

# An Introduction to the **Cosmic Microwave Background**

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#### astro8405: The Cosmic Microwave Background

Aktionen 🗸

This course intends to give you a modern and up-to-date introduction to the science and experimental techniques relating to the Cosmic Microwave Background. No prior knowledge of cosmology is necessary, your prerequisite are a basic understanding of electrodynamics and thermal physics and some familiarity with Python programming.

# Lecture 3:

# Spectral distortion and Recombination physics

#### What we aim to learn today



# Recap: Cosmology basics

$$H^{2} = \left(\frac{\dot{a}}{a}\right)^{2} = \frac{8\pi\mathcal{G}}{3c^{2}}\rho_{\rm R} + \frac{8\pi\mathcal{G}}{3c^{2}}\rho_{\rm M} - \frac{kc^{2}}{a^{2}} + \frac{\Lambda}{3}$$

The density of radiation, matter, spatial curvature, and a cosmological constant changes with the scale factor of the universe as *a*<sup>-4</sup>, *a*<sup>-3</sup>, *a*<sup>-2</sup>, and *a*<sup>0</sup>, respectively.

Radiation dominationMatter domination
$$\frac{\dot{a}^2}{a^2} \propto a^{-4}$$
,  $a(t) \propto t^{1/2}$ ,  $H(t) = \frac{1}{2t}$  $\frac{\dot{a}^2}{a^2} \propto a^{-3}$ ,  $a(t) \propto t^{2/3}$ ,  $H(t) = \frac{2}{3t}$ 

#### **Matter-radiation equality**

 $\frac{\rho_m}{\rho_r} \propto \frac{a^{-3}}{a^{-4}} = a, \quad \text{so we can find this scale factor of matter-radiation equality by} \\ \text{knowing the matter and radiation densities today (when a = 1)} \qquad a_{\text{eq}} = \frac{\rho_{r0}}{\rho_{m0}}.$ 

$$\rho_{\gamma} = \frac{\pi^2}{30} g_{\star,eff} T_{\gamma 0}^4 = 7.8 \times 10^{-34} \,\mathrm{g/cm^3}$$

$$\rho_{m0} = \frac{3\Omega_m H_0^2}{8\pi G} = 1.879 \times 10^{-29} \Omega_m h^2 \,\mathrm{g/cm^3}.$$

 $\Omega_m h^2 = \Omega_b h^2 + \Omega_{dm} h^2$ . From WMAP:  $\Omega_m h^2 = 0.128 \pm 0.008$ .

$$a_{\rm eq} = 4.15 \times 10^{-5} (\Omega_m h^2)^{-1}.$$

$$1 + z_{\rm eq} = 2.4 \times 10^4 \Omega_m h^2,$$
  

$$\approx 3070$$

#### Time scales vs expansion rate

To understand the relative importance of different scattering process, we need to compare their characteristic time-scales with the expansion time-scale of the universe.

Whenever an interaction rate  $\Gamma$  drops below the expansion rate of the universe  $\Gamma < H$ we can consider the corresponding reaction "frozen". It becomes negligible.

We will also see this later for recombination and decoupling: When the recombination rate drops below the expansion rate, recombination freezes and the ionization fraction remains constant. When the Thomson scattering rate of photons on electrons falls below the expansion rate, photons become free to propagate without further scattering.

#### Matter-radiation equality and thermal decoupling

The universe becomes matter-dominated after redshift z~3000, when the densities of matter and radiation was equal. The process of Compton scattering, however, keeps the kinetic energy of matter in equilibrium with the CMB photons *far after* the cosmic recombination by scattering with the *residual electrons*, down to redshift z~200. Thus, "thermal decoupling" means the decoupling of matter and *temperatures,* and it is important for neutral H 21cm observation.



From Chluba (2018), arXiv:1806.02915

From Zaroubi (2012), arXiv:1206.0267

# Comptonization and Compton cooling



The energy exchange between electrons and photons is carried out by Compton scattering. One can distinguish two different effects, Comptonization and Compton cooling, depending upon whether energy is flowing from the electrons to the photons, or otherwise. The effectiveness of these processes are determined by the timescales  $t_{ey}$  and  $t_{ye}$ .

The time-scale in which electrons transfer energy to photons is the *Comptonization* time-scale:

$$t_{\rm e\gamma} = \frac{t_{\rm T}}{4\theta_{\rm e}} \simeq 4.9 \times 10^5 t_{\rm T} \left[\frac{1+z}{1100}\right]^{-1} \simeq 1.2 \times 10^{29} (1+z)^{-4} \, {\rm sec} \, .$$

where  $t_T$  is the Thomson scattering time-scale and  $\theta_e$  is the dimensionless electron temperature (see last lecture). Comparing this to the Hubble expansion time-scale, H(t), we found that at redshift  $z_K = 5x10^4$  the Comptonization process is inefficient, and that ends the era of µ-distortion.

The time-scale where electrons are heated by photons is much shorter! This is because every electron has roughly 1.9x10<sup>9</sup> photons to scatter with. This is the *Compton-cooling* time-scale:

$$t_{\gamma e} = \frac{\rho_{\text{th}}}{\rho_{\gamma}} t_{e\gamma} \simeq \frac{3N_{\text{H}}(1 + f_{\text{He}} + X_{e})}{8\rho_{\gamma}/(m_{\text{e}}c^{2})} t_{\text{T}} \simeq 0.31 t_{\text{T}} (1 + z)^{-1} \simeq 7.3 \times 10^{19} (1 + z)^{-4} \text{ sec} \,.$$

This is about 1.6x10<sup>9</sup> times shorter than the Comptonization time, meaning that photons and baryons (i.e. matter) remain in good thermal contact until very late, to about z~200.

### Recap: $\mu$ and y distortions



## Recap: $\mu$ and y distortions



#### The BE-spectrum & µ-distortion

When many scatterings occur, the photon spectrum is driven towards an equilibrium via the Comptonization process, and the result is a Bose-Einstein (BE) spectrum:



The spectral distortion is more prominent at lower frequencies:

 $n_{\rm BE} = \frac{1}{{\rm e}^{x_{\rm e}+\mu_0}-1} \approx \frac{1}{{\rm e}^{x_{\rm e}}-1} - \frac{G(x_{\rm e})}{x_{\rm e}}\mu_0 + O(\mu_0^2).$ 

$$\Delta n_{\rm BE} \approx \frac{\mu e^x}{(e^x - 1)^2} \left(\frac{x}{2.19} - 1\right)$$

This second form is suitable for calculating spectral intensities, which we get by expanding around the reference blackbody temperature for small energy injection ( $\mu \ll 1$ )

$$(T_e - T)/T \approx \frac{\pi^2}{18\zeta(3)}\mu_0 \approx 0.456\,\mu_0$$

$$\frac{\Delta I_{\nu}}{I_{\nu}^{\rm bb}} \approx \mu_0 \, \frac{x e^x}{(e^x - 1)} \left[ 0.456 - \frac{1}{x} \right]$$

The zero-crossing frequency (for  $\mu$ =0.5):  $v_{\rm cr} \approx 124 \,\text{GHz} (1 - 0.304 \,\mu \ln \mu)$  $\approx 158 \,\text{GHz}$ 

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 $v_{\rm cr} \approx 124 \,\mathrm{GHz} \left(1 - 0.304 \,\mu \ln \mu\right)$ 

 $\approx 158 \, \mathrm{GHz}$ 

# Compton-y distortion

Compton-y distortion is created when scattering between electrons and photons are inefficient in causing an energy exchange. This is typically the case when the electron and photon temperatures are vastly different, so that electrons practically don't change energy after scattering. The energy exchange is parametrized by the Compton *y*-parameter, and we typically have y <<1.

The Kompaneets equation (right) is the general equation describing the Comptonization problem, which can be solved analytically for the limiting case of a small y-parameter ( $\Delta \tau \ll 1$ ):

$$\frac{\partial n}{\partial \tau} \equiv \frac{\theta_{\rm e}}{x^2} \frac{\partial}{\partial x} x^4 \left[ \frac{\partial}{\partial x} n + \frac{T_{\gamma}}{T_{\rm e}} n(1+n) \right],$$

$$\Delta n \approx \frac{\Delta \tau \,\theta_{\rm e}}{x^2} \frac{\partial}{\partial x} x^4 \left[ \frac{\partial}{\partial x} n_{\rm bb} + \frac{T_{\gamma}}{T_{\rm e}} n_{\rm bb} (1+n_{\rm bb}) \right] \approx \frac{\Delta \tau \,(\theta_{\gamma} - \theta_{\rm e})}{x^2} \frac{\partial}{\partial x} x^4 n_{\rm bb} (1+n_{\rm bb})$$

$$\approx \Delta \tau \,(\theta_{\gamma} - \theta_{\rm e}) \left[ 4x n_{\rm bb} (1+n_{\rm bb}) - x^2 n_{\rm bb} (1+n_{\rm bb}) (1+2n_{\rm bb}) \right]$$

$$\approx \Delta \tau \,(\theta_{\rm e} - \theta_{\gamma}) G(x) \left[ x \frac{e^x + 1}{e^x - 1} - 4 \right] \equiv \Delta \tau \,(\theta_{\rm e} - \theta_{\gamma}) \, Y_{\rm SZ}(x),$$

$$G(x) \equiv x e^x / (e^x - 1)^2$$

$$Y_{\rm SZ}(x) = G(x) \left[ x \, \frac{{\rm e}^x + 1}{{\rm e}^x - 1} - 4 \right] \approx \begin{cases} -\frac{2}{x} & {\rm for} & x \ll 1 \\ x(x - 4){\rm e}^{-x} & {\rm for} & x \gg 1. \end{cases}$$



# y-distortion special case: SZ effect

The Compton y-parameter depends on the number of scatterings (dependence on optical depth,  $\tau$ ) and the net energy transfer per scattering,  $\Delta v/v \simeq 4(\theta_e - \theta_\gamma)$ ,

$$y = \int_0^\tau \frac{k(T_e - T_\gamma)}{m_e c^2} \,\mathrm{d}\tau' = \int_0^t \frac{k(T_e - T_\gamma)}{m_e c^2} \sigma_\mathrm{T} N_e c \,\mathrm{d}t' \quad \overset{\sim}{\underset{\sim}{\longrightarrow}}$$

In the local universe we have  $T_{\gamma} \ll T_e$ such that the Compton y-parameter is simply proportional to the line of sight integral of the electron pressure (y << 1):

$$y = \int_0^\tau \frac{kT_{\rm e}}{m_{\rm e}c^2} \,\mathrm{d}\tau' \approx \theta_{\rm e}\,\tau$$



This is the thermal Sunyaev-Zeldovich effect, first studied by Sunyaev & Zeldovich (1968), for the scattering of CMB photons by thermal electrons inside galaxy clusters.

$$\frac{\Delta T_{\text{CMB}}}{T_{\text{CMB}}} \approx y \left( x \frac{e^x + 1}{e^x - 1} - 4 \right) = y f(x). \qquad \Delta I_v \approx I_0 y \frac{x^4 e^x}{(e^x - 1)^2} \left( x \frac{e^x + 1}{e^x - 1} - 4 \right) \equiv I_0 y g(x)$$

# Recombination and Decoupling

# Chronology of the universe

Event	Time since big bang	Redshift	Temperature (kT)
Inflation	10 <sup>-34</sup> s (?)		
QCD phase transition	20 µs	10 <sup>12</sup>	150 MeV
Neutrino decoupling	1 s	6 x 10 <sup>9</sup>	1 MeV
e-/e+ annihilation	6 s	2 x 10 <sup>9</sup>	500 keV
Big Bang Nucleosynthesis	3 min	4 x 10 <sup>8</sup>	100 keV
Matter-radiation equality	60 kyr	3400	0.75 keV
Recombination	260 - 380 kyr	1400 - 1100	0.33 - 0.26 eV
Decoupling of photons	380 kyr	1100	0.23 - 0.28 eV
Reionization	100 - 400 Myr (?)	10 -30 (?)	2.6 - 7.0 meV
Dark energy-matter equality	9 Gyr	0.4	0.33 meV
Today	13.7 Gyr	0	0.23 meV

# Brief summary of the BBN

- The first prediction of Big Bang Nucleosynthesis came from Gamow (The Alpher, Bethe, Gamow paper from 1948), which also conjectured the thermal radiation bath and the high photon-to-baryon ratio. But they erroneously predicted that all elements are cooked up during the BBN.
- ◆ BBN mainly produces He<sup>4</sup> (with small amounts of other elements, up to Li<sup>7</sup>). The nucleosynthesis is determined by the high photon temperature, the density is not high enough to run the triple-alpha reaction (3 <sup>4</sup>He → <sup>12</sup>C). The high density condition in met inside stellar interiors later.
- Higher element production is effective at temperature around 0.1 MeV. This is due to the *deuterium bottleneck*. The binding energy of D is 2.2 MeV and there are enough higher-energy photons to cause D to disintegrate, until the universe cools down to below ~100 keV.
- The abundance of D, He<sup>3</sup>, He<sup>4</sup>, and Li<sup>7</sup> are essentially fixed at the end of first three minutes. Since D is destroyed inside stars, its observation provides a lower limit on primordial D abundance and an upper limit on the photon-to-baryon ratio.

Brief summary of the BBN



## Recombination and reionization

At high redshift the Universe was completely ionized. As it expanded and cooled down, it went through several stages of recombination, starting with HeIII $\rightarrow$ HeII recombination (z ~ 7000), HeII $\rightarrow$ HeI recombination (z ~ 2500), and ending with the recombination of hydrogen (z ~ 1100). At low redshift (z ~ 10) the Universe eventually got reionized by the first sources of radiation that appeared in the Universe.



## Recombination and reionization

How do we know the universe was neutral? Mainly because from the absorption spectra of high-redshift quasars, whose photons bluewards of the Lyman alpha are completely absorbed. Even a very small amount of neutral hydrogen can completely absorb the Ly-α photons.



The total amount of ionized medium is also strongly constrained by the observed CMB temperature anisotropy power. In fact, CMB data gives a very stringent limit on the total optical depth of the universe.

 $\tau = 0.054 \pm 0.007$  (*Planck* 2018 result)



# The physics of recombination



At redshift z~1400, the electron number density of the universe is roughly n<sub>e</sub> ~ 500 cm<sup>-3</sup>, a number very similar to the compact star-forming regions in the Galaxy. However, the major difference is the huge bath of the CMB photons *at the same temperature as the electrons* (T<sub>e</sub> = T<sub>cmb</sub> ~ 3800 K), which makes radiative processes far more important than collisional processes. Also, unlike Galactic HII regions, the universe has no boundary, so the evolution of radiation is a *time-dependent* problem, rather than a *spatial* problem.

# Recombination temperature & redshift



The temperature of last scattering depends very weakly on the cosmological parameters (weaker than logarithmically).

# It is mainly determined by the ionization potential of hydrogen and the baryon-to-photon ratio.

n<sub>e</sub> ~ 500 cm<sup>-3</sup> (roughly same as Galactic HII regions)

 $T_e = T_r = 2970 \text{ K} = 0.26 \text{ eV}$  $z_r = T_r/T_{CMB} - 1 = 1090$ 

The reason for this delay is the huge specific entropy of our universe:  $N_{\gamma}/N_e \sim 2 \times 10^9$ 

#### Recombination temperature & redshift

# >the CMB number density dominates over that of baryons by $\sim 2 \times 10^9$ (valid for all times).

$$\eta \equiv n_{\gamma}/n_b \approx 1.6 \times 10^9 \left(\frac{\Omega_b h^2}{0.022}\right)^{-1}$$

This ratio implies that recombination occurs at *substantially* lower temperature than the binding energy  $\mathbf{Q} = \mathbf{13.6} \ \mathbf{eV}$  of the Hydrogen atom, at  $\mathbf{T} = \mathbf{0.26} \ \mathbf{eV}$  only, or at a redshift of  $z_* \approx 10^3$ .

This follows from Saha's ionization equilibrium equation, which in terms of ionization fraction  $X \equiv n_e/n_H$  (here  $n_e = n_p$ ) and photon-to-baryon ratio  $\eta$  reads as

$$\frac{1-X}{X^2} = 3.84 \eta \left(\frac{kT}{m_e c^2}\right)^{3/2} \exp\left(\frac{Q}{kT}\right) = S, \quad X = \frac{-1+\sqrt{1+4S}}{2S}$$

## Recombination temperature & redshift



**I**onization fraction  $X = n_e/n_H$ 



Once  $Q_{13.6 \text{ eV}}/kT$  reaches a certain level, recombination proceeds rapidly. If we define the moment of recombination when X = 1/2, we get

$$kT_{rec} = 0.323 \ eV \Rightarrow T_{rec} = 3740 \ K, \quad 1 + z_{rec} = \frac{1}{a_{rec}} = \frac{T_{rec}}{T_{CMB,0}} = \frac{3740 \ K}{2.73 \ K} \approx 1371$$

#### Precise calculation for recombination



In the above plot, hydrogen recombination in Saha equilibrium is shown against more precise numerical calculation with RECFAST (recombination is aided by the H-atom  $2s \rightarrow 1s$  two-photon decay). The features seen before hydrogen recombination are due to helium recombination.

Recombination is not an instantaneous process, but nevertheless proceeds rapidly ( $\chi$ =0.9 to  $\chi$ =0.1 in a time interval  $\Delta$ t=70,000 y). Its redshift is practically independent of the standard cosmological parameters (but **its duration** must be computed to better than 1% precision for accurately knowing the CMB power spectrum).

## Recap: Hydrogen recomb. lines



(Slide: Jens Chluba)



(Slide: Jens Chluba)



#### Routes to the ground state ?

- direct recombination to 1s
  - Emission of photon is followed by immediate re-absorption

No

#### (Slide: Jens Chluba)



#### Routes to the ground state ?

- direct recombination to 1s
  - Emission of photon is followed by immediate re-absorption
- recombination to 2p followed by Lyman-α emission
  - medium optically thick to Ly- $\alpha$  phot.
  - many resonant scatterings
  - escape very hard (p ~10<sup>-9</sup> @ z ~1100)

#### (Slide: Jens Chluba)



#### Routes to the ground state ?

•	direct recombination to 1s	
	- Emission of photon is followed by	> No
	immediate re-absorption	
•	recombination to 2p followed by Lyman- $lpha$ emission	
	<ul> <li>medium optically thick to Ly-α phot.</li> <li>many resonant scatterings</li> <li>escape very hard (<i>p</i> ~10<sup>-9</sup> @ <i>z</i> ~1100)</li> </ul>	
•	recombination to 2s followed by 2s two-photon decay	
	<ul> <li>2s → 1s ~10<sup>8</sup> times slower than Ly-α</li> <li>2s two-photon decay profile → maximum at v ~ 1/2 v<sub>α</sub></li> </ul>	
	- immediate escape	

#### 2s→1s continuum spectrum

Since the two photons can share energies between them, the resulting spectrum is a continuum and not a line spectrum (i.e. it is spread over a broad frequency range). The corresponding change in the recombination spectrum (fig. on right) is also observable over the entire CMB frequency range.



#### The recombination process



# Epoch of decoupling

The photon Thomson scattering rate is controlled by how many electrons are available. The scattering rate  $\Gamma$  is thus a function of ionization fraction:

$$\Gamma = \frac{c}{\lambda} = n_e(z)\sigma_e c = X(z)n_{B,0}(1+z)^3\sigma_e c = 4.4 \times 10^{-21}X(z)(1+z)^3 s^{-1}$$

For free streaming of electrons, this scattering rate should at least be of the order of Hubble time, H(t). Now recombination and photon decoupling takes place in the matter dominated era, so from the Friedmann equation:

 $\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} = \Omega_{m,0} (1+z)^3 \Rightarrow H = 1.24 \times 10^{-18} (1+z)^{3/2}$ Setting  $\Gamma \sim H$  (condition for photon free streaming), we get  $1 + z_{dec} = \frac{43.0}{X(z_{dec})^{2/3}} \Rightarrow z_{dec} = 1130$ 

Actually, when  $\Gamma \sim H$ , the system is not in equilibrium (Saha equation not valid). A more precise calculation yields

 $Z_{dec} \approx 1100, T_{dec} \approx 3000K, t_{dec} \approx 380,000 \text{ yrs}$ 

# Epoch of decoupling



From Chluba (2018), arXiv:1806.02915

# **Recombination & Decoupling**

To summarize, we define the epoch of recombination when the ionization fraction of the Universe reaches  $X_e=0.5$ , and the epoch of decoupling when the Thomson scattering becomes inefficient (typical time between scattering exceeds the expansion time, given by  $H^{-1}$ ).

 $Z_{dec} \approx 1100$ t<sub>dec</sub>  $\approx 380,000$  yrs

Although  $z_{dec}$  isn't far from  $z_{rec}$  ( $z \approx 1100$  from  $z \approx 1300$ ), the ionization fraction decreases significantly between recombination and decoupling:

 $\chi_{e}(z_{rec}) \approx 0.5 \rightarrow \chi_{e}(z_{dec}) \approx 0.1.$ 

This shows that a large degree of neutrality is necessary for the universe to become transparent to photon propagation.

After decoupling the photons stream freely. We can start seeing the primordial temperature fluctuations!

# Thickness of the last scattering surface



The **visibility function** is defined as the probability density that a photon is last scattered at redshift z:  $g(z) \sim exp(-\tau) d\tau/dz$ 

Probability distribution is well described by Gaussian with mean  $z \sim 1100$ and standard deviation  $\delta z \sim 80$ .

## Importance of the visibility function



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## Importance of the visibility function



# Spectral features from recombination

The known physics of recombination predicts several other spectral features on the CMB, and **failure** to detect those will create big theoretical challenge!



Cosmological Time in Years

# Spectral features from recombination

The signatures of each of these recombination lines of H (and helium too) should be present as tiny modifications on top of the CMB spectrum (because the number of thermal photons still dominates by far). The lines in the Wien part of the spectrum (≥300 GHz) are confused with the cosmic infrared background (CIB), e.g. the redshifted Ly-α line.



# Spectral features from recombination



Frequency dependent modulation in the CMB **temperature** from the H I and He II recombination epoch, after subtracting the mean recombination spectrum. This signal is unpolarized and **same in all directions on the sky**.

# Rayleigh scattering of the CMB



http://en.wikipedia.org/wiki/Rayleigh\_scattering

Thomson Scattering



#### Rayleigh Scattering



(Frequency dependent) Rayleigh scattering cross section :  

$$\sigma_R(\nu) = \sigma_T \left[ \left( \frac{\nu}{\nu_{eff}} \right)^4 + \frac{638}{243} \left( \frac{\nu}{\nu_{eff}} \right)^6 + \frac{1299667}{236196} \left( \frac{\nu}{\nu_{eff}} \right)^8 + \dots \right]$$

 $\sigma_T$ : Thomson scattering cross section  $v_{eff} \sim 3.1 \times 10^3 GHz$  See Lewis 2 ٠

See Lewis 2013 and Alipour et al. 2015

# Rayleigh scattering of the CMB

Rayleigh scattering allows us to probe yet another epoch of the early universe using CMB photons, hence giving additional leverage in constraining cosmological models. The bulk of Rayleigh scattering takes place shortly after the recombination (when the photon and neutral hydrogen densities are still high) and is easily modelled with the existing CMB anisotropy Boltzmann codes. The challenge is to detect the signal at high frequencies (>300 GHz).



#### Questions?



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