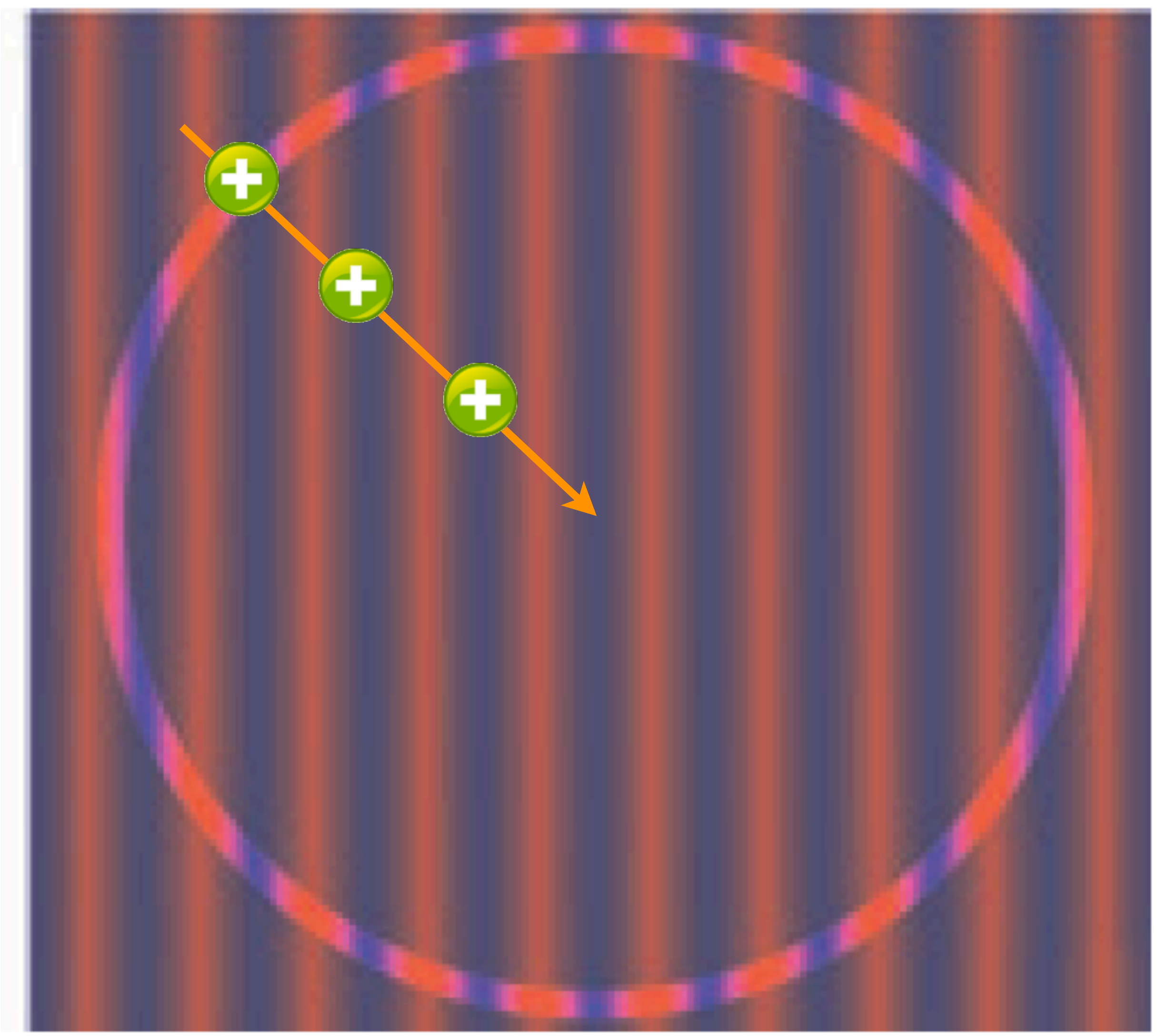
Scalar perturbations

$$\begin{aligned}
&\dot{\Delta}_T + ik\mu\Delta_T = \dot{\Phi} - ik\mu\Psi \\
&+\dot{\tau}\left[-\Delta_T + \Delta_{T_0} + i\mu v_B + \frac{1}{2}P_2(\mu)\Pi\right] \\
&\dot{\Delta}_P + ik\mu\Delta_P = \dot{\tau}\left[-\Delta_P + \frac{1}{2}\{1 - P_2(\mu)\}\,\Pi\right]
\end{aligned}$$

$$C_\ell = (4\pi)^2 \int k^2 dk P(k) |\Delta_{T\ell}(k, \eta_0)|^2$$

CMBFast: Seljak & Zaldarriaga (1996)

Calculation of the C_l-s (codes like CMBFast & CAMB)



- We know how the intensity distribution for a single k-mode looks like for any given instant
- Choose one single k-mode and evolve that from before the recombination until today (coupled & linearized Boltzmann and Einstein equations)
- Translate the contribution of that k-mode into the angular power spectrum (C_l) by line of sight projection (Limber approximation)
- Average over all possible phases, and then finally, sum up the contributions from all the k-modes

Fitting cosmology from CMB data

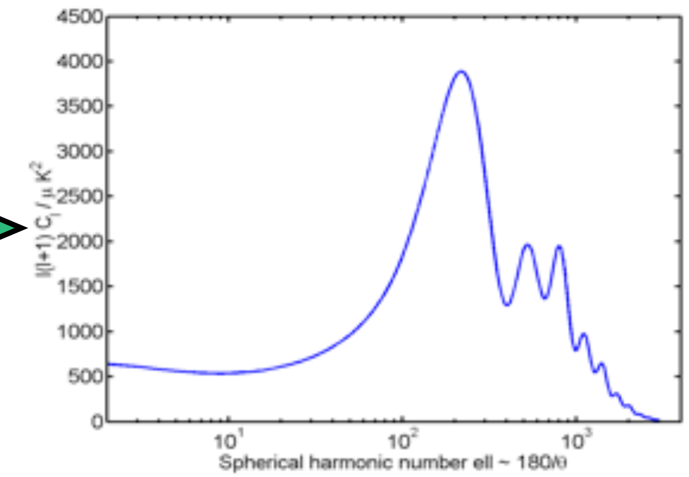
INPUT

Draw random parameters for cosmological model:
Ω_m, Ω_Λ, σ_8, H_0, ..

Boltzmann solvers

powerful cosmological codes (CMBFAST, CAMB)

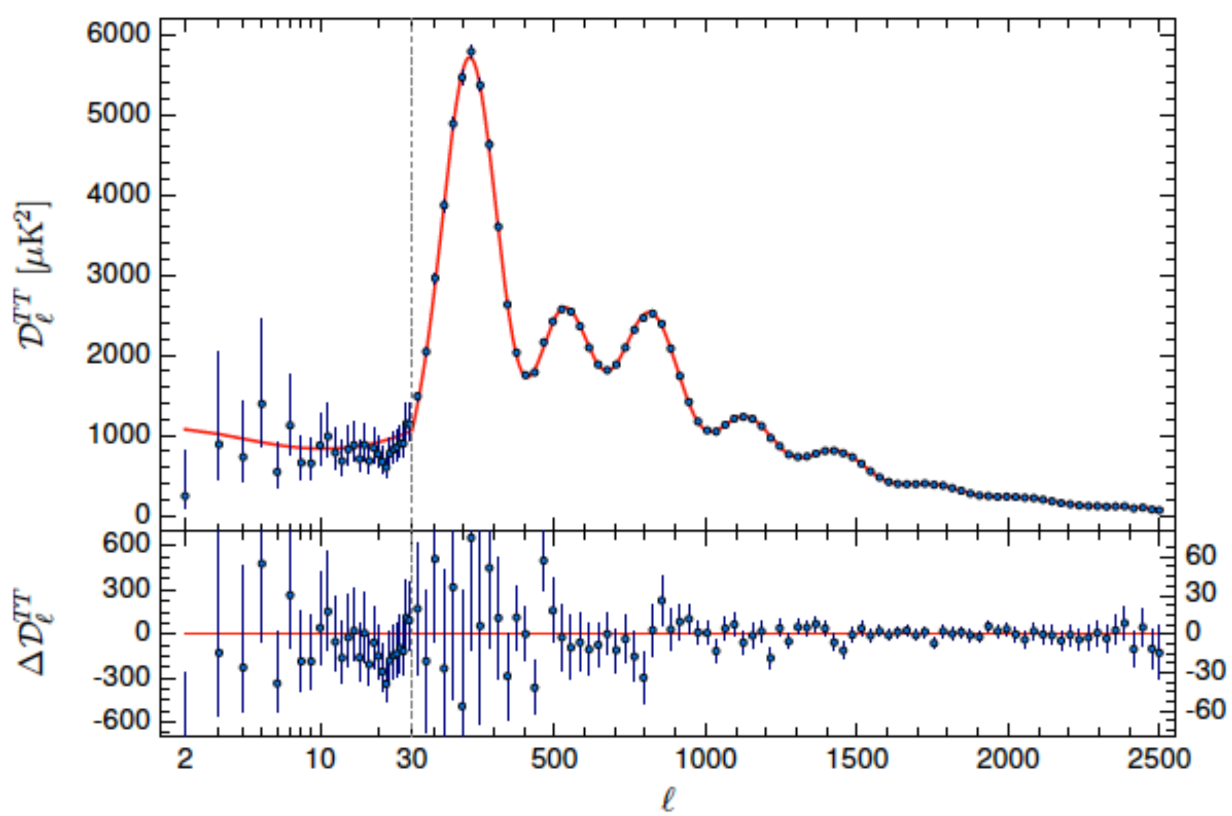
OUTPUT



Fit to data

Run an MCMC chain and compute the parameter likelihoods

Publish papers!



Questions?

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