astro8405

An Introduction to the Cosmic Microwave Background

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eCampus | Lernplattform der Universität Bonn

astro8405: The Cosmic Microwave Background

Aktionen

This course intends to give you a modern and up-to-date introduction to the science and experimental techniques relating to the Cosmic Microwave Background. No prior knowledge of cosmology is necessary, your prerequisite are a basic understanding of electrodynamics and thermal physics and some familiarity with Python programming.

Lecture 7:

Polarization Anisotropies

The E- and B-modes

What we aim to learn here

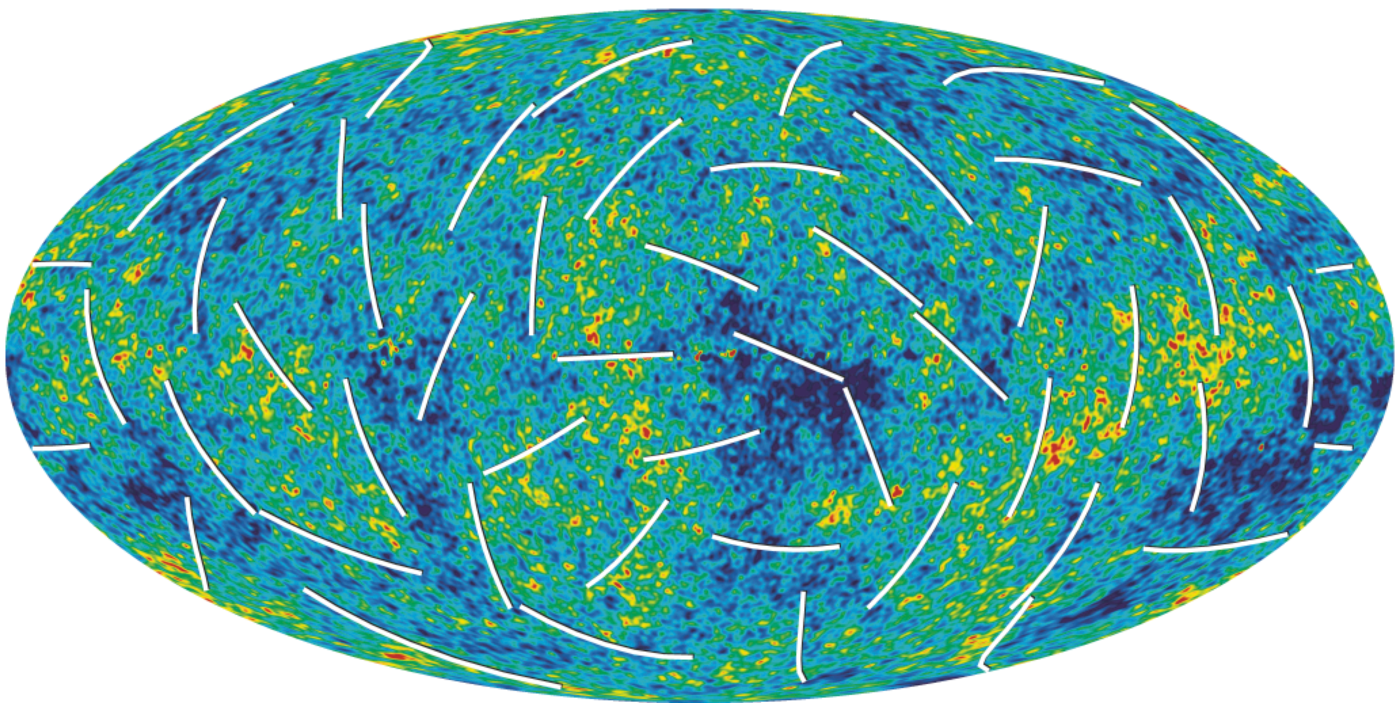
➡ In today's lecture:

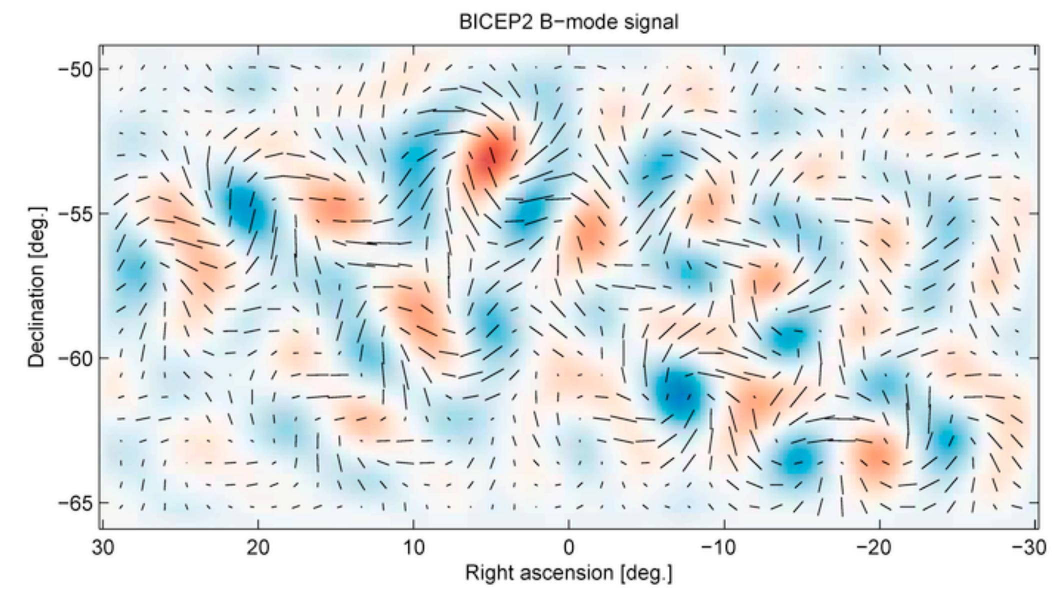
- ◉ The use of E- and B-modes for CMB polarization
- ◉ Origin of CMB polarization, density modes
- ◉ E-mode power spectrum and reionization "bump"

➡ In next week's lecture:

- ◉ Gravity waves and tensor perturbations
- ◉ The search for inflationary gravity waves via B-mode

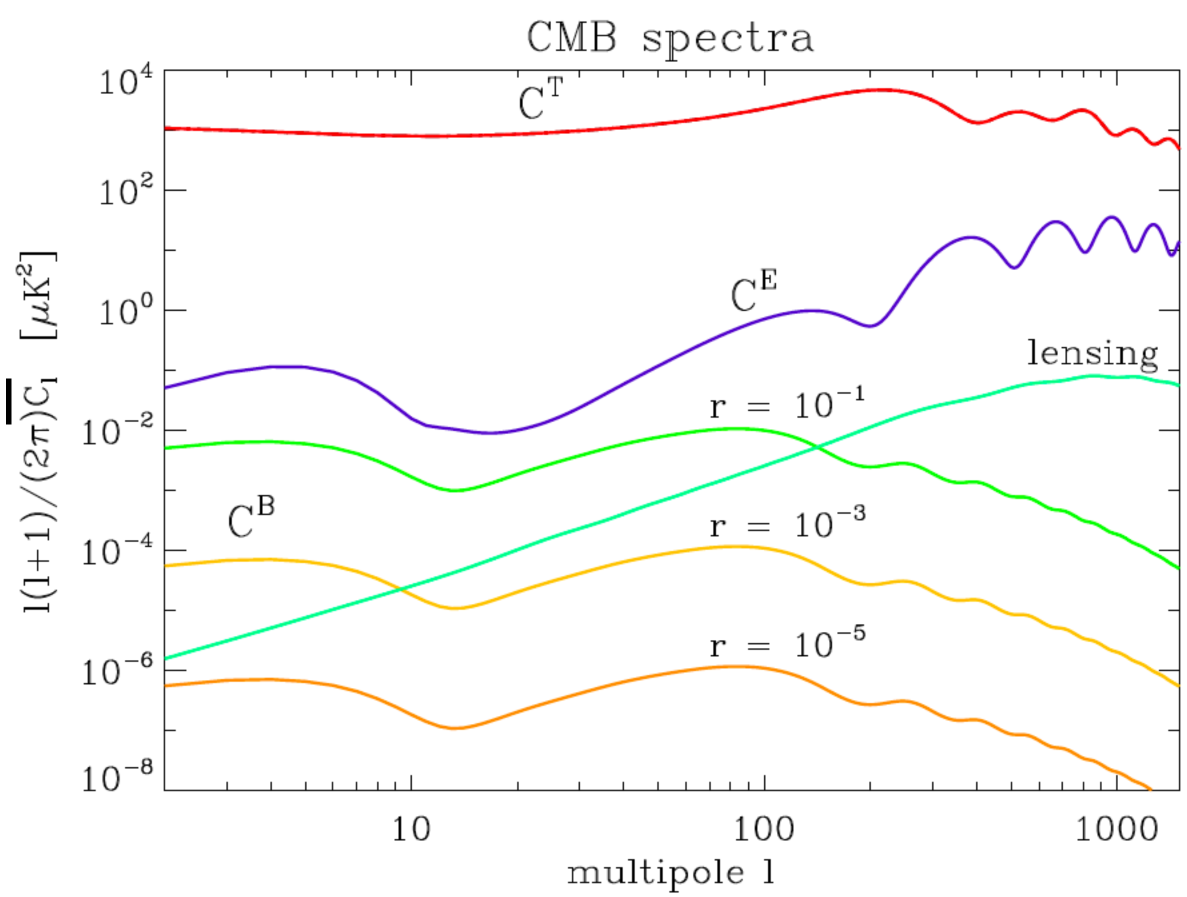
CMB is weakly polarized



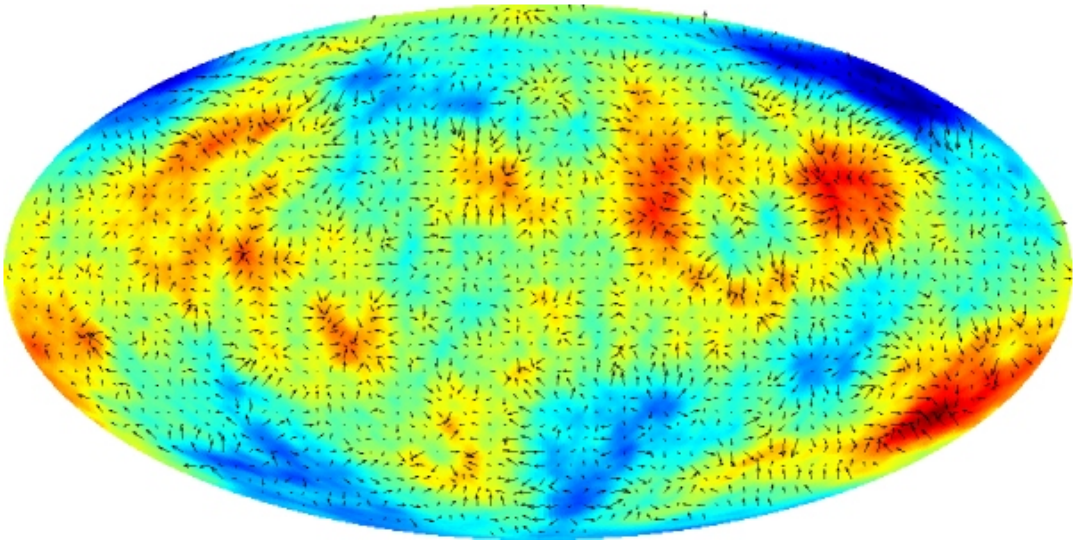


The CMB has a weak (~few percent) linearly polarized component. This depends on the small quadrupole at the epoch of last scattering, and also the fact that only few photons having the time to scatter in an optically thin medium.

As seen from right, the anisotropy in the polarized signal (C_l^{EE}) is only about ~1% at large scales.



Quantifying the polarization



One could describe the polarization by means of standard Stokes Q and U parameters, but that will make their value dependent of the choice of X- and Y-axes.

$$S_0 = I = E_x^2 + E_y^2$$
$$S_1 = Q = E_x^2 - E_y^2$$
$$S_2 = U = 2E_x E_y \cos\delta$$
$$S_3 = V = 2E_x E_y \sin\delta$$

$$I = E_X^2 + E_Y^2 = E_{0^\circ}^2 + E_{90^\circ}^2$$
$$Q = E_X^2 - E_Y^2 = E_{0^\circ}^2 - E_{90^\circ}^2$$
$$U = E_{45^\circ}^2 - E_{-45^\circ}^2$$
$$V = E_{LCP}^2 - E_{RCP}^2$$

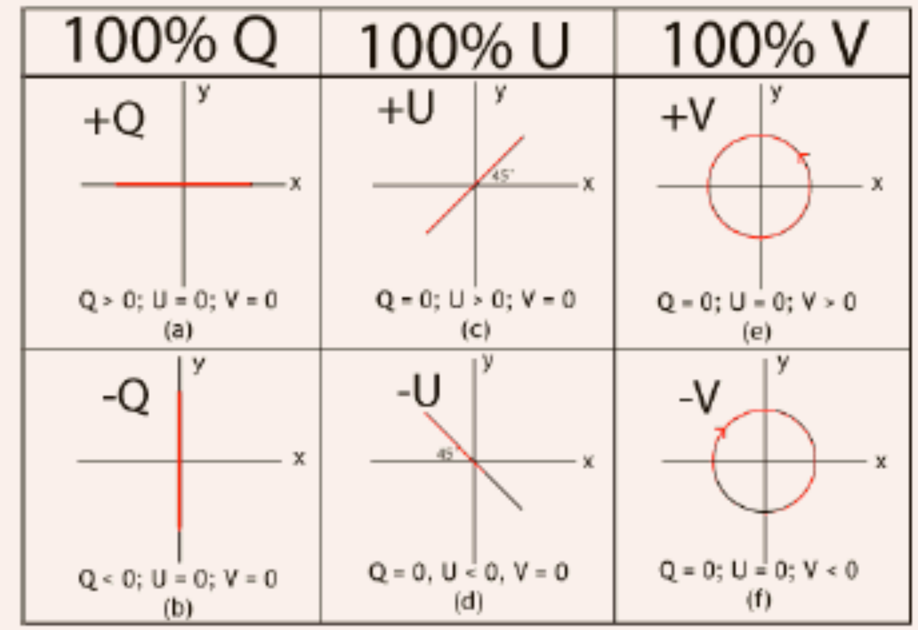
Polarization fraction

$$p = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}$$

Polarization angle

$$\alpha \equiv \frac{1}{2}\tan^{-1}\frac{U}{Q}$$

Stokes parameters formalism



$$\left\{ \begin{array}{c} I \\ Q \\ U \\ V \end{array} \right\}$$

- ★ *I*, intensity
- ★ *Q*, *U*, linear polarization
- ★ *V*, circular polarization

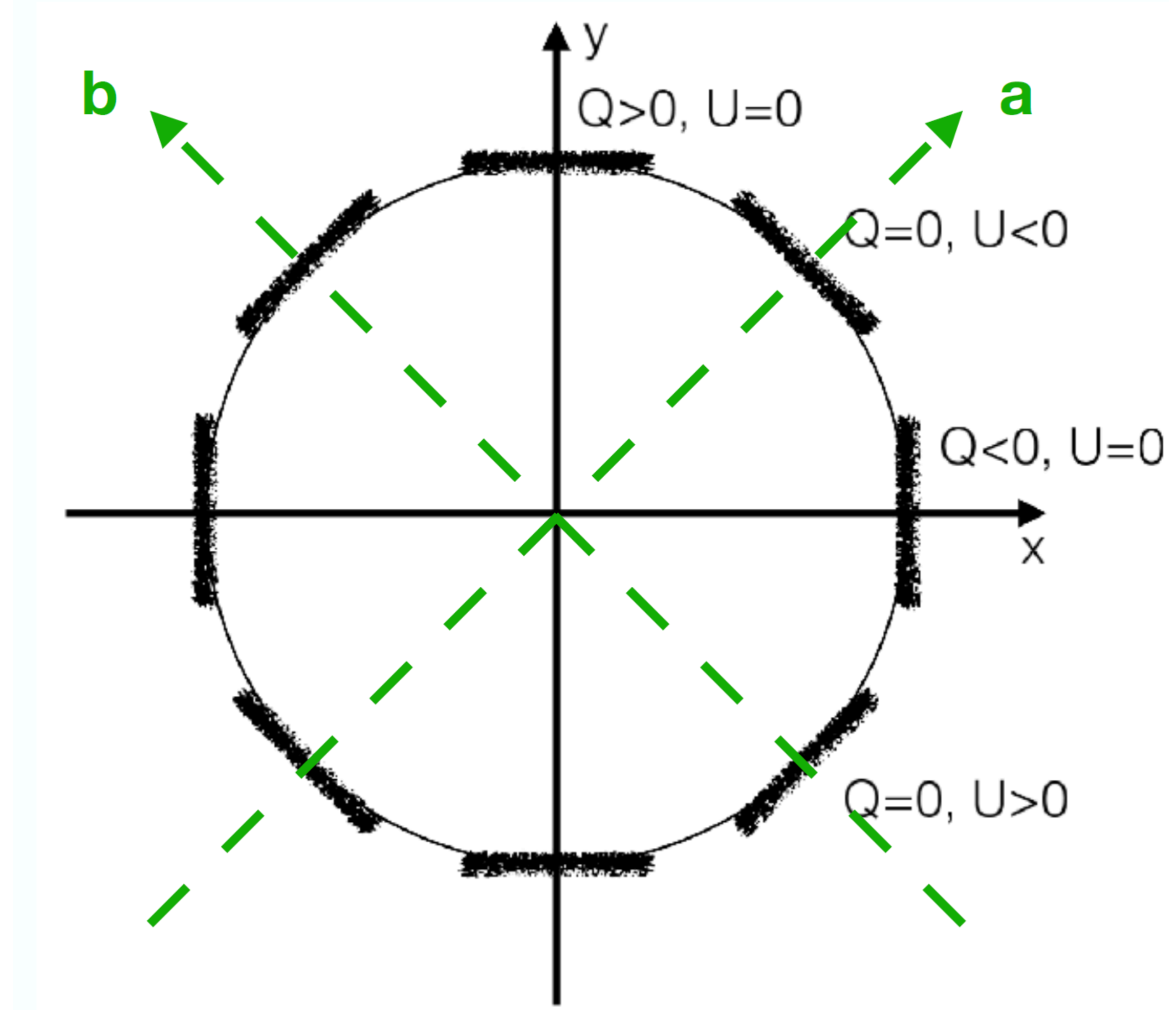
★in the case of the CMB, $V = 0$

Polarization of the CMB

$$Q \propto E_x^2 - E_y^2$$
$$U \propto E_a^2 - E_b^2$$

Under (x,y) -> (x',y'):

$$Q \longrightarrow \tilde{Q}$$
$$U \longrightarrow \tilde{U}$$



$$\begin{pmatrix} \tilde{Q} \\ \tilde{U} \end{pmatrix} = \begin{pmatrix} \cos 2\varphi & \sin 2\varphi \\ -\sin 2\varphi & \cos 2\varphi \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix}$$

We need to define the polarization vectors in a coordinate-independent way!

$$\tilde{Q} + i\tilde{U} = \exp(-2i\varphi)(Q + iU)$$

To do that, we write the polarization as a complex vector, which behaves as a spin-2 field. We then choose special spin-2 spherical harmonic coefficients which has the rotation term in-built, which cancels out the rotation of $Q + iU$ vector.

$$Q(\boldsymbol{\theta}) + iU(\boldsymbol{\theta}) = \int \frac{d^2\ell}{(2\pi)^2}\, a_{\boldsymbol{\ell}} \exp(i\boldsymbol{\ell}\cdot\boldsymbol{\theta})$$

where we write the coefficients as(*)

$$a_{\boldsymbol{\ell}} = -{}_2a_{\boldsymbol{\ell}} \exp(2i\phi_{\ell})$$

Formalism for E and B modes

Stokes Q,U parameters are not rotationally invariant: under rotation of ψ degrees we get

$$I' = I \qquad V' = V$$

$$Q' = Q\cos(2\psi) - U\sin(2\psi) \qquad U' = U\cos(2\psi) + Q\sin(2\psi)$$

or in a compact form $Q' \pm iU' = e^{\pm 2i\psi}(Q \pm iU)$

i.e., (Q ± iU) transforms like a spin-2 variable under rotation.

Therefore, we go to Fourier-space and use spin-weighted spherical harmonics, which are invariant under rotation. In general, a *spin-s spherical harmonics function* transforms under rotation as

$$_sY_{\ell m} \rightarrow e^{\pm s i\psi}\, _sY_{\ell m}(\hat{n})$$

For all-sky decomposition

$$(Q \pm iU)(\hat{n}) = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{+\ell} a_{\ell m}^{\pm 2}\ _{\pm 2}Y_{\ell m}(\hat{n}) = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{+\ell}(a_{E,\ell m} \pm i a_{B,\ell m})\ _{\pm 2}Y_{\ell m}(\hat{n})$$

Here $a_{lm}^{\pm 2}$ are decomposition into positive and negative helicity, which are used to define the E- and B-modes

$$a_{E,\ell m} = \frac{1}{2}(a_{\ell m}^{+2} + a_{\ell m}^{-2}) \qquad a_{B,\ell m} = \frac{-i}{2}(a_{\ell m}^{+2} - a_{\ell m}^{-2})$$

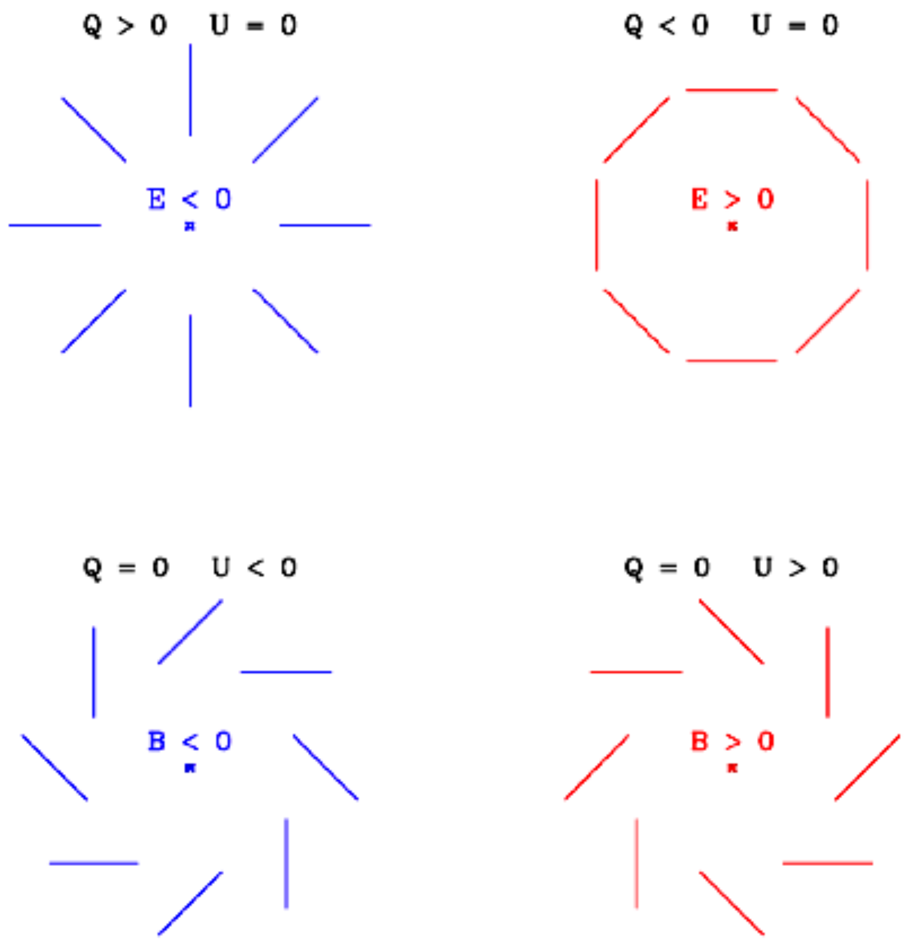
E and B mode characteristics

Two flavours of CMB polarization:

Density perturbations: parity-symmetric, "E-mode"
Gravity waves: parity-asymmetric, "B-mode"

E-mode
("gradient-like")

B-mode
("curl-like")

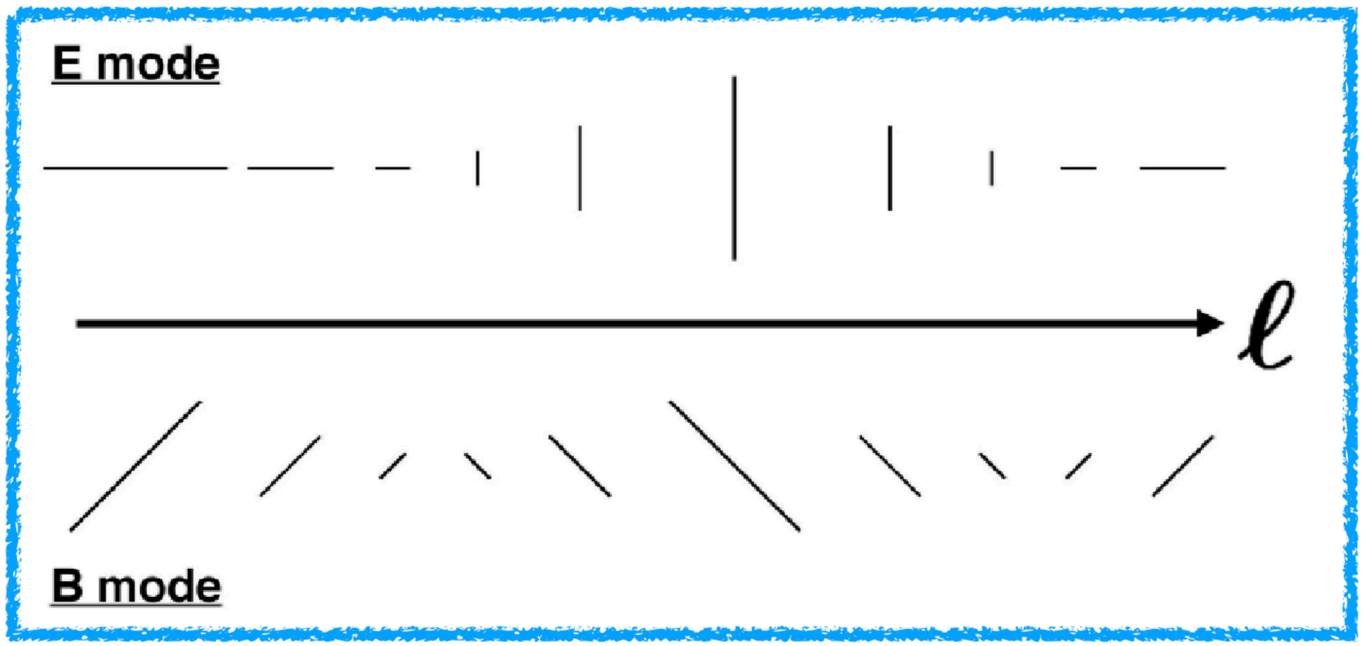


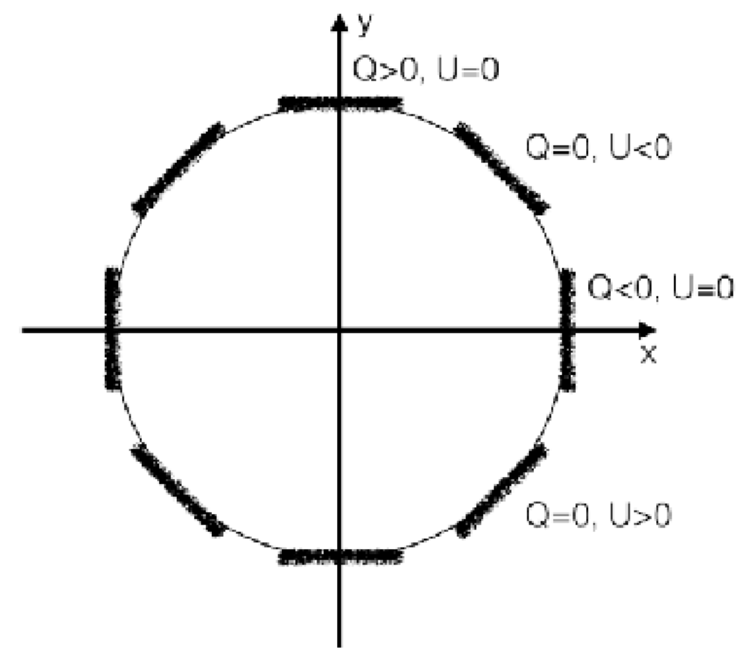
- We can break down the polarization field into two components which we call E and B modes. This is the spin-2 analog of the gradient/curl decomposition of a vector field.
- E modes are generated by density (scalar) perturbations via Thomson scattering.
- Additional vector modes are created by vortical motion of the matter at recombination – this is small
- B modes are generated by gravity waves (tensor perturbations) at last scattering or by gravitational lensing (which transforms E modes into B modes along the line of sight to us) later on.

Geometric meaning of E and B modes

E- and B-modes are therefore nothing but Fourier transforms of the Stokes Q and U parameters with a rotation (in Fourier space)

$$Q(\theta) = \Re\left[E_\ell \exp(i\ell\theta)\right]$$
$$U(\theta) = \Re\left[B_\ell \exp(i\ell\theta)\right]$$

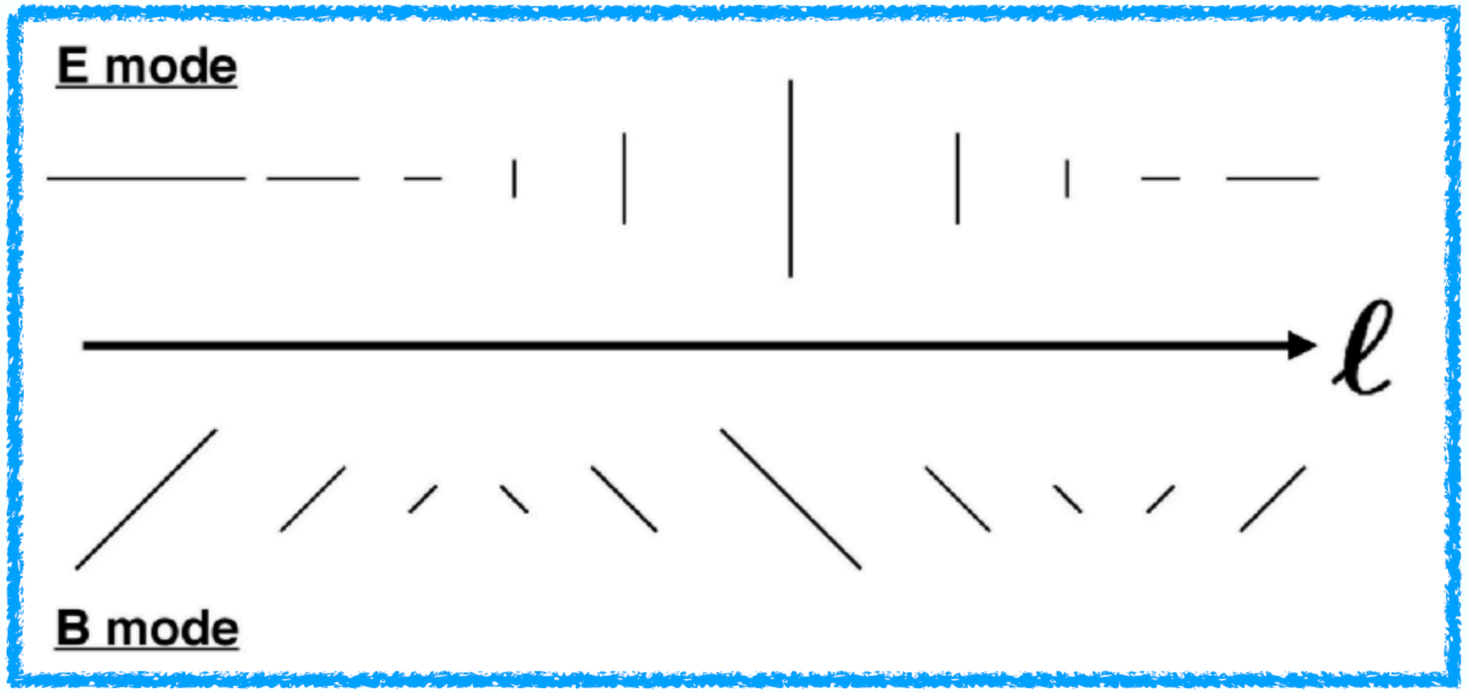




- **E mode**: Stokes Q, defined with respect to ℓ as the x-axis
- **B mode**: Stokes U, defined with respect to ℓ as the y-axis

E and B modes have different parities

$$Q(\boldsymbol{\theta}) \pm iU(\boldsymbol{\theta}) = \int \frac{d^2\ell}{(2\pi)^2} \, (E_{\boldsymbol{\ell}} \pm iB_{\boldsymbol{\ell}}) \exp(\pm 2i\phi_\ell + i\boldsymbol{\ell}\cdot\boldsymbol{\theta})$$



- **E mode**: Polarisation directions parallel or perpendicular to the wavevector
- **B mode**: Polarisation directions 45 degree tilted with respect to the wavevector

E and B modes: 2D vector analogy

The Helmholtz's Theorem on Vector Fields

Helmholtz's theorem is also called as the fundamental theorem of vector calculus. It is stated as

"A sufficiently smooth, rapidly decreasing vector field in three dimensions can be decomposed into the sum of a solenoidal (divergence-less) vector field and an irrotational (curl-less) vector field."

The theorem is also called as Helmholtz decomposition, it breaks a vector field into two *orthogonal* components.

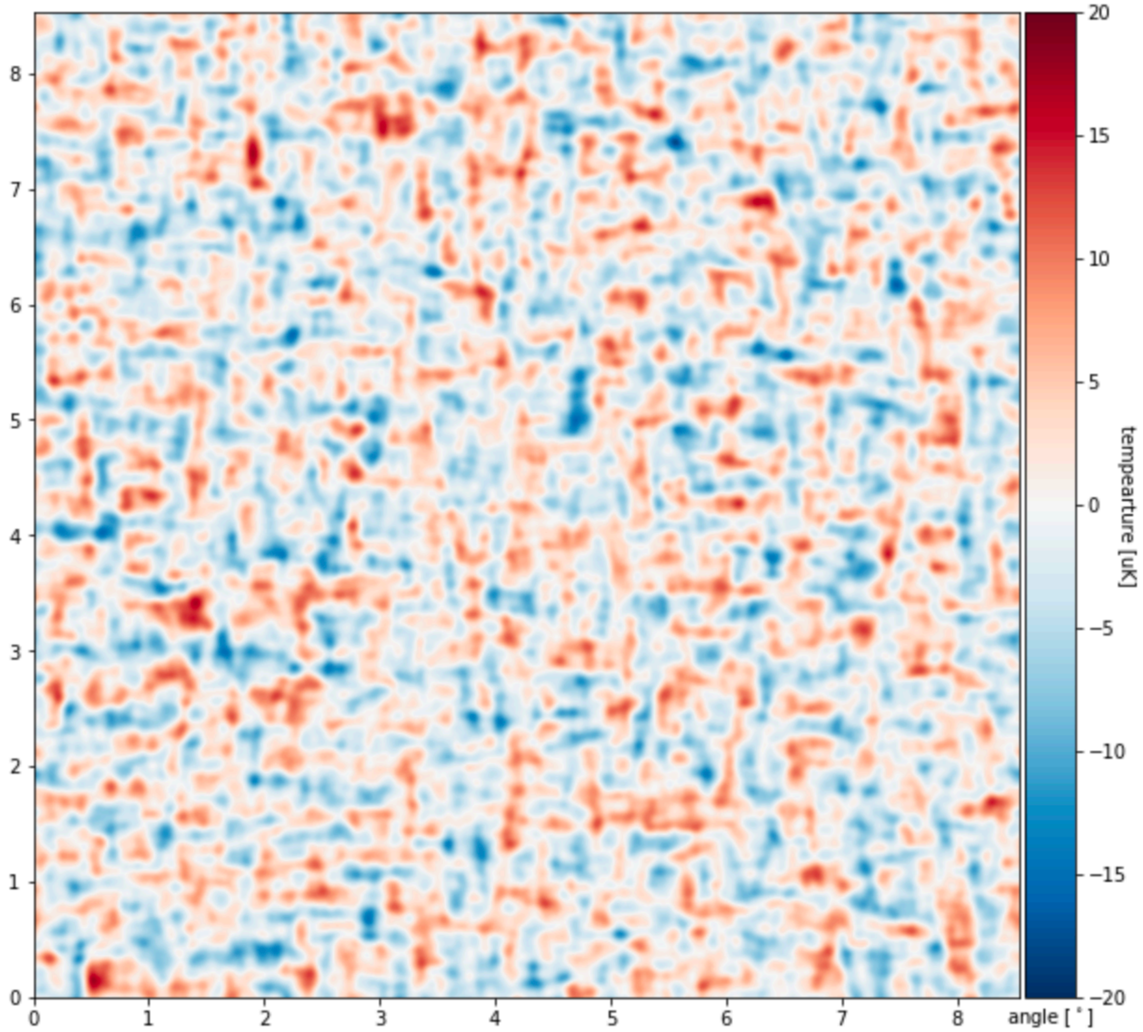
$$\mathbf{F} = -\nabla\Phi + \nabla \times \mathbf{A}$$

Please be aware of the distinction of E- and B-modes with the actual E/B fields in electrodynamics (and the somewhat unfortunate nomenclature)! There, the electic field vector $\boldsymbol{E}$ has odd parity, and the magnetic field pseudovector $\boldsymbol{B}$ has off parity. For the polarization vectors, E_l has even parity, and B_l has odd parity. This parity difference greatly simplifies their separation from the actual sky measurements.

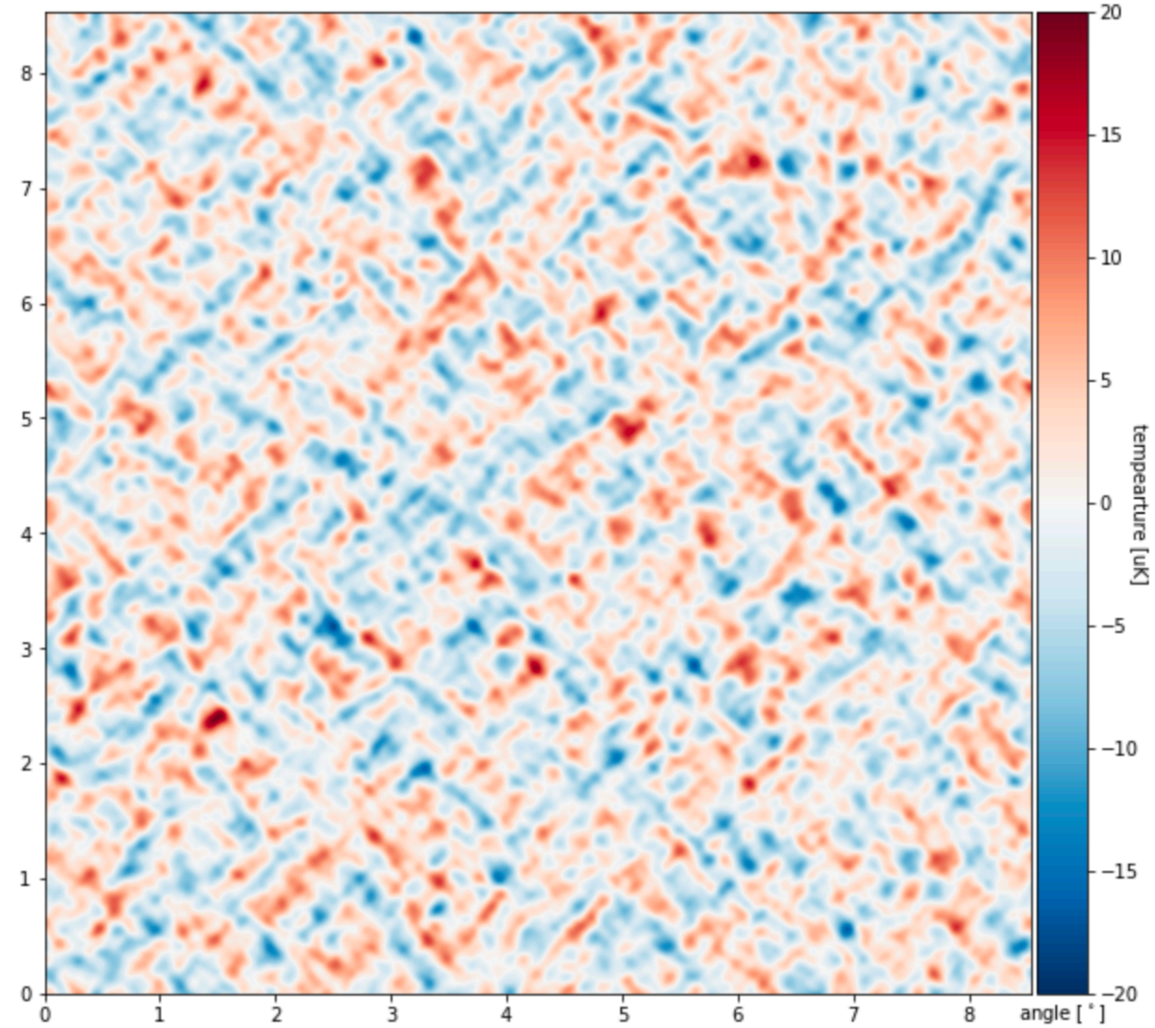
Patterns of CMB polarization

The relation between E-B and Q-U maps is given by a position dependant rotation in fourier space. We introduce an angle $\psi = \arctan \frac{k_Y}{k_X}$, where k_X and k_Y are the wave numbers in fourier space. With this angle in had the relation between E-B and Q-U is:

$$\tilde{Q} = \tilde{E} \cos 2\psi - \tilde{B} \sin 2\psi$$
$$\tilde{U} = \tilde{E} \sin 2\psi + \tilde{B} \cos 2\psi$$



Q

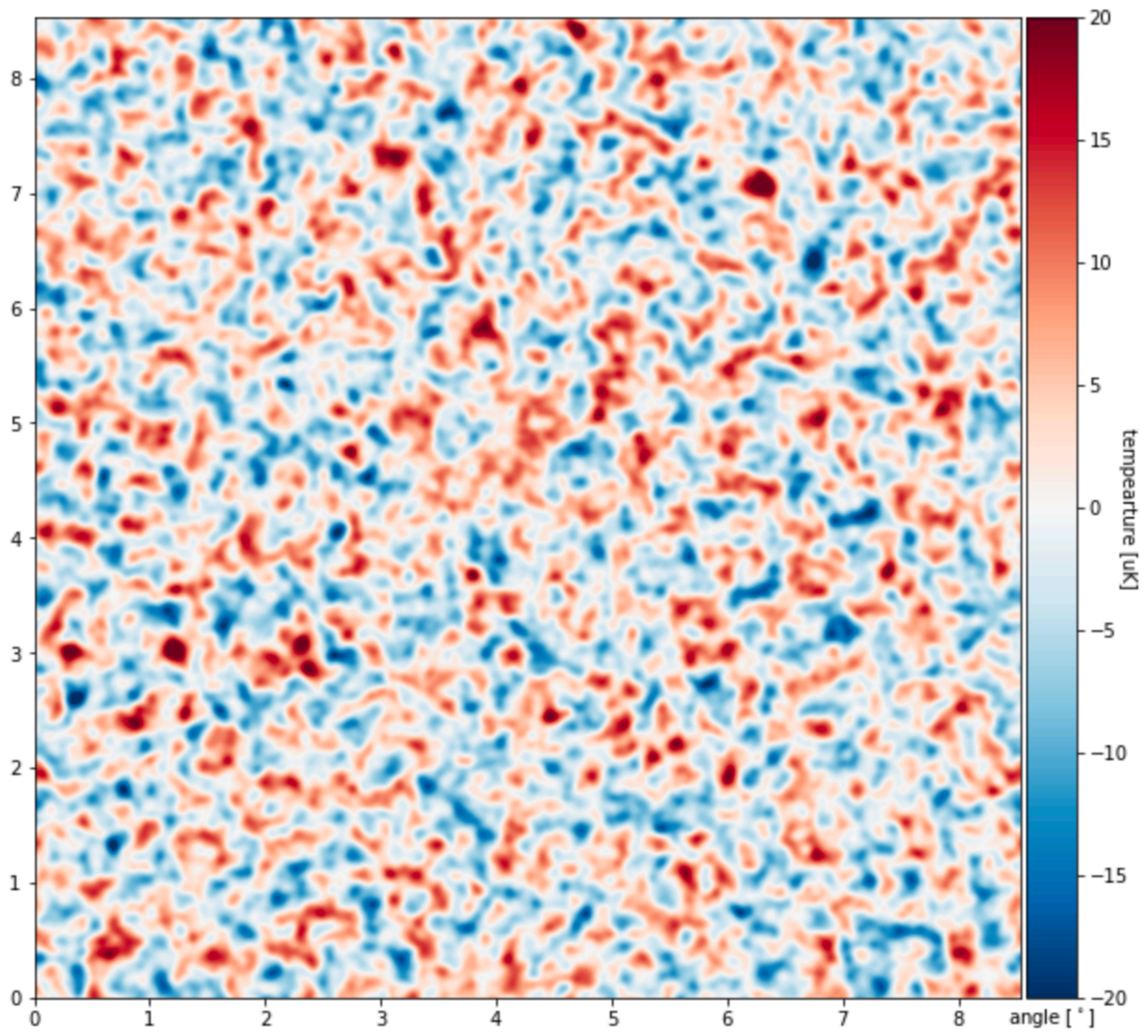


U

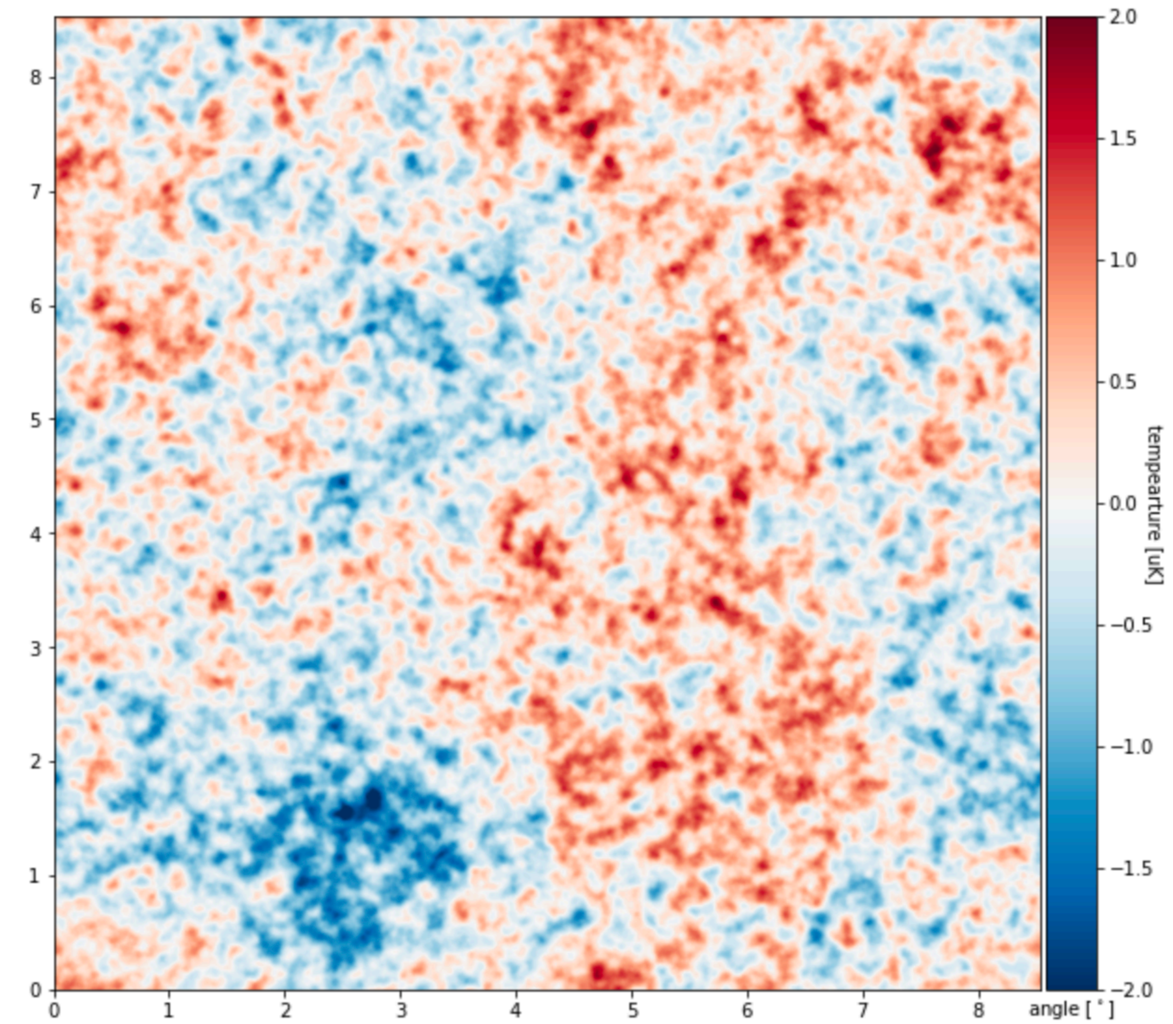
Patterns of CMB polarization

The relation between E-B and Q-U maps is given by a position dependant rotation in fourier space. We introduce an angle $\psi = \arctan \frac{k_Y}{k_X}$, where k_X and k_Y are the wave numbers in fourier space. With this angle in had the relation between E-B and Q-U is:

$$\tilde{Q} = \tilde{E}\cos 2\psi - \tilde{B}\sin 2\psi$$
$$\tilde{U} = \tilde{E}\sin 2\psi + \tilde{B}\cos 2\psi$$

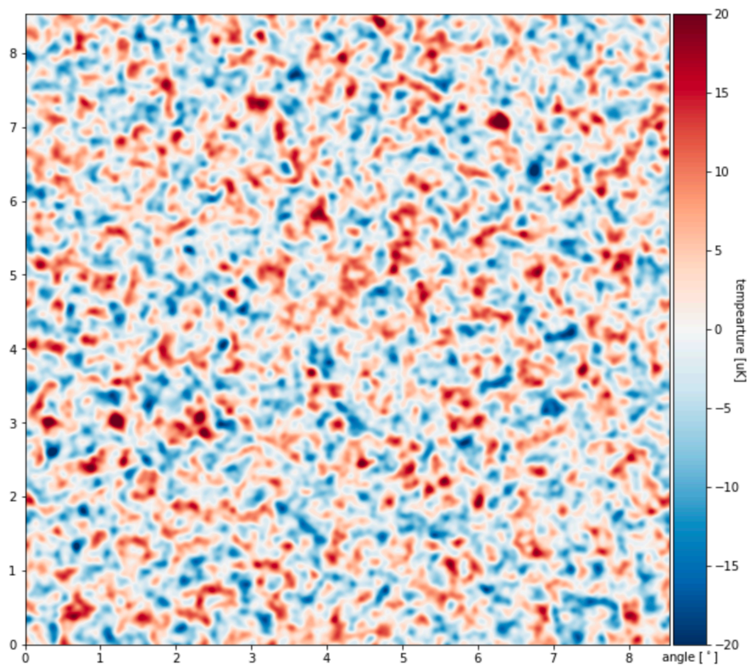


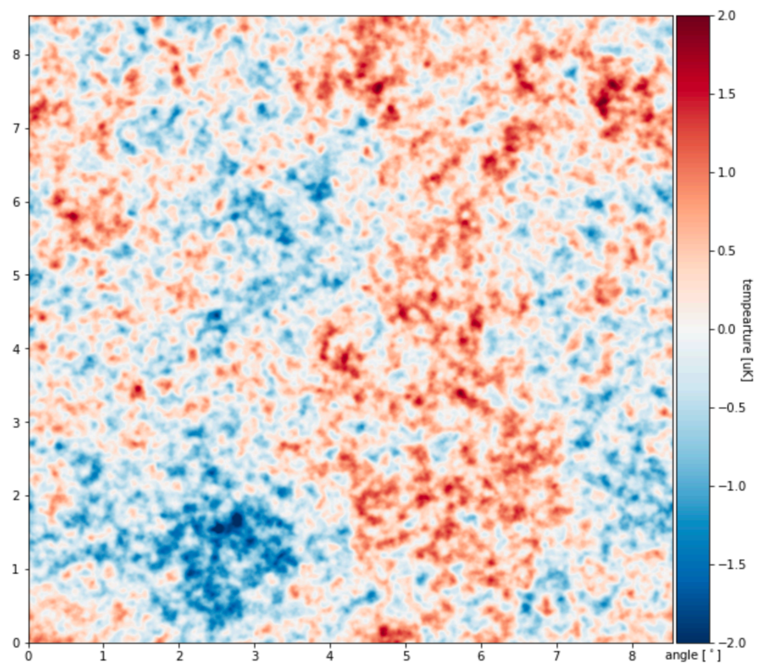
E



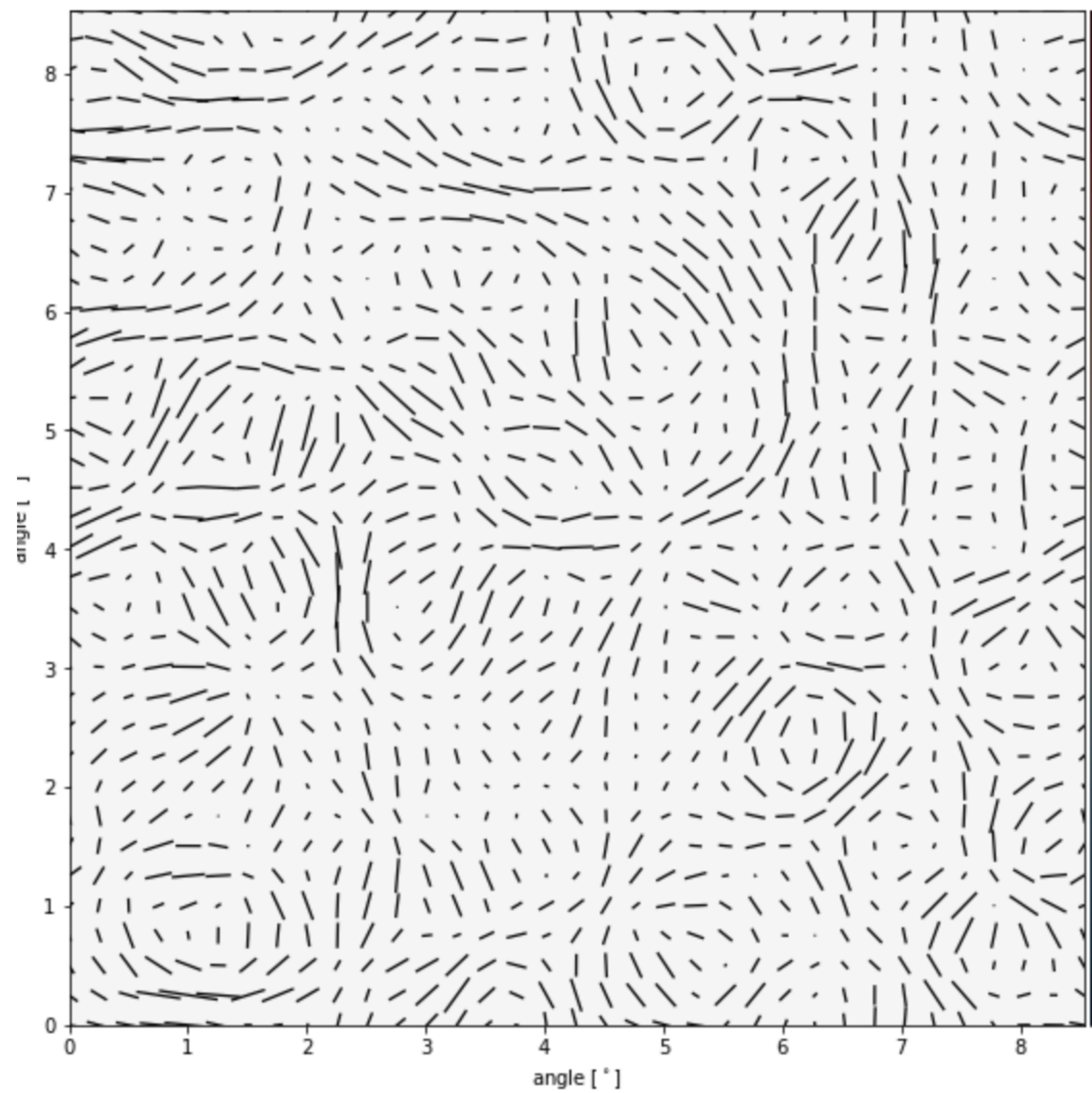
B

Patterns of CMB polarization





Below is the same part of a CMB polarized sky as is described by the E- and B-mode maps on left, but now showing the amplitude and direction of polarization.



Splitting the CMB polarization in E- and B-modes have two major advantages: **(i) They have different parities, so they can be measured independently, and (ii) they are generated by distinct physical processes!**

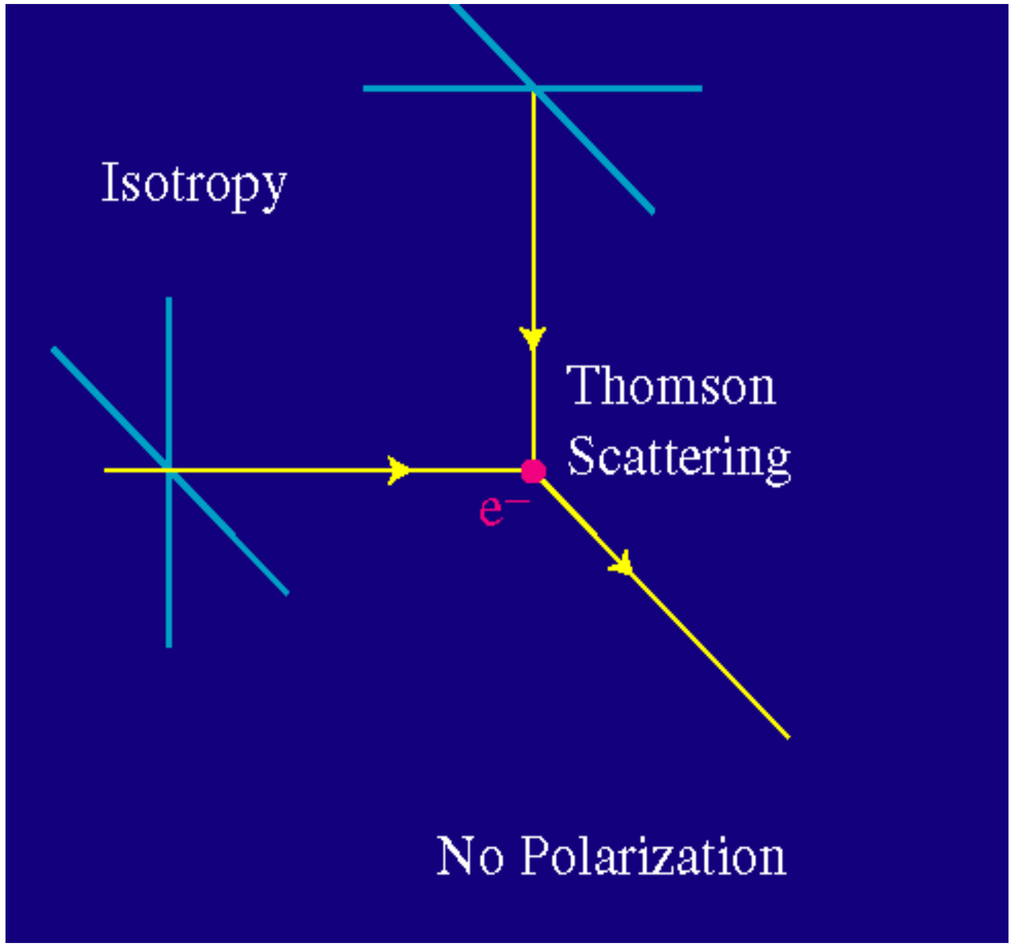
Quadrupole + Thomson scattering

Polarization is induced by Thomson scattering, either at decoupling or during a later epoch of reionization.

For scattering at $\Theta=\pi/2$ only one orthogonal component of the initially unpolarized radiation field will get scattered.

$$P(\theta,\phi) \propto 1 - \cos^2\theta$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{4\pi m c^2}\right)^2 |\hat{\epsilon}\cdot\hat{\epsilon}'|^2$$



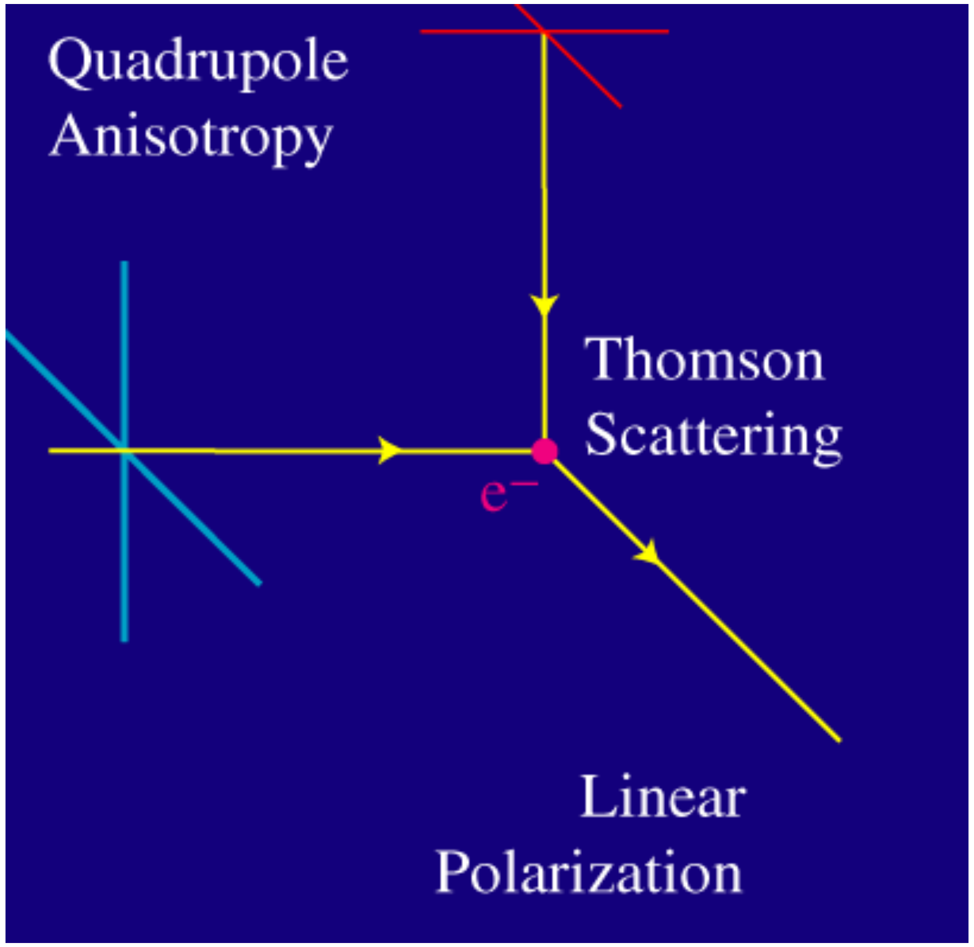



Photo Credit: TALEX


Photo Credit: TALEX
horizontally polarised


Photo Credit: TALEX

What causes the CMB quadrupole anisotropy?

Two things:

"Normal" CDM (scalar modes): Density perturbations at $z \approx 1100$ lead to velocities that create local quadrupoles seen by scattering electrons.

=> E-mode polarization ("parity-even")

Gravity waves (tensor modes): create local quadrupoles seen by the scattering electrons.

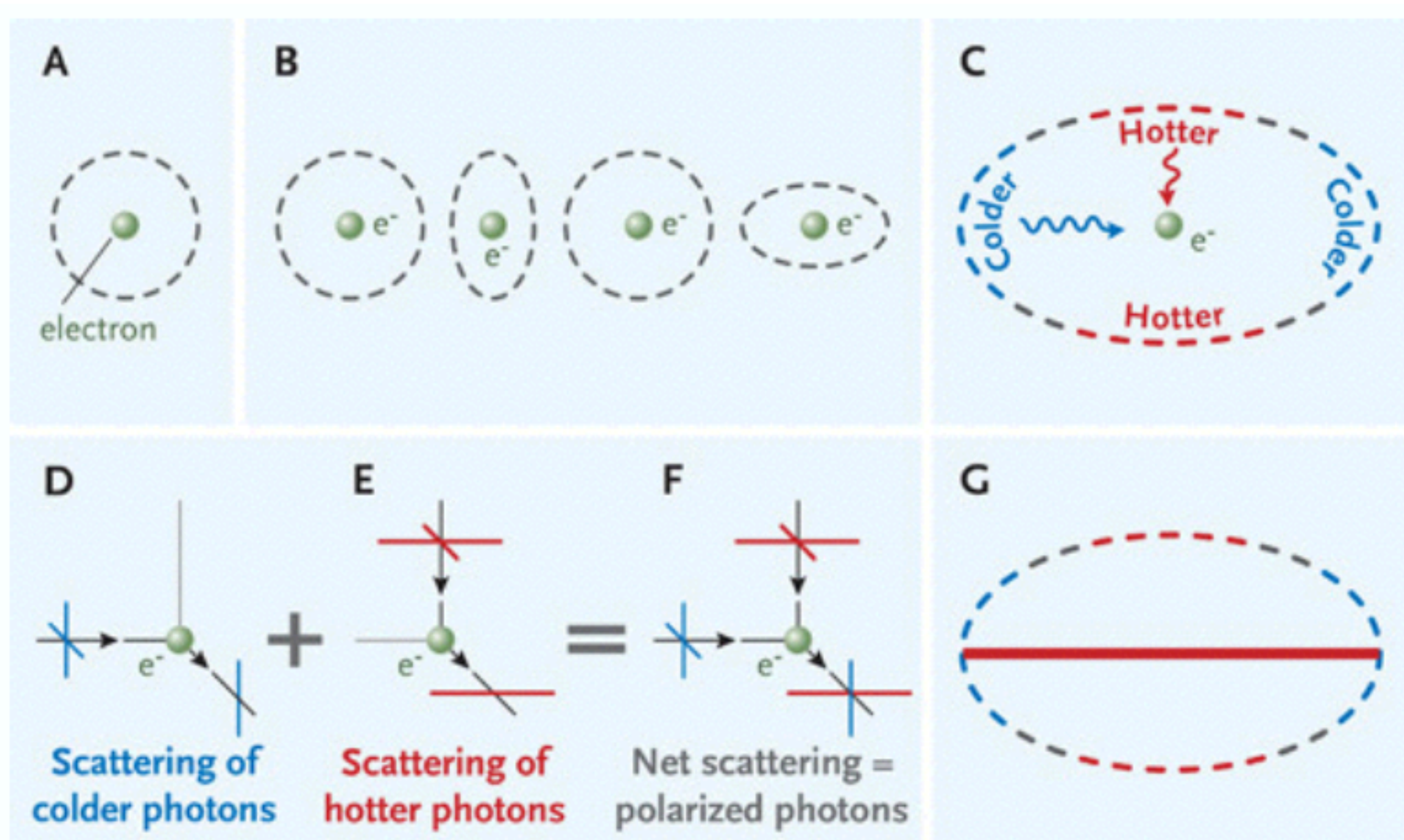
=> B-mode polarization ("parity-odd")

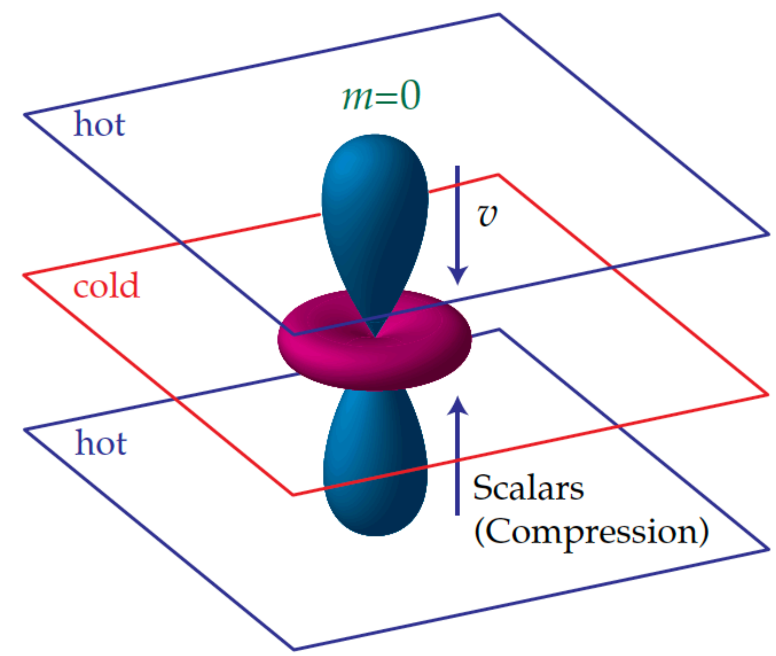
The problem of understanding the polarization pattern of the CMB thus reduces to understanding the quadrupole temperature fluctuations at the *moment of last scattering*.

From velocity gradients to E-mode polarization

Velocity gradients in the photon-baryon fluid lead to a quadrupole component of the intensity distribution, which, through Thomson scattering, is converted into polarization

(See Zaldarriaga, astro-ph/0305272)





When gravity overwhelms pressure, matter flows towards the overdense regions. But these overdense regions are also colder to start with, as photons must climb out of the potential well. Hence flows are established from hot to cold regions *locally*, and these velocity gradients create the **primordial E-mode signal**.

Polarization patterns on the last scattering surface

We saw that polarization pattern created at the last scattering can only come from a quadrupole temperature anisotropy present at that epoch.

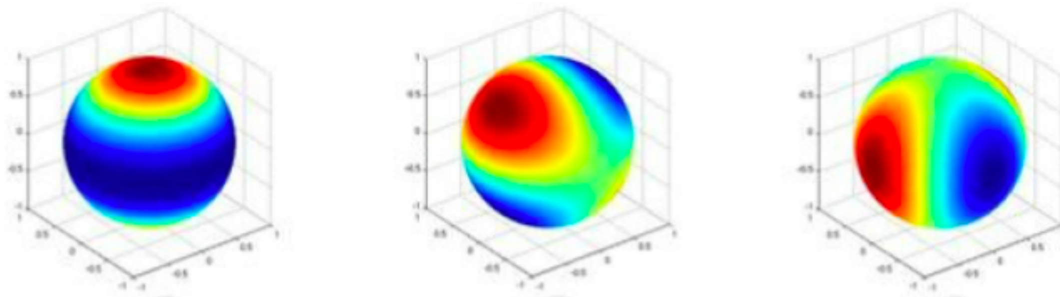
In terms of multipole decomposition of a radiation field in terms of spherical harmonics, $Y_{lm}(\theta,\phi)$, the five quadrupole moments are represented by ***$l = 2$ and $m = 0, \pm1, \pm2$.***

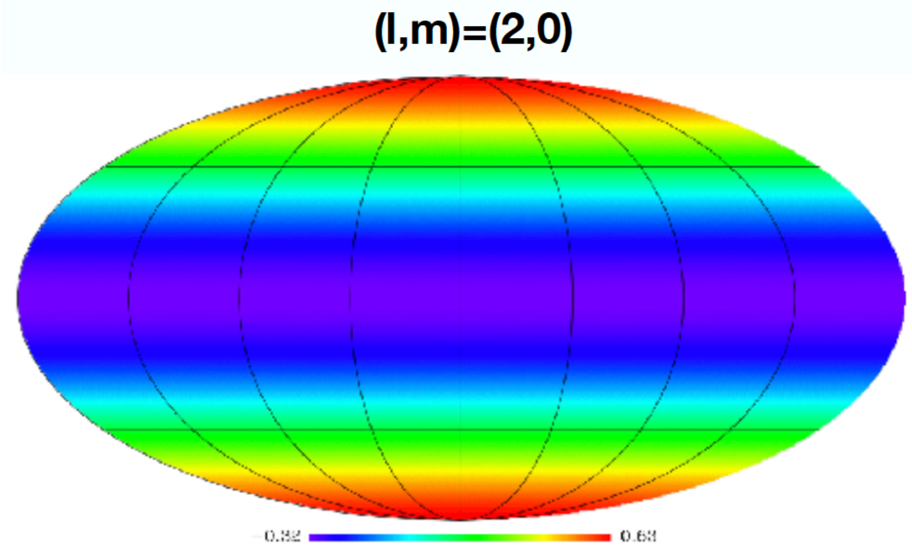
The orthogonality of the spherical harmonics guarantees that no other moment can generate polarization from Thomson scattering!

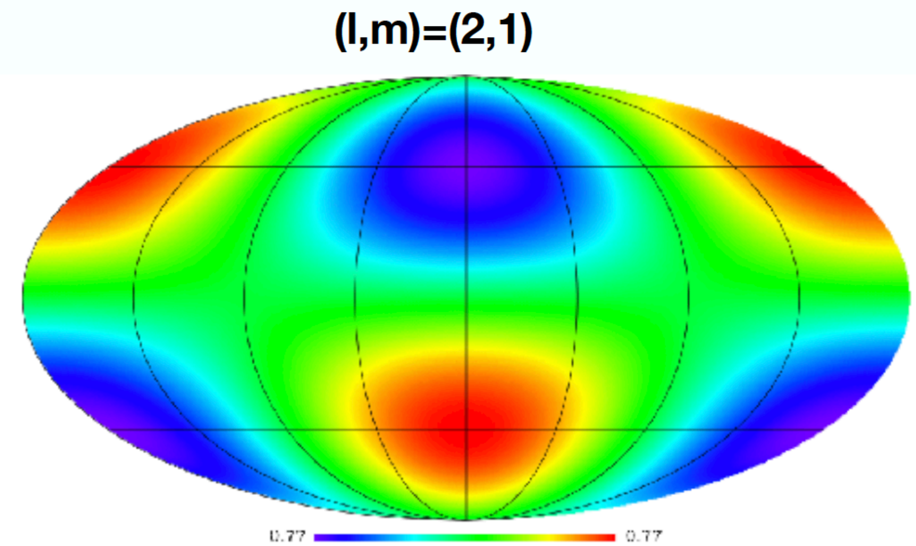
The problem of understanding the polarization pattern of the CMB thus reduces to understanding the quadrupolar temperature fluctuations at the epoch of last scattering.

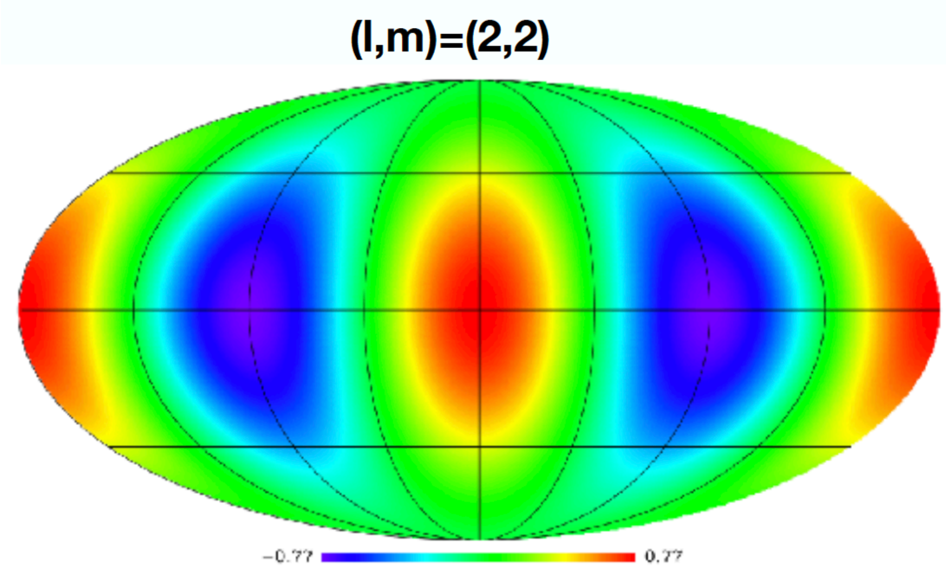
Polarization patterns

$\ell = 2$



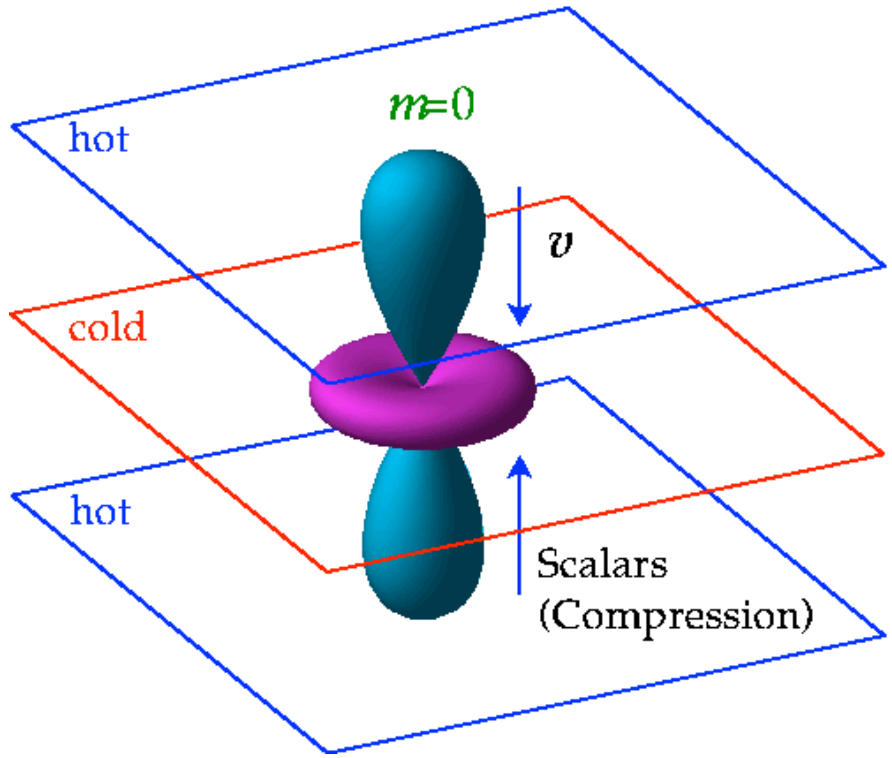


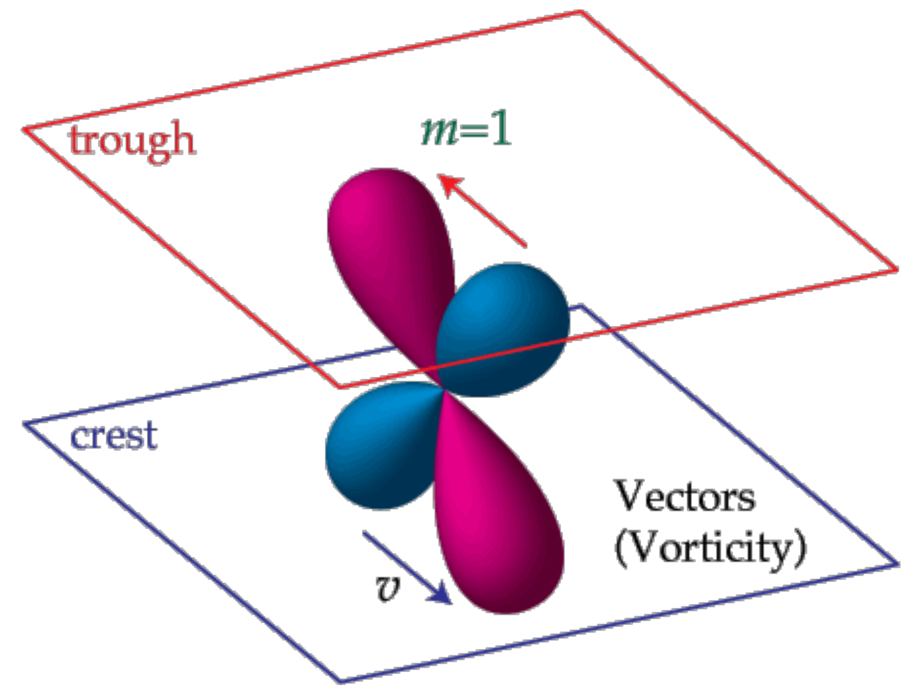


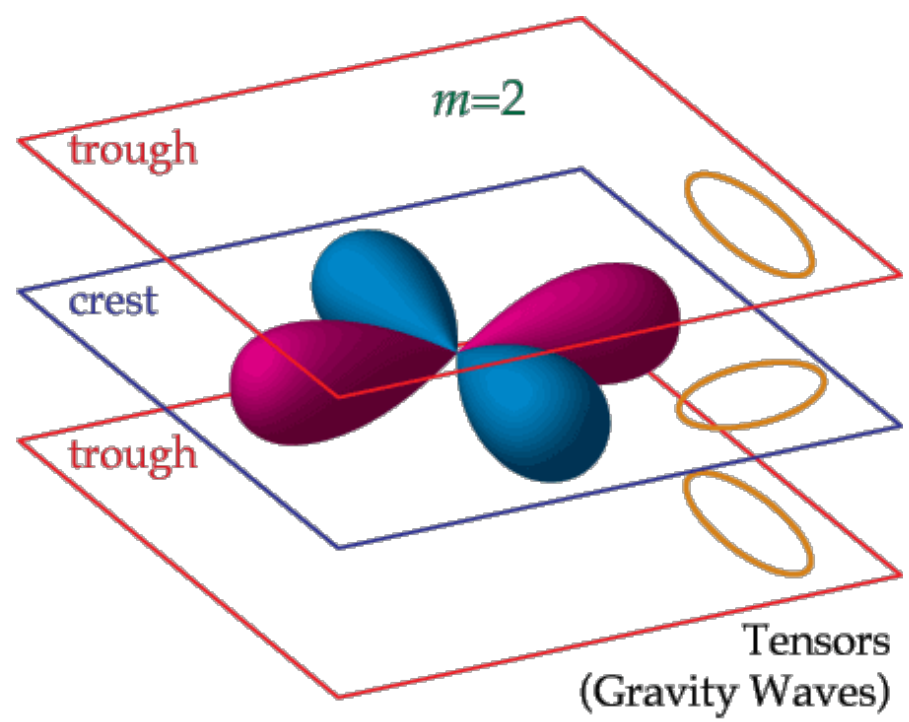


Local quadrupole temperature anisotropy seen from an electron

Polarization patterns





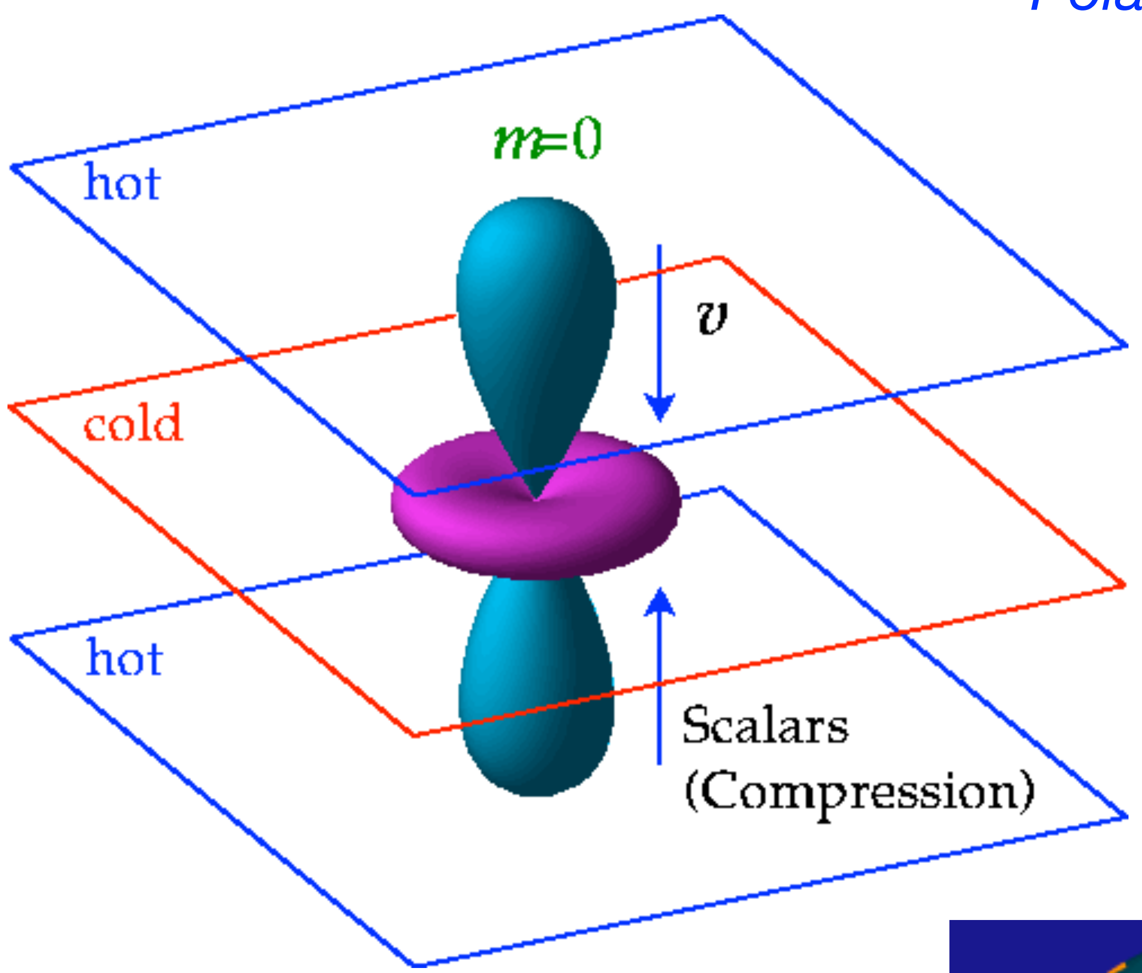


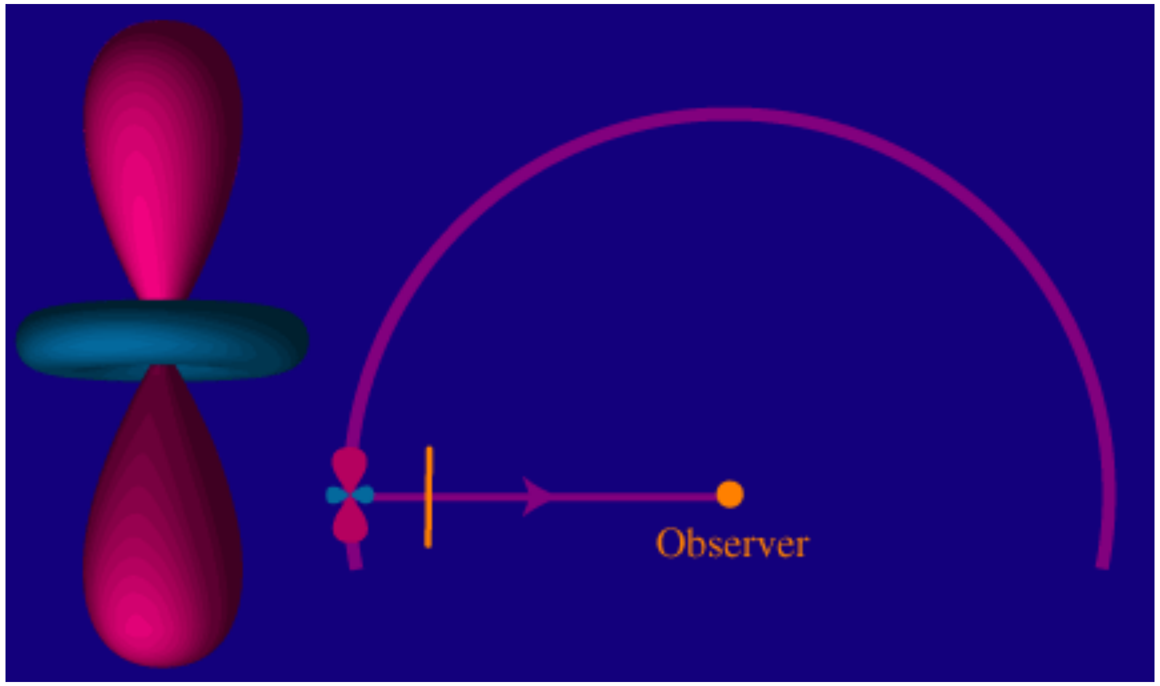
There are three sources to the quadrupole temperature anisotropy at recombination:

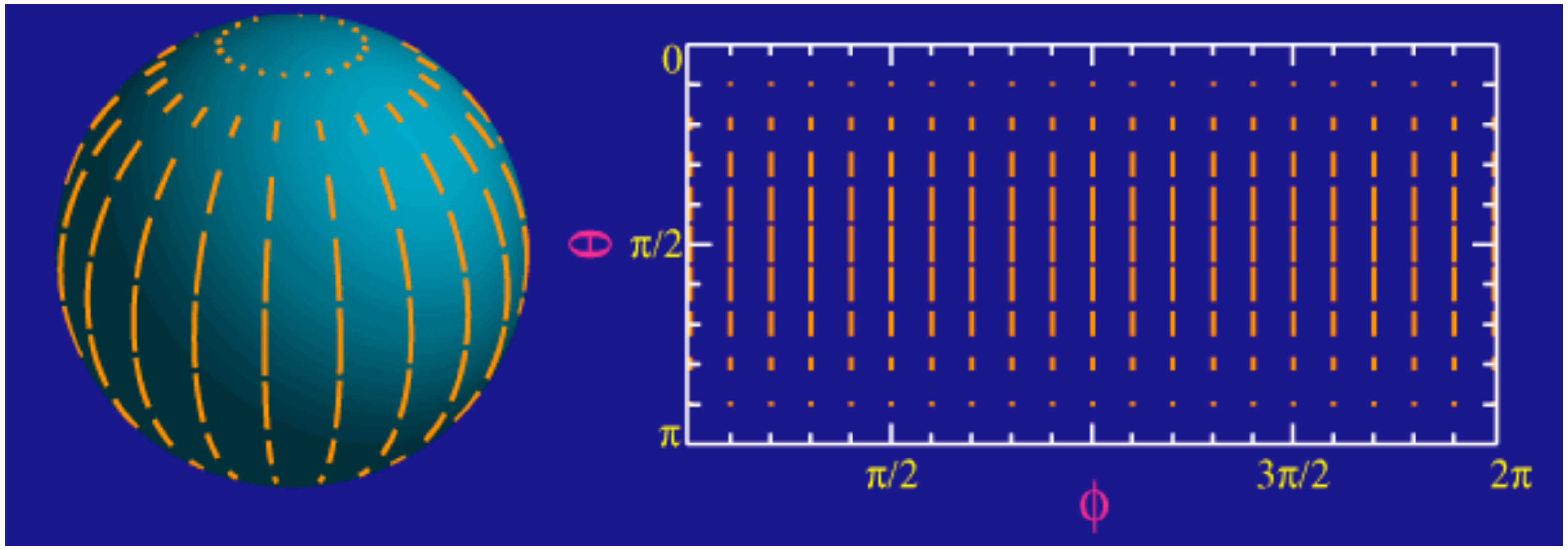
- scalers (m=0) for velocity perturbation
- vectors (m=1) for vorticity *(negligible)*
- tensors (m=2) for gravity waves

Visualization of the polarization pattern

Polarization pattern is a projection of quadropole moments

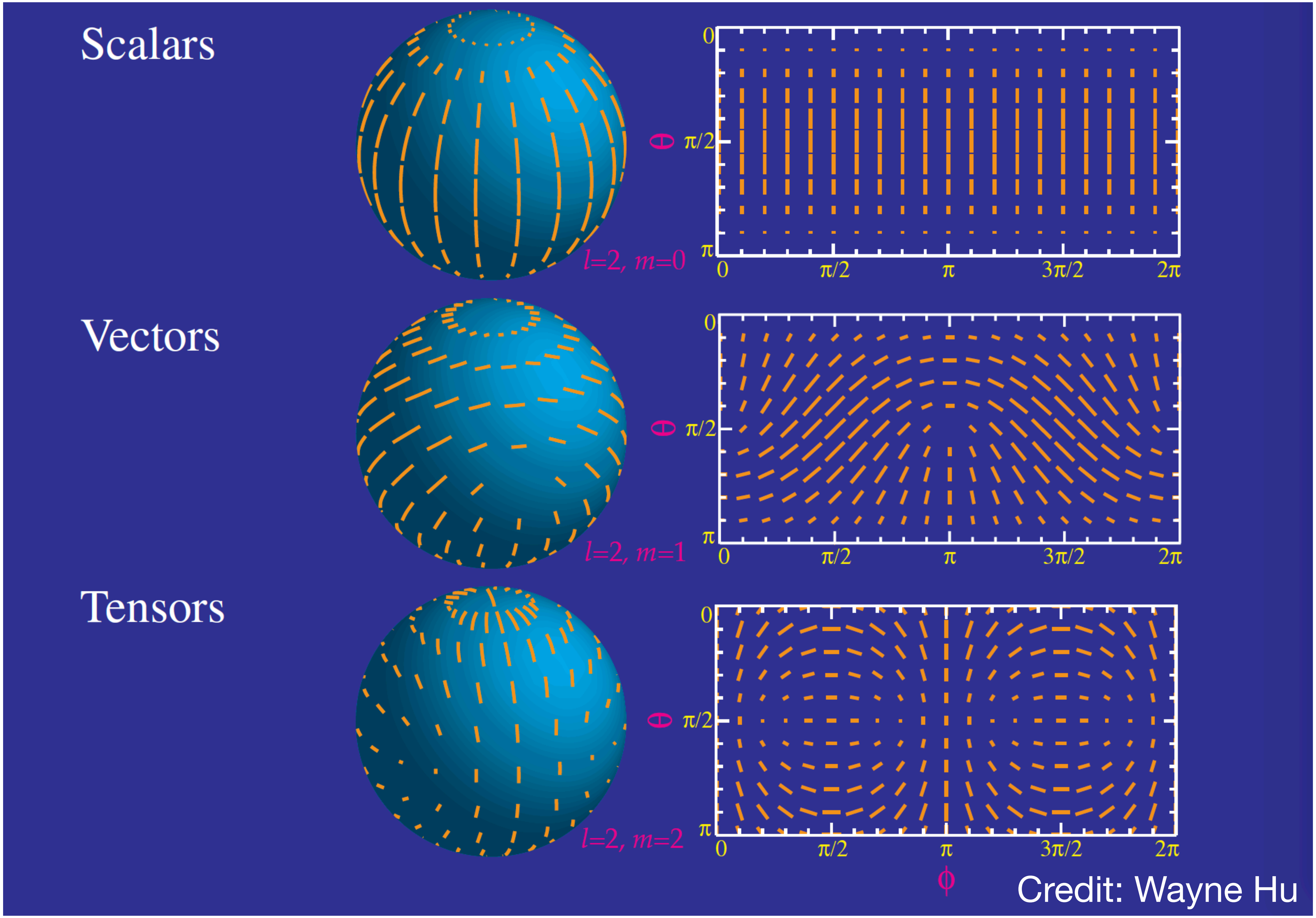






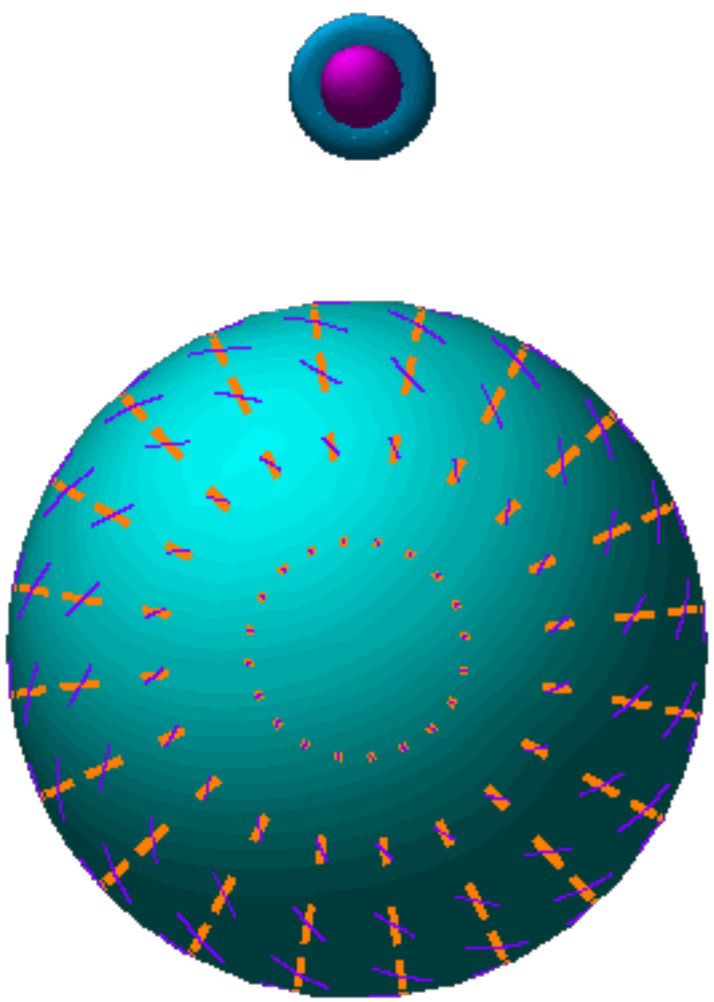
The scaler quadrupole moment, l=2, m=0. Note the azimuthal symmetry in the transformation of this quadrupole anisotropy into linear polarization.

Polarization patterns

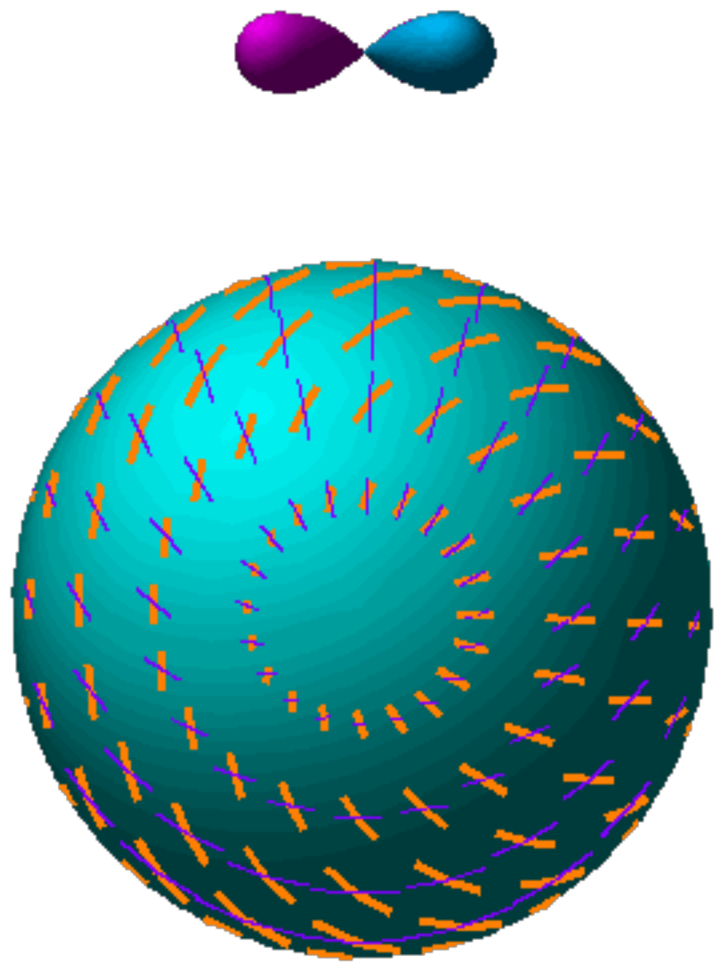


Polarization patterns

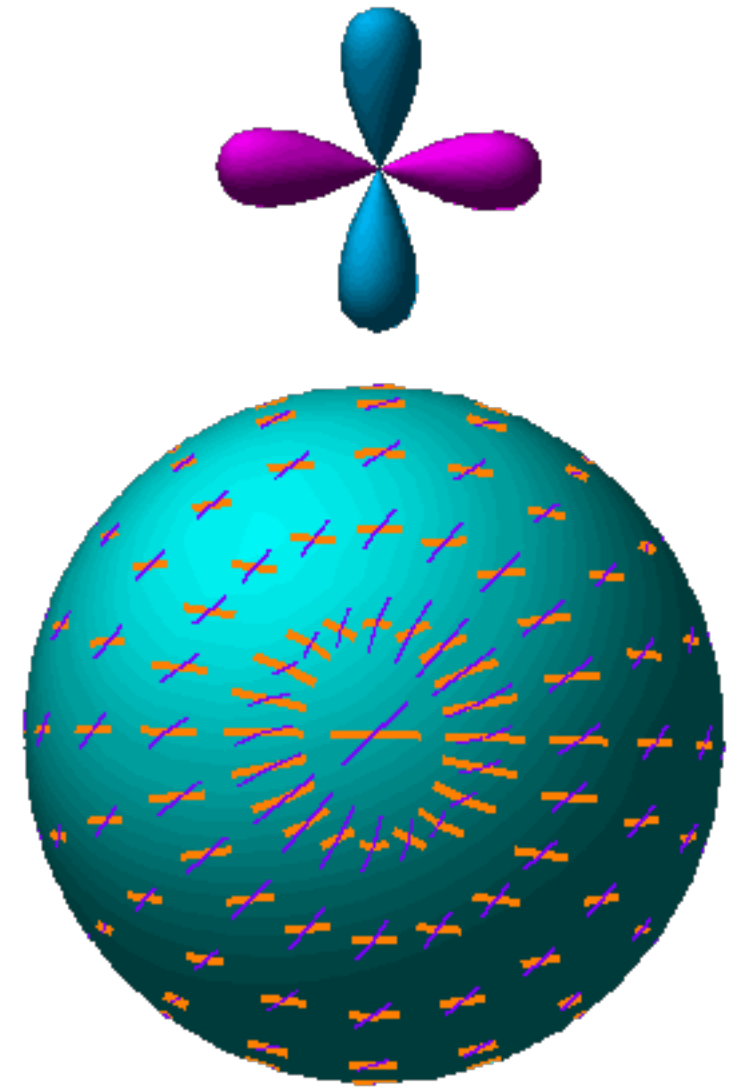
Animations by Wayne Hu. Thick and thin lines are E and B-mode patterns.



Scaler mode
(l=2, m=0)

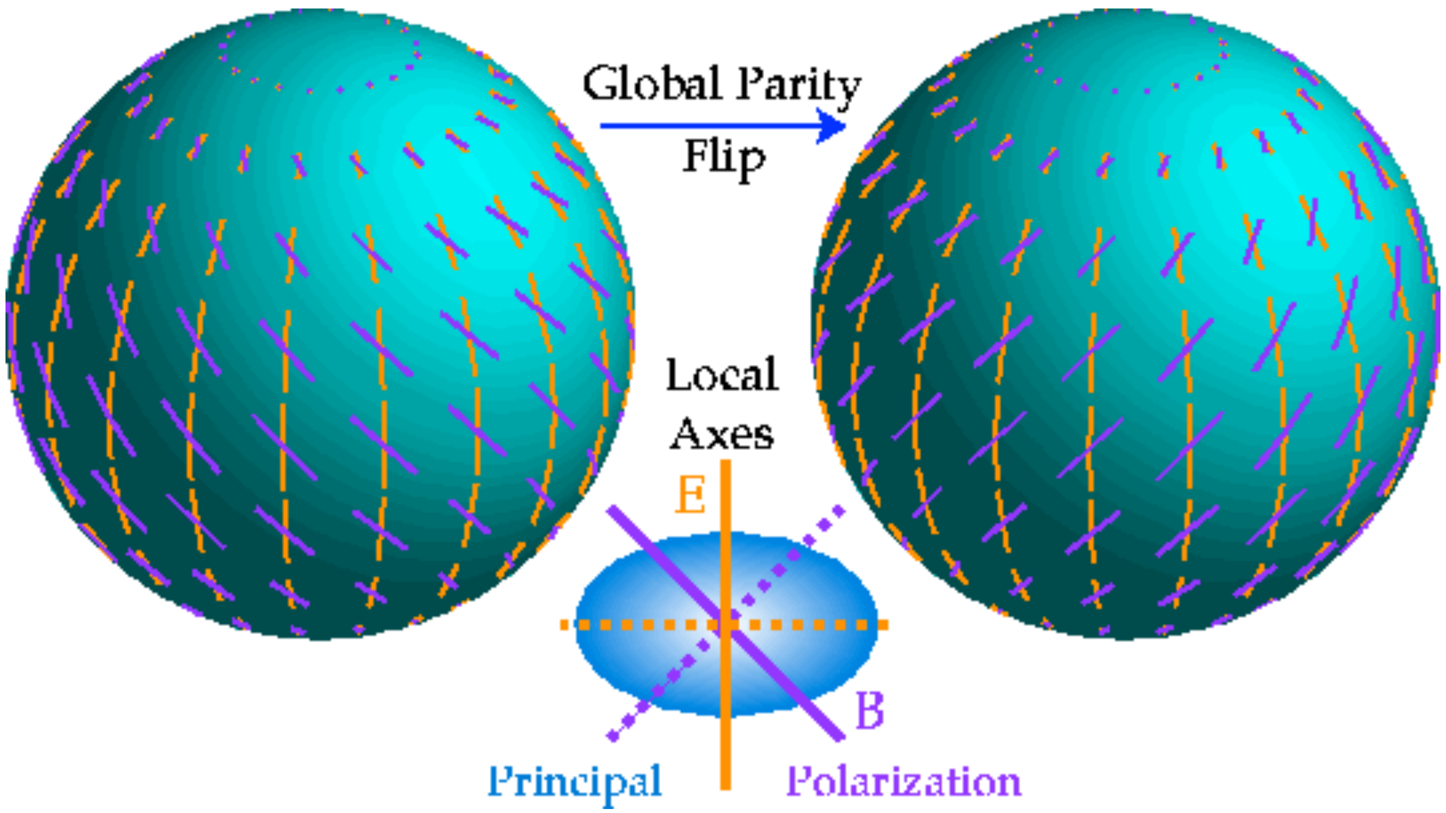


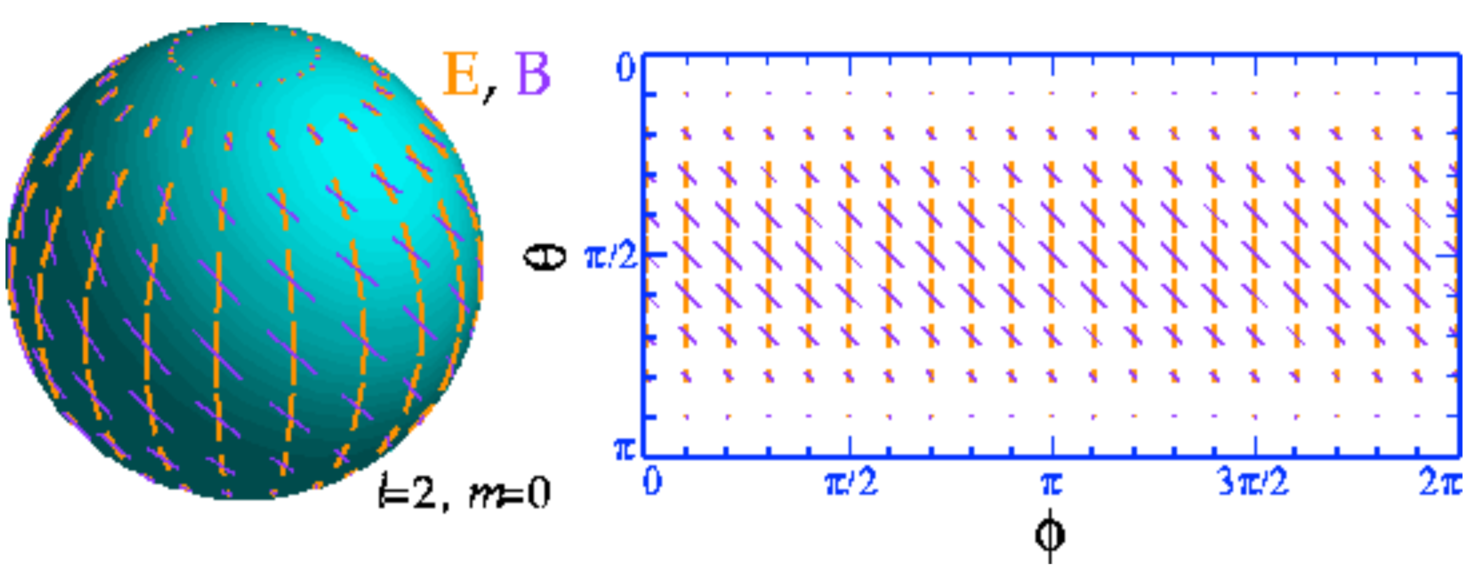
Vector mode
(l=2, m=±1)
(negligible)

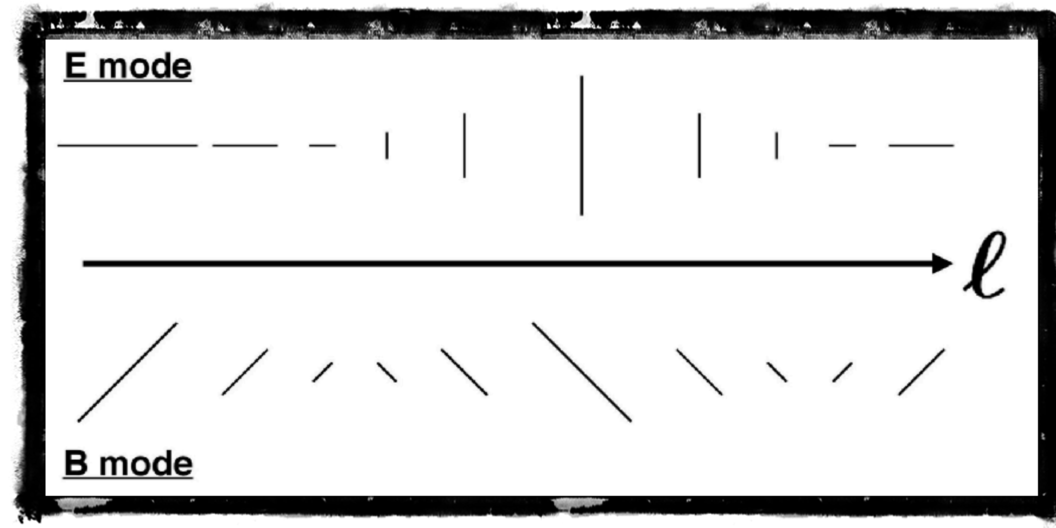


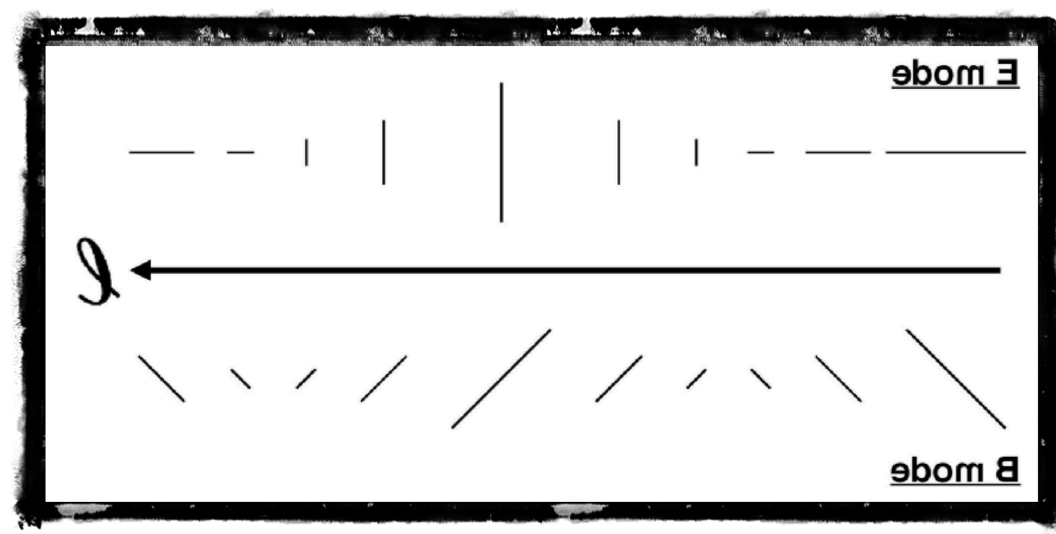
Tensor mode
(l=2, m=±2)

Parity of E & B modes on a sphere



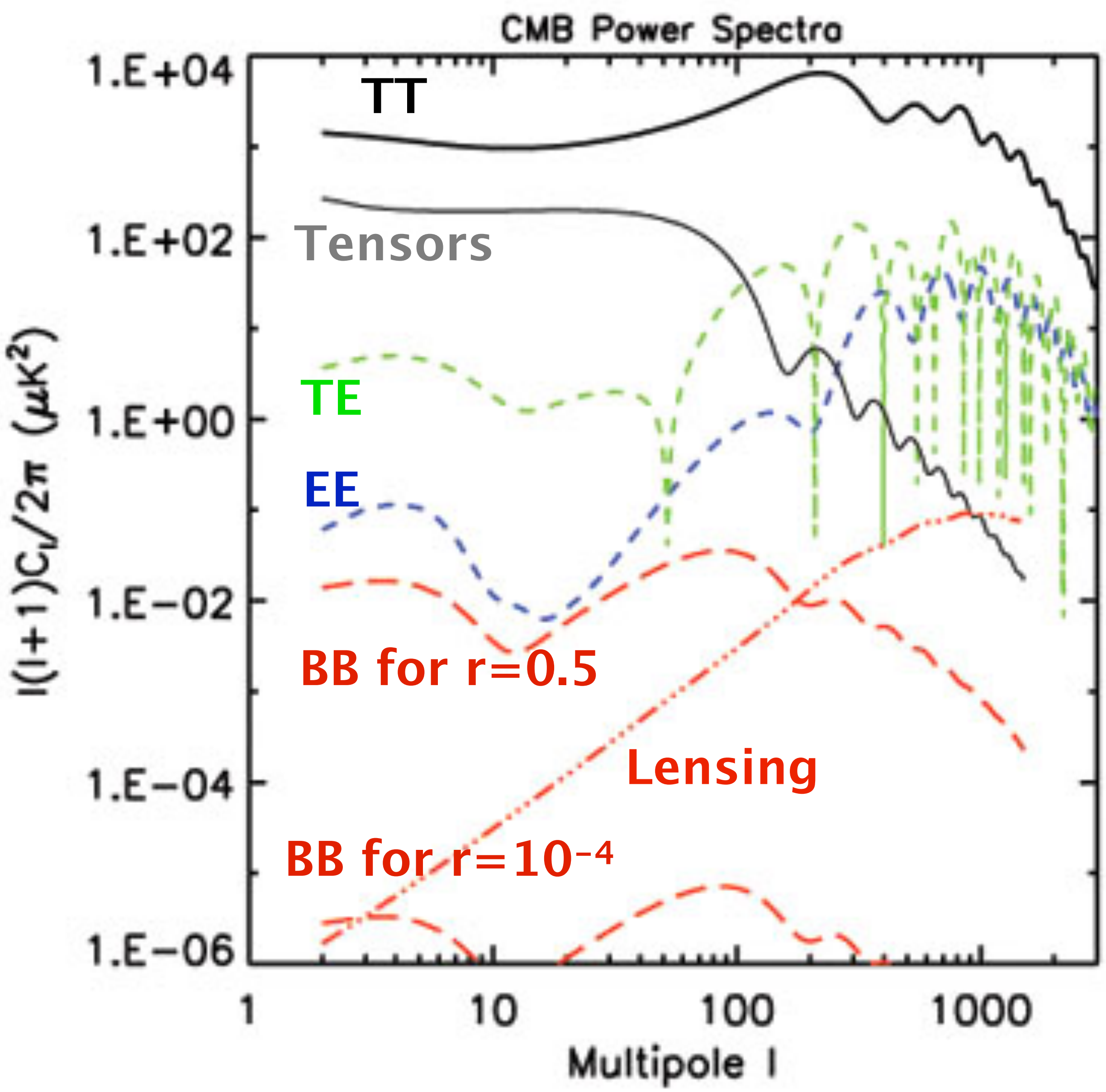






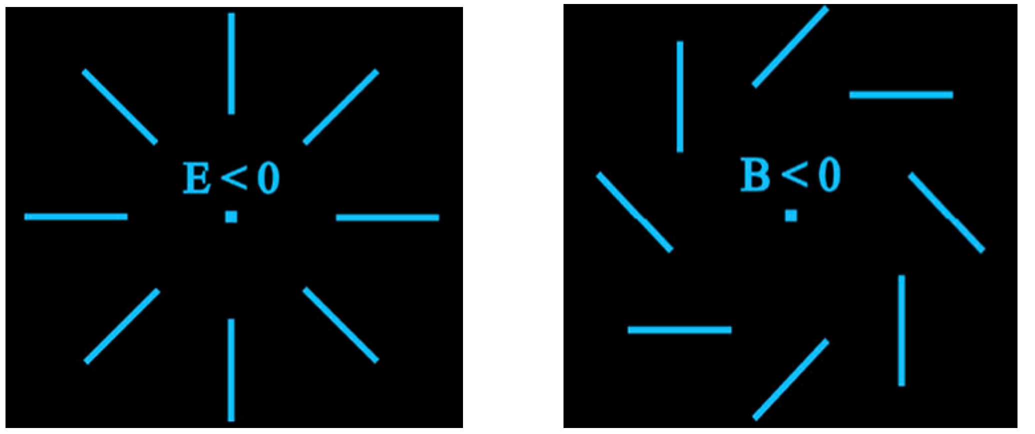
- **E mode**: Parity even
- **B mode**: Parity odd

Parity determines the number of observable power spectra



r = T/S: Tensor to scaler ratio, generated by the primordial gravity waves at last scattering

E & B modes have different reflection properties ("parities"):

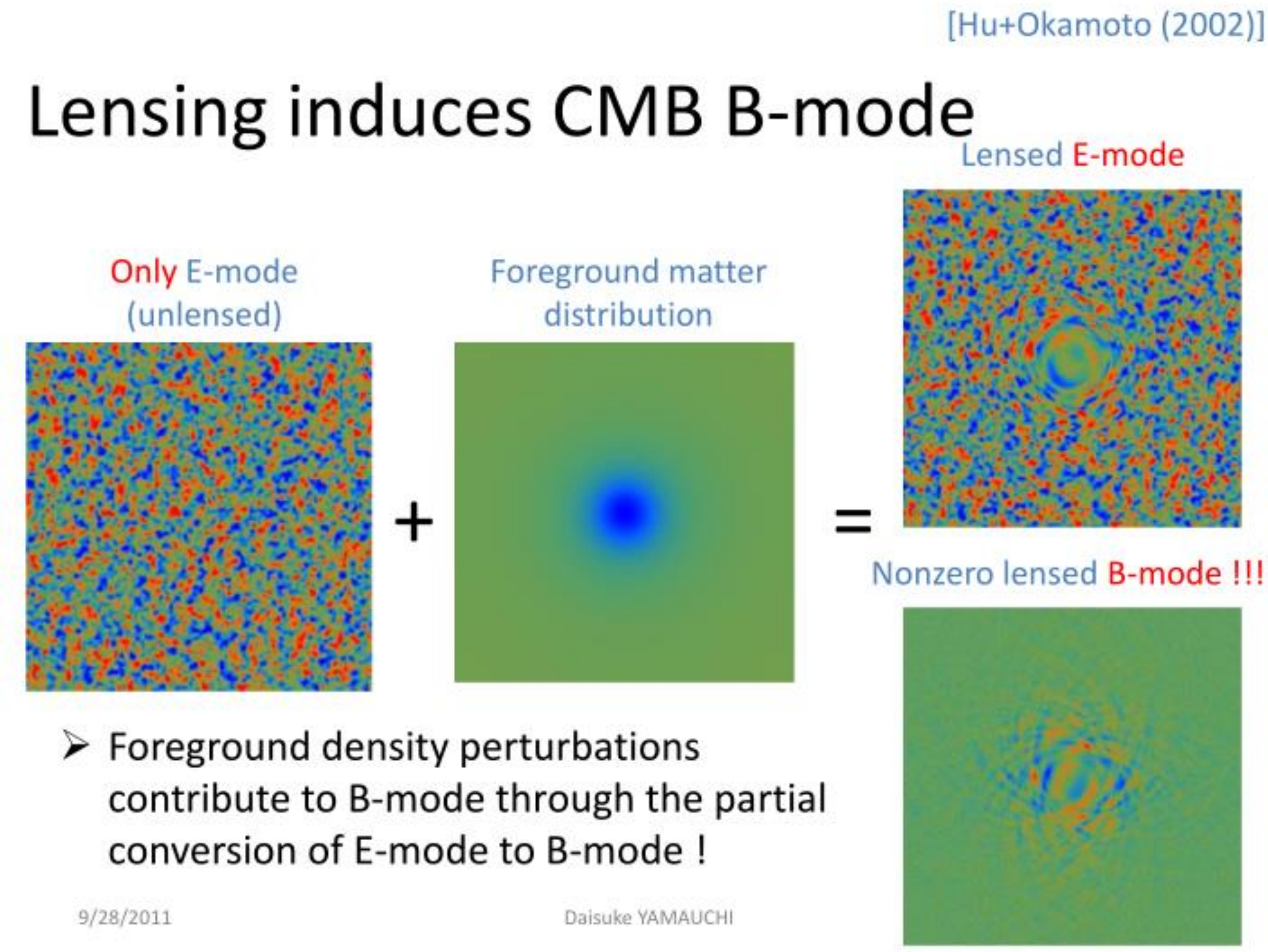


Parity: $(-1)^l$ for E and $(-1)^{l+1}$ for B (here l=2) ⇒ **B has negative parity**

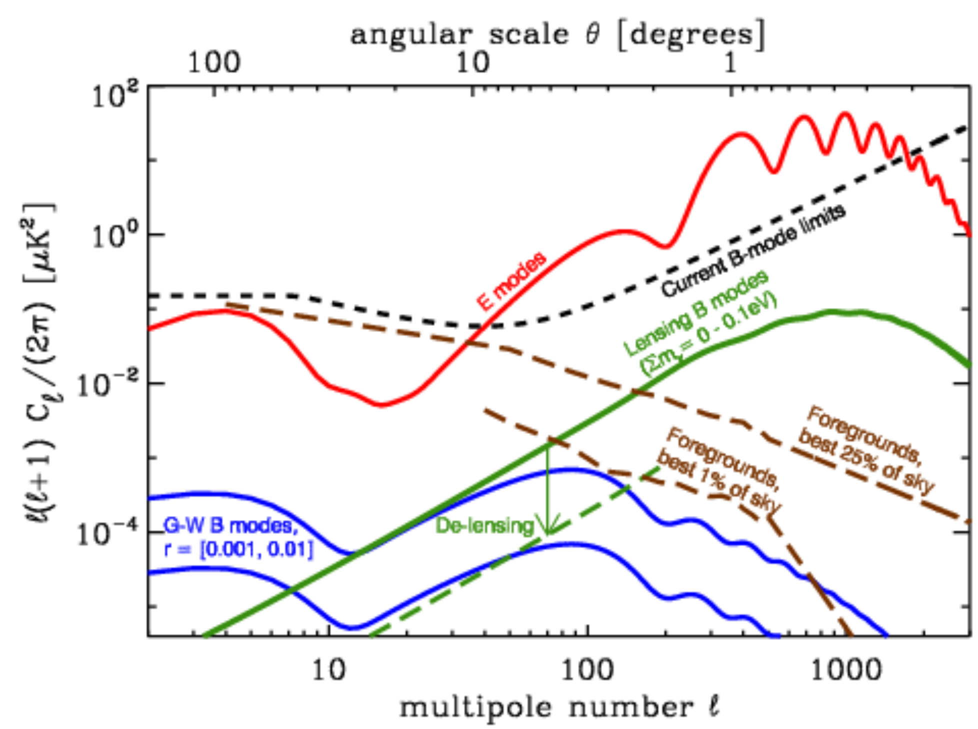
The cross-correlation between B and E or B and T vanishes (unless there are parity-violating interactions), because B has opposite parity to T or E.

We are therefore left with 4 fundamental observables. This also helps with their separate measurement.

Effect of gravitational lensing

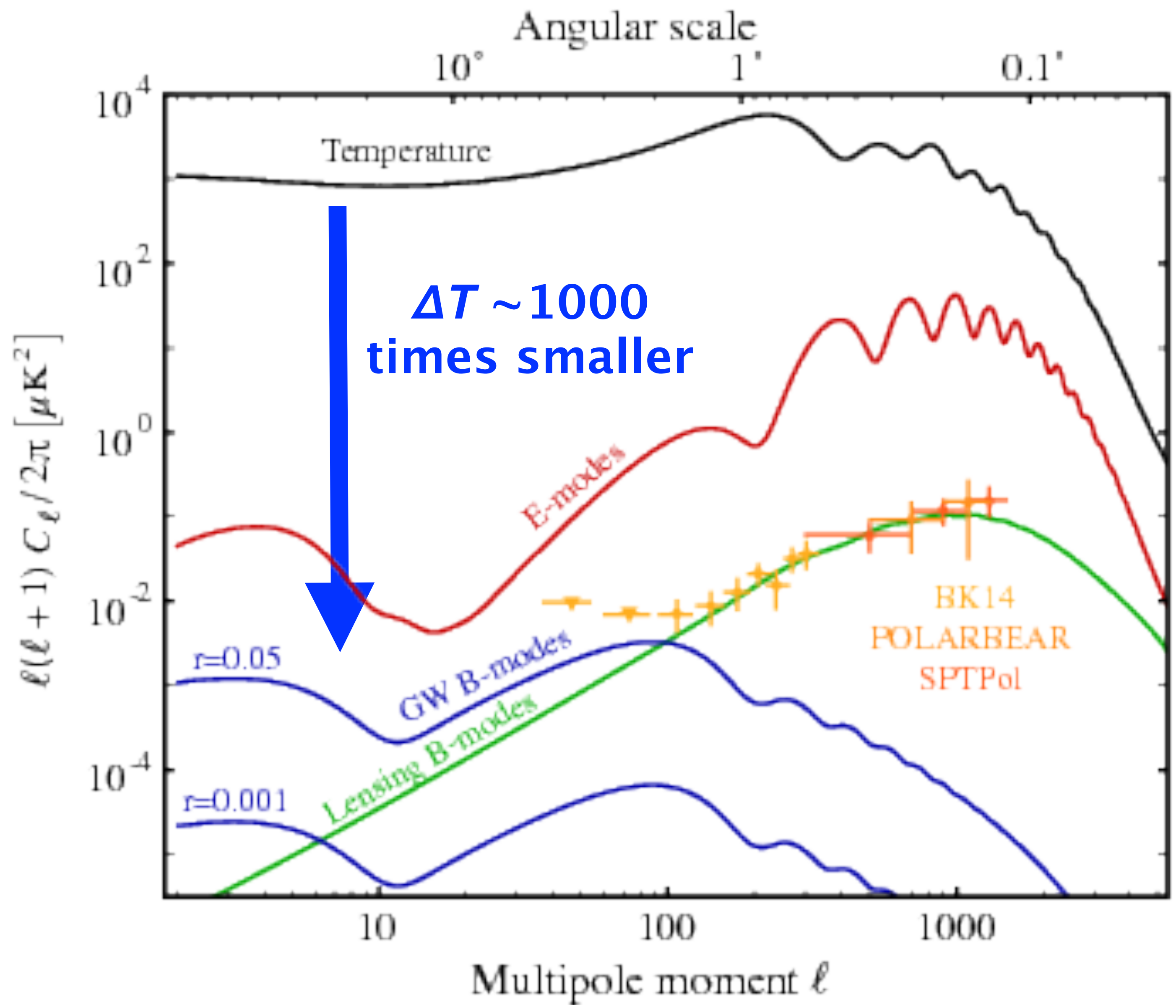


Need to de-lens to get to the primordial B-mode



We will visit CMB–lensing in more details later!

Detecting polarization is difficult!



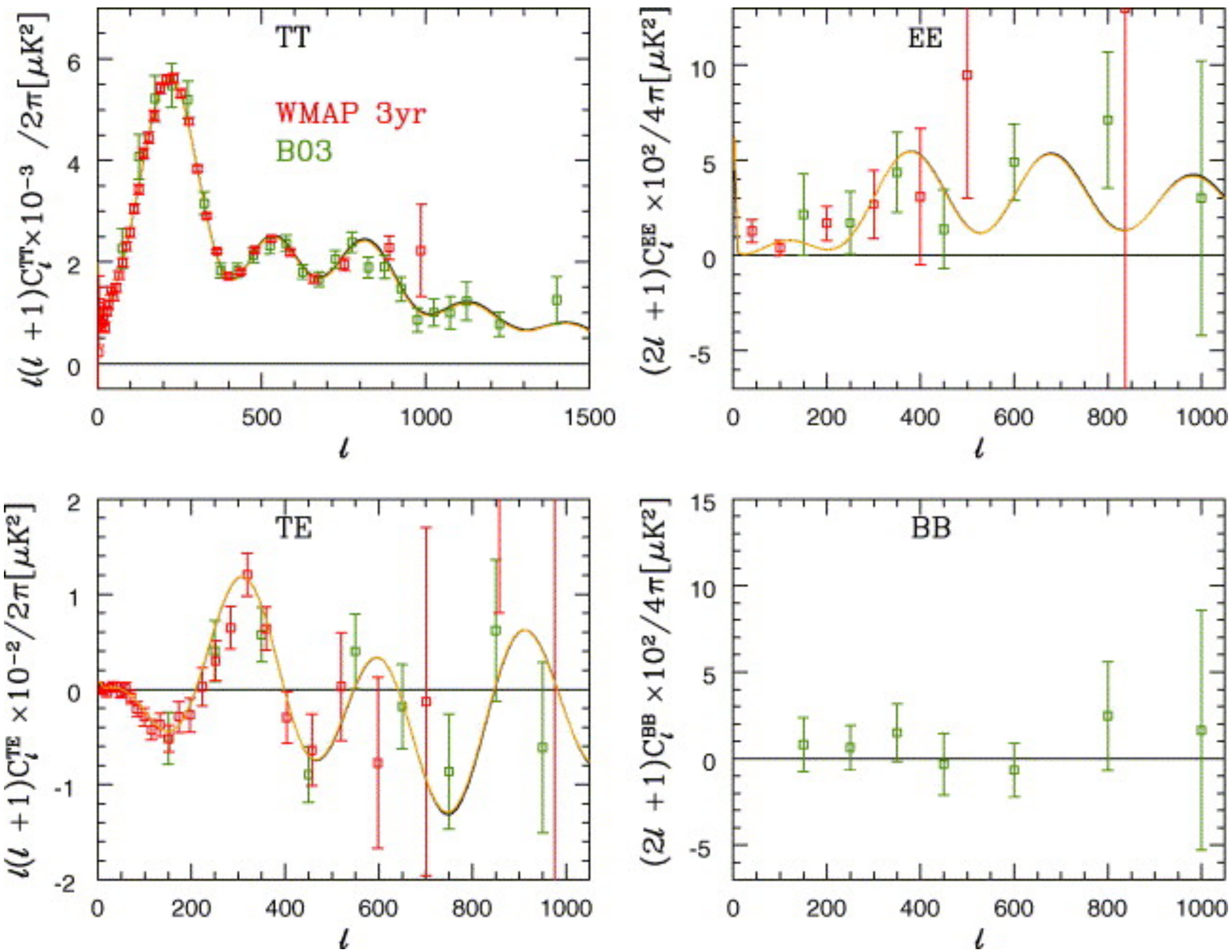
Polarization signal amplitude is much smaller than the temperature, since it requires a scattering event and hence can only be produced in optically thin condition (any subsequent scattering will cancel the polarization signature).

We are getting a snapshot of the quadrupole anisotropy from the moment of last scattering.

Power spectra of CMB temperature anisotropies (black), grad polarization (red), and curl polarization due to the GWB (blue) and due to the lensing of the grad mode (green), all assuming a standard CDM model with T/S = 0.28. The dashed curve indicates the effects of reionization on the grad mode for $\tau = 0.1$.

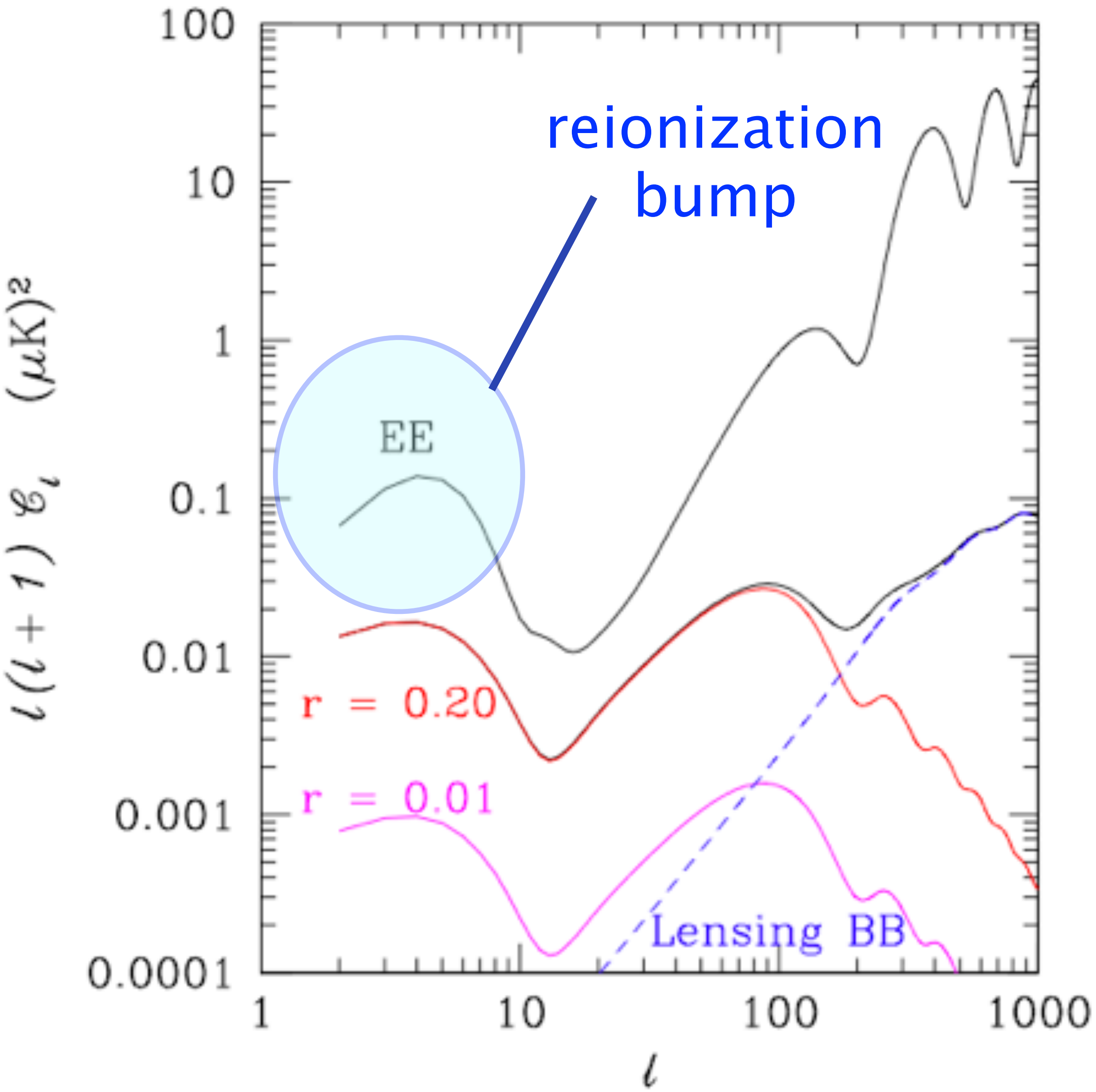
Shape of the polarization power spectra

The polarization power also exhibits acoustic oscillations since the quadrupole anisotropies that generate it are themselves formed from the acoustic motion of the fluid.



The EE peaks are out of phase with TT peaks because these E-mode primordial polarization anisotropies are sourced by the fluid velocity (hence *roughly* in-phase with velocity maxima).

Shape of the polarization power spectra



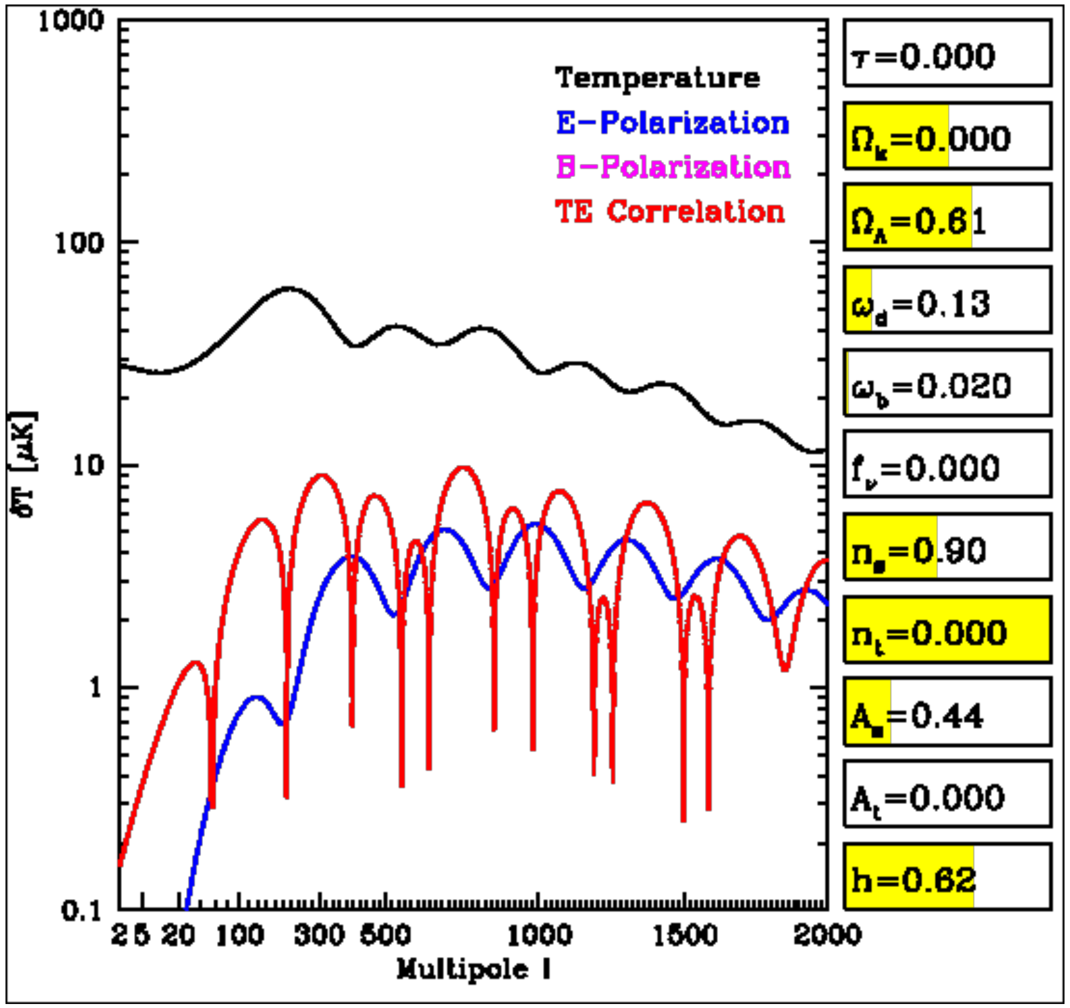
- Primordial E-mode signal peaks at small scales (high l), corresponding to the width of the epoch of last scattering
- The primordial B-mode signal (due to a stochastic background of gravitational waves) dominates primarily at large angular scales
- On very large angular scales (>5°), the E- and B-mode polarization signals are dominated by the *secondary fluctuations imprinted by reionization*
- The lens-generated signal grows at smaller scales (turning E modes into B modes!)

Shape and amplitude of EE are predicted by ΛCDM.

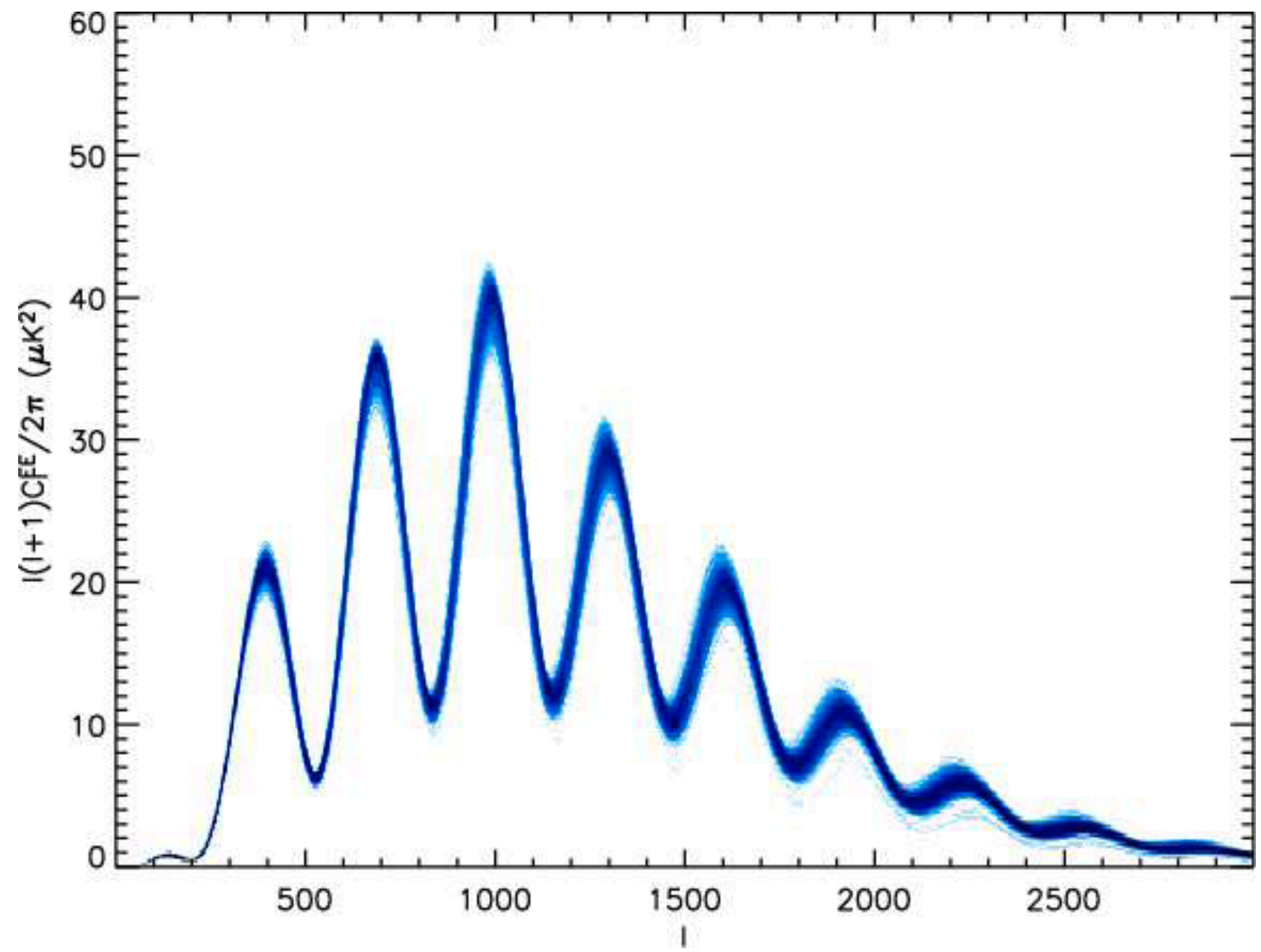
Shape of BB is predicted from "scale-invariant gravity waves".

Amplitude of BB is model dependent, and not really constrained from standard ΛCDM. Measuring this amplitude would provide a direct handle of the energy scale of inflation!

EE power spectrum



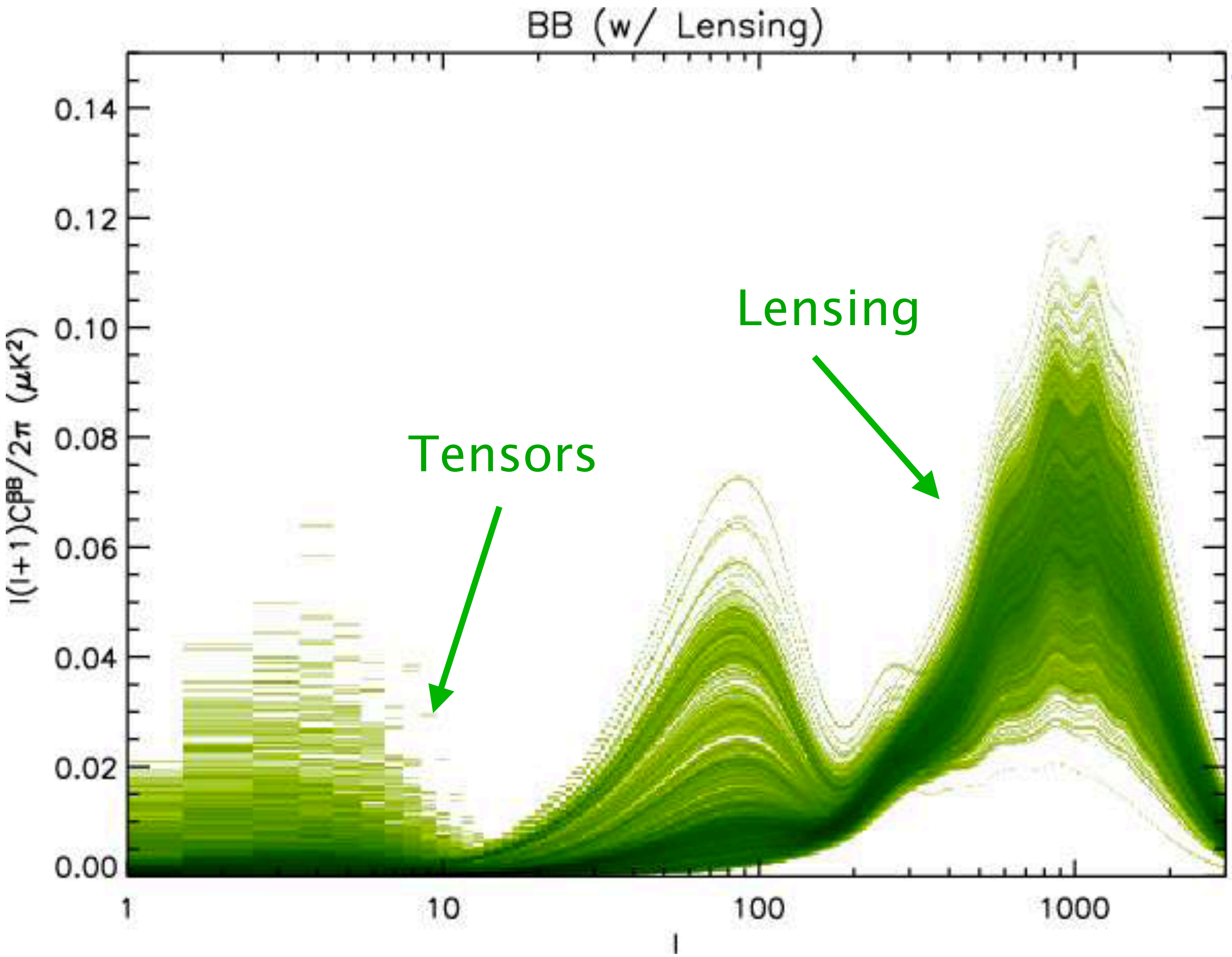
simulations from K. Vanderlinde



The intermediate to small scale EE polarization signal is sensitive only to the physics at the epoch of last scattering (unlike TT which can be modified).

The EE spectrum is already well constrained from the cosmological models, but it provides additional checks and helps to break some degeneracies. **Plus, it gives a more accurate measurement of the reionization optical depth.**

BB spectrum uncertainties

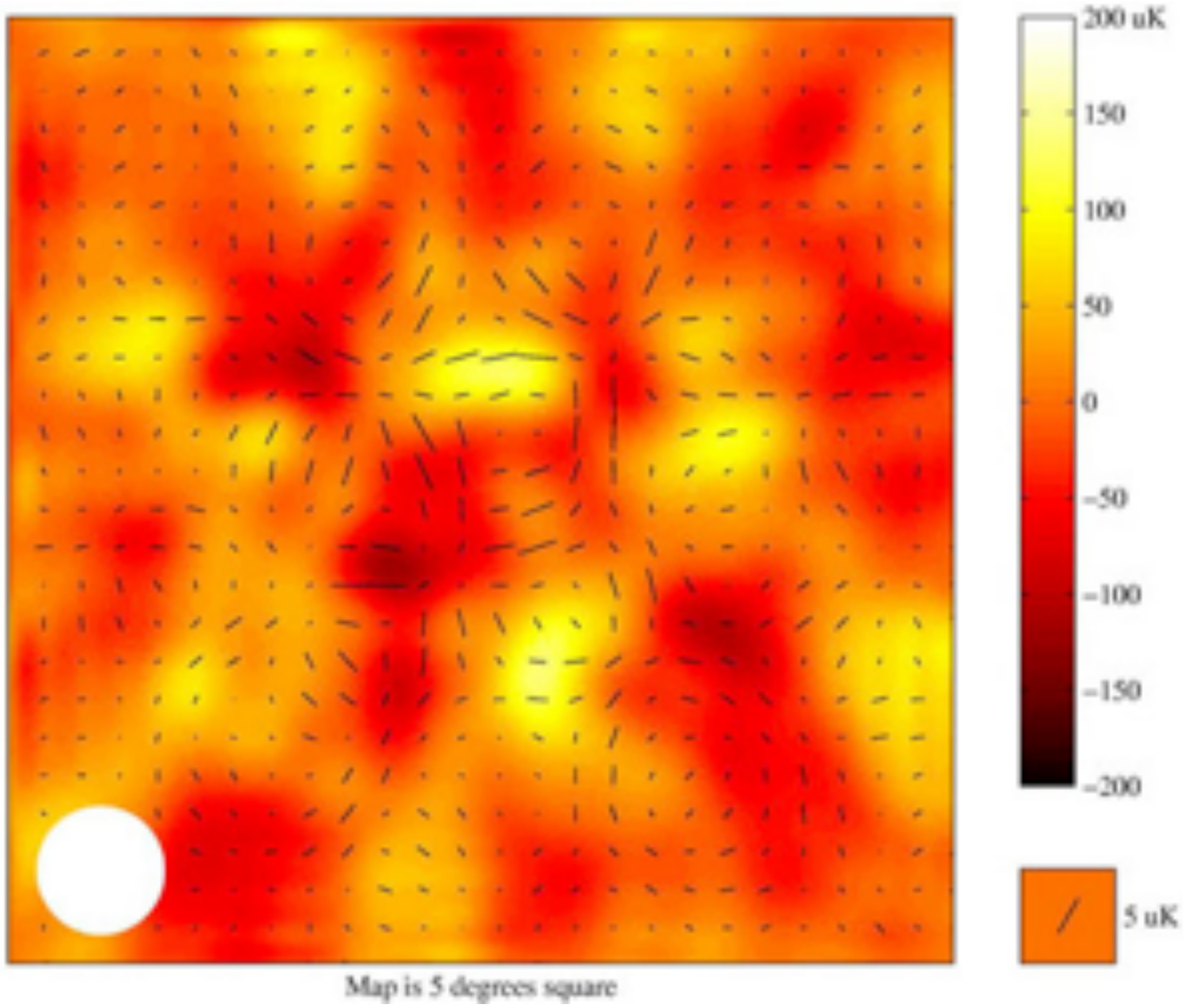


Credit: K. Vanderlinde

BB mode can tell us about a lot of new physics (energy scale at inflation, neutrino mass, etc.), but its prediction is still very uncertain (i.e. the prediction of the primordial signal; the lensing part is well modelled since we have a good estimation of the lensing potential across whole sky).

Detection of the E-mode polarization

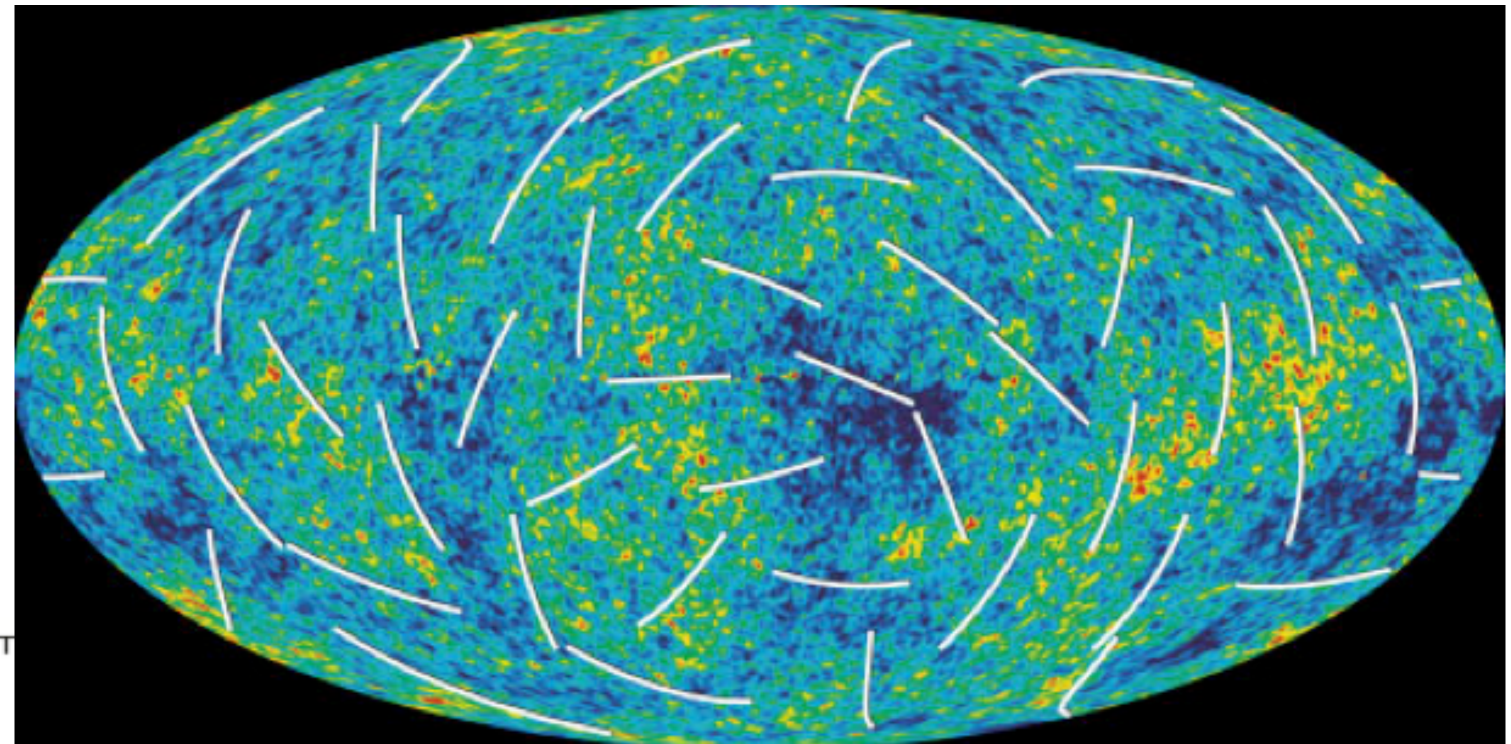
- The DASI experiment at the South Pole was the first to detect E-mode CMB polarization
- It was followed by WMAP's measurement of $C^{TE}(l)$ for $l<500$
- Both the BOOMERANG and the CBI experiments have reported measurements of C^{TT}, C^{TE}, C^{EE} and a non-detection of B modes
- E-mode has also been measured by CAPMAP and Maxipol
- B-mode polarization has not been detected yet (current noise level for ground-based experiment is below 1 μK in Q and U))

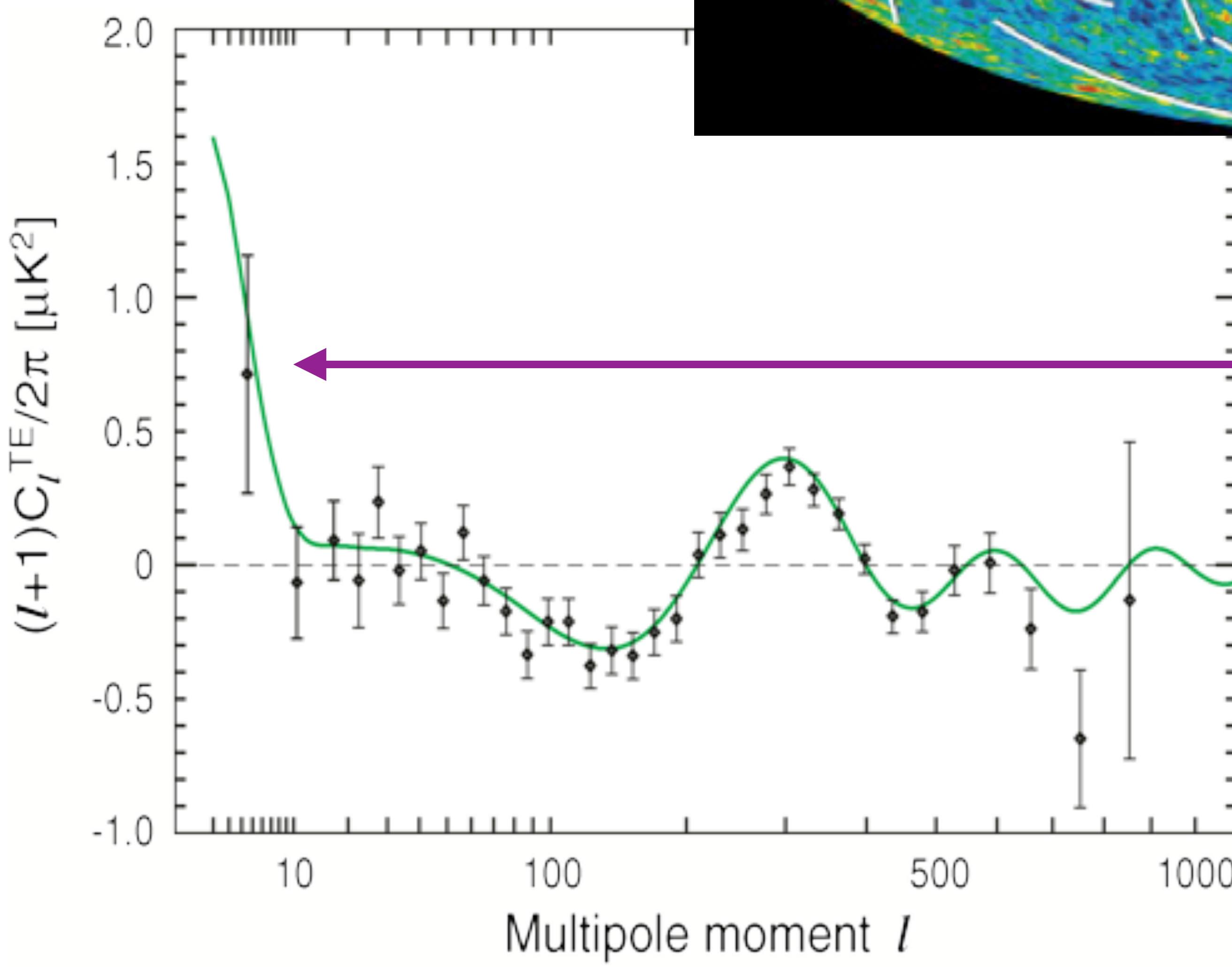


DASI collaboration, 2002

E-mode generated during reionization

First measurements from WMAP satellite (Kogut et al. 2003)

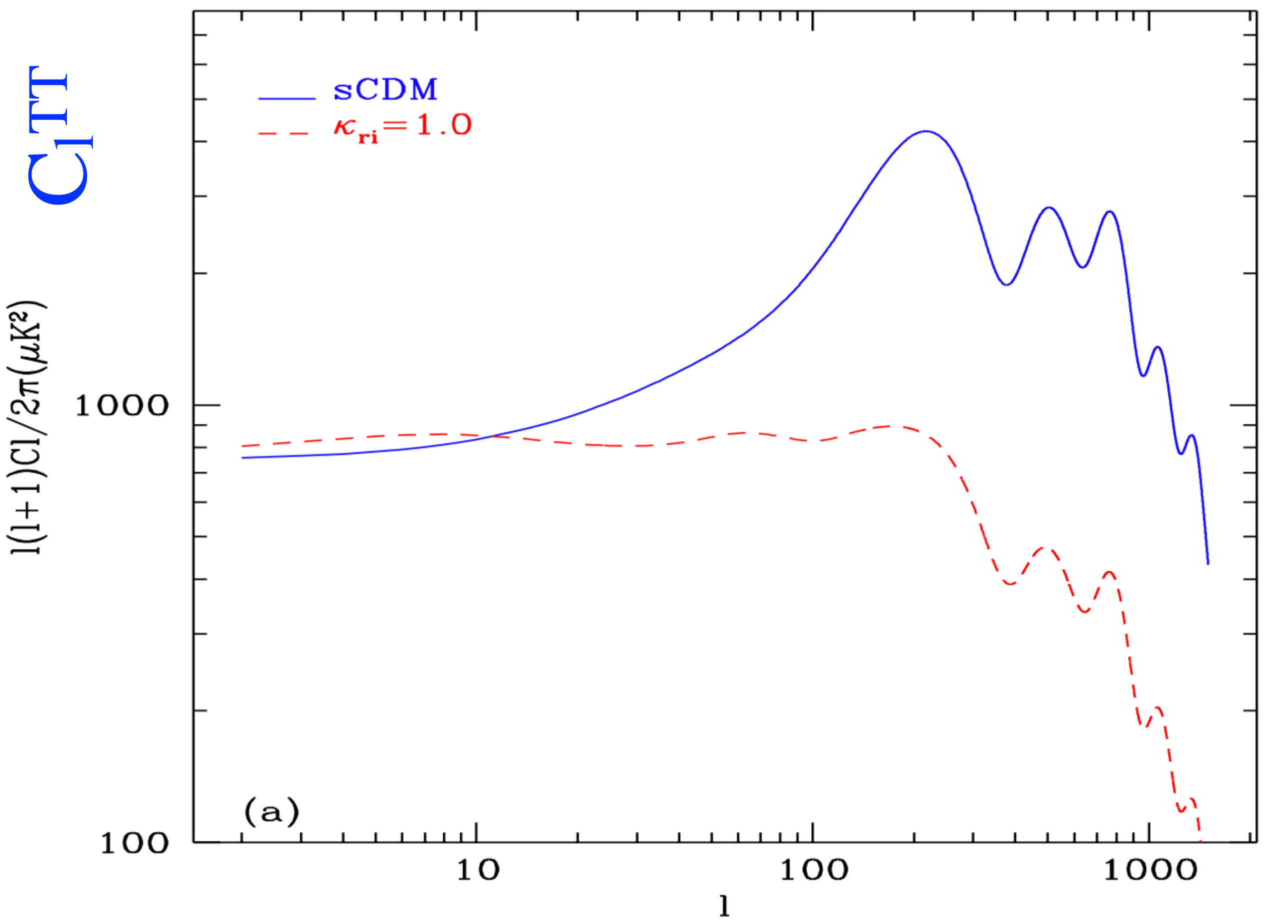


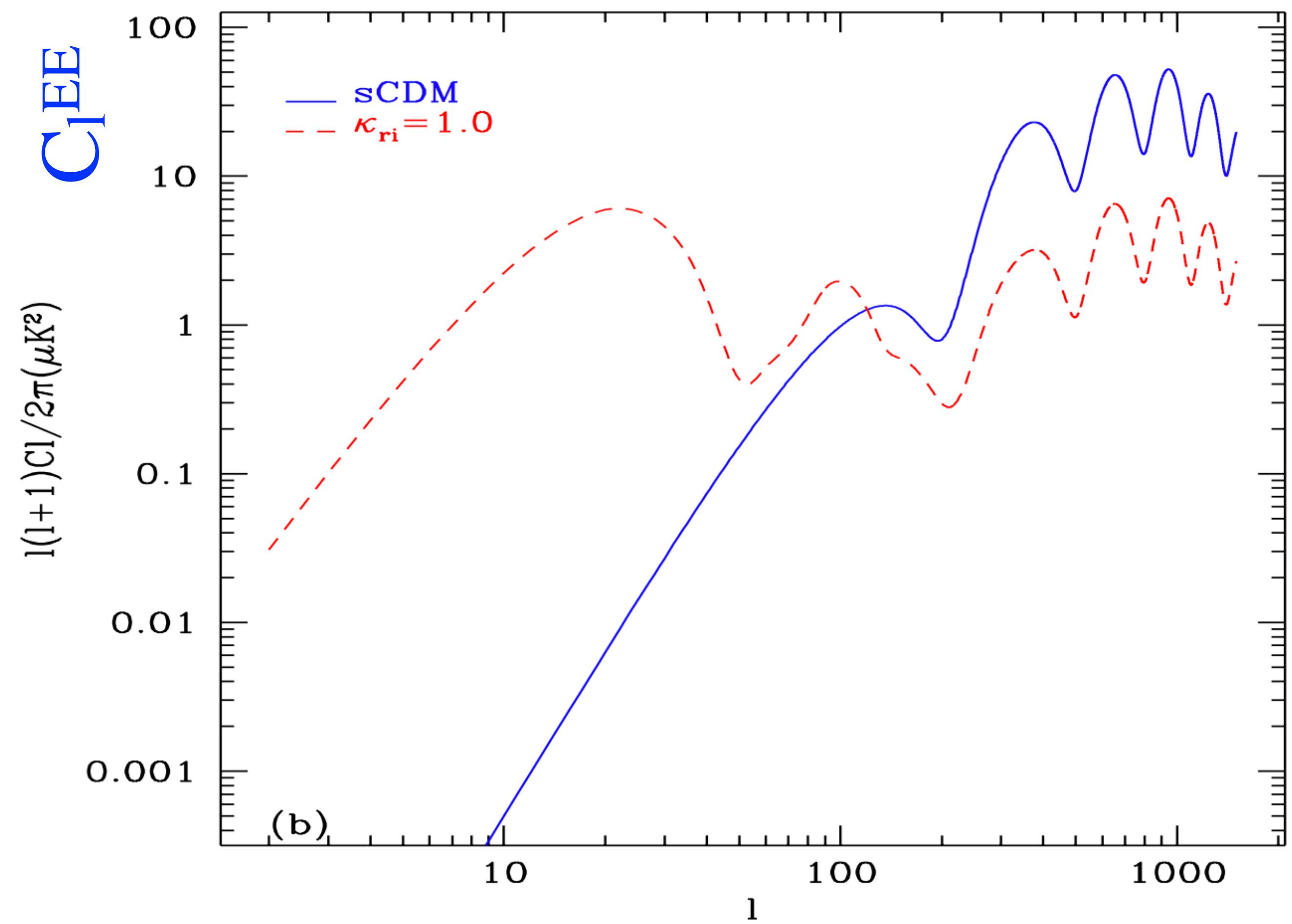


Re-scattering of CMB during and after reionization adds to the polarized power on large angular scales, caused by coherent motion of electrons. The large scales correspond to the horizon scale (H^{-1}) at the time of reionization (~5°).

This “bump” exceeds the cosmic variance limit for EE and TE spectrum.

Reionization "bump" in polarization





These figures from a paper by **Zaldarriaga (1996)** show the effect of reionization on the temperature (TT, on left) and polarization (EE, on right) power spectra, compared to a "standard" CDM model without reionization (using a very large optical depth, for illustration). The relative change in the E-mode power is very large, making it an ideal target for measurement and constraining the reionization history of the universe.

Reionization "bump" in polarization

Bump in low-ℓ spectra

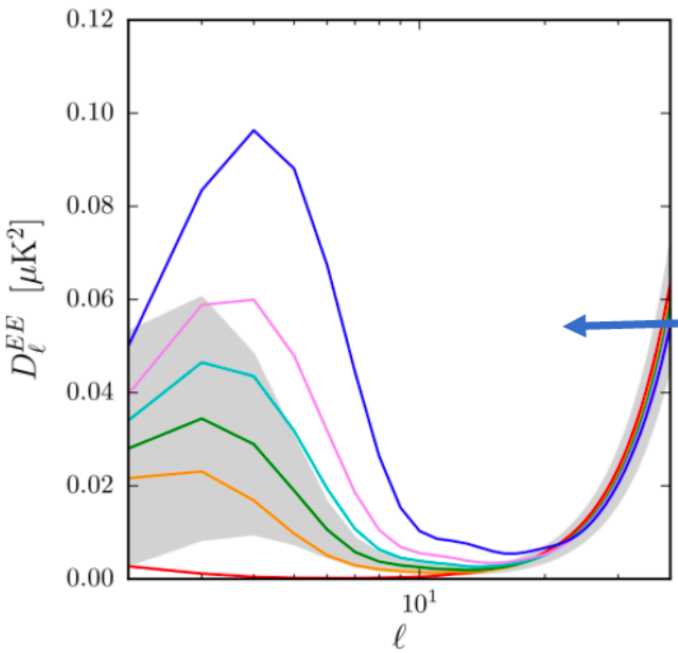
Height $\Delta_E \to \Delta_E \, \tau \quad C_\ell^{EE} \propto \tau^2 \quad C_\ell^{TE} \propto \tau$

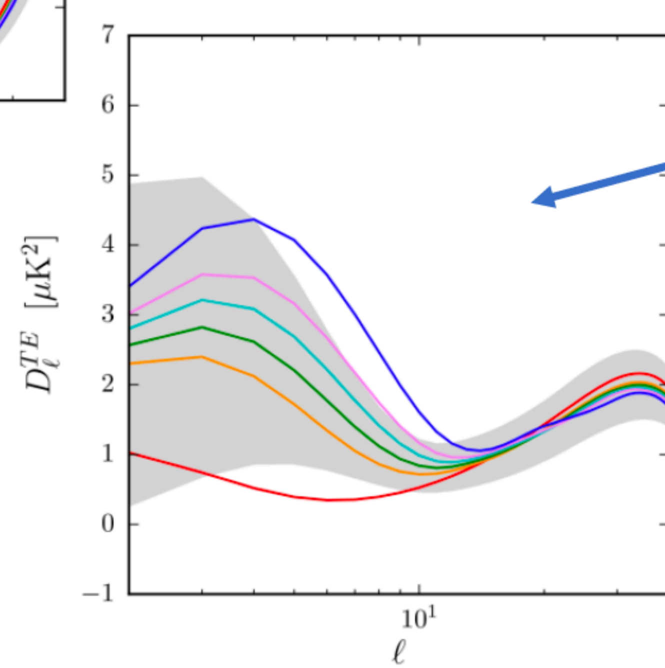
The reionization bump is practically independent of other cosmological parameters, it basically gives a measure of the integrated optical depth.

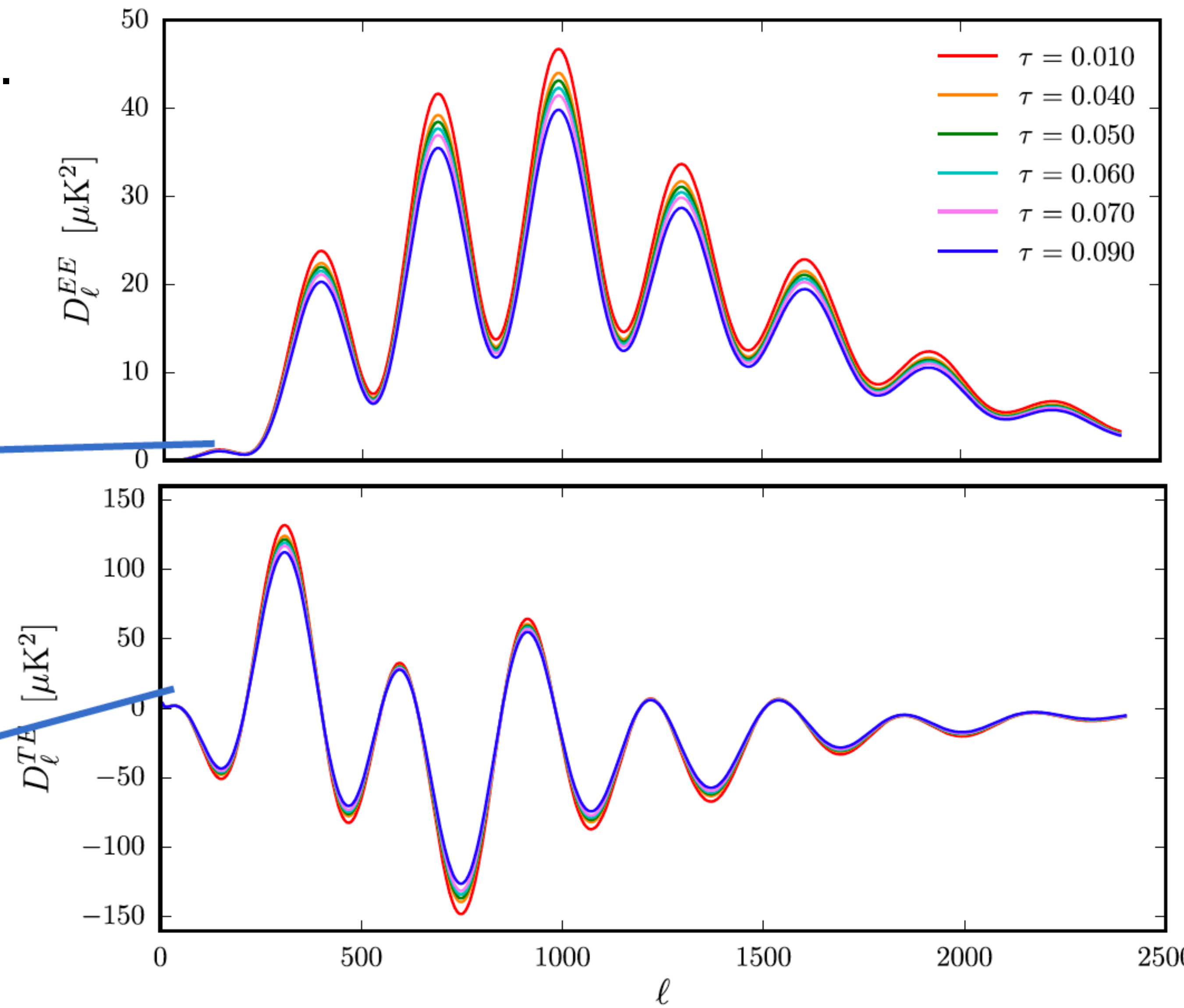
Figures from Planck collaboration (2016)

$\tau = 0.054 \pm 0.007$

(Planck coll. 2018)

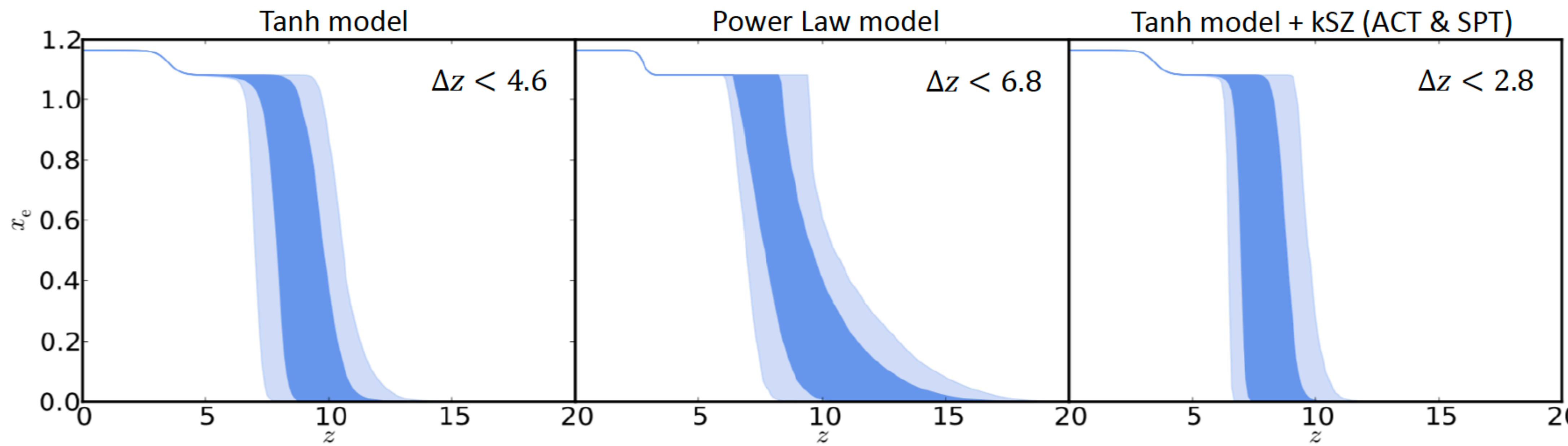






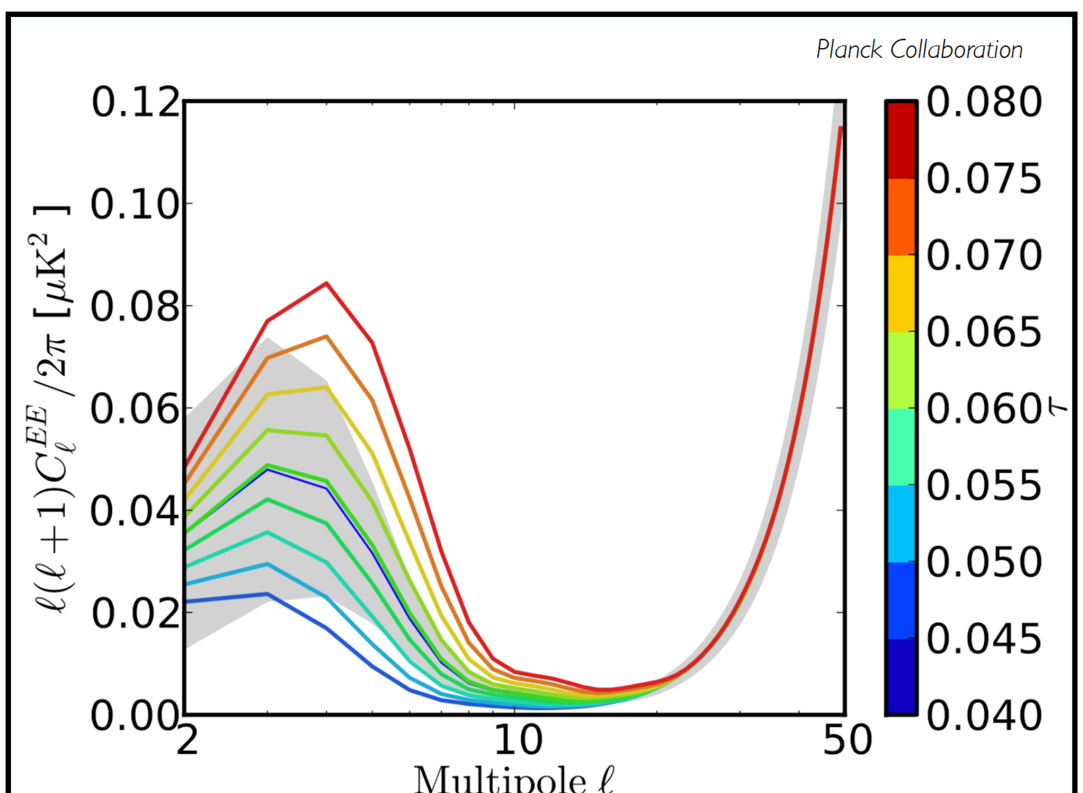
Grey bands denote the cosmic variance limit (f_{sky}=0.65) for Planck data

Optical depth of the universe



Constraints still model dependent. **Data disfavour early onset of reionization.**

Planck collaboration (2016)



- Planck Collaboration XLVII (2016).
- $\tau = 0.058 \pm 0.012$ (assuming instantaneous reionization).
- Redshift of reionization is model dependent: $z_{re} \sim 8.5$. Complementary to 21cm studies.

Questions?

Feel free to email me or ask questions in our eCampus Forum