astro8405

An Introduction to the

Cosmic Microwave Background

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astro8405: The Cosmic Microwave Background

Aktionen

This course intends to give you a modern and up-to-date introduction to the science and experimental techniques relating to the Cosmic Microwave Background. No prior knowledge of cosmology is necessary, your prerequisite are a basic understanding of electrodynamics and thermal physics and some familiarity with Python programming.

Lecture 5:

Temperature Anisotropies:

Creation of the Primary Anisotropies

Astrophysics (astro-ph/9511148)

[Submitted on 30 Nov 1995]

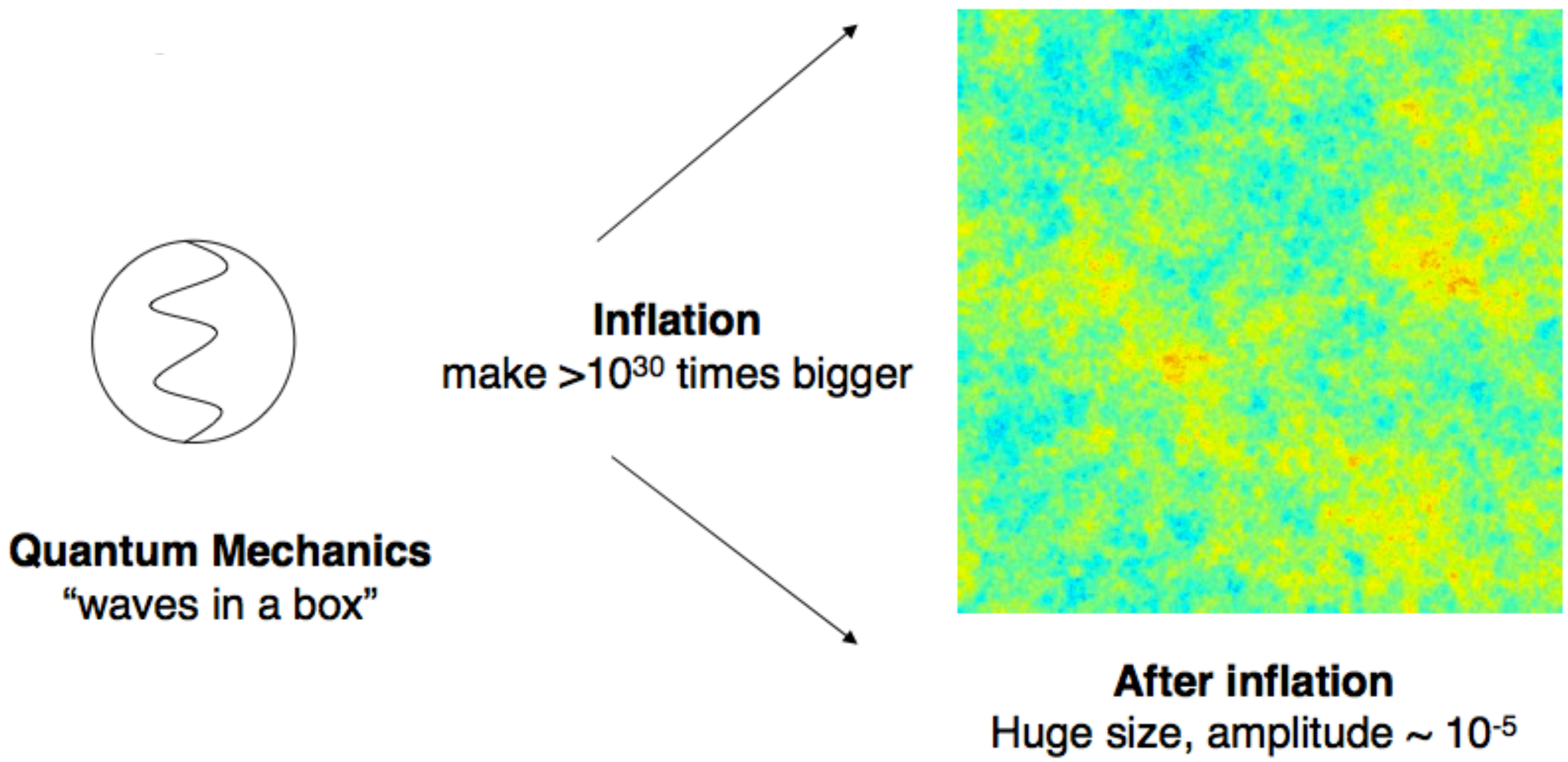
Doppler peaks and all that: CMB anisotropies and what they can tell us

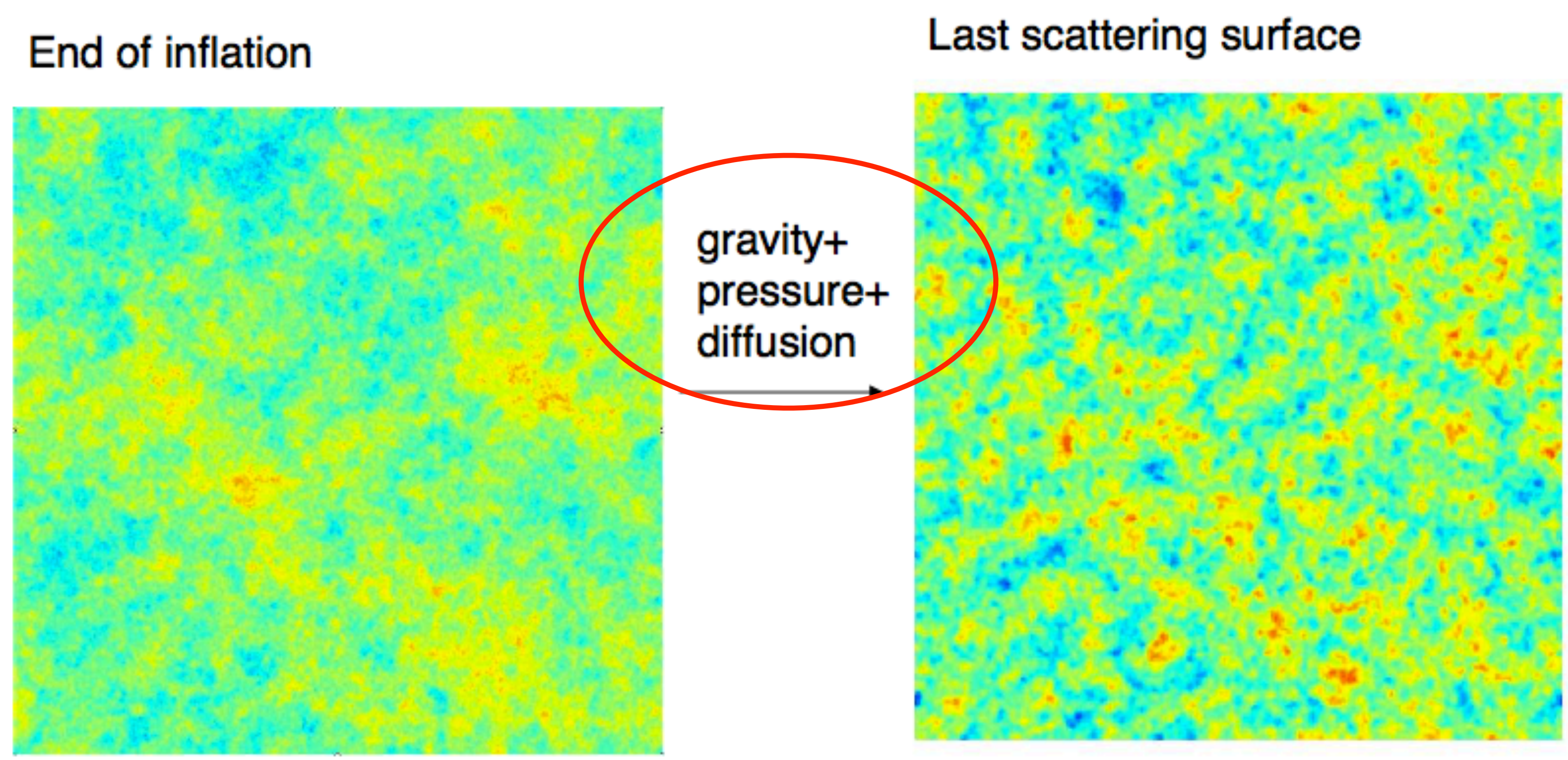
Max Tegmark

Table 1. Sources of temperature fluctuations.

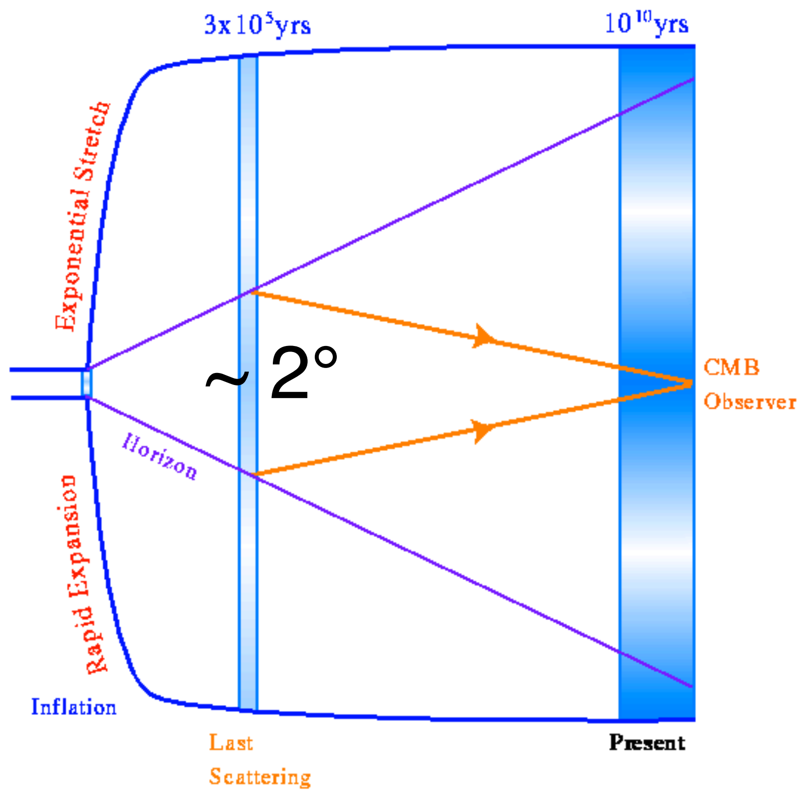
PRIMARY	Gravity	
	Doppler	
	Density fluctuations	
	Damping	
	Defects	Strings
		Textures
SECONDARY	Gravity	Early ISW
		Late ISW
		Rees-Sciama
		Lensing
	Local reionization	Thermal SZ
		Kinematic SZ
	Global reionization	Suppression
		New Doppler
		Vishniac
“TERTIARY” (foregrounds & headaches)	Extragalactic	Radio point sources
		IR point sources
	Galactic	Dust
		Free-free
		Synchrotron
	Local	Solar system
		Atmosphere
		Noise, *etc.*

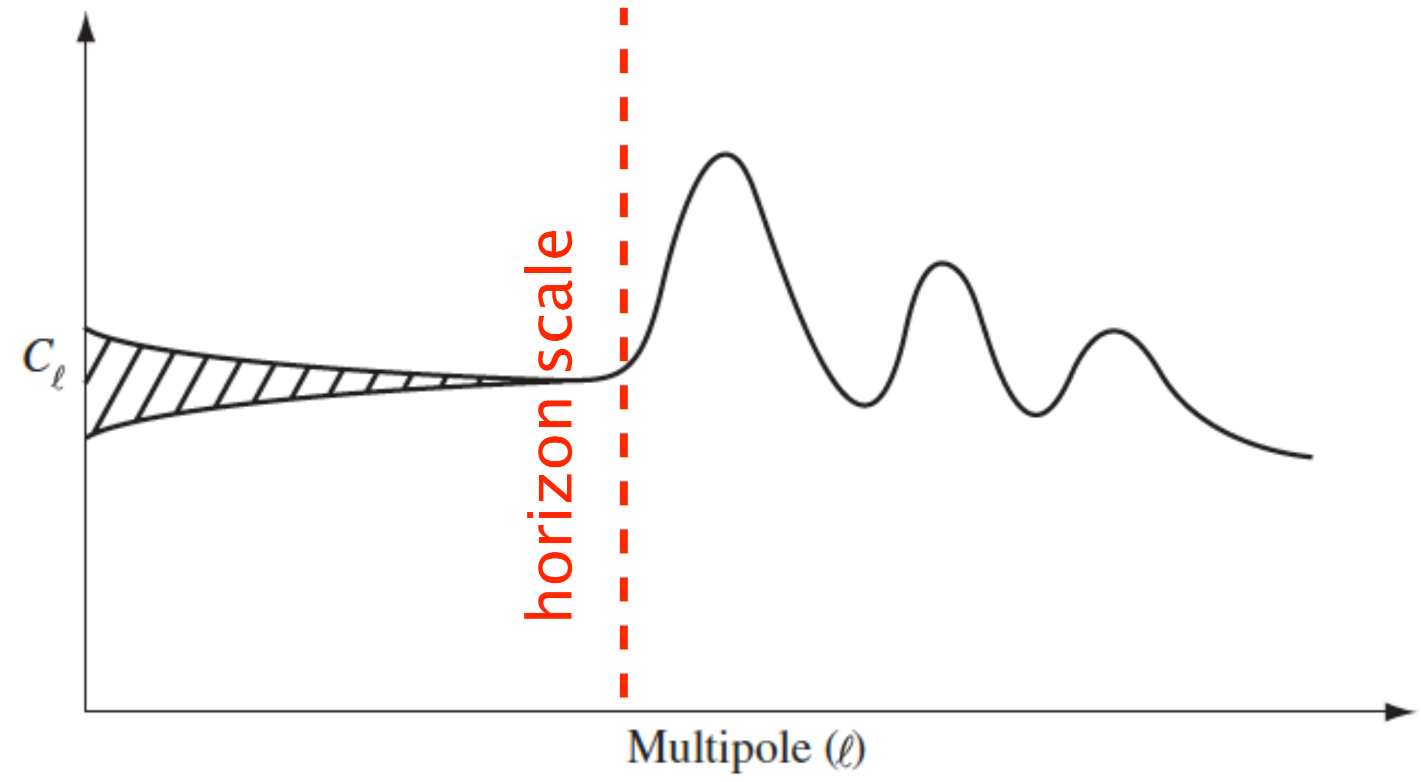
Inflation again! Origin of primary ΔT

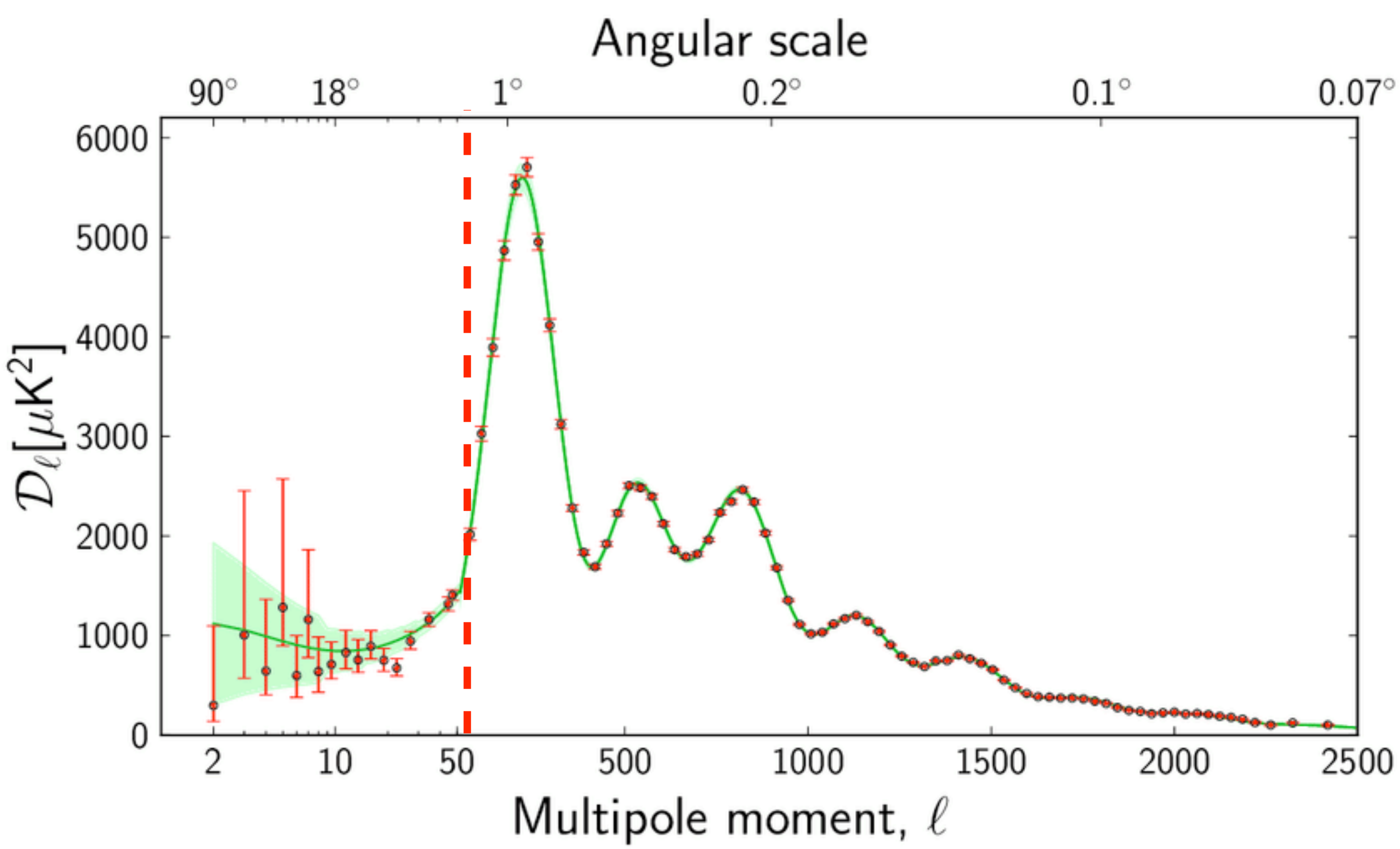




Demarkation at the horizon scale







We learned that the horizon scale at recombination was roughly 2 degrees. We also see there is a distinct demarkation of the power at this scale in the C_l.

The power at larger scales is not zero, there is a roughly uniform level of power (l^2C_l). What causes this?

Density perturbations from inflation: Power spectrum

The scaler perturbations are Gaussian, so all information about them is contained in the two-point correlation function:

$$\langle \mathcal{R}(\mathbf{k})\mathcal{R}^*(\mathbf{k}')\rangle = \frac{P(k)}{(2\pi)^3}\delta(\mathbf{k}-\mathbf{k}')$$

The mean square value of the initial perturbation amplitude is

$$\langle \mathcal{R}^2(\mathbf{x})\rangle = \langle \int e^{i\mathbf{kx}} R(\mathbf{k}) d^3k \int e^{-i\mathbf{k}'\mathbf{x}} R^*(\mathbf{k}') d^3k' \rangle = \int d^3k \frac{P(k)}{(2\pi)^3} = \int_0^\infty \frac{dk}{k}\mathcal{P}(k)$$

Where $\mathcal{P}(k) = k^3 P(k)/(2\pi^2)$ is the dimensionless power spectrum, and is approximated as follows:

$$\mathcal{P}_s(k) = A_s \left(\frac{k}{k_*}\right)^{n_s-1}$$

In the 1960's, Zel'dovich and Harrison independently predicted the flat spectrum of perturbations (i.e. $n_s = 1$). But now we know the spectrum is *slightly red.* The WMAP5 values for a fixed $k_* = 500\ \mathrm{Mpc}^{-1}$ are:

$$\boxed{\begin{aligned} A_s &= (2.46 \pm 0.09)\cdot 10^{-9}, \\ n_s &= 0.960 \pm 0.014. \end{aligned}}$$

Density perturbations from inflation

Inflation occurs if the universe is filled with a scalar field φ, which has non-vanishing scalar potential V(φ). The homogeneous field φ then satisfies the equation

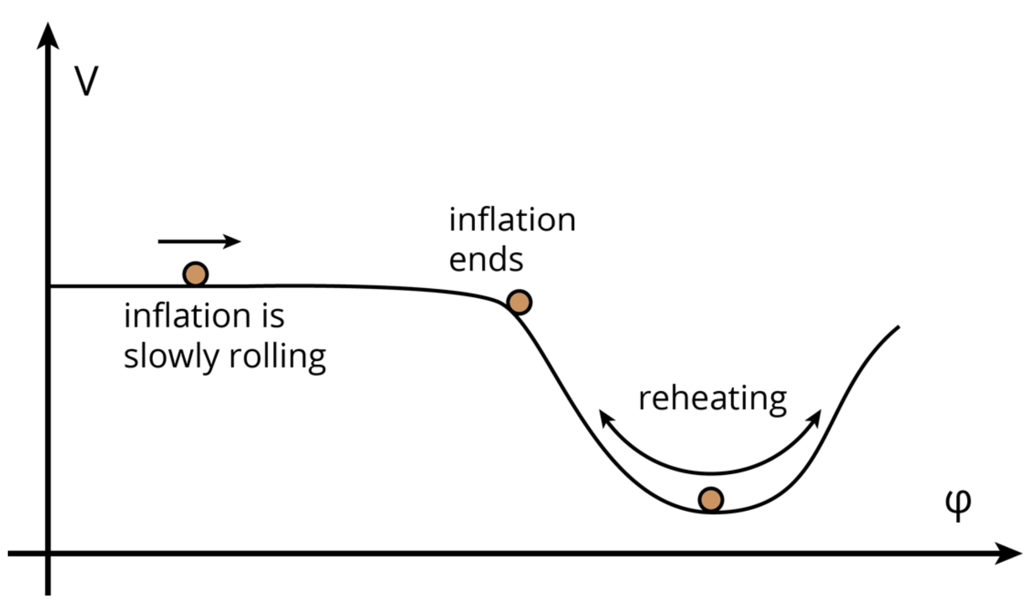
$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{dV}{d\varphi}. \qquad a(t) \propto \exp\left(\int H dt\right), \quad H \approx \text{const.}$$

For a relatively flat potential (dV/dφ small), the acceleration term can be neglected. The Friedmann equation in this case is $H^2 = 8\pi/3G\, V(\varphi)$. So if φ varies slowly, then V(φ) and thus H also varies slowly, and the parameters of inflation are almost time independent *(slow-roll inflation)*.

At the end of inflation, the initial (inflation potential) perturbations get re-processed into density perturbations. If the inflation parameters are time independent, then density perturbations are *k*-independent as they exit the inflation ($\delta\varphi \sim H$) and the resulting power spectrum is flat.

However, the parameters are not *exactly* time-independent at inflation, so the predicted value of the spectral tilt (n_s – 1) is small but non-zero. It can be positive or negative, depending on the scalar potential V(φ). In particular, it is negative for the simplest power-law potentials like

$$V(\varphi) = \frac{m^2}{2}\varphi^2 \quad \text{or} \quad V(\varphi) = \frac{\lambda}{4}\varphi^4.$$

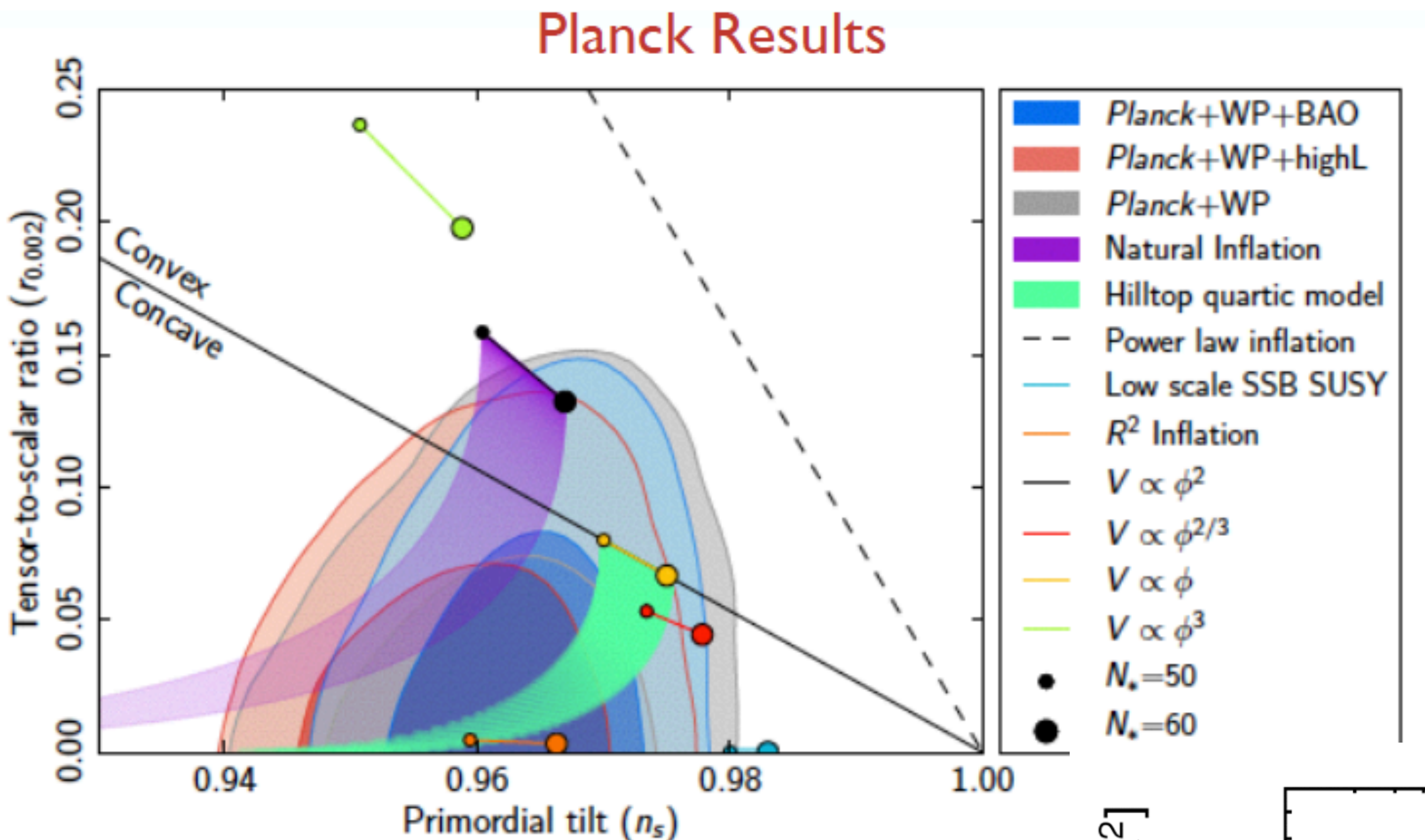


For the case of slow-roll inflation,

$$\boxed{n_s - 1 = -3M_{\rm pl}^2\left(\frac{V'}{V}\right)^2 + 2M_{\rm pl}^2\frac{V''}{V}}$$

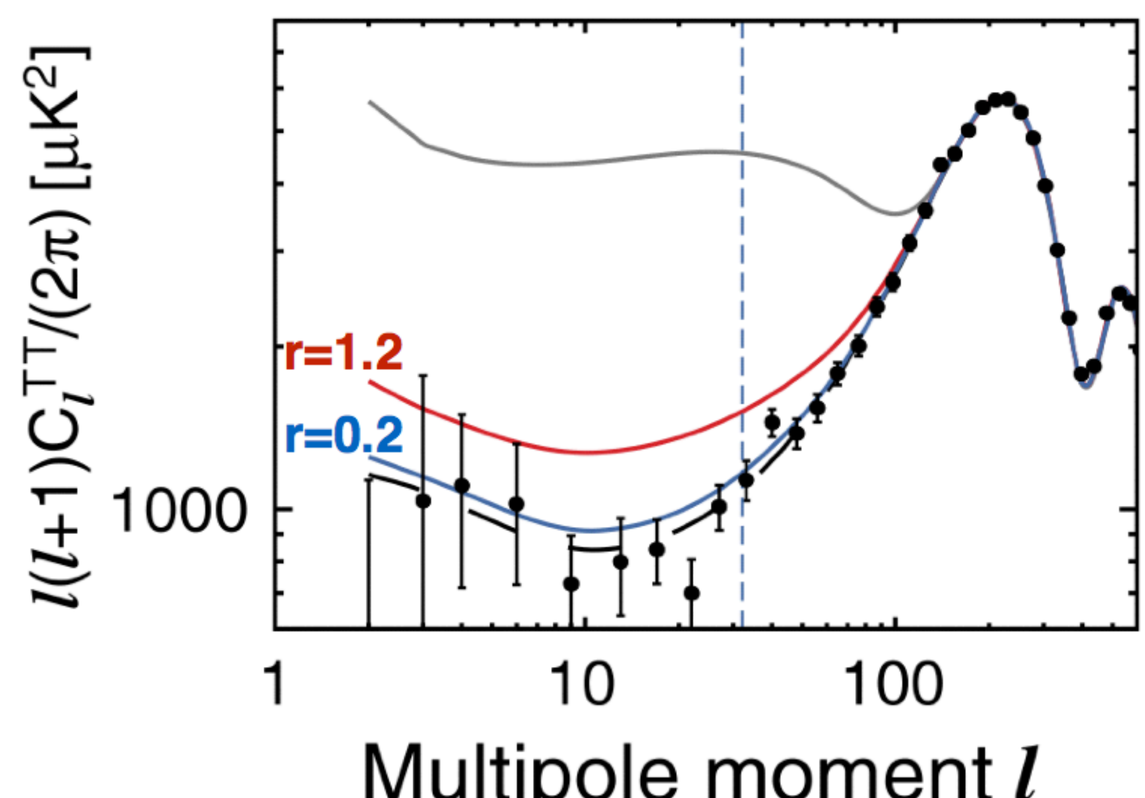
with $' \equiv \frac{\partial}{\partial\phi}$.

Planck limits from TEMPERATURE data

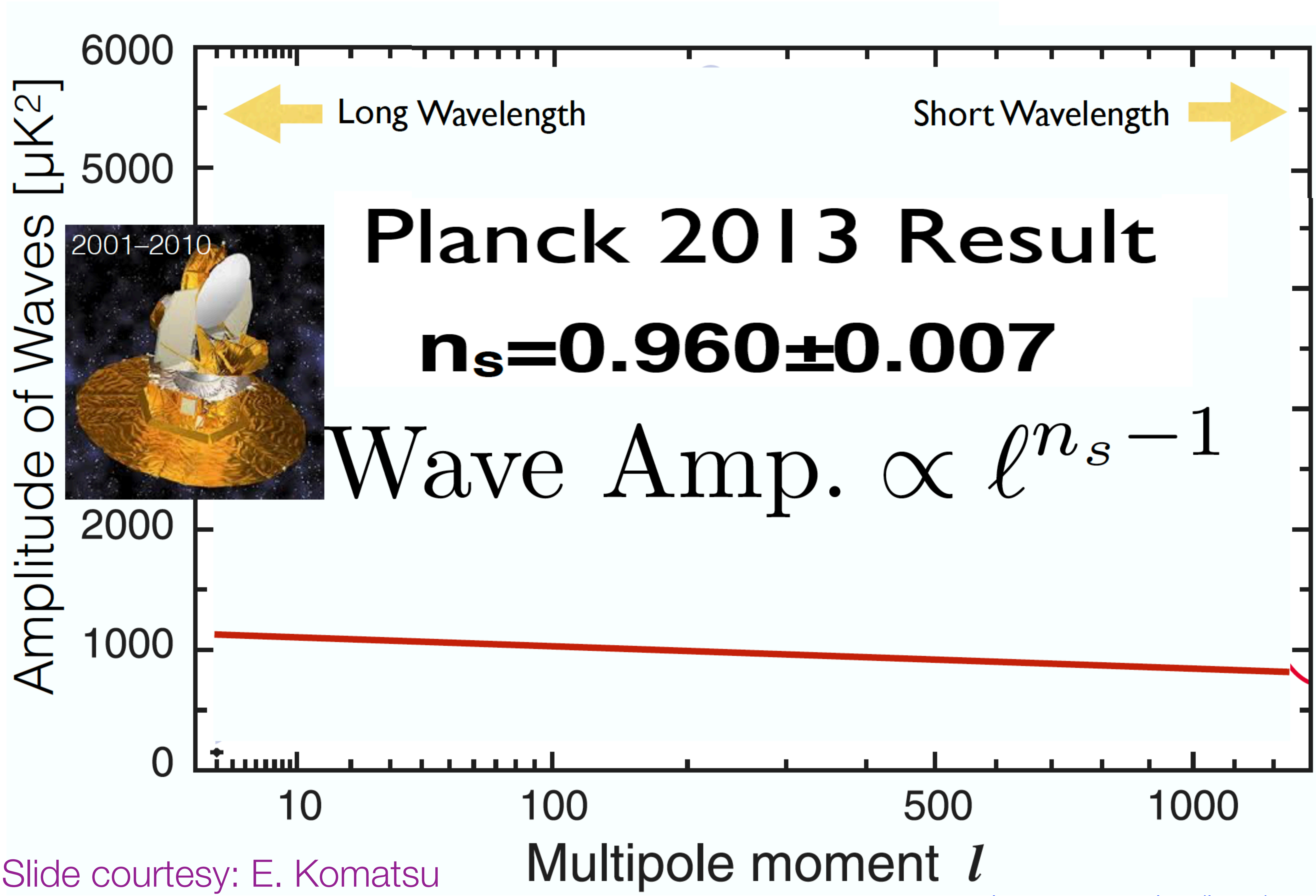


No experiments so far have measured the B-mode of polarization (later in the class), so there is only an upper limit on the tensor-to-scalar ratio, r.

However, from precise TT measurements, *Planck* could constrain the slope of the primordial spectrum, n_s, and hence constrain inflation. This already excludes a lot of parameter space for inflationary models.



Planck limits from TEMPERATURE data



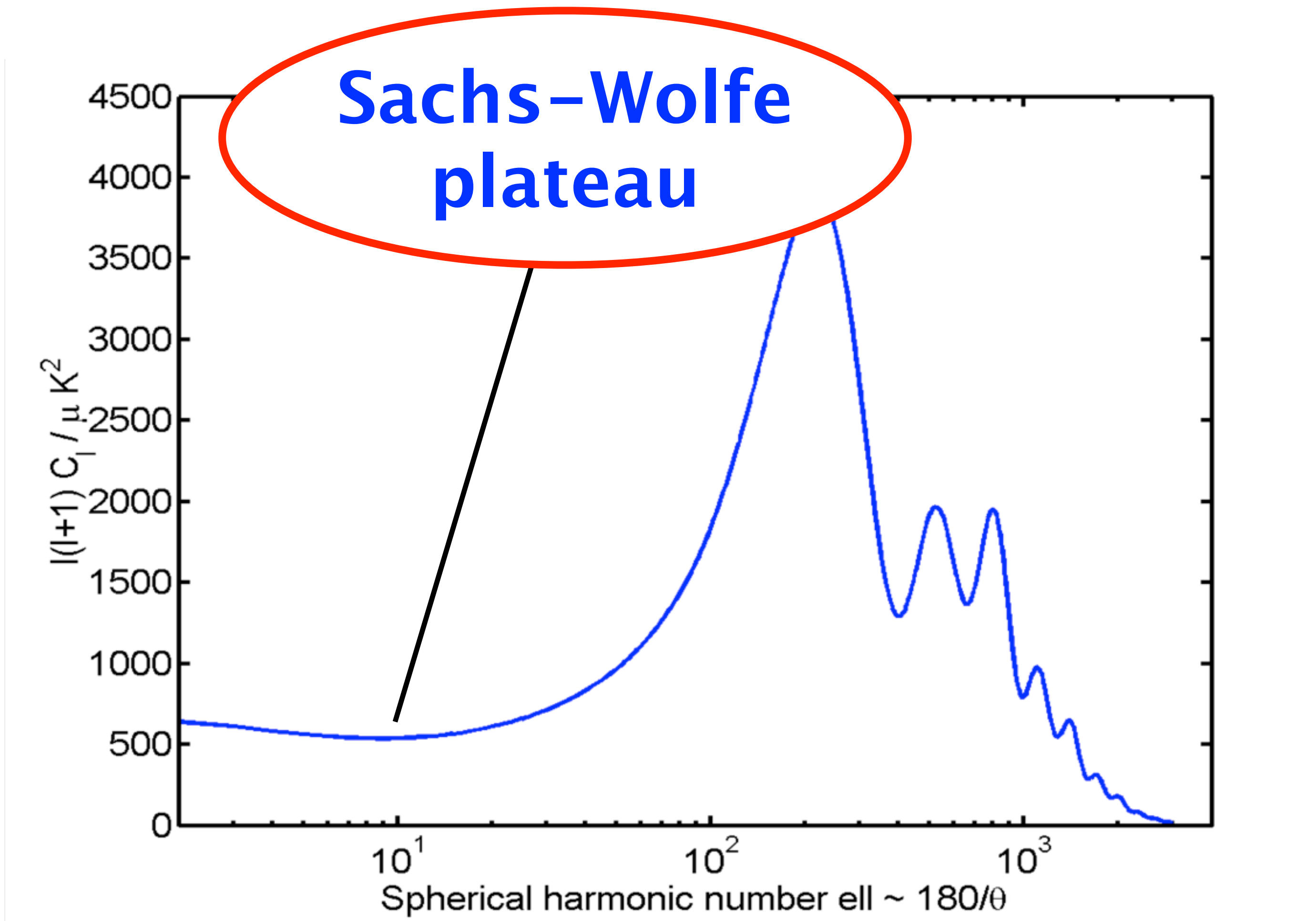
Slide courtesy: E. Komatsu

Sachs–Wolfe effect

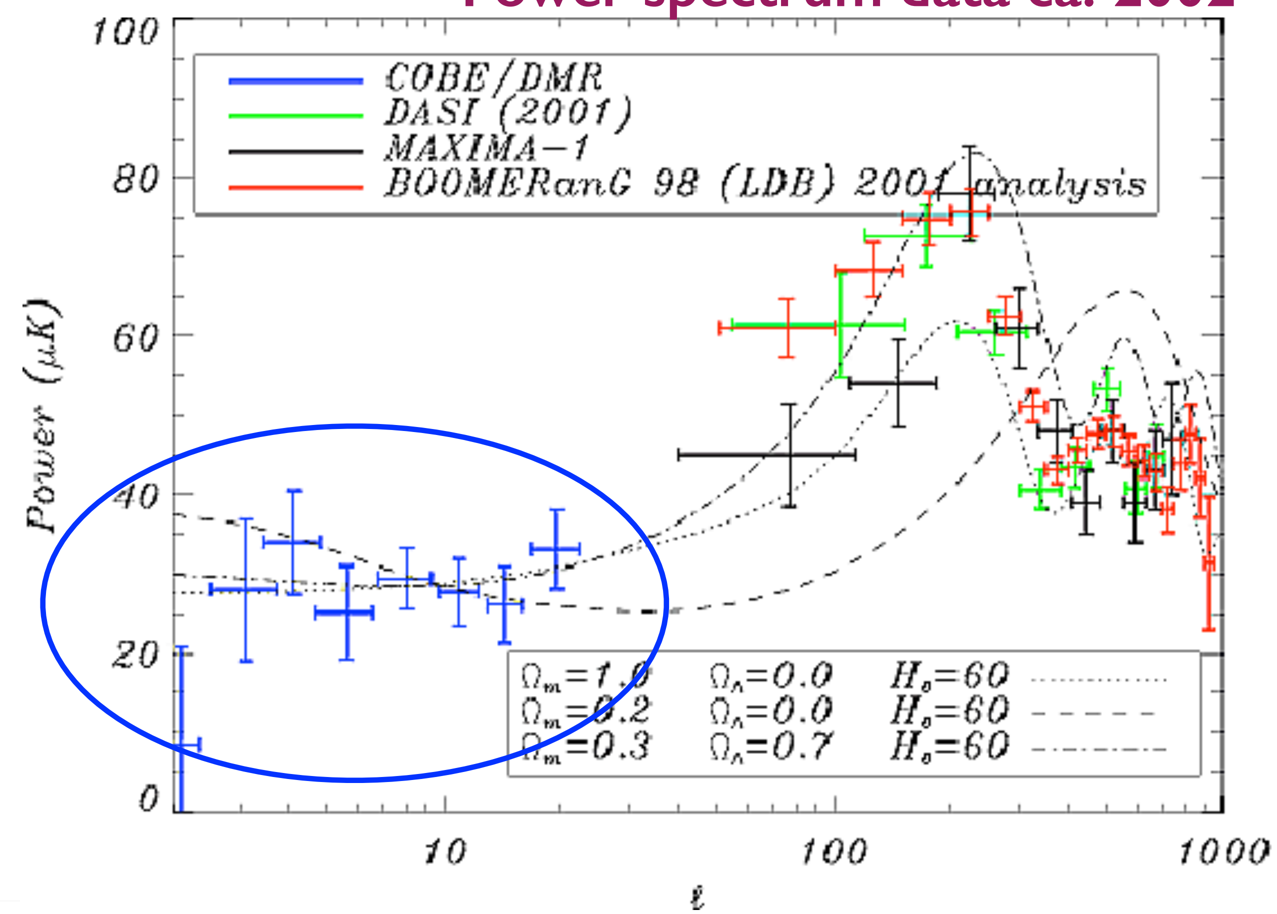
Following from the inflationary prediction of a (nearly) scale-independent primordial density power spectrum, the Sachs-Wolfe effect produces a flat power spectrum: $C_l^{SW} \sim const / l(l+1)$. This was predicted by Sachs & Wolfe in 1967.

This is the main effect creating temperature fluctuations at θ ≳ 2° scales.

At smaller angles, perturbations enter the horizon and they have time to evolve from their initial state. This evolution is driven by the baryons.



Power spectrum data ca. 2002



Sachs–Wolfe effect amplitude

For an independent random variable $\mathcal{R}_\mathbf{k}$ and its power-spectrum $\mathcal{P}_\mathcal{R}(k)$:

$$C_\ell \equiv \frac{1}{2\ell+1}\sum_m \langle |a_{\ell m}|^2\rangle$$

$$= \frac{4\pi}{25}\int_0^\infty \frac{dk}{k}\mathcal{P}_\mathcal{R}(k) j_\ell(kx)^2 \,,$$

the final result for an arbitrary primordial power spectrum $\mathcal{P}_\mathcal{R}(k)$.

The integral can be done for a power-law power spectrum, $\boxed{\mathcal{P}(k) = A^2 k^{n-1}.}$ In particular, for a scale-invariant ($n = 1$) primordial power spectrum,

$$\mathcal{P}_\mathcal{R}(k) = \text{const.} = A^2 \,,$$

we have

$$C_\ell = A^2 \frac{4\pi}{25}\int_0^\infty \frac{dk}{k} j_\ell(kx)^2 = \frac{A^2}{25}\frac{2\pi}{\ell(\ell+1)} \,,$$

since

$$\int_0^\infty \frac{dk}{k} j_\ell(kx)^2 = \frac{1}{2\ell(\ell+1)} \,.$$

We can write this as

$$\frac{\ell(\ell+1)}{2\pi} C_\ell = \frac{A^2}{25} = \text{const.} \quad \text{(independent of } \ell)$$

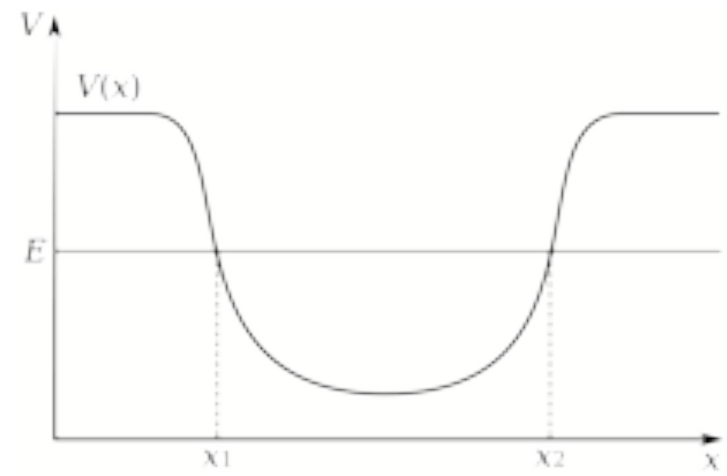
(Lecture notes by Hannu Kurki_Suonnio, Uni Helsinki)

For power-law index of scale-independent primary density perturbations, n_s=1 (Zel'dovich, Harrison ~1970), the Sachs-Wolfe effect produces a flat power spectrum: C_l^{SW} ~ const / l(l+1).

Sachs-Wolfe effect $\Delta T/T$

We expect $\Delta\nu/\nu \sim \Delta\, T/T \sim \Phi/c^2$

Additional effect of time dilation (see next slide) while the potential evolves gives a factor 1/3 instead (e.g. White & Hu 1997):



$$\frac{\Delta T}{T} \sim \frac{1}{3}\frac{\Delta\Phi}{c^2}$$

The temperature fluctuations due to the so-called Sachs-Wolfe effect are due to two competing effects: (1) the redshift experienced by the photon as it climbs out of the potential well toward us and (2) the delay in the release of the radiation, leading to less cosmological redshift compared to the average CMB radiation.

The first contribution leads to a redshift of the order of:

$$\frac{\delta T_1}{T} = \frac{\delta\Phi}{c^2}$$

Sachs–Wolfe effect $\Delta T/T$

The second contribution (redshift due to time delay) can be computed roughly as the following. Because of general relativity, the proper time goes slower inside the potential well than outside. The cooling of the gas in this potential well thus also goes slower, and it reaches 3000 K at a later time relative to the average Universe.

The time delay (in terms of global time t) is:

$$\frac{\delta t}{t} = -\frac{\delta\Phi}{c^2} \tag{8.7}$$

This means that 3000 K is reached at a slightly larger (global) scale parameter $a + \delta a > a$. Since in the Einstein-de-Sitter Universe we have $a \propto t^{2/3}$ we can write

$$\frac{\delta a}{a} = \frac{2}{3}\frac{\delta t}{t} = -\frac{2}{3}\frac{\delta\Phi}{c^2} \tag{8.8}$$

Now, from that point $a = (a_{\mathrm{cmb}} + \delta a)$ until today $a = 1$ the redshift due to expansion is less by:

$$\frac{\delta z}{z} = -\frac{\delta a}{a} \tag{8.9}$$

which leads to a positive contribution to the temperature fluctuation δT that we observe today:

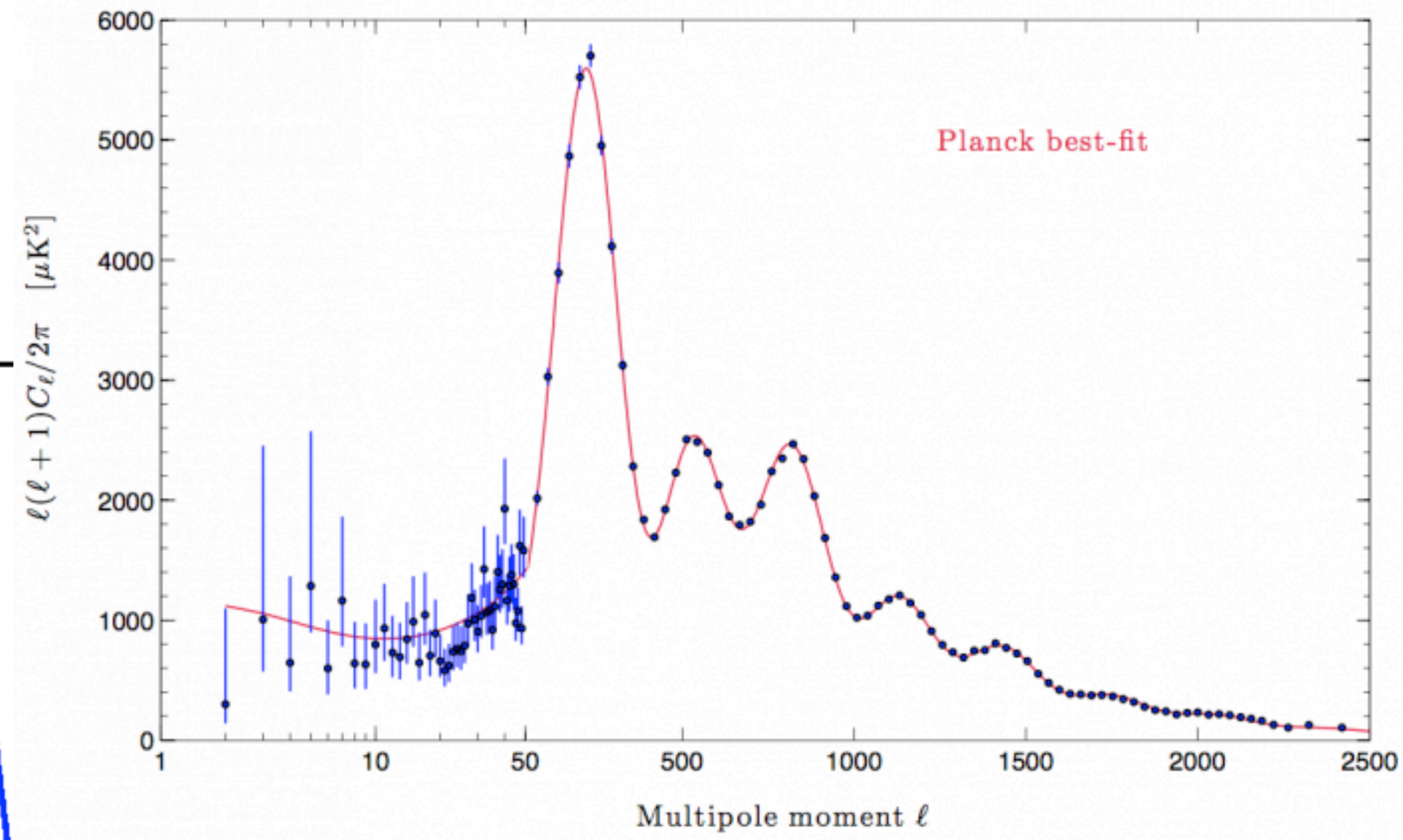
$$\frac{\delta T_2}{T} = -\frac{\delta z}{z} = \frac{\delta a}{a} = -\frac{2}{3}\frac{\delta\Phi}{c^2} \tag{8.10}$$

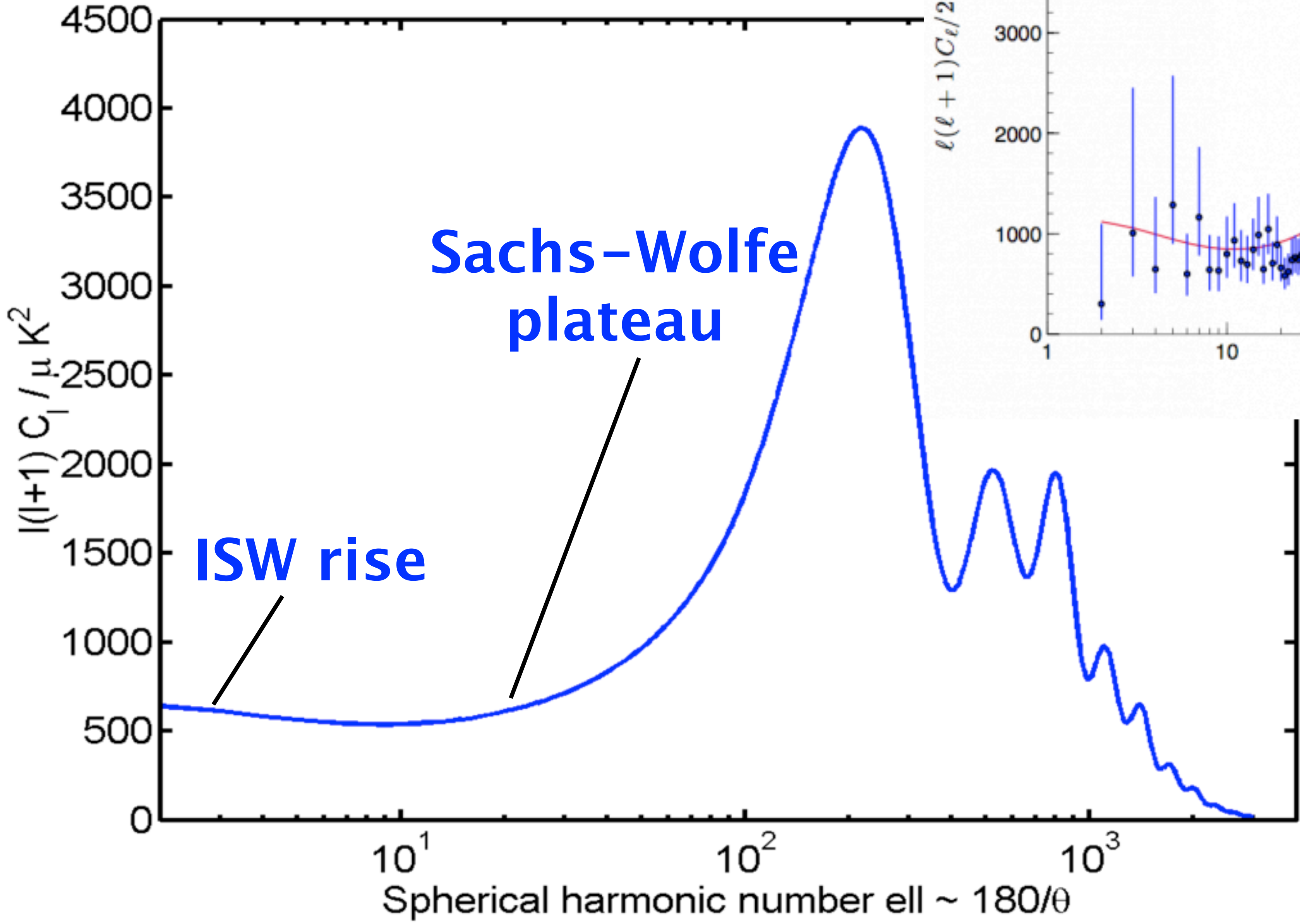
The total is the sum of both contributions:

$$\frac{\delta T}{T} = \frac{\delta T_1}{T} + \frac{\delta T_2}{T} = \frac{1}{3}\frac{\delta\Phi}{c^2} \tag{8.11}$$

Power at very low-ℓ: ISW effect

What is the reason behind the small amount of increase in anisotropy power at very low multipoles?





Integrated Sachs-Wolfe effect

The ISW effect is caused by the interaction of the CMB photons with the large-scale structure after their decoupling.

Temperature anisotropies due to density change and associated gravitational potential (scaler perturbations) along the direction **n** can be represented as:

Large-scale (linear) anisotropies:

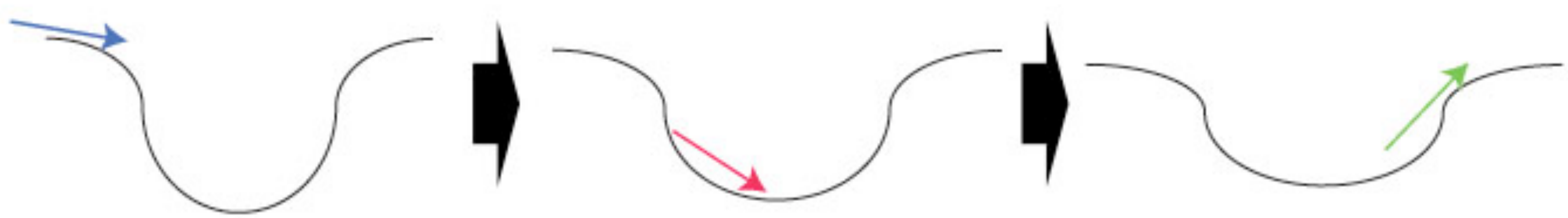
$$\frac{\Delta T}{T}(\vec{n}) \approx \phi_e(\vec{n}) + \int_e^o \frac{\partial \phi}{\partial t} dt + \vec{n} \cdot (\vec{v}_o - \vec{v}_e) + \left(\frac{\Delta T}{T}(\vec{n})\right)_e$$

SW	ISW	Doppler	Acoustic Oscillations
(grav.pot @ decoupling)	(grav.pot evolution)	(motion emitters @ dec)	(photon-baryon @ dec)

Photon enters well at a certain energy

Photon gains energy on its path into the gravitation well

Photon loses less energy than it gained on the way out of the shallower well

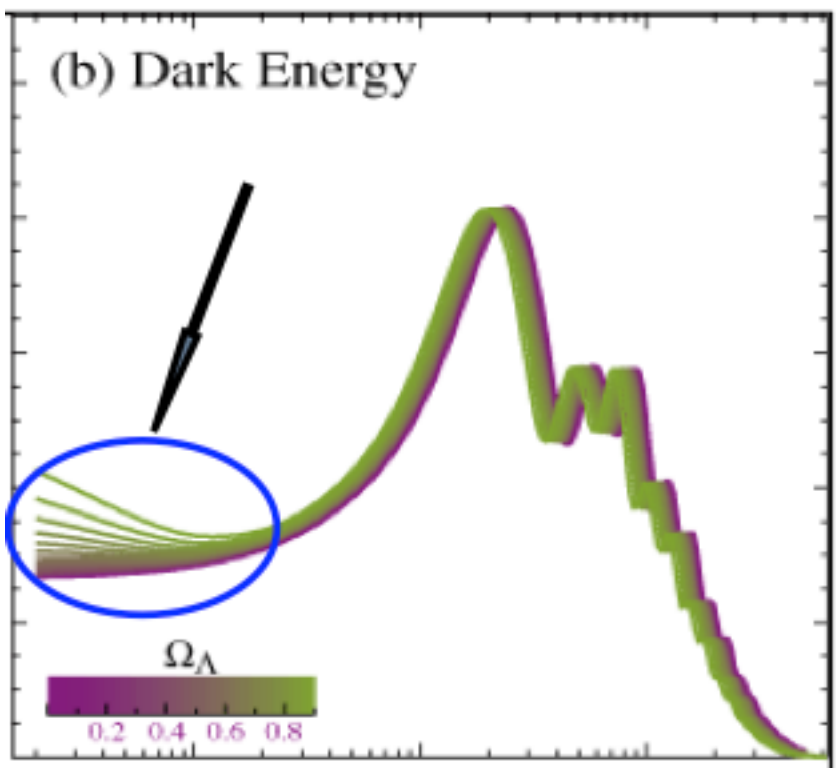


Gravitational well of galaxy supercluster - the depth shrinks as the universe (and cluster) expands

Integrated Sachs–Wolfe effect

Gravitational potential changes along the photon path after recombination:

$$\frac{\delta T}{T_0}(\hat{\boldsymbol{n}}) = -\frac{2}{c^2}\int_0^{r_{\rm LSS}} \mathrm{d}r\, \frac{\partial \phi(r,\hat{\boldsymbol{n}})}{\partial r}$$



- From Poisson's equation: $-k^2\, a^{-2}\phi_k = 4\pi G \rho_m(z)\, \delta_k$, with a the scale factor, so

$$\phi_k = -\frac{4\pi G}{k^2}\rho_m(z=0)\, \frac{\delta_k}{a}$$

- In a $\Omega_m = 1$ Universe, $\delta_k \propto a$ and $\phi_k = const$
- In a Λ Universe, the growth of potentials is *suppressed* compared to the critical case:

$\Rightarrow$ **positive correlation between ϕ_k and $\delta T/T_0$**

ISW effect as Dark Energy probe

The ISW effect constraints the dynamics of acceleration, be it from dark energy, non-flat geometry, or non-linear growth.

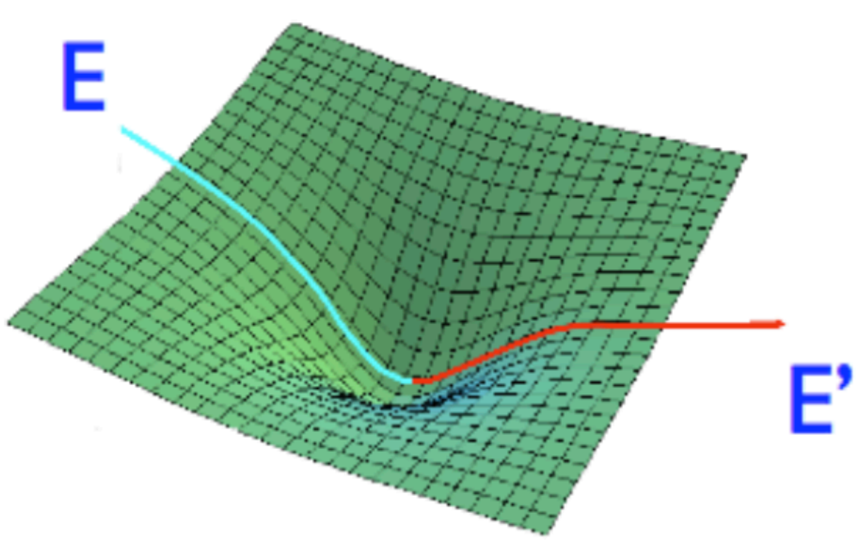
In the absence of curvature, measurement of ISW is measurement of DE.

EdS universe : $\delta \propto t^{2/3} \propto a$ for $\delta << 1$

$\Omega_m = 1, \Omega_\Lambda = 0, \Omega_k = 0$

ϕ=const (*linear growth = expansion rate)*

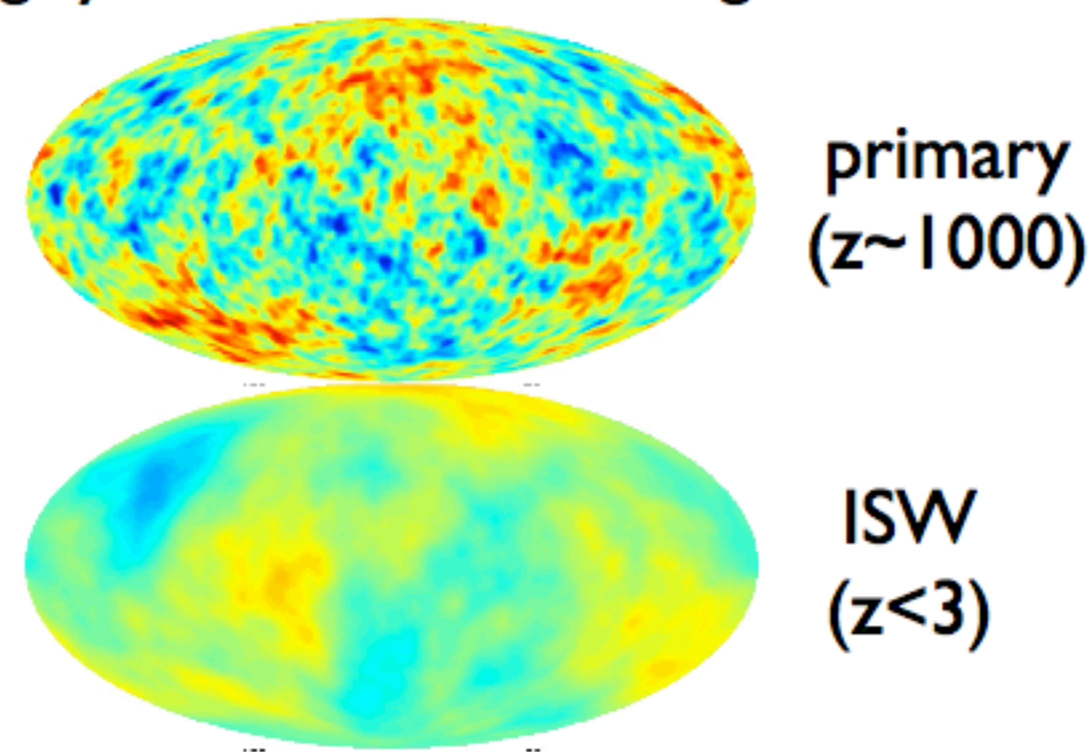
E '=E ⟹ No ISW effect

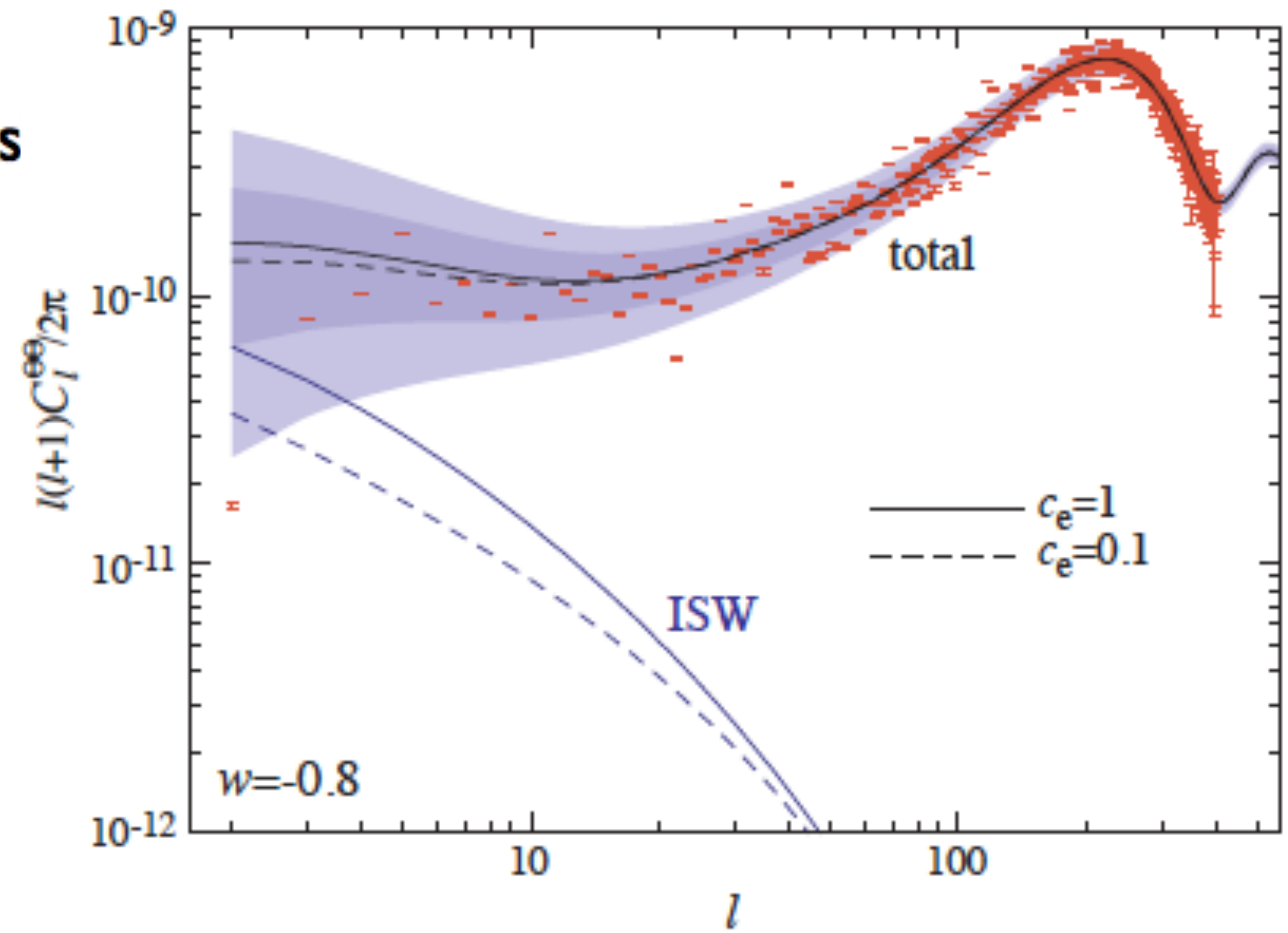


- Linear regime - Integrated Sachs-Wolfe effect (ISW) (Sachs & Wolfe 1967)
- Non-linear regime - Rees-Sciama effect (RS) (Rees & Sciama 1968)

But, we have the cosmic variance problem

Primary anisotropies (SW+acoustic peaks largely dominate the ISW signal



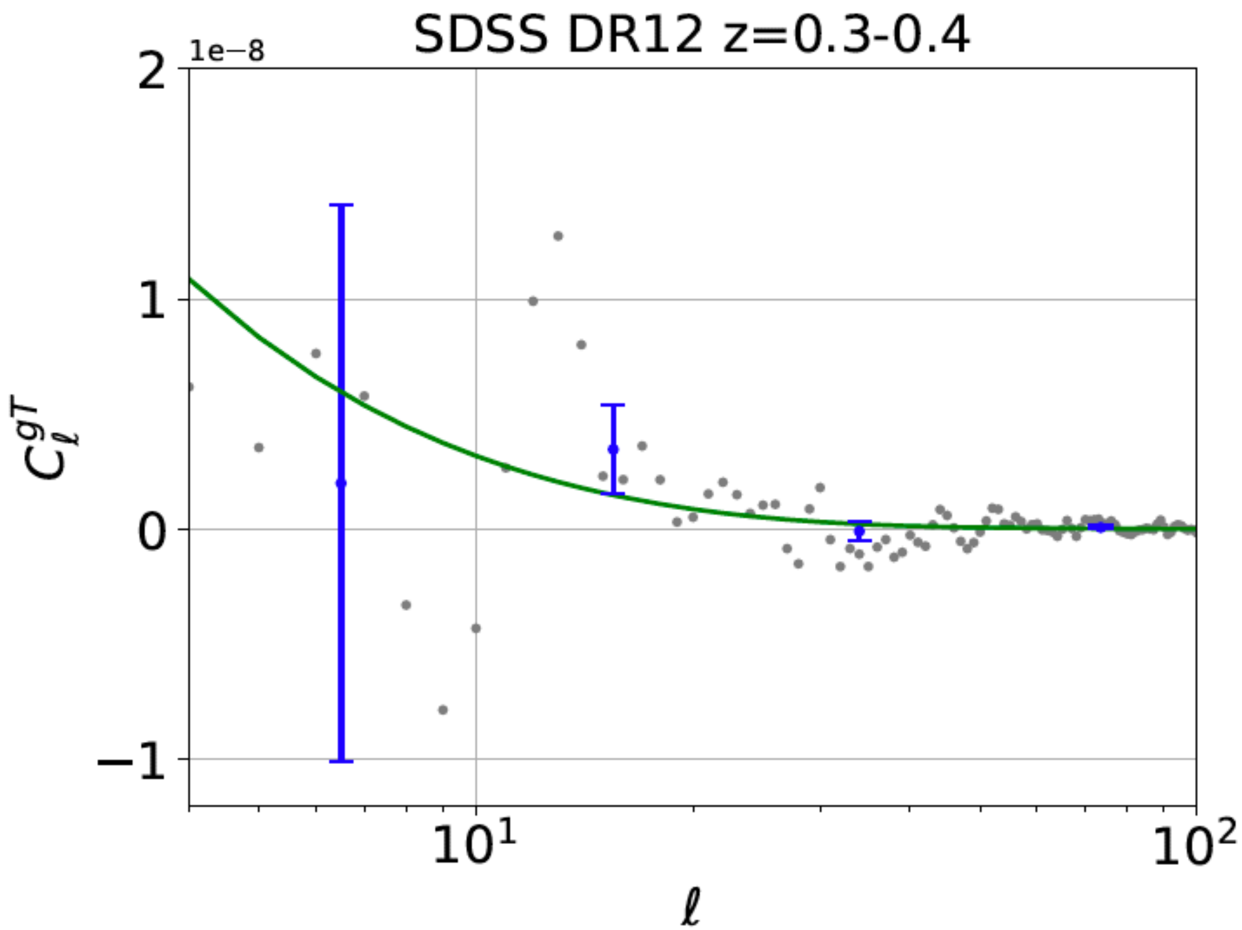


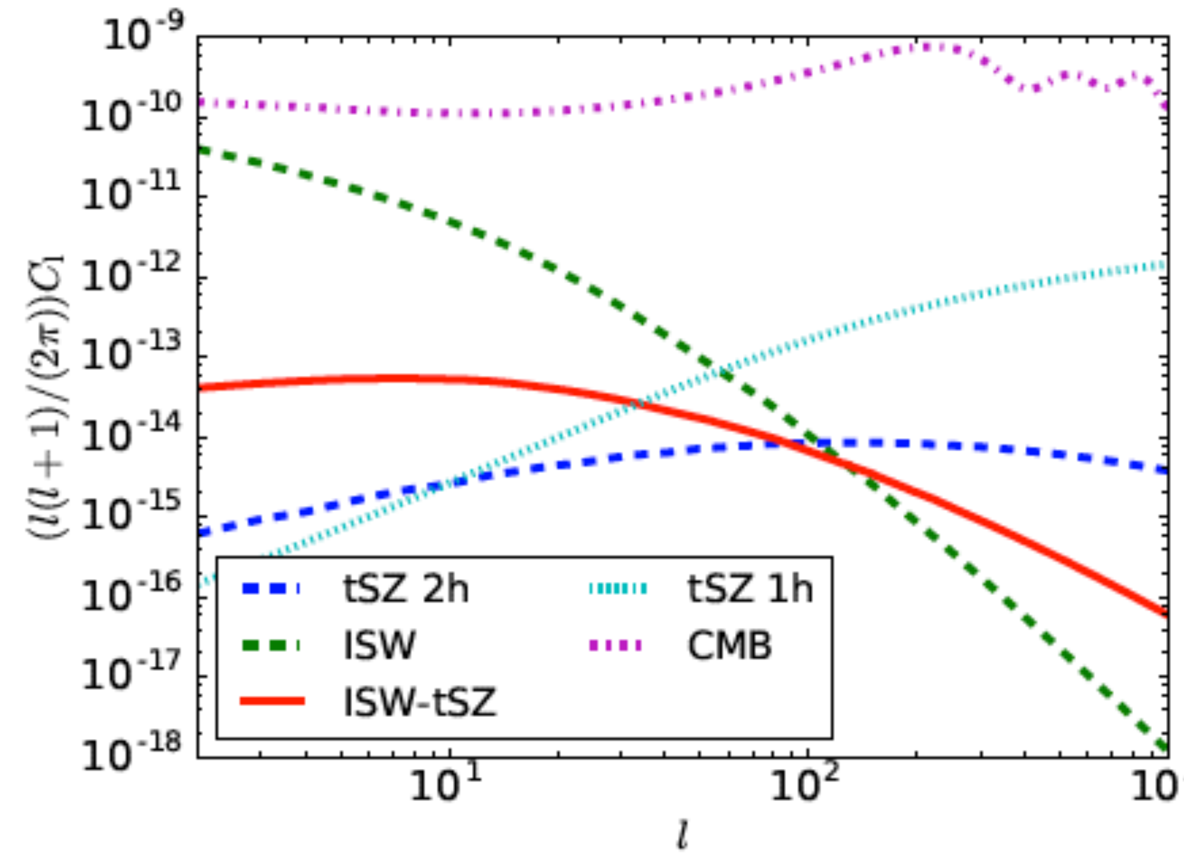
Large-scale anisotropies are dominated by cosmic variance (impossible to extract the ISW signal from CMB maps alone)

Solution: Cross-correlate with other probes of dark energy, which has large sky coverage (e.g. optical, X-ray or radio surveys of galaxies, tSZ signal)

Solution: cross-correlate with others

thermal SZ effect—ISW cross-correlation
(Creque-Serbinowski et al. 2016)



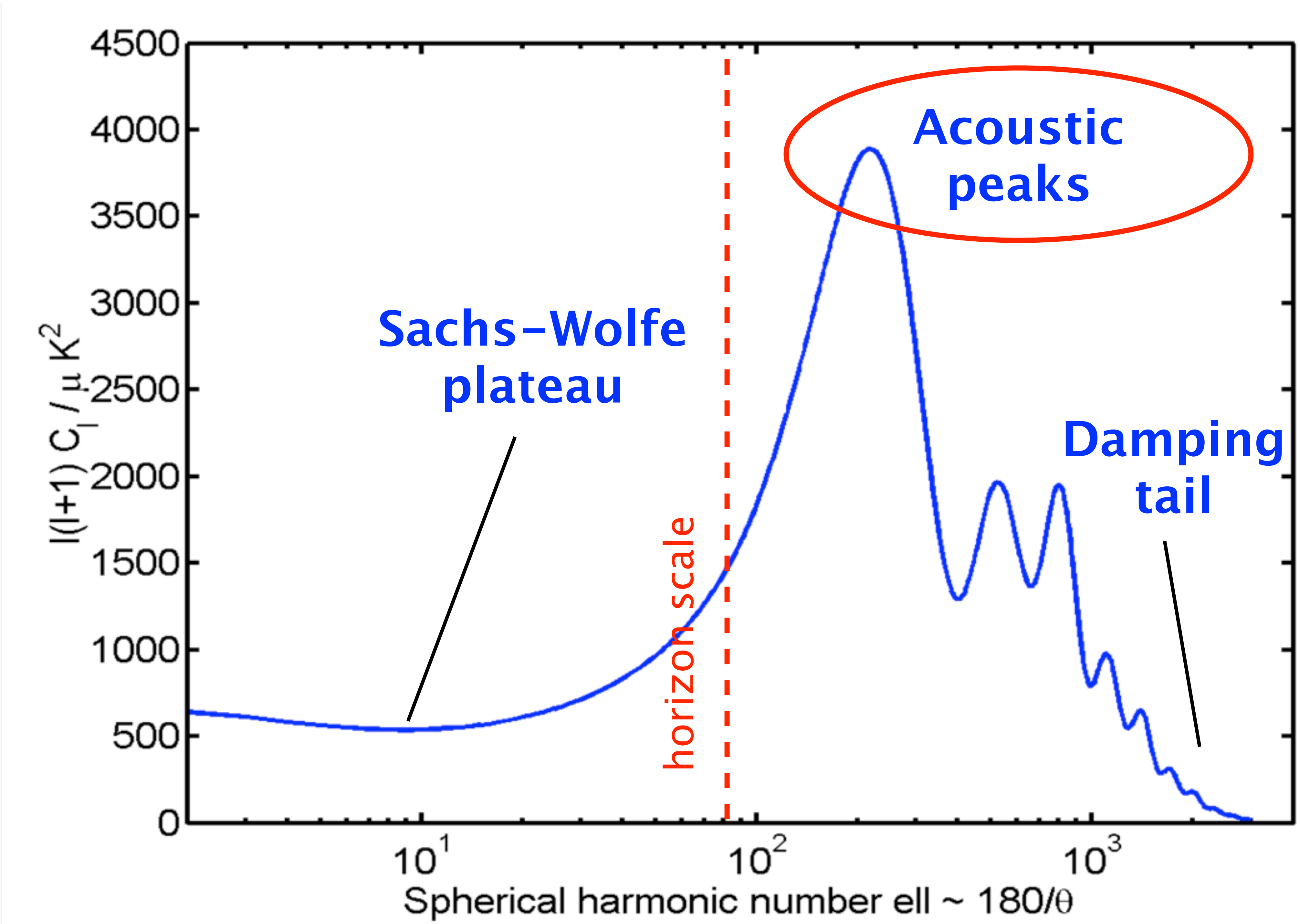


Two examples. The plot above shows cross power spectrum of SDSS galaxies and the CMB temperature (from Stölzner et al. 2018). ISW detection at roughly 4.5σ.

FIG. 1. The ISW power spectrum $C_l^{\rm isw}$ (green) and the two-halo contribution $(y, 2h)$ to C_l^{yy} (blue) are shown in dashed lines, while C_l^{yT} is shown in solid red. The one-halo contribution $(y, 1h)$ to C_l^{yy} is dotted. The CMB power spectrum is shown dot-dashed for comparison.

Power spectrum: primary anisotropies

Photon-baryon fluid interactions within the horizon



Sources of primary anisotropies

Quantum density fluctuations in the dark matter were amplified by inflation.
Gravitational potential wells (or "hills") developed, baryons fell in (or moved away).

Various related physical processes affected the CMB photons:

- **Perturbations in the gravitational potential (Sachs-Wolfe effect):** photons that last scattered within high-density regions have to climb out of potential wells and are thus redshifted

- **Intrinsic adiabatic perturbations (the acoustic peaks):** in high-density regions, the coupling of matter and radiation will also compress the radiation, giving a higher temperature. This creates sound-like waves.

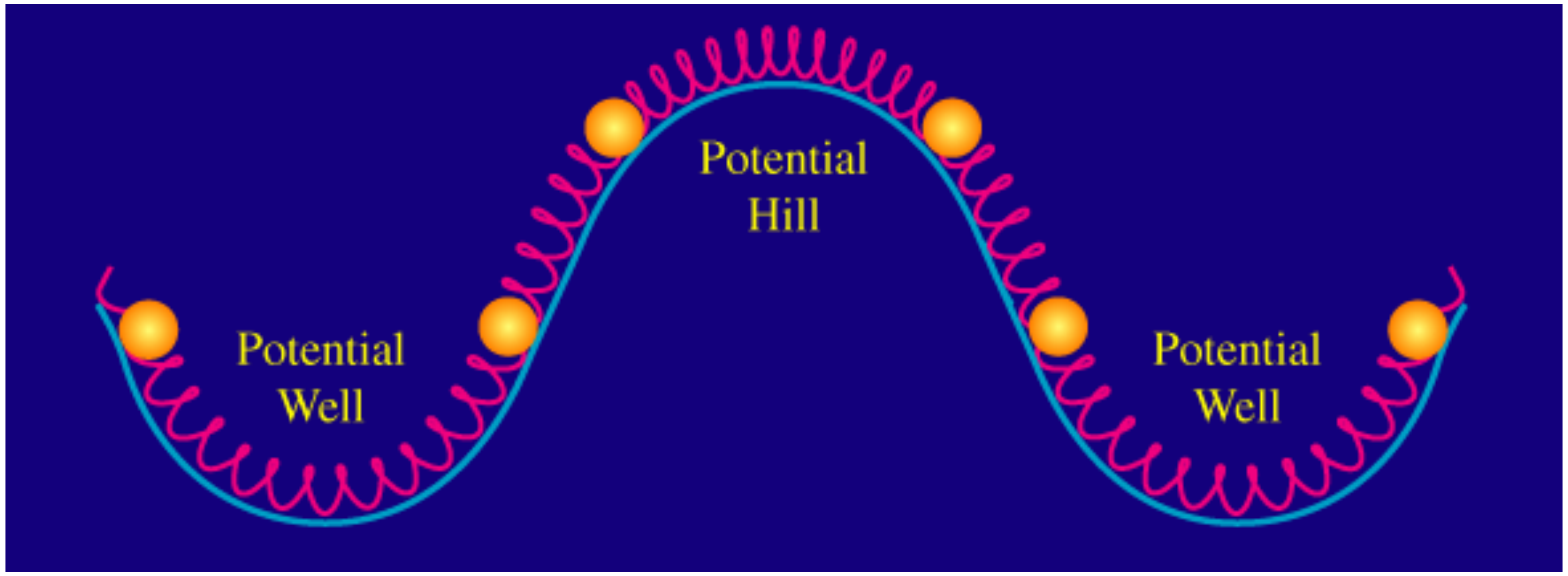
- **Velocity perturbations (the Doppler troughs):** photons last-scattered by matter with a non-zero velocity along the line-of-sight will receive a Doppler shift

Acoustic oscillations

- Dark matter already created the perturbations in gravitational potential
- Baryons fall into these potential wells: Photon baryon fluid heats up
- Radiation pressure from photons resists collapse, overcomes gravity, expands: Photon-baryon fluid cools down
- Oscillating cycles exist on all scales. All of these standing waves stop oscillating at recombination (when photons and baryons decouple).

Credit: Wayne Hu

Springs: photon pressure



Balls: baryon mass

Acoustic peaks

Oscillations took place on all scales. We see temperature features from modes which had reached the extrema

- Maximally compressed regions were hotter than the average
  Recombination happened later, corresponding photons experience less red-shifting by Hubble expansion: HOT SPOT

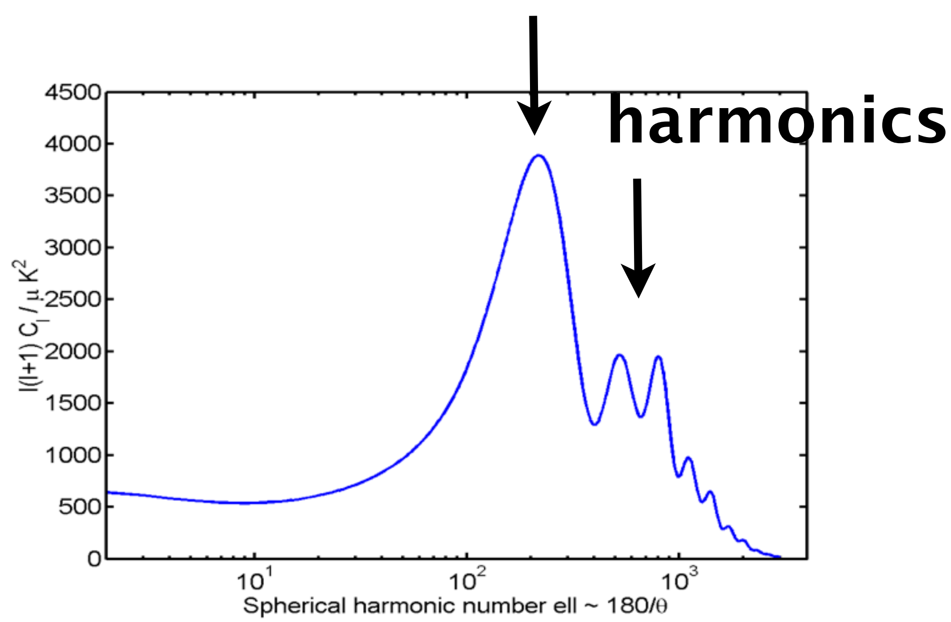
- Maximally rarified regions were cooler than the average
  Recombination happened earlier, corresponding photons experience more red-shifting by Hubble expansion: COLD SPOT

Harmonic sequence, like waves in pipes or strings:

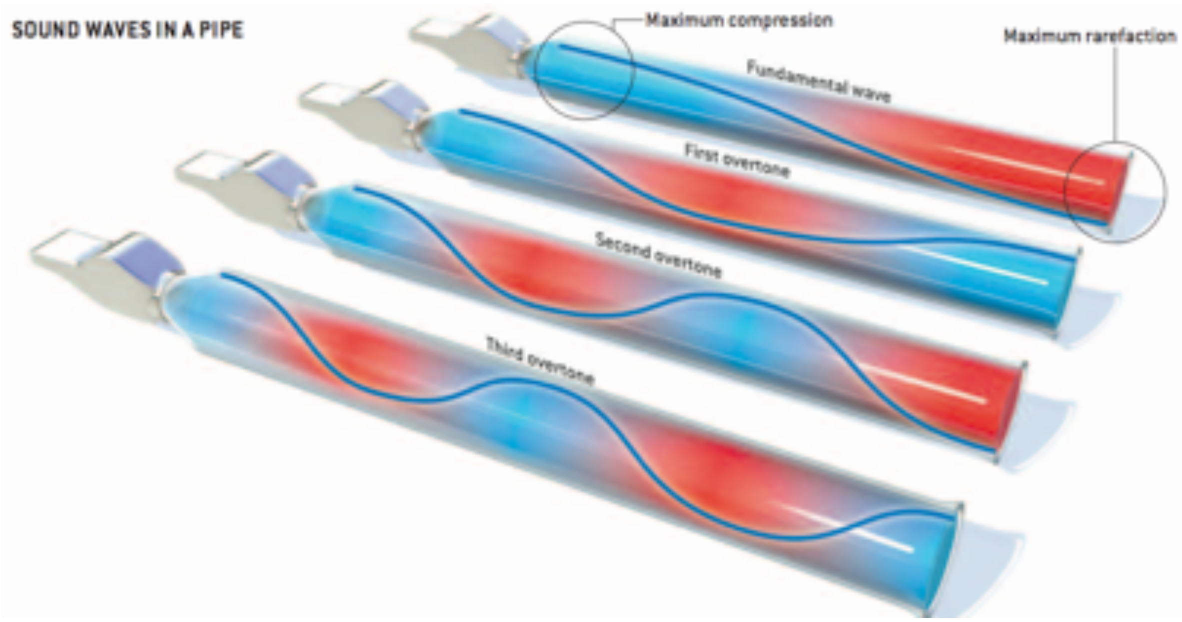
2nd harmonic: mode compresses and rarifies by the time of recombination
3rd harmonic: mode compresses, rarifies, compresses

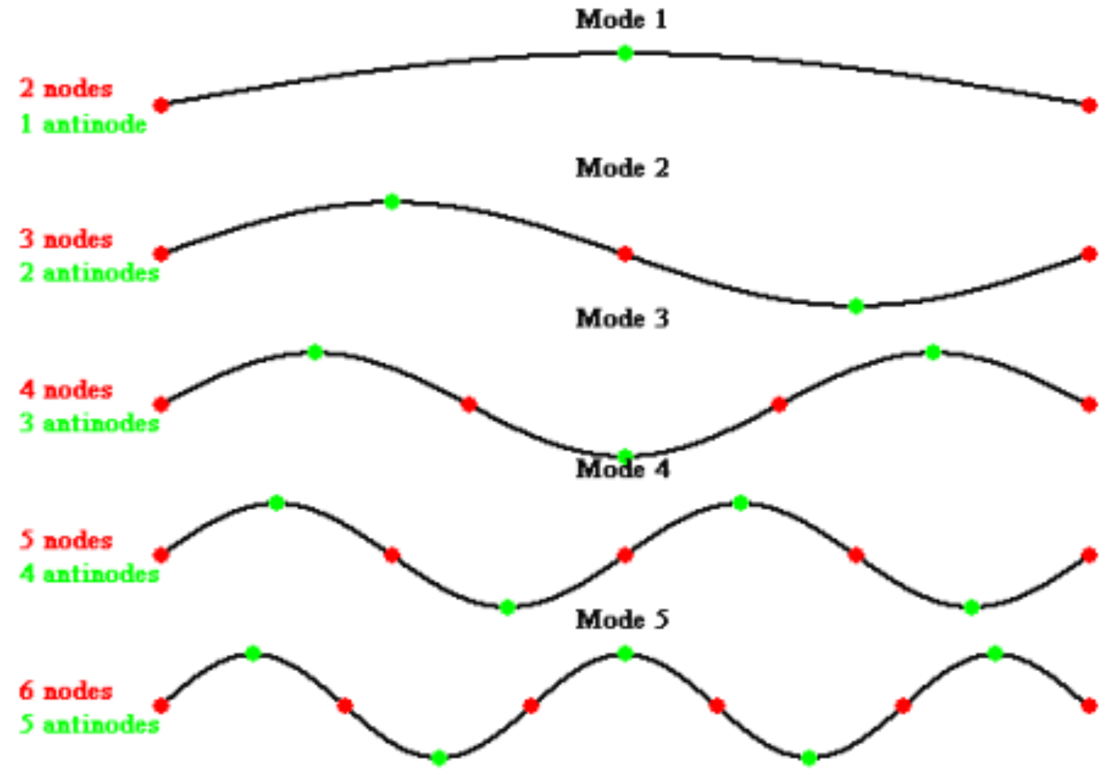
➡ 2nd, 3rd, .. peaks

1st peak



Harmonic sequence





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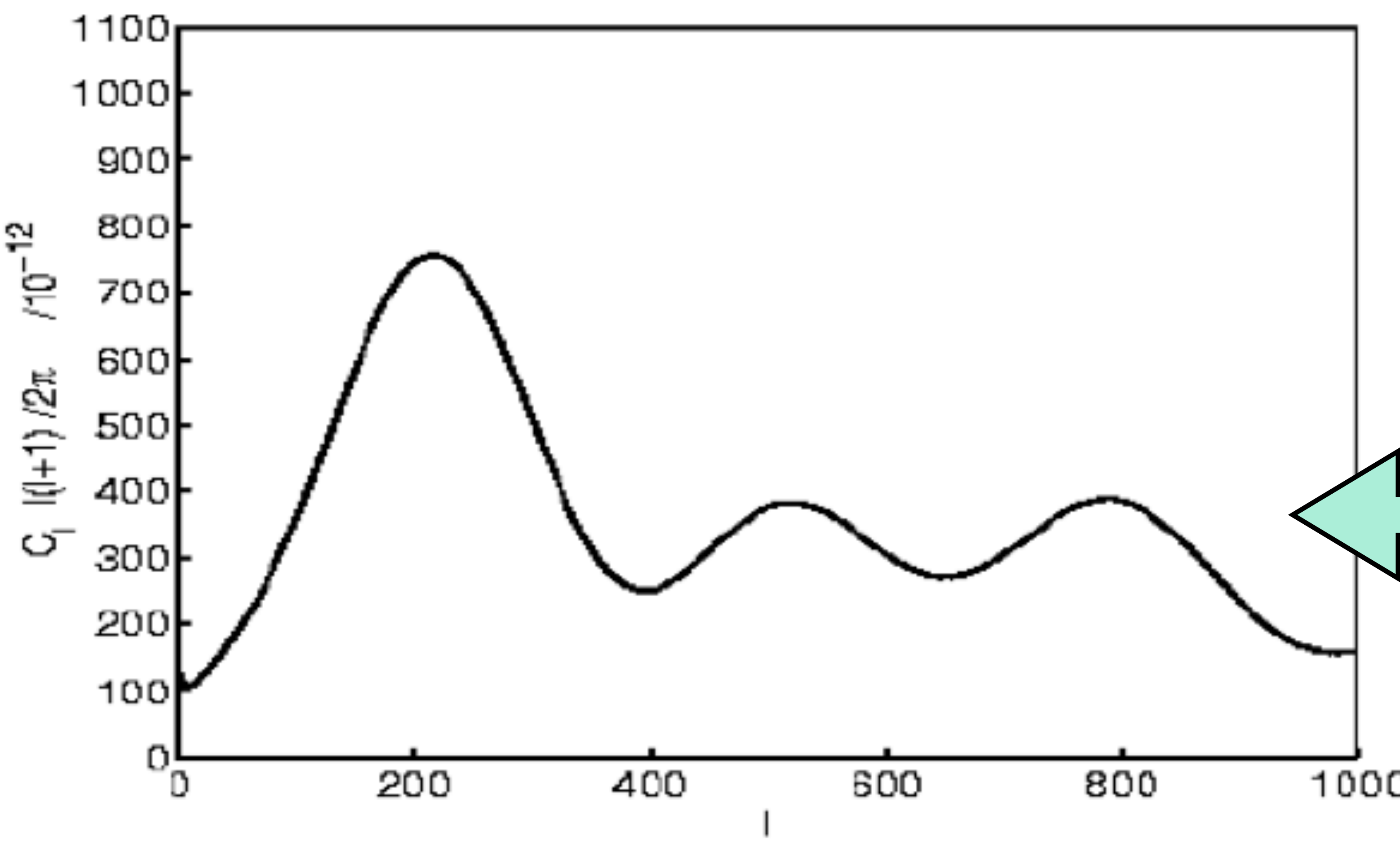


Modes with half the wavelengths oscillate twice as fast ($\nu = c/\lambda$).

Peaks are equally spaced in ℓ

Acoustic oscillations theory

Equations of motion for self-gravitating non-relativistic gas
(here photon-baryon fluid):

Continuity eqn.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

Euler eqn.

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{\nabla p}{\rho} - \nabla \Phi$$

Poisson eqn.

$$\nabla^2 \Phi = 4\pi G \rho$$

From these three we get
the perturbation equation
for the photon gas

For radiation-dominated era, set $p=(1/3)\varrho c^2$

$$\begin{aligned} \dot{\delta} + \tfrac{4}{3}\nabla \cdot \vec{u} &= 0 \\ \dot{\vec{u}} + 2H\vec{u} &= -\frac{\frac{1}{4}c^2 \nabla \delta + \nabla \delta \Phi}{a^2} \\ \nabla^2 \delta \Phi &= 8\pi G \rho_0 a^2 \delta \end{aligned}$$

$$\frac{3}{4}\ddot{\delta} + \frac{3}{2}H\dot{\delta} = \frac{c^2 \nabla^2 \delta}{4a^2} + 8\pi G \rho_0 \delta$$

We want to solve this for δ.

Acoustic oscillations theory

(perturbation equation from previous slide)

$$\frac{d^2\delta}{dt^2} + 2H\frac{d\delta}{dt} = \frac{c^2}{3a^2}\nabla^2\delta + \frac{32}{3}\pi G\rho_0\delta \qquad (8.12)$$

Now let us introduce the *conformal time* τ, defined by

$$\tau = \int_0^t \frac{dt'}{a(t')}$$

We can now write the second derivative $d^2\delta/dt^2$ as

$$\frac{d^2\delta}{dt^2} = \frac{1}{a^2}\frac{d^2\delta}{d\tau^2} + H\frac{d\delta}{dt}$$

so Eq. (8.12) becomes

$$\frac{d^2\delta}{d\tau^2} + 3Ha\frac{d\delta}{d\tau} = \frac{c^2}{3}\nabla^2\delta + \frac{32}{3}\pi a^2 G\rho_0\delta$$

Acoustic oscillations theory

In Fourier space, where ω this time belongs to the *conformal* time τ, we thus obtain the dispersion relation:

$$\omega^2 - 3Ha\dot{\omega} = \frac{c^2}{3}k^2 - \frac{32}{3}\pi a^2 G\rho_0 \tag{8.16}$$

If we assume that we have large enough k and ω that we can ignore both the gravitational term on the right and the term proportional to H on the left, then we arrive at:

$$\omega^2 = \frac{c^2}{3}k^2 \tag{8.17}$$

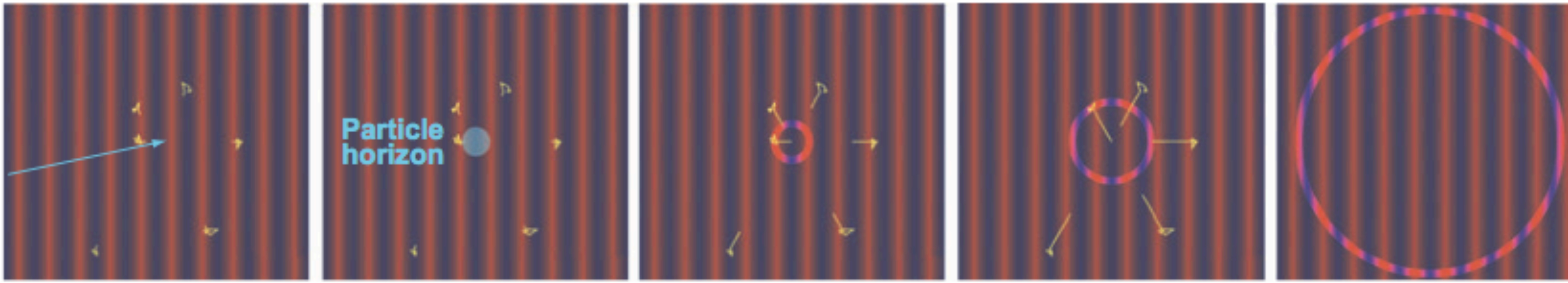
This means that we have solutions of the form

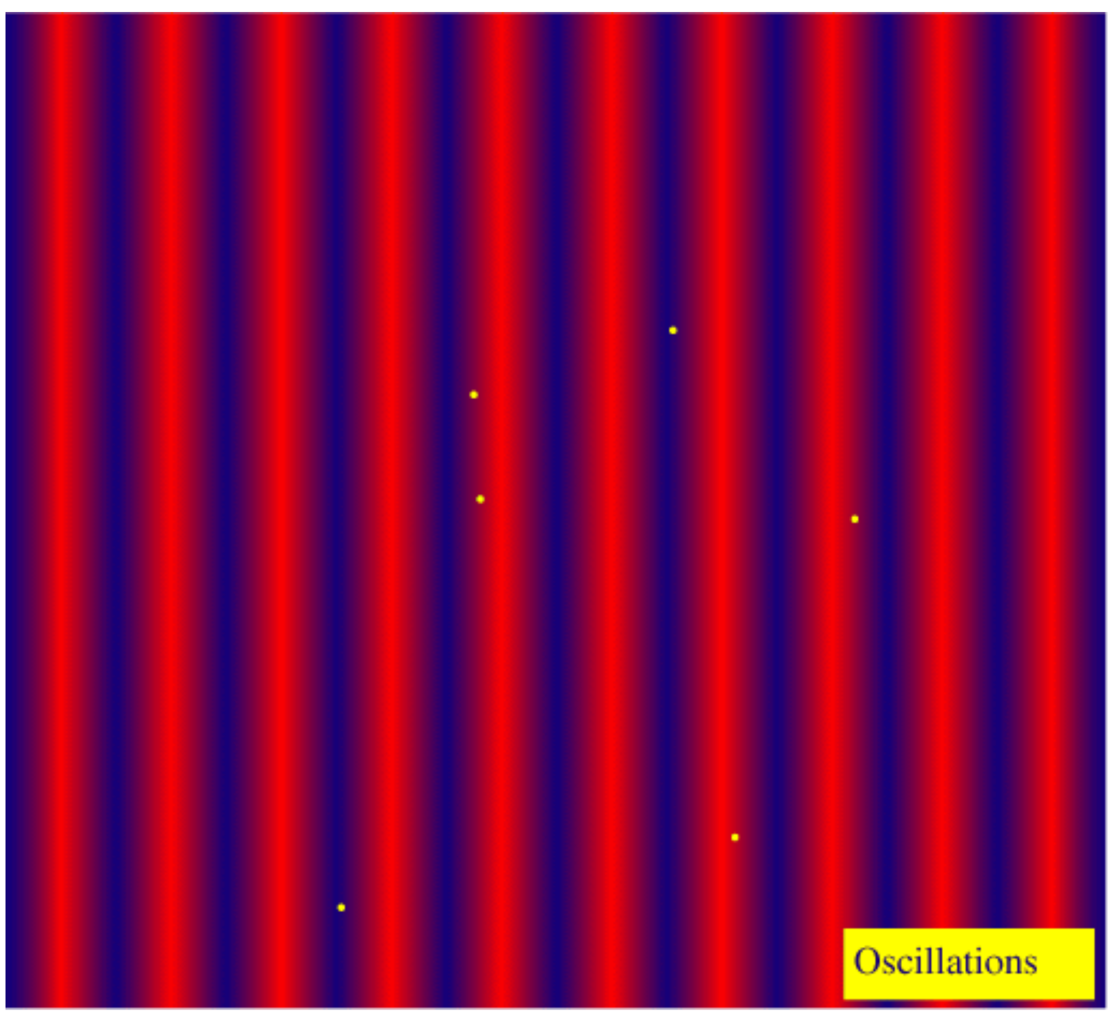
$$\delta(x,\tau) = \delta_0 \cos(kx+\varphi)\cos\left(\frac{c}{\sqrt{3}}k[\tau - \tau_{\mathrm{start}}(k)]\right) \tag{8.18}$$

The last equation is a standing wave with an interesting property: its phase is fixed by the factor in parenthesis. This means that for every wave number k we know precisely what the phase of the oscillating standing wave at the time of the CMB release is. For some modes this phase may be $\pi/2$, in which case the density fluctuation has disappeared by the time of CMB release, but the motion is maximum. For others the density fluctuation may be near maximum (phase 0 or π). *This gives the distinct wavy pattern in the power spectrum of the CMB.*

Angular variations

Density fluctuation on the sky from a single *k* mode, and how it appears to an observer at different times:





These frames show one super-horizon temperature mode just after decoupling, with representative photons last scattering and heading toward the observer at the center. *(From left to right)* Just after decoupling; the observer's particle horizon when only the temperature monopole can be detected; some time later when the quadrupole is detected; later still when the 12-pole is detected; and today, a very high, well-aligned multipole from just this single mode in k space is detected.

Animation by Wayne Hu

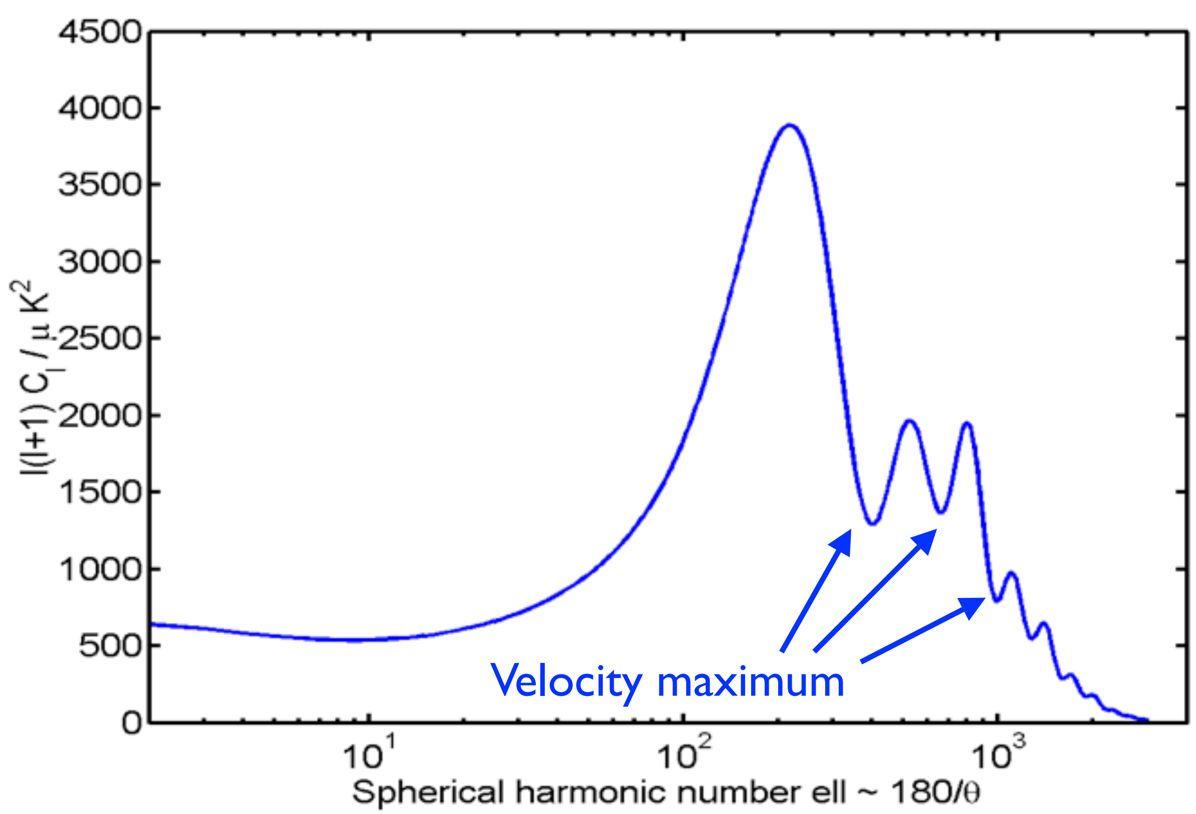
The valleys: Doppler (velocity) effect

Times in between maximum compression/rarefaction, modes reached maximum velocity

This produced temperature enhancements via the Doppler effect (non-zero velocity along the line of sight)

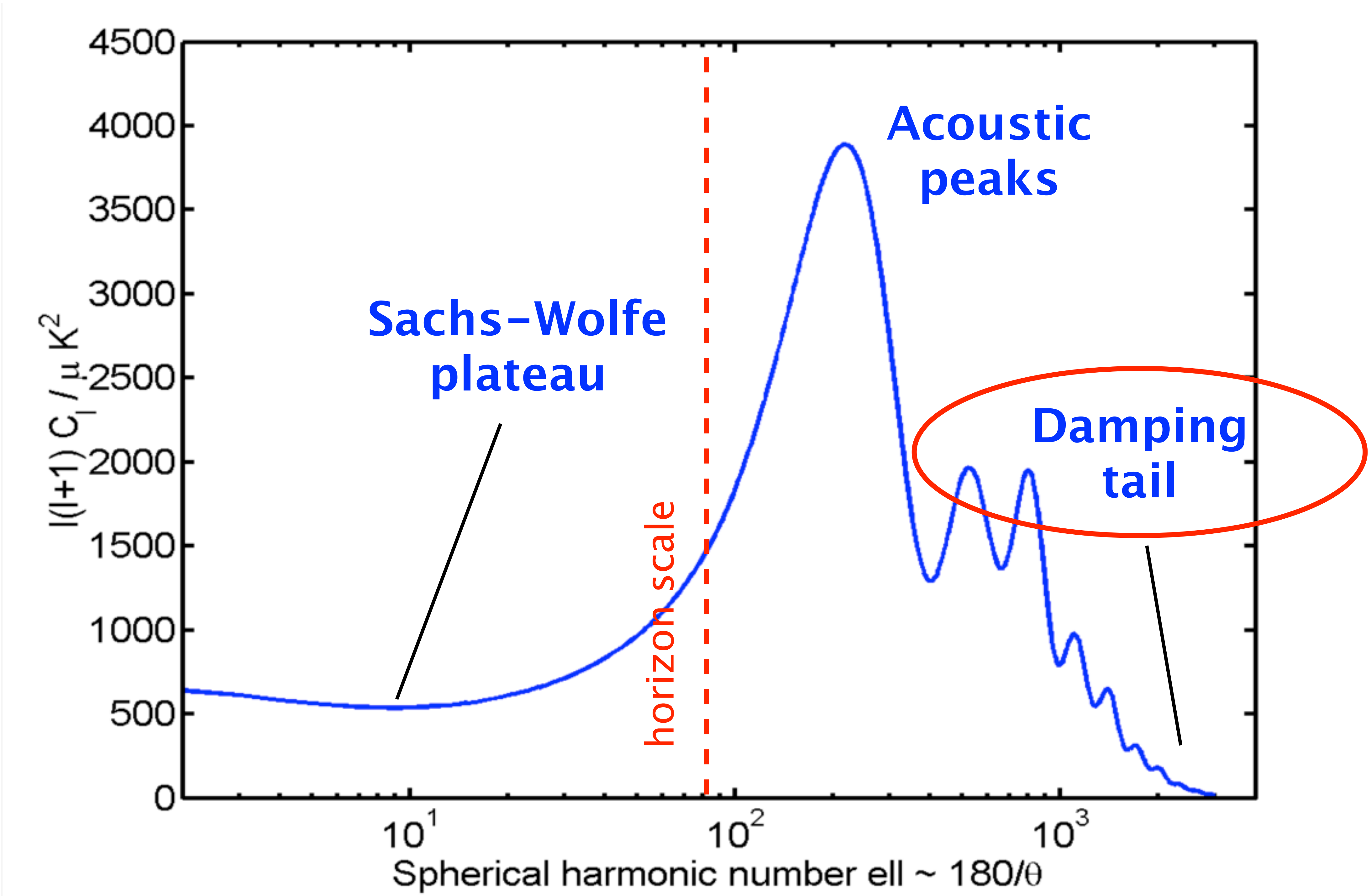
<u>This contributes power in between the peaks</u>

➡ **Power spectrum does not go to zero** (it does at very high l-s)



Power spectrum: primary anisotropies

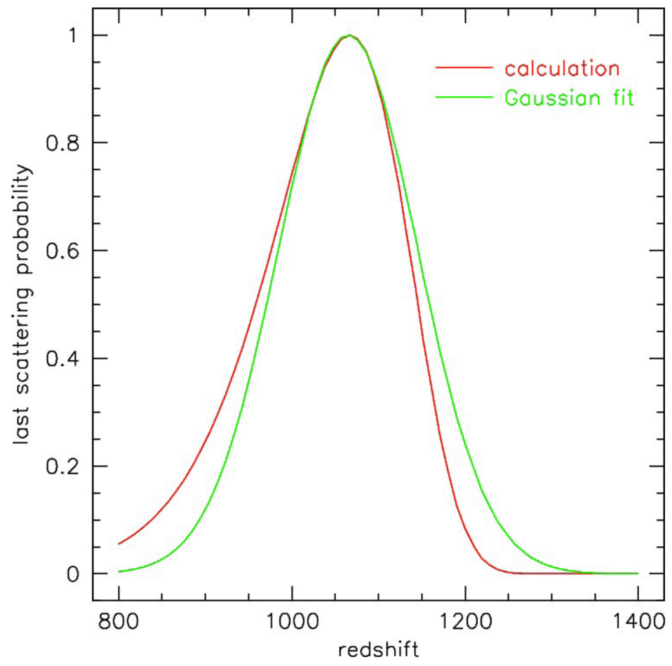
Mixing of anisotropies at the time of last scattering



Damping and diffusion

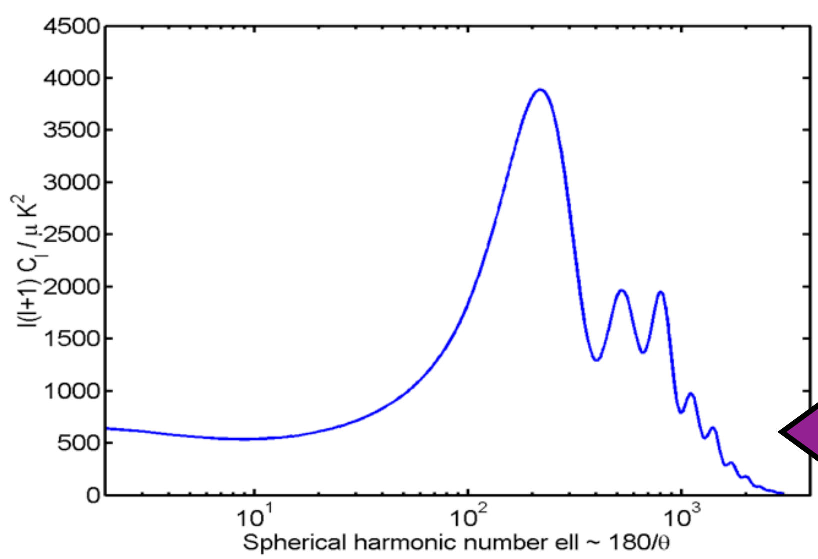
- Photon diffusion suppresses fluctuations in the baryon-photon plasma. This is known as diffusion damping or Silk damping (Silk 1968).

- During recombination the mean free path of photons increase rapidly! These photons tend to travel between the cold and hot spots and homogenize the medium. Density perturbations with wavelengths which are shorter than the photon mean free path are damped (the hot and cold parts mix together).



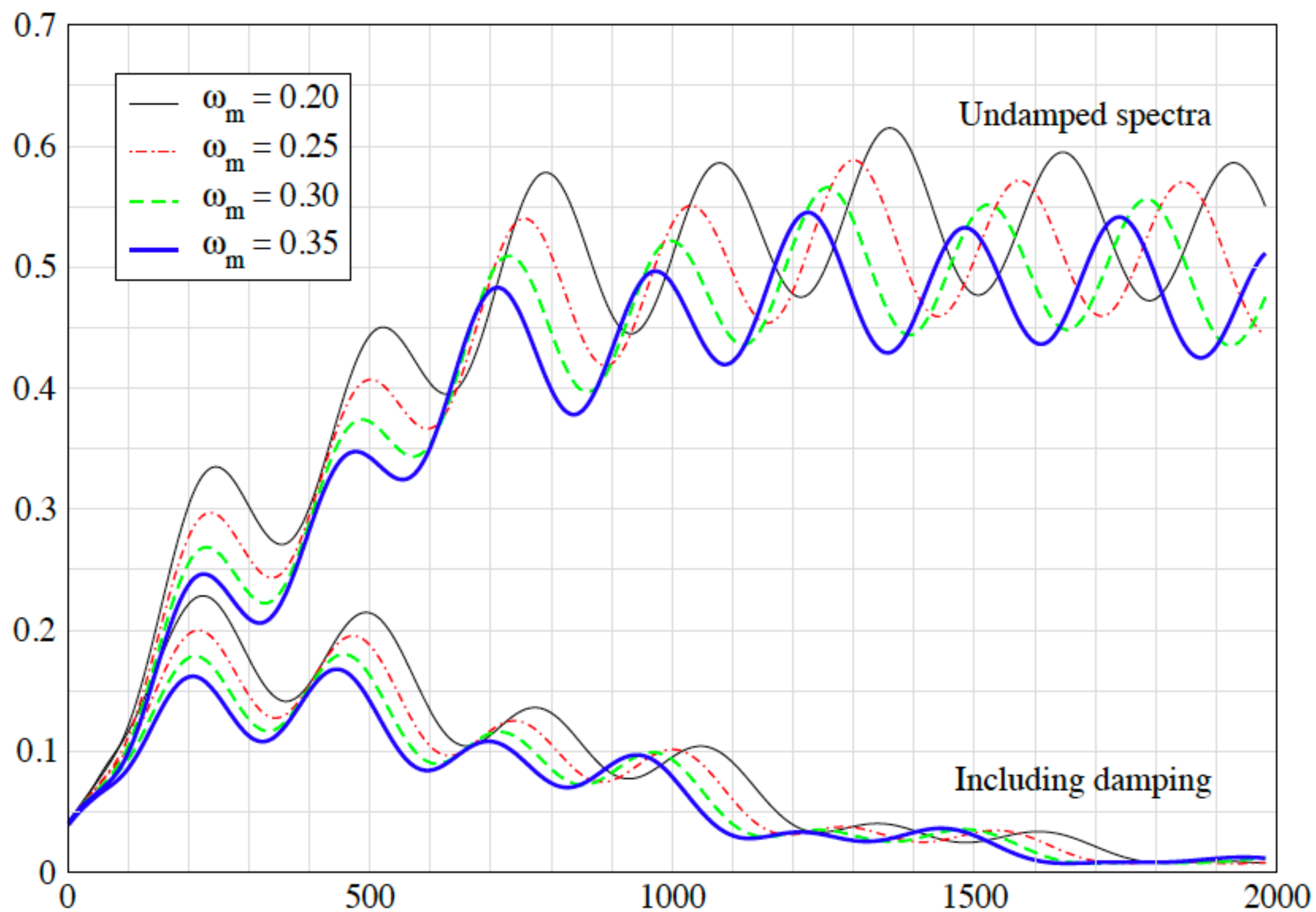
Thickness of the LSS is comparable to the oscillation scales

In the power spectrum scales below ~10 arcmin ($l > \sim 10^3$) are damped.



Power falls off

Impact of the diffusion damping



The figure on left shows the dramatic changes on the temperature anisotropy spectrum imposed by diffusion damping, for four different values of matter content in the universe, with baryonic density ω_b=0.01 (fig. from R. Keskitalo, master's thesis U. Helsinki).

Diffusion damping (or silk damping) is nothing but photons carrying energy from one part of the fluid to another, which acts towards homogenizing the fluid temperature over a scale that is comparable to the mean free path of the photons at the time of recombination.

The C_l spectrum is damped as e^{-k^2/k_D^2}, where k_D is the damping scale.

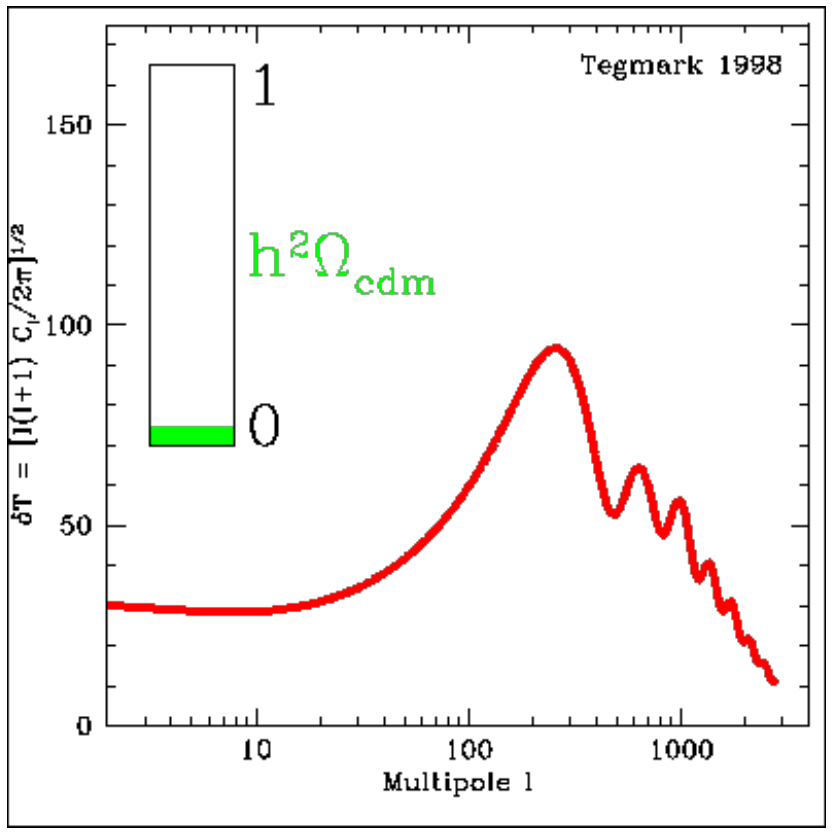
$$\ell_D \sim k_D d_A^c(t_{\rm dec})$$

$$k_D^{-1} \sim \frac{1}{\text{few}} \frac{a_0}{a} \sqrt{\frac{\lambda_\gamma(t_{\rm dec})}{H_{\rm dec}}}$$ from random walk

ℓ_D ~1500 ⇒ θ ~ 7 arcmin

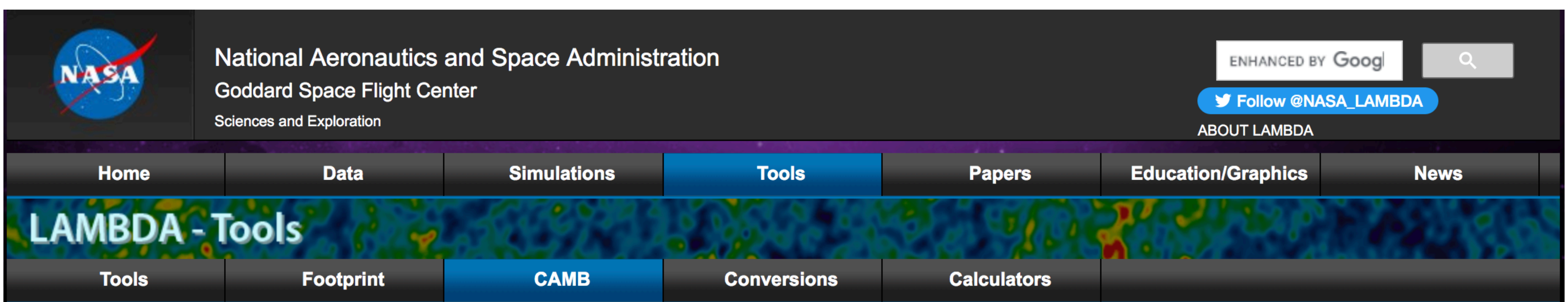
Exercise (next week)

Learn how the CMB power spectrum changes with respect to selected cosmological parameters, using an online CMB power-spectrum calculator like CAMB.



Select any one parameter (say Ω_m) and try to make a gif-animation with 10-15 steps

https://lambda.gsfc.nasa.gov/toolbox/camb_online.html



Questions?

Feel free to email me or ask questions in our eCampus Forum