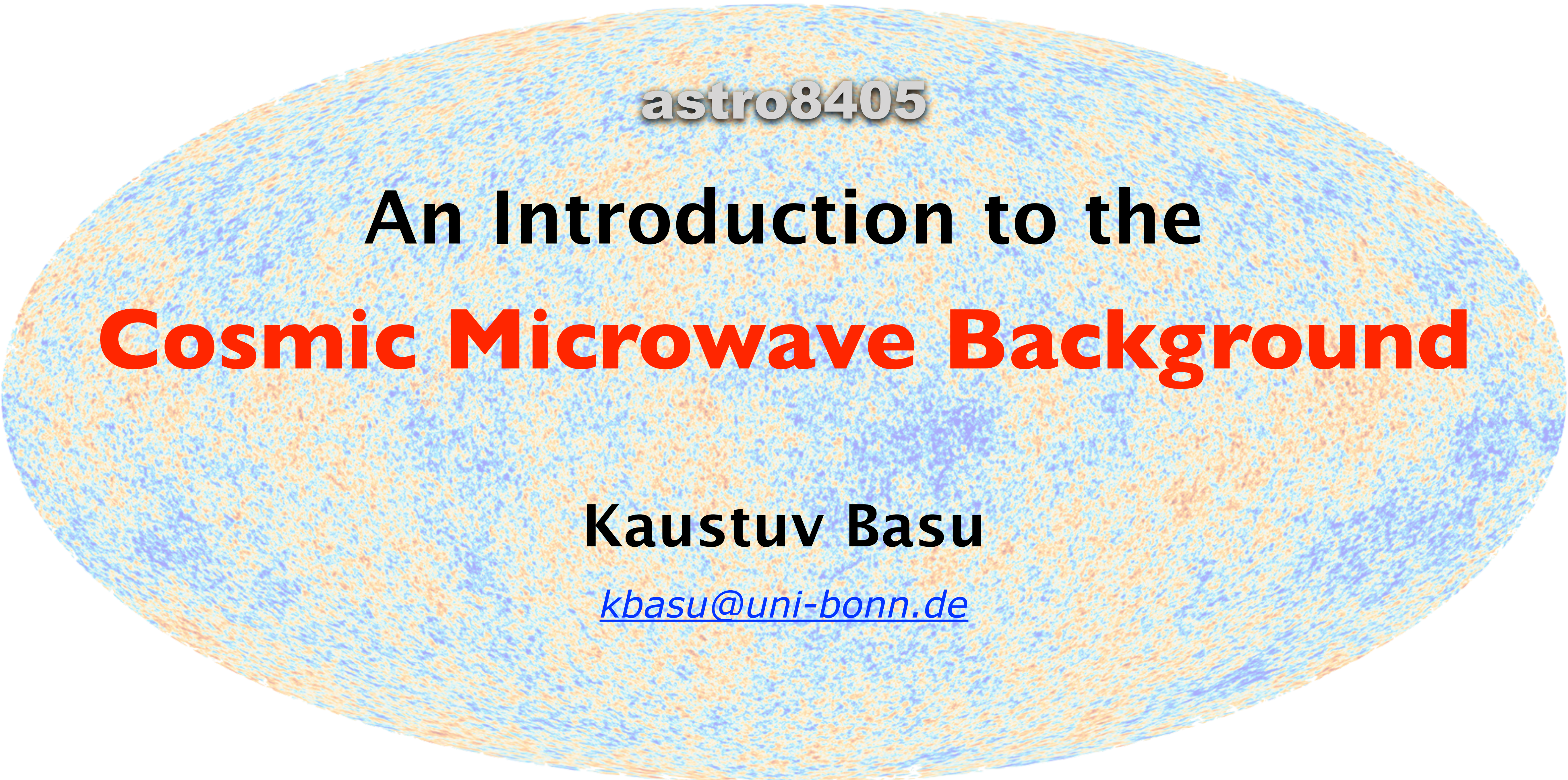

astro8405

An Introduction to the Cosmic Microwave Background

Kaustuv Basu

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eCampus | Lernplattform der Universität Bonn

astro8405: The Cosmic Microwave Background

Aktionen

This course intends to give you a modern and up-to-date introduction to the science and experimental techniques relating to the Cosmic Microwave Background. No prior knowledge of cosmology is necessary, your prerequisite are a basic understanding of electrodynamics and thermal physics and some familiarity with Python programming.

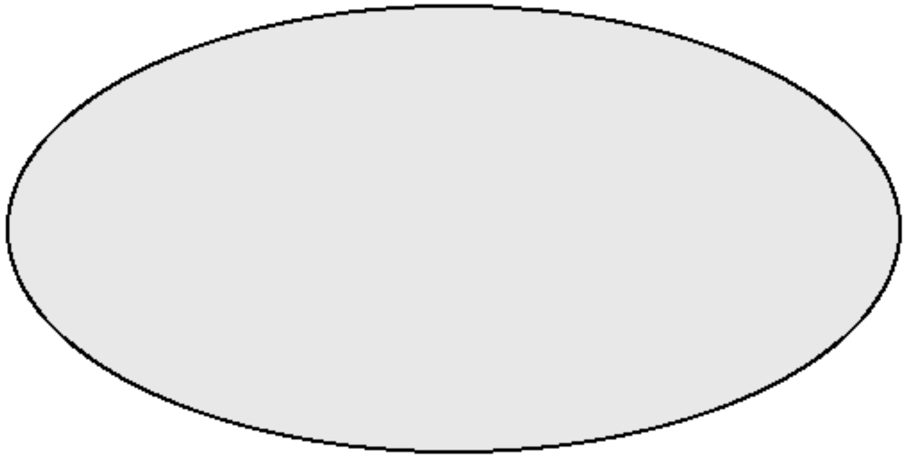
Lecture 3:

Thermal spectrum of the CMB and its distortions

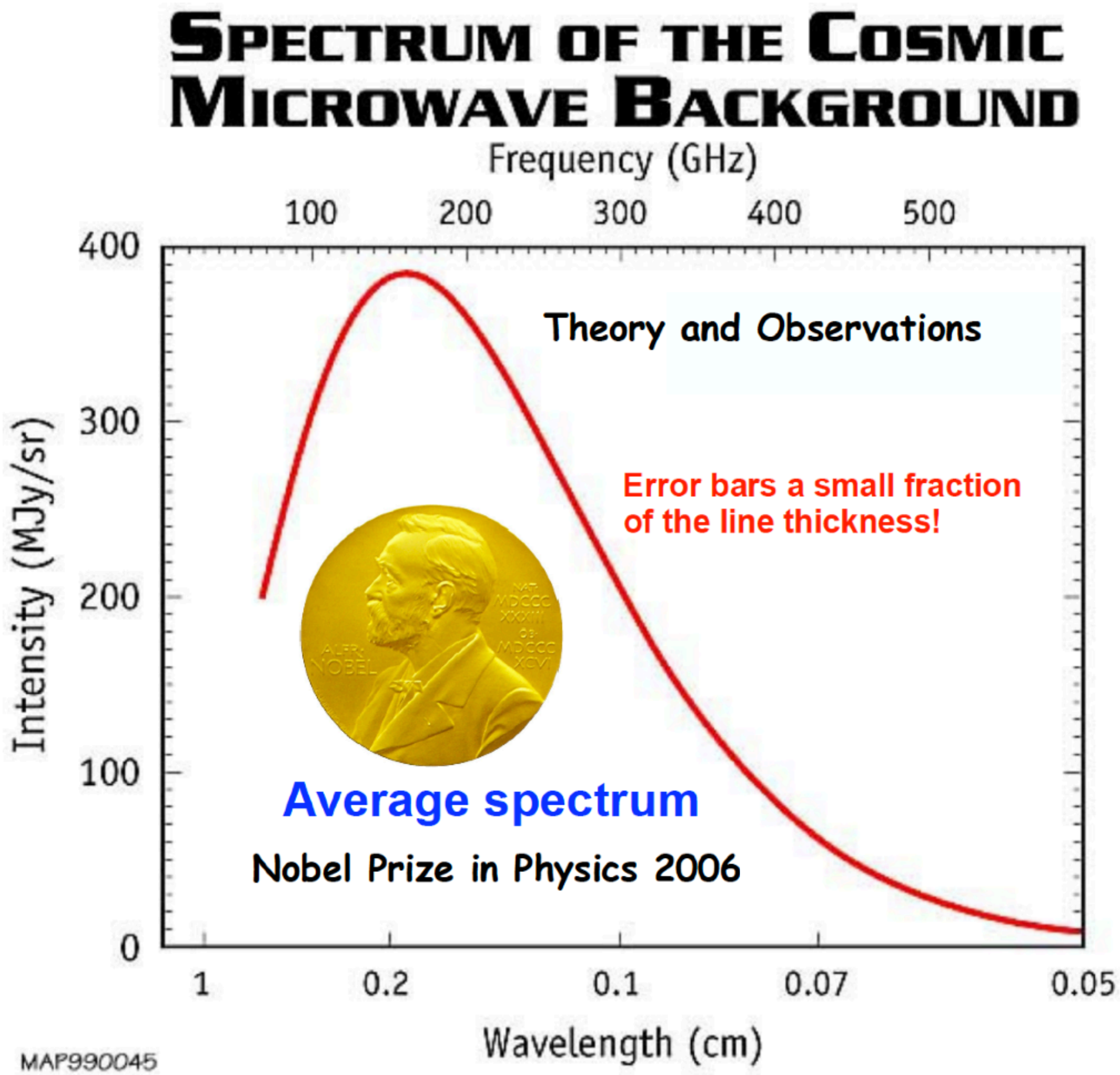
What we aim to learn today

- ➡ The blackbody spectrum of the CMB
    - ◉ Why the spectrum is a blackbody? what created it?
    - ◉ Why it has remained a blackbody?
- ➡ μ and y-type distortions on the blackbody spectrum
    - ◉ What are these spectral distortions? When are they created?
    - ◉ Have we detected those? Why should we care?

The near-uniform blackbody



CMB temperature map on a scale of 0 K (black) to 3 K (white), made by Ned Wright



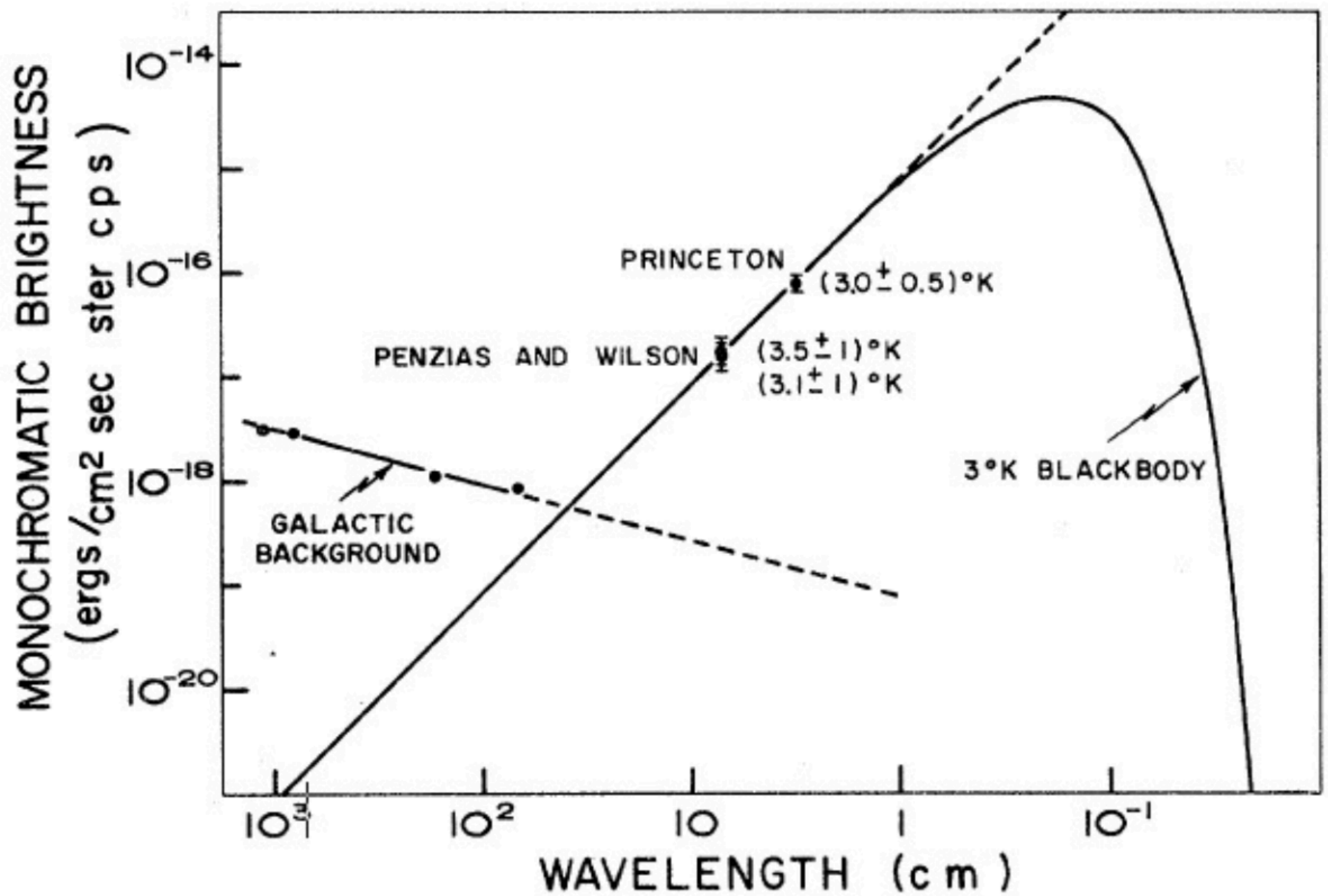
First evidence of blackbody spectrum

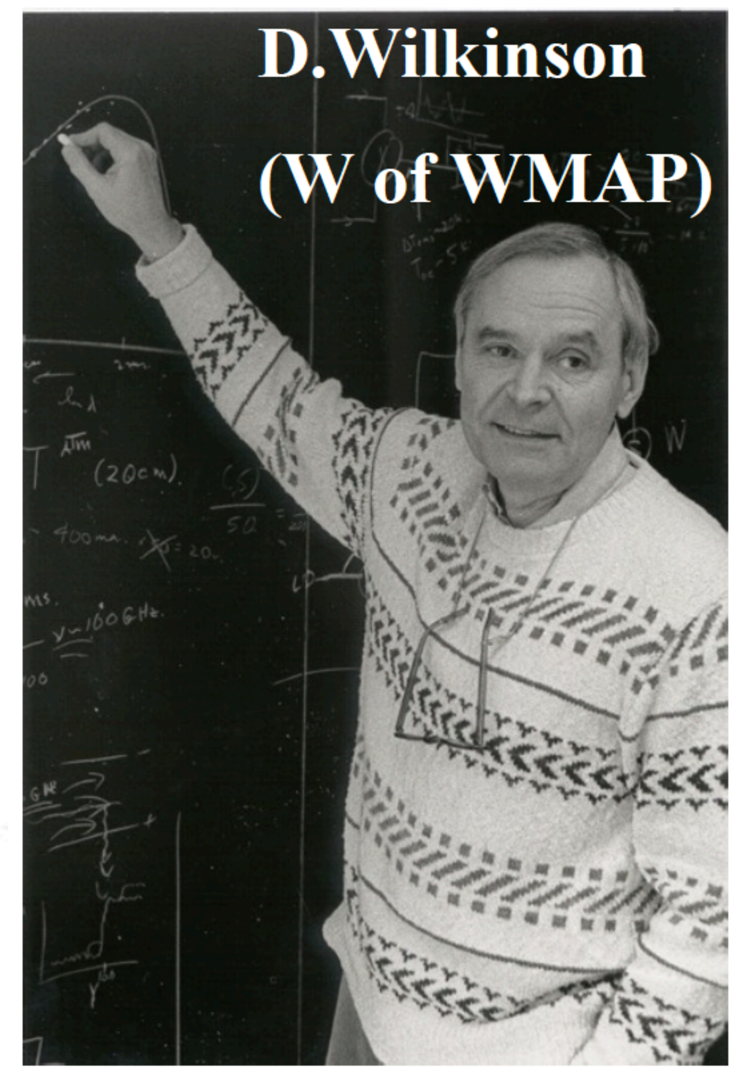
COSMIC BACKGROUND RADIATION AT 3.2 cm—SUPPORT FOR COSMIC BLACK-BODY RADIATION*

P. G. Roll† and David T. Wilkinson

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey

(Received 27 January 1966)





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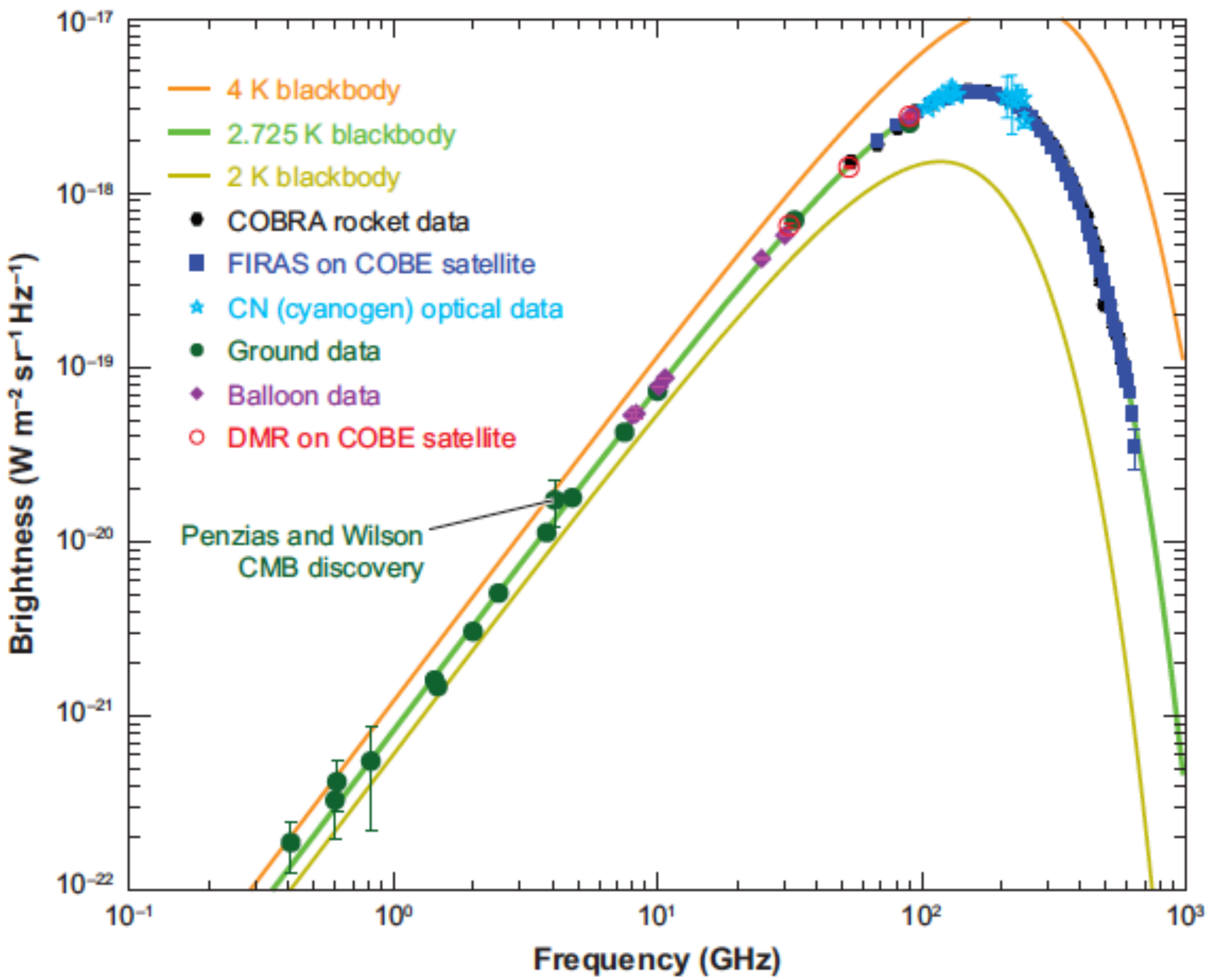
Using $B_\nu(T) = \frac{2\nu^2}{c^2} k_{\mathrm{B}} T$
see later..

Measurement of T_{CMB}

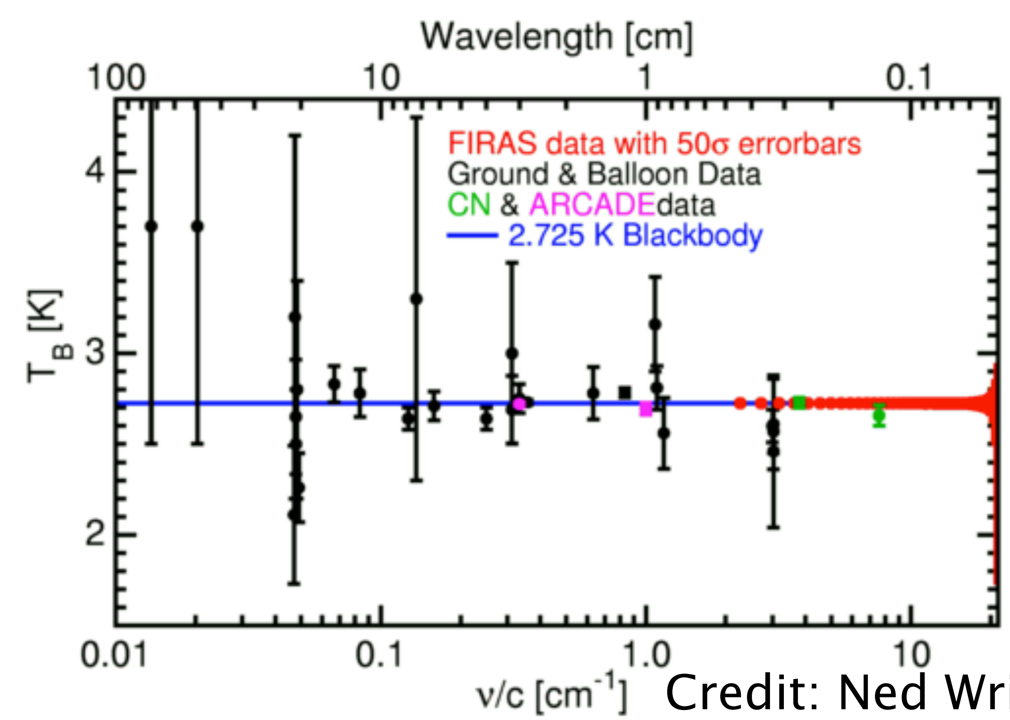
Credit: D. Samtleben



Measured blackbody spectrum of the CMB, with fit to various data

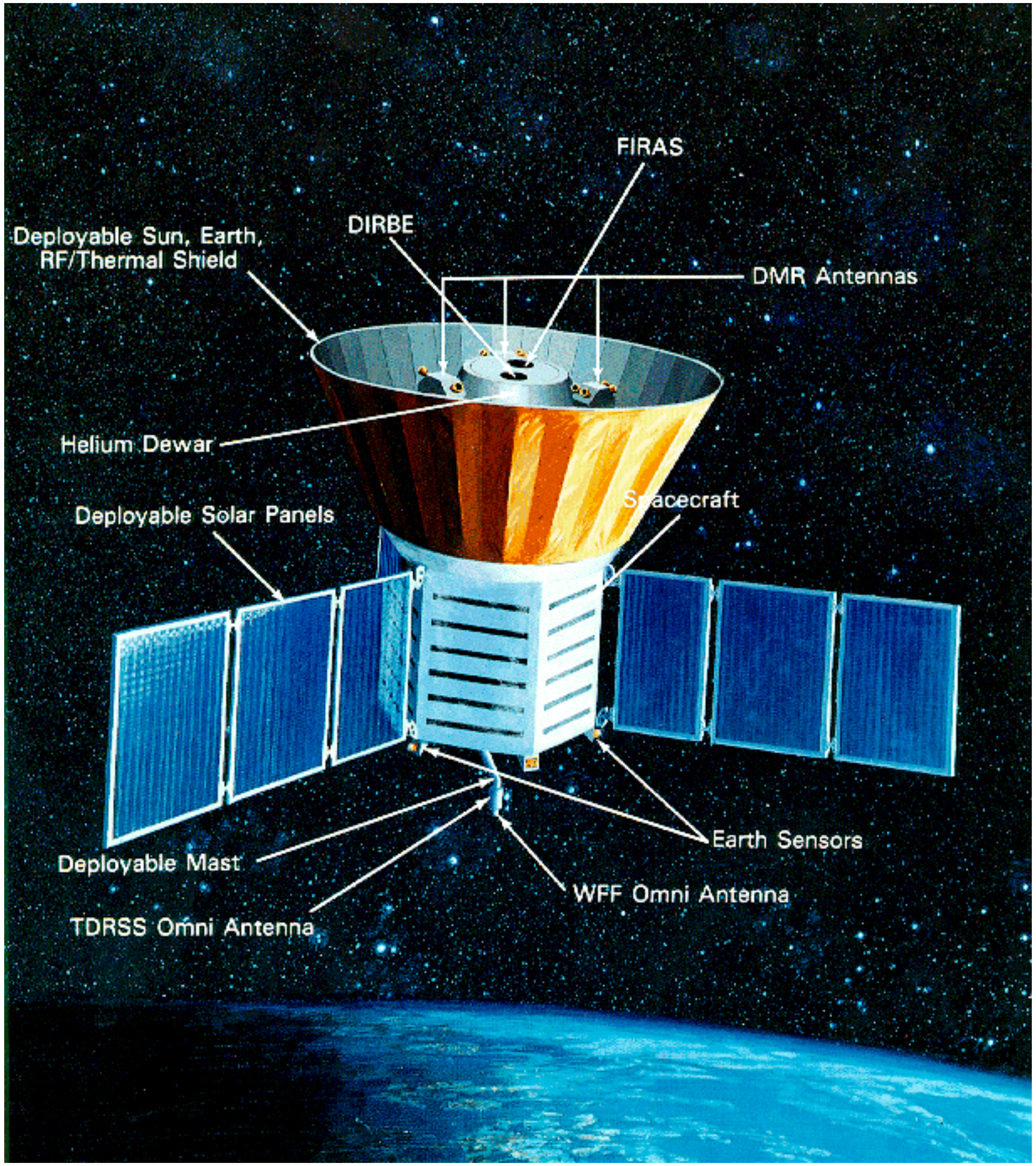
Ground and balloon based experiments have been measuring CMB temperature for decades with increasing precision.

But it was realized that one has to go to the stable thermal environment of outer space to get a really accurate measurement and observe in the Wien part of the spectrum.



Credit: Ned Wright

COBE satellite



Credit: NASA

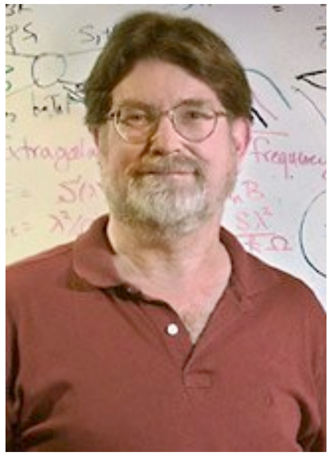
Launched on Nov. 1989 on a Delta rocket.

DIRBE: Measured the absolute sky brightness in the 1–240 μm wavelength range, to search for the Infrared Background

FIRAS: Measured the spectrum of the CMB, finding it to be an almost perfect blackbody with $T_0 = 2.725 \pm 0.002$ K

DMR: Found "anisotropies" in the CMB for the first time, at a level of 1 part in 10^5

2006 Nobel prize in physics

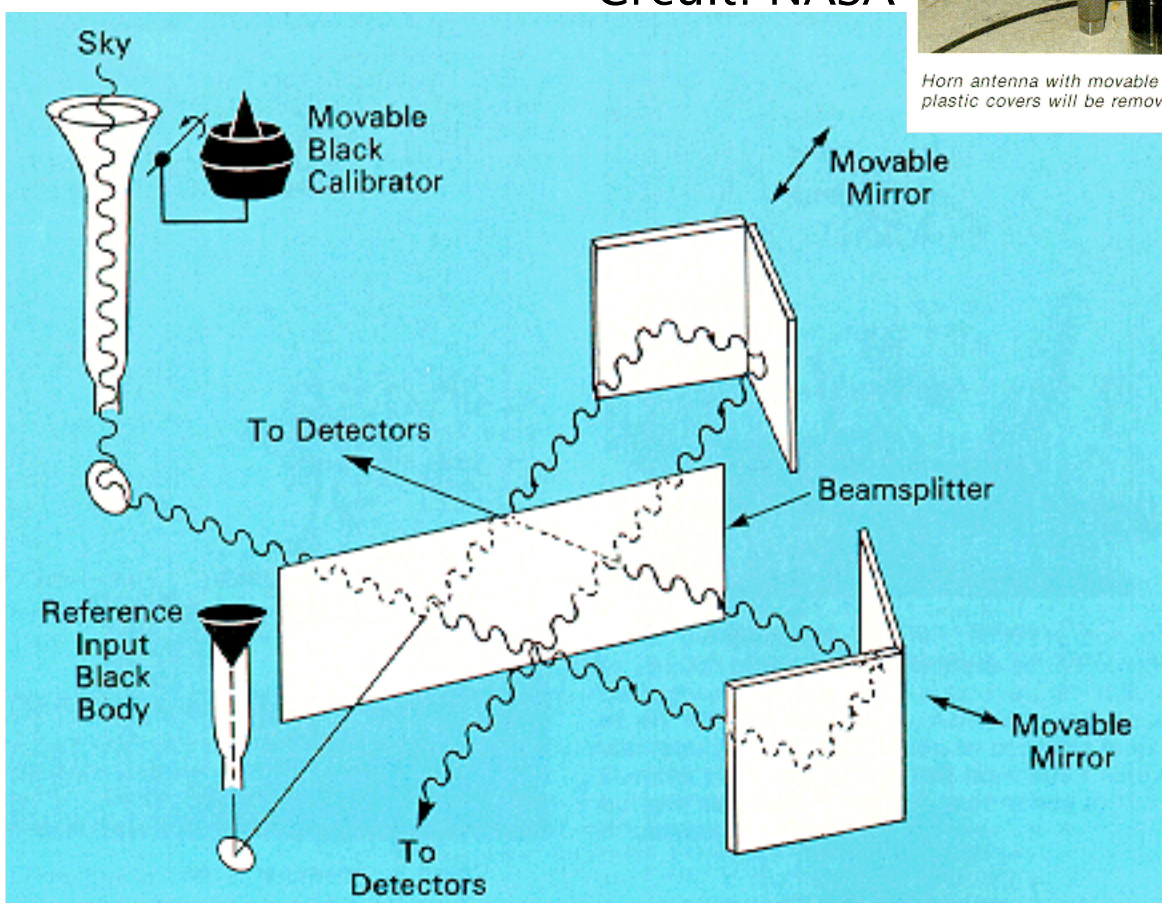


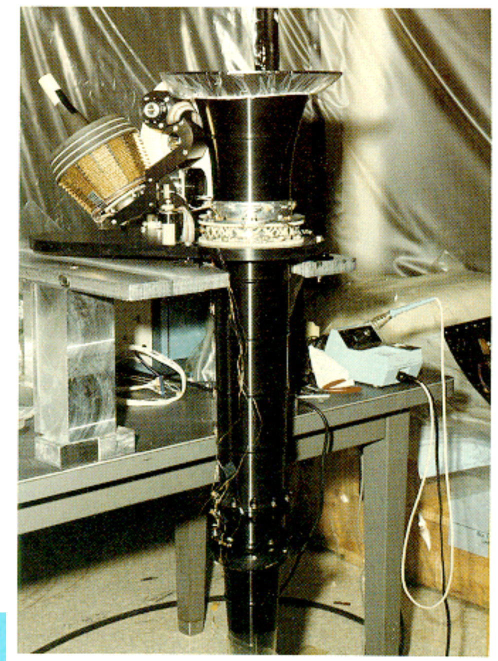
FIRAS on COBE

Far Infra-Red Absolute Spectrophotometer

A differential polarizing Michelson interferometer

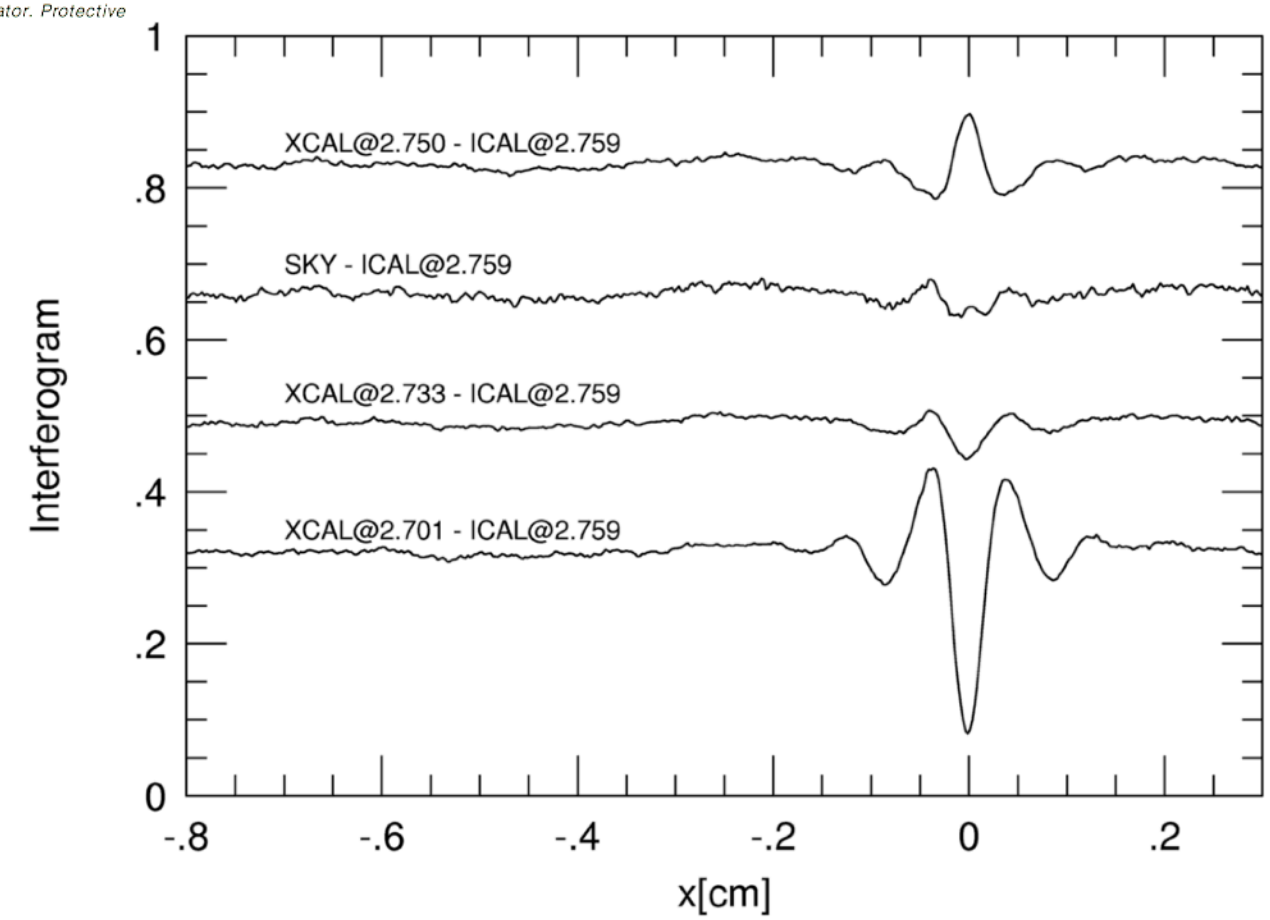
Credit: NASA





Horn antenna with movable calibrator. Protective plastic covers will be removed.

- One input is either the sky or a blackbody, other is a well-calibrated blackbody
- Zero output when the two inputs are equal
- A movable-mirror mechanism adjusts the path difference against changing calibrator temperature
- The residual is the measurement!



The CMB blackbody

Mather et al. (1994)
Fixsen et al. (1996)

MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND SPECTRUM BY THE *COBE*[1] FIRAS INSTRUMENT

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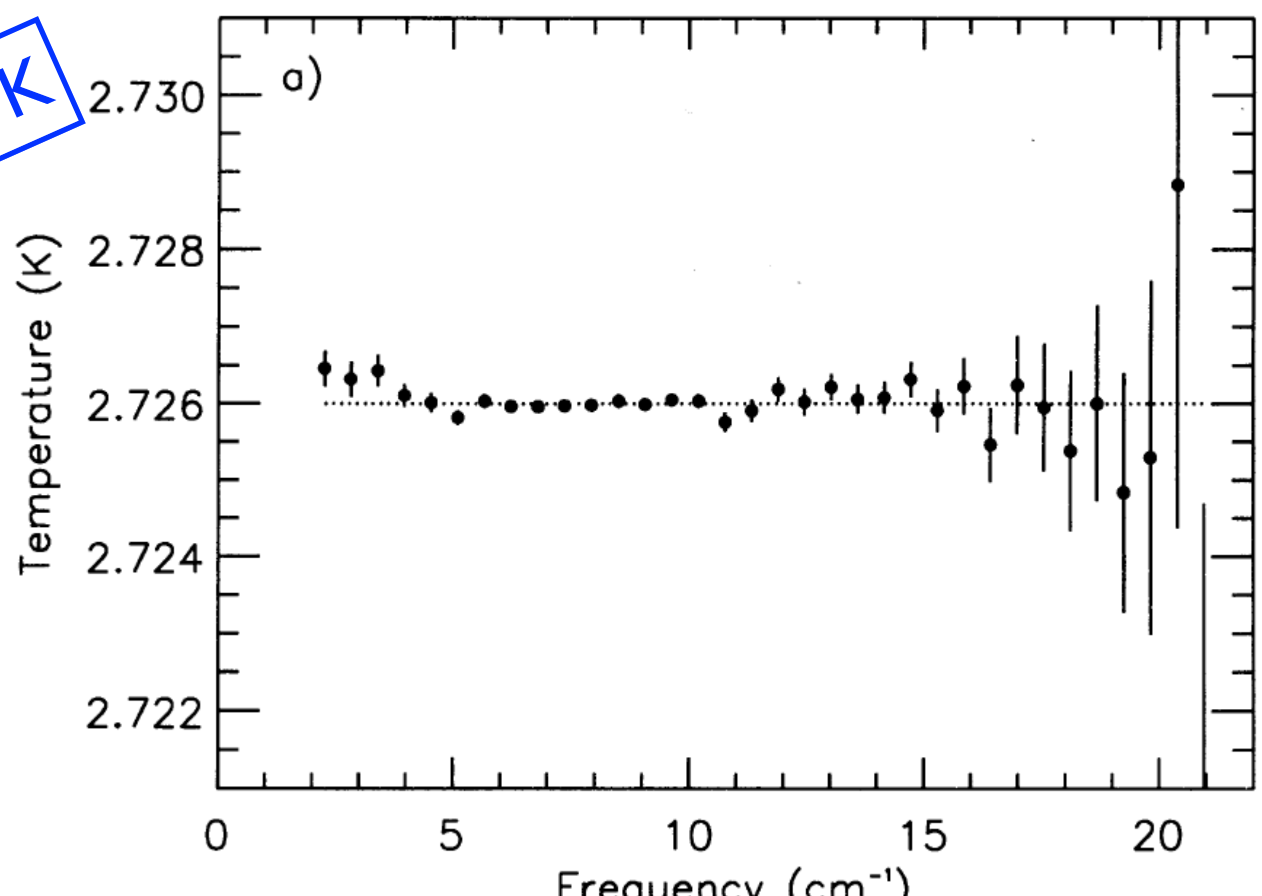
Received 1993 February 5; accepted 1993 July 21

ABSTRACT

The cosmic microwave background radiation (CMBR) has a blackbody spectrum within 3.4×10^{-8} ergs $\mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}$ cm over the frequency range from 2 to 20 cm^{-1} (5–0.5 mm). These measurements, derived from the FIRAS instrument on the *COBE* satellite, imply stringent limits on energy release in the early universe after $t \sim 1$ year and redshift $z \sim 3 \times 10^6$. The deviations are less than 0.03% of the peak brightness, with an rms value of 0.01%, and the dimensionless cosmological distortion parameters are limited to $|y| < 2.5 \times 10^{-5}$ and $|\mu| < 3.3 \times 10^{-4}$ (95% confidence level). The temperature of the CMBR is 2.726 ± 0.010 K (95% confidence level systematic).

Subject headings: cosmic microwave background — cosmology: observations — early universe

$\boxed{T_0 = 2.7255 \pm 0.0006\ \mathrm{K}}$



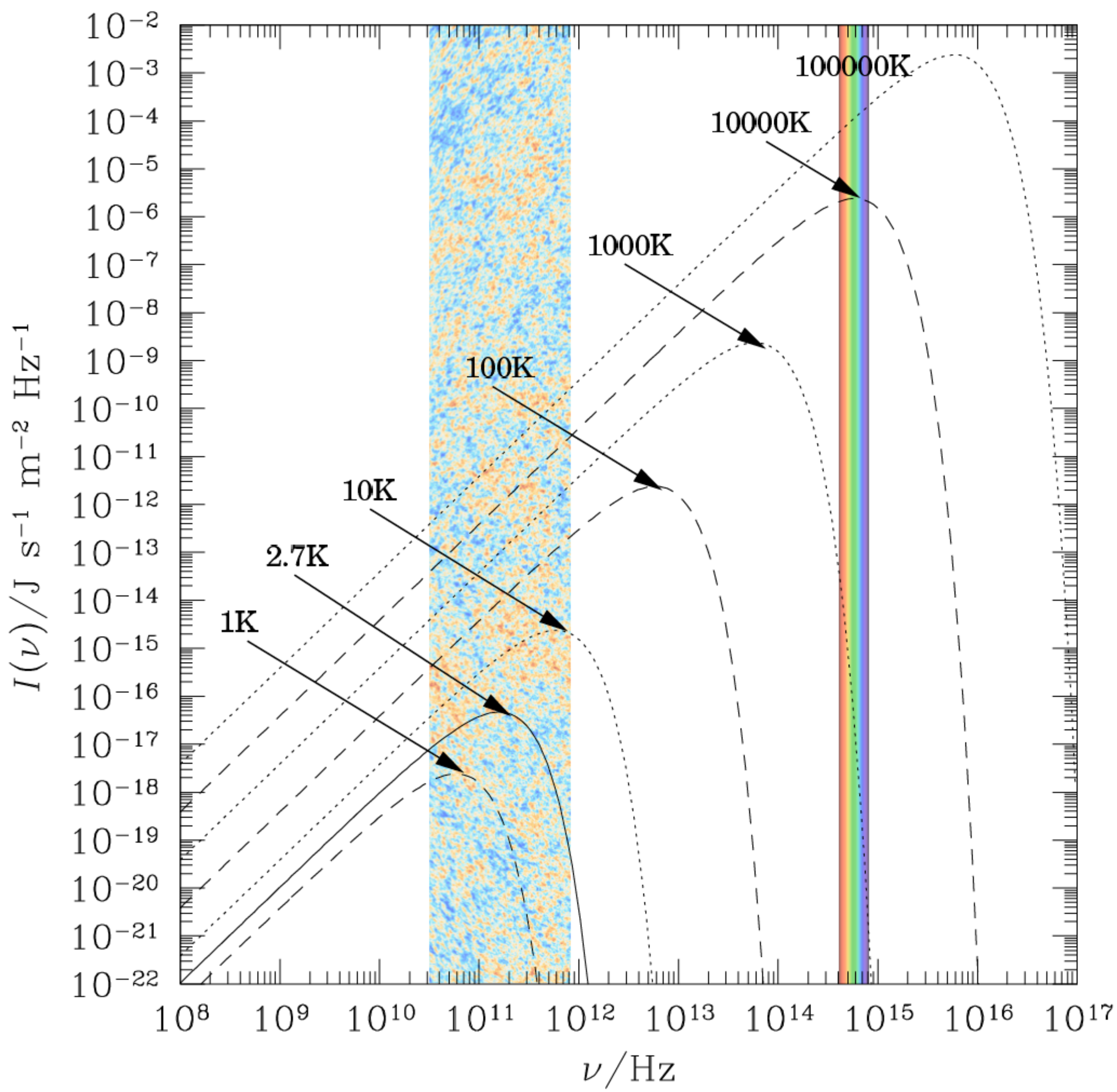
Note: **CMB is cooling !**

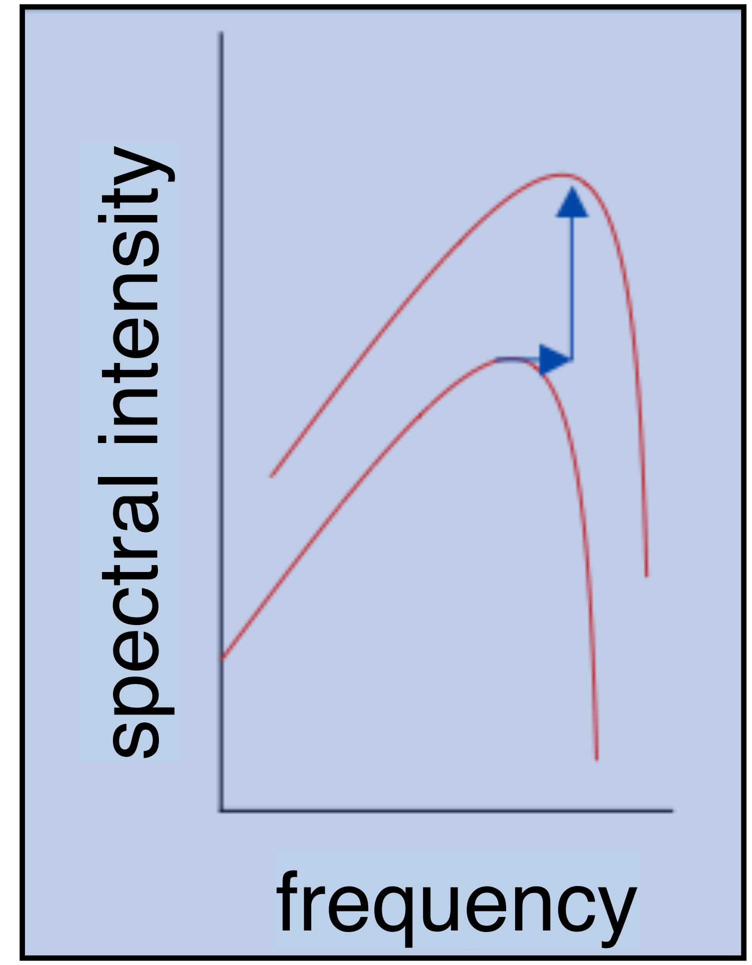
From $T(t) = T_0/a(t)$

$$\dot{T}_0 = H_0 T_0 = -0.20\ \mathrm{nK\ yr}^{-1}.$$

See Abitbol et al. (2019) for actual possibilities of measuring this cooling

The CMB blackbody





The blackbody shape is preserved at all redshifts if expansion is purely adiabatic. The frequency shifts by *1/a* and energy density drops by *1/a⁴*.

Cosmology basics

- Static (i.e., non-expanding) Euclidean space
  - In Cartesian coordinates $\boldsymbol{x} = (x, y, z)$

$$ds^2 = dx^2 + dy^2 + dz^2$$

- Homogeneously expanding Euclidean space
  - In Cartesian **comoving** coordinates $\boldsymbol{x} = (x, y, z)$

$$ds^2 = \underbrace{\boxed{a^2(t)}}_{\text{“scale factor”}}(dx^2 + dy^2 + dz^2)$$

The Friedmann-Robertson-Walker (FRW) metric defines the ‘distance element’ in an expanding universe

[Ref. Cosmology lectures]

$$\boxed{\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - a^2(t)\left[\mathrm{d}\chi^2 + f_K^2(\chi)\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)\right]}$$

$$f_K(\chi) \equiv \begin{cases} K^{-1/2}\sin(K^{1/2}\chi) & (K > 0) \\ \chi & (K = 0) \\ (-K)^{-1/2}\sinh[(-K)^{1/2}\chi] & (K < 0) \end{cases}$$

The function $H(t)$ is called the *Hubble parameter* , or the *Hubble expansion rate*.
Its value at the present time, H_0 , is the Hubble constant.

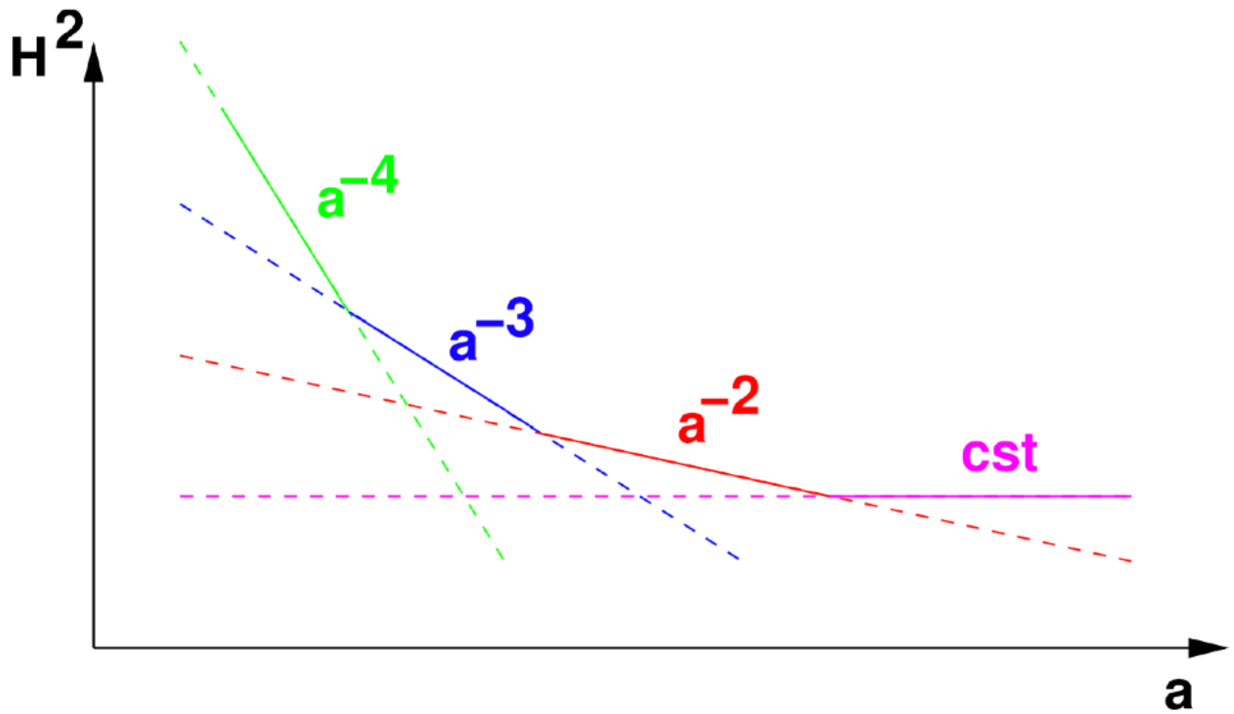
$$H(t) = \dot{a}/a$$

$$\mathrm{d}t = \frac{\mathrm{d}a}{\dot{a}} = \frac{\mathrm{d}a}{a\,H}\ ;$$

$$1 + z = \frac{1}{a}\ .$$

$$\boxed{\frac{H^2}{H_0^2} = \Omega_r\,a^{-4} + \Omega_m\,a^{-3} + \Omega_k\,a^{-2} + \Omega_\Lambda}$$

Cosmology basics



$$\boxed{\frac{H^2}{H_0^2} = \Omega_r a^{-4} + \Omega_m a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda}$$

The inverse of the Hubble rate gives the typical expansion time-scale of the universe

$$t_{\rm exp} = H^{-1}$$

To derive this equation for the dynamics of the universe, you would need General Relativity, whose solution (for a homogeneous, isotropic universe) is given by the famous Friedmann equations.

These are a series of equations, one connecting the scale factor a(t) of the universe with the spatial curvature and the homogeneous energy density ρ(t), and the others for conservation equations for different components of ρ(t). Putting together the contributions from different components, one gets the above-mentioned form of Friedmann equation.

$$\boxed{H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\mathcal{G}}{3c^2}\rho_{\rm R} + \frac{8\pi\mathcal{G}}{3c^2}\rho_{\rm M} - \frac{kc^2}{a^2} + \frac{\Lambda}{3}}$$

$$\Omega_{\rm M} = \frac{8\pi G}{3H_0^2c^2}\rho_{\rm M}$$

etc.

Cosmology basics

$$\boxed{H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi\mathcal{G}}{3c^2}\rho_{\rm R} + \frac{8\pi\mathcal{G}}{3c^2}\rho_{\rm M} - \frac{kc^2}{a^2} + \frac{\Lambda}{3}}$$

The density of radiation, matter, spatial curvature, and a cosmological constant changes with the scale factor of the universe as a^{-4}, a^{-3}, a^{-2}, and a^{0}, respectively.

Radiation domination

$$\frac{\dot{a}^2}{a^2} \propto a^{-4}, \qquad a(t) \propto t^{1/2}, \qquad H(t) = \frac{1}{2t}$$

Matter domination

$$\frac{\dot{a}^2}{a^2} \propto a^{-3}, \qquad a(t) \propto t^{2/3}, \qquad H(t) = \frac{2}{3t}$$

Matter-radiation equality

$$\frac{\rho_m}{\rho_r} \propto \frac{a^{-3}}{a^{-4}} = a,$$

so we can find this scale factor of matter-radiation equality by knowing the matter and radiation densities today (when $a = 1$)

$$a_{\rm eq} = \frac{\rho_{r0}}{\rho_{m0}}.$$

$$\rho_\gamma = \frac{\pi^2}{30} g_{\star,eff} T_{\gamma 0}^4 = 7.8 \times 10^{-34}\,\mathrm{g/cm^3}.$$

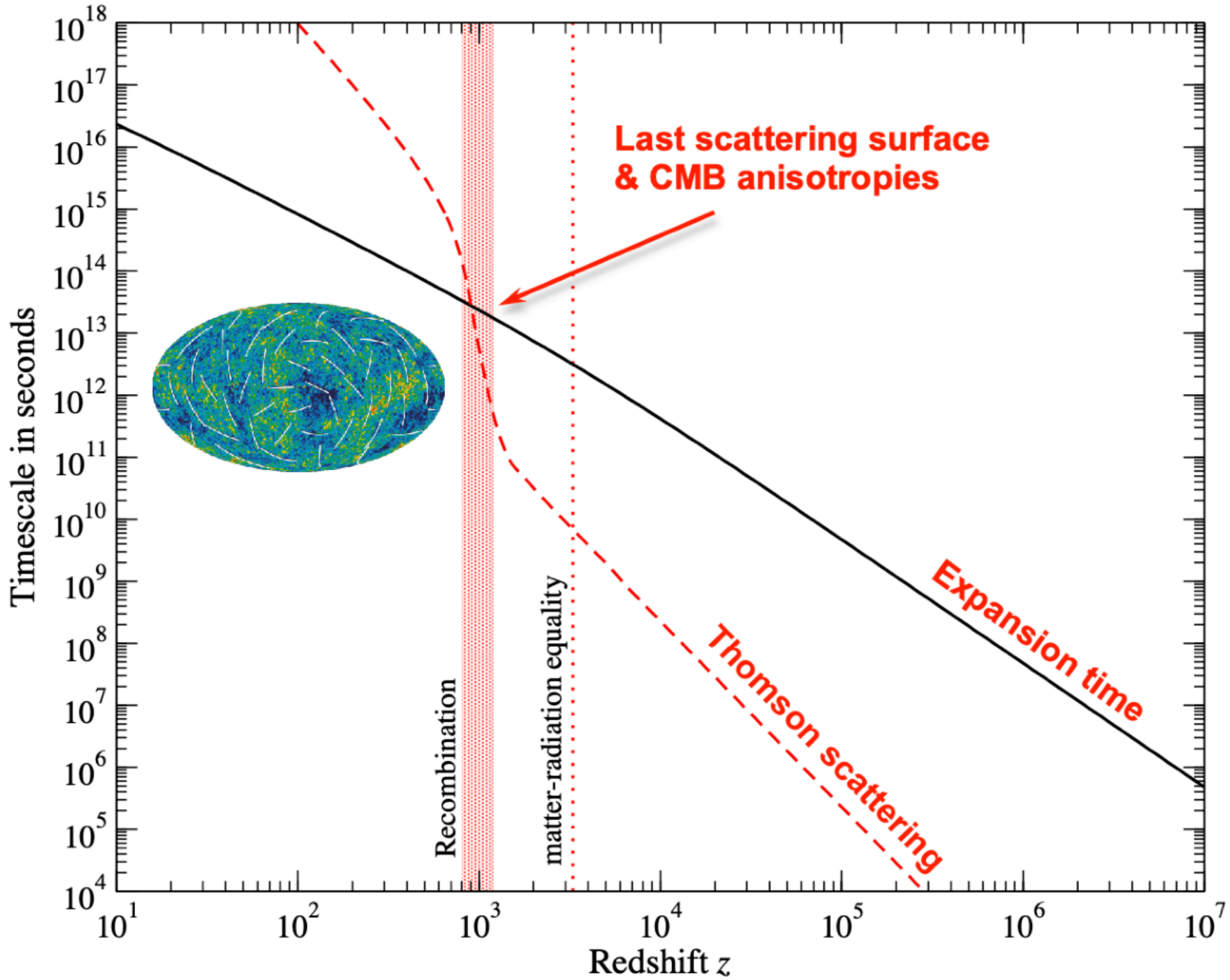
$$\rho_{m0} = \frac{3\Omega_m H_0^2}{8\pi G} = 1.879 \times 10^{-29}\Omega_m h^2\,\mathrm{g/cm^3}.$$

$\Omega_m h^2 = \Omega_b h^2 + \Omega_{dm} h^2$. From WMAP: $\Omega_m h^2 = 0.128 \pm 0.008$.

$$a_{\rm eq} = 4.15 \times 10^{-5} (\Omega_m h^2)^{-1}.$$

$$1 + z_{\rm eq} = 2.4 \times 10^4 \Omega_m h^2,$$
$$\approx 3070$$

Cosmology basics



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$$\approx 3070$$

Properties of blackbody radiation

$$\boxed{B_\nu(T) = \frac{2h}{c^2}\frac{\nu^3}{e^{h\nu/kT} - 1}.}$$

$$[B_\nu(T)] = \text{ergs sec}^{-1}\,\text{cm}^{-2}\text{Hz}^{-1}\text{sr}^{-1} = 10^{17}\,\text{MJy sr}^{-1}.$$

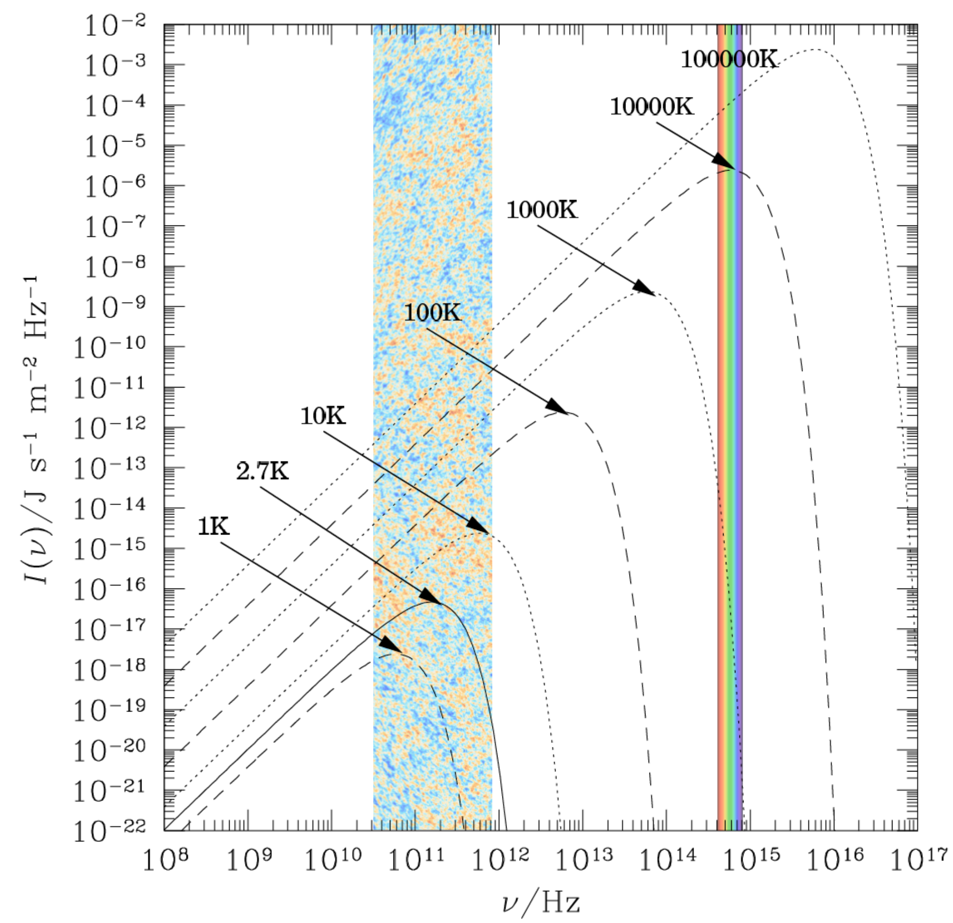
Planck's law determines the specific intensity of photons (i.e., the power flowing orthogonal to a unit surface area per unit solid angle per unit frequency). The specific energy density of photons is $U_\nu = B_\nu/c$.

At low frequencies, we have the Rayleigh-Jeans limit

$$B_\nu(T) \overset{h\nu \ll kT}{\approx} \frac{2\nu^2}{c^2}kT \propto \nu^2 T,$$

Whereas at the high-frequency limit we have the Wien law

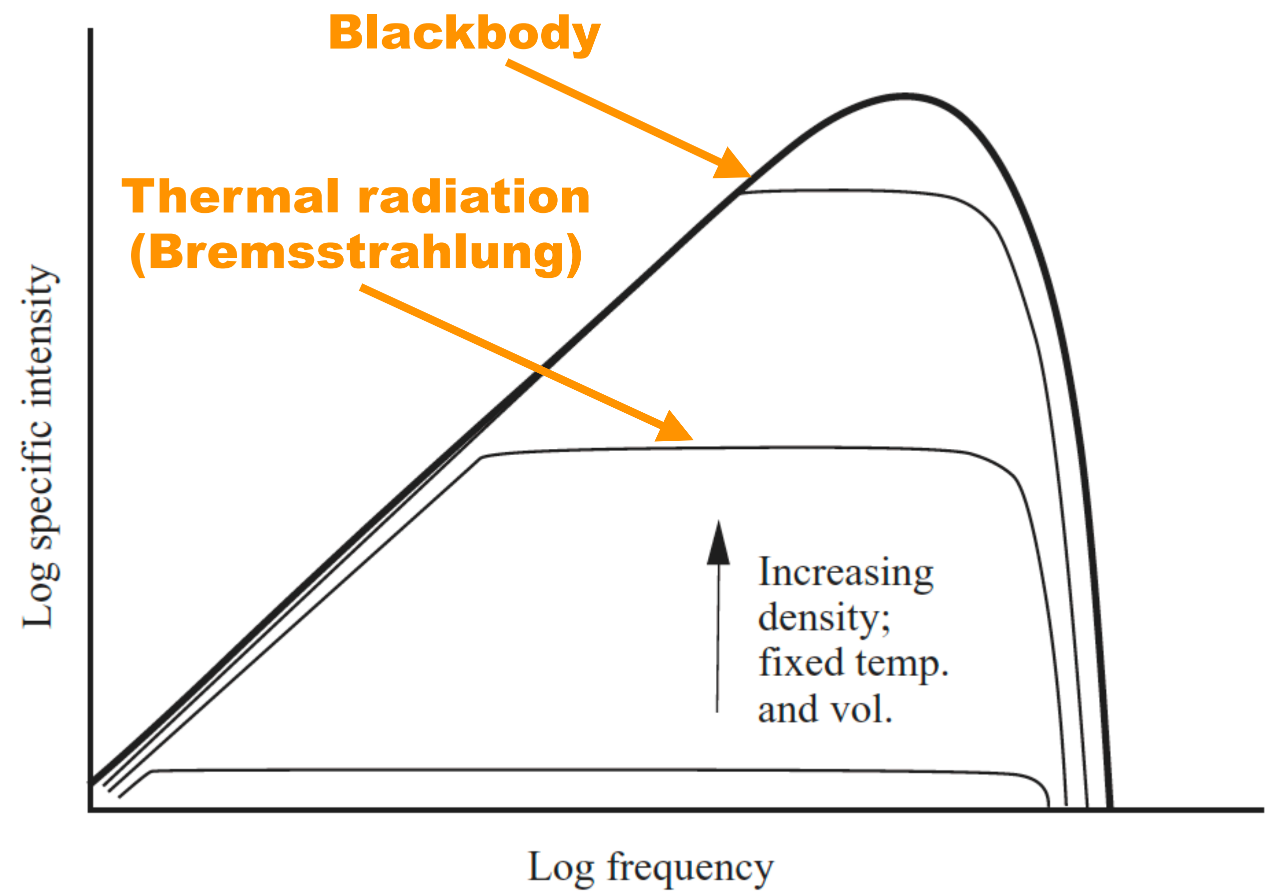
$$B_\nu(T) \overset{h\nu \gg kT}{\approx} \frac{2h\nu^3}{c^2}e^{-h\nu/kT}.$$

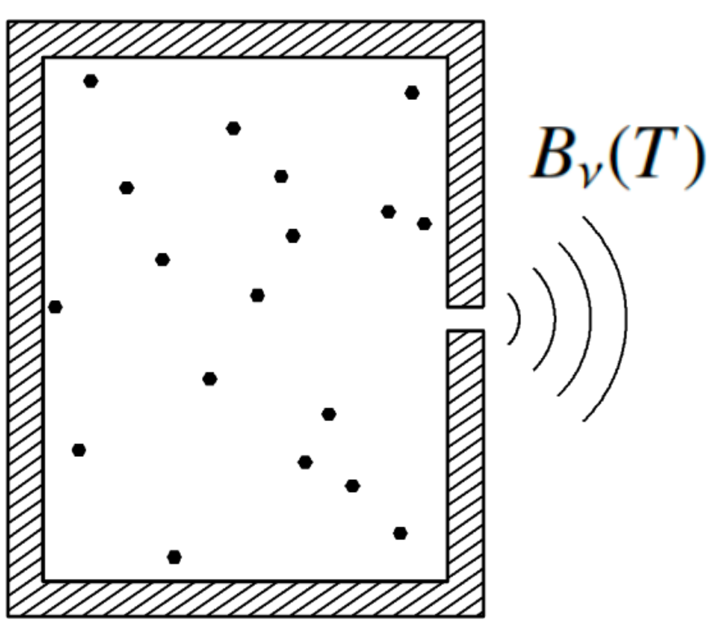


Properties of blackbody radiation

Blackbody radiation arises when matter in thermodynamic equilibrium is optically thick, thereby photons scatter many times before encountering an observer. Under such conditions, the particle and photons share their kinetic energies.

The CMB provides such a condition. The container wall (the last scattering surface) and the radiation are at the same temperature, as in the famous “container with a cavity” (*Hohlraum*).





This figure shows the so-called “blackbody limit” of thermal radiation, as particles are added to an optically thin gas holding the volume and temperature constant. The thermal bremsstrahlung curve rises upwards and reaches the blackbody curve, but first only at the low-frequencies! The emission becomes optically thick first at low frequencies, the break is known as the low-frequency cutoff.

Properties of blackbody radiation

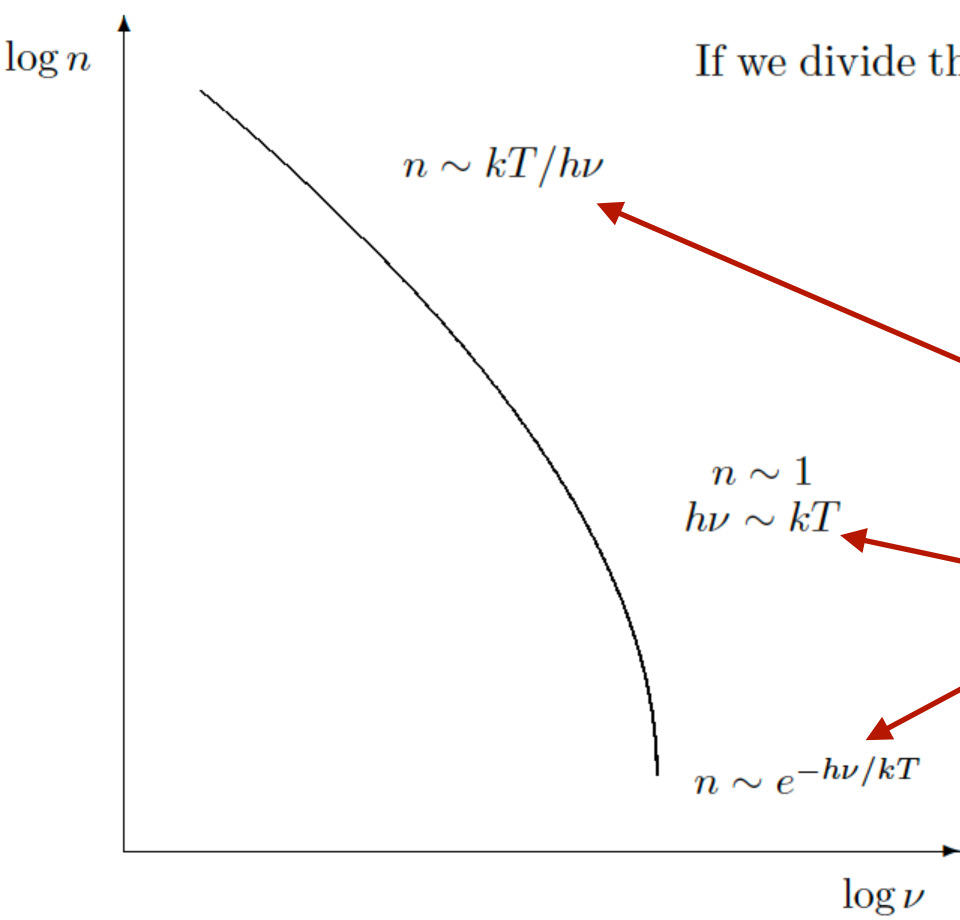
The photon occupation number of a blackbody radiation field represents the mean number of photons for every specific state of energy, and is give by

$$n_{\mathrm{Pl}} = \frac{c^3 U_\nu(T)}{2h\nu^3} = \frac{c^2 B_\nu(T)}{2h\nu^3} = \frac{1}{e^{h\nu/kT} - 1} = \frac{1}{e^x - 1},$$

$\bar{U}_\nu = B_\nu/c$ is the specific energy density

$x = h\nu/kT$ ($x = 1$ corresponds to $\nu \approx 56.8\,\mathrm{GHz}$).

Defined for $T = T_{CMB} = 3.726$ K



If we divide the mean energy $\bar{E}$ by the energy per photon $h\nu$ we obtain the *mean occupation number*

$$\bar{n} = \frac{1}{e^{\beta h\nu} - 1} \tag{5.24}$$

- $h\nu \ll kT$: expanding the exponential as a Taylor series the leading order term is $n \simeq kT/h\nu \gg 1$. In this 'Raleigh-Jeans' region the occupation number is large — each state contains a large number of photons, and photon discreteness effects are negligible. The occupation number diverges as $\nu \to 0$, but the mean energy remains finite $\bar{E} \simeq kT$.
- $h\nu \simeq kT$: this is the characteristic photon energy, and the occupation number is of order unity.
- $h\nu \gg kT$: the $+1$ in the denominator is negligible and $\bar{n} \simeq \exp(-h\nu/kT)$ and becomes exponentially small.

Properties of blackbody radiation

The photon occupation number of a blackbody radiation field is

$$n_{\rm Pl} = \frac{c^3 U_\nu(T)}{2h\nu^3} = \frac{c^2 B_\nu(T)}{2h\nu^3} = \frac{1}{\mathrm{e}^{h\nu/kT}-1} = \frac{1}{\mathrm{e}^x - 1}, \qquad x = h\nu/kT \ (x = 1 \text{ corresponds to } \nu \approx 56.8\,\mathrm{GHz}).$$

Photon **energy density** and **number density** for blackbody radiation:

$$\rho_\gamma = \int U_\nu \,\mathrm{d}\nu\,\mathrm{d}\Omega = \frac{2}{c^3}\int E\,\nu^2 n_\nu\,\mathrm{d}\nu\,\mathrm{d}\Omega \equiv \int E\,f\,\mathrm{d}^3p, \qquad N_\gamma = \int \frac{U_\nu}{h\nu}\,\mathrm{d}\nu\,\mathrm{d}\Omega = \frac{2}{c^3}\int \nu^2 n_\nu\,\mathrm{d}\nu\,\mathrm{d}\Omega \equiv \int f\,\mathrm{d}^3p.$$

Blackbody photon energy density

$$\rho_\gamma^{\rm Pl} = \frac{2h}{c^3}\int \frac{\nu^3}{\mathrm{e}^x-1}\,\mathrm{d}\nu\,\mathrm{d}\Omega = \frac{8\pi h}{c^3}\left(\frac{kT}{h}\right)^4\int\frac{x^3\,\mathrm{d}x}{\mathrm{e}^x-1} = \frac{8\pi^5(kT)^4}{15\,c^3h^3}$$
$$= a_R T^4 \approx 5.10\times10^{-7}\, m_{\rm e}c^2\mathrm{cm}^{-3}\left(\frac{T}{2.725\mathrm{K}}\right)^4 \approx 0.26\,\mathrm{eV}\left(\frac{T}{2.725\mathrm{K}}\right)^4$$

Photon number density

$$N_\gamma^{\rm Pl} = \frac{2}{c^3}\int\frac{\nu^2}{\mathrm{e}^x-1}\,\mathrm{d}\nu\,\mathrm{d}\Omega = \frac{8\pi}{c^3}\left(\frac{kT}{h}\right)^3\int\frac{x^2\,\mathrm{d}x}{\mathrm{e}^x-1} = \frac{16\pi\zeta_3(kT)^3}{c^3h^3}$$
$$= b_R T^3 \approx 410\,\mathrm{cm}^{-3}\left(\frac{T}{2.725\mathrm{K}}\right)^3,$$

Properties of blackbody radiation

Blackbody entropy

We can apply the standard laws of thermodynamics to blackbody radiation:

$$T\,\mathrm{d}S_\gamma = \mathrm{d}E_\gamma + P_\gamma\,\mathrm{d}V = \mathrm{d}(V\rho_\gamma^{\mathrm{Pl}}) + \frac{\rho_\gamma^{\mathrm{Pl}}}{3}\,\mathrm{d}V = V\,\mathrm{d}\rho_\gamma^{\mathrm{Pl}} + \frac{4}{3}\rho_\gamma^{\mathrm{Pl}}\,\mathrm{d}V = V\,\mathrm{d}(a_R T^4) + \frac{4}{3}(a_R T^4)\,\mathrm{d}V,$$

where S_γ is the photon entropy, $E_\gamma = V\rho_\gamma^{\mathrm{Pl}}$ the total internal energy and $P_\gamma = \rho_\gamma^{\mathrm{Pl}}/3$ the photon pressure

$$\mathrm{d}S_\gamma = V\,4a_R T^2 + \frac{4}{3}(a_R T^3)\,\mathrm{d}V = V\,\mathrm{d}\left(\frac{4}{3}a_R T^3\right) + \frac{4}{3}(a_R T^3)\,\mathrm{d}V = \mathrm{d}\left(\frac{4}{3}\,a_R T^3 V\right) \Longrightarrow \frac{S_\gamma}{V} = \frac{4}{3}\frac{\rho_\gamma^{\mathrm{Pl}}}{T}.$$

This expression implies that for *adiabatic changes* of a blackbody ($S_\gamma \equiv$ const) we have $V\,T^3$ = const. This just means that if you increase the temperature by a factor of f you need to decrease the confining volume by a factor of $1/f^3$, as we already argued above. Recasting this expression in terms of PV^γ = const we find the adiabatic index of photons $\gamma = 4/3$. For comparison, a monoatomic ideal gas has $\gamma = 5/3$ and for diatomic idea gases one has $\gamma = 7/5$.

Blackbody radiation in adiabatic expansion

At constant S we have $\quad T \propto V^{-1/3} \quad$ and $\quad P \propto T^4 \propto V^{-4/3}$

The *adiabatic equation of state* for black-body radiation radiation is therefore

$$PV^\gamma = \text{constant}$$

with *adiabatic index* $\gamma = 4/3$.

What is needed to *keep* a blackbody?

Blackbody radiation is fully characterized by only one parameter: its temperature T. Let's suppose the temperature is increased to T' by adding some energy:

$$\epsilon = \Delta\rho_\gamma/\rho_\gamma^{\mathrm{Pl}}(T) \equiv (T'/T)^4 - 1,$$

$$\frac{\Delta T}{T} = (1+\epsilon)^{1/4} - 1 \approx \frac{1}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma^{\mathrm{Pl}}}.$$

To maintain the blackbody radiation, $N \propto T^3$, one then needs to add photons:

$$\frac{\Delta N_\gamma}{N_\gamma^{\mathrm{Pl}}} = (T'/T)^3 - 1 = (1+\epsilon)^{3/4} - 1 \approx 3\frac{\Delta T}{T} \quad \Longrightarrow \quad \frac{\Delta N_\gamma}{N_\gamma^{\mathrm{Pl}}} \equiv \frac{3}{4}\frac{\Delta\rho_\gamma}{\rho_\gamma^{\mathrm{Pl}}}$$

However, this is necessary but not sufficient condition! These photons need to be redistributed in energy to keep the blackbody spectrum. In the early universe, the photon creation part is taken care of by double-Compton scattering and bremsstrahlung processes, and the Compton scattering takes care of the energy redistribution.

What is needed to *keep* a blackbody?

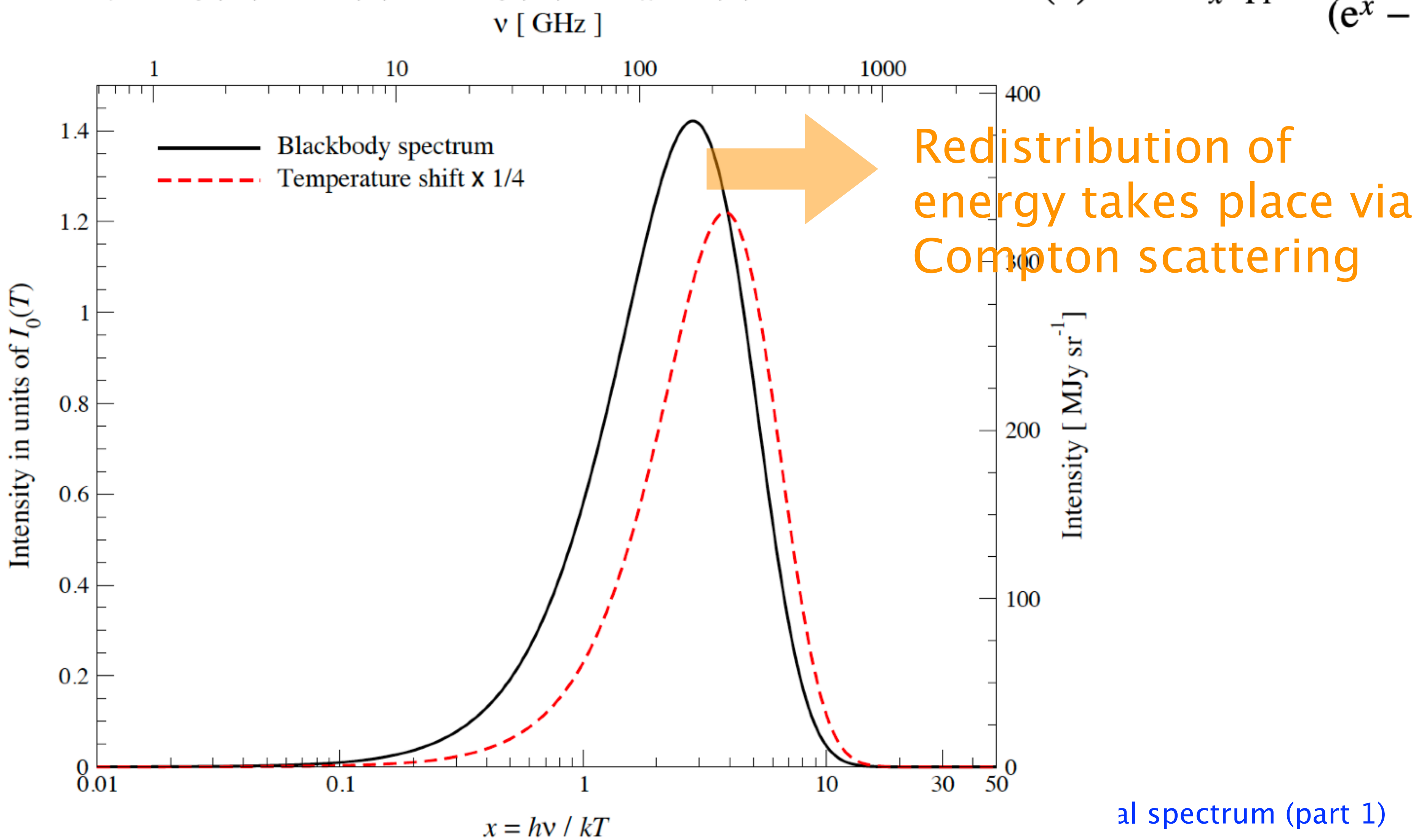
To keep a blackbody spectrum at a new temperature T' the photon occupation number must change as the following:

$$\Delta n_\nu = n_{\rm Pl}(T') - n_{\rm Pl}(T) = \frac{1}{e^{x'}-1} - \frac{1}{e^{x}-1} = -x\partial_x n_{\rm Pl}\,\frac{\Delta T}{T} + O(\Delta T/T)^2 = \frac{x e^x}{(e^x-1)^2}\,\frac{\Delta T}{T} + O(\Delta T/T)^2$$

with $x' = x\,T/T'$

The figure shows a blackbody spectrum and the spectrum of a temperature shift: $T\partial_T B_\nu = I_0(T)\,x^3 G(x) = -I_0(T)\,x^4\partial_x n_{\rm Pl}(x).$

$$G(x) = -x\partial_x n_{\rm Pl} = \frac{x\,e^x}{(e^x-1)^2}$$



Blackbody in an expanding universe

We saw for blackbody radiation the entropy is proportional to the photon number and the entropy change follows the condition

$$dS_\gamma = V\,4a_R T^2 + \frac{4}{3}(a_R T^3)\,dV = V\,d\left(\frac{4}{3}a_R T^3\right) + \frac{4}{3}(a_R T^3)\,dV = d\left(\frac{4}{3}\,a_R T^3 V\right)$$

This means that in adiabatic expansion ($S_\gamma \equiv const$), we will have $VT^3 = const$.

This is consistent with the expression $PV^\gamma = const$ with photon adiabatic index $\gamma = 4/3$.

Thus, the temperature goes down with the inverse of radius, i.e. the scale factor of the universe:

$$\boxed{T(t) = T_0/a(t)}$$

(The electron temperature will go down as $1/R^2$, due to electron's adiabatic index 5/3)

Since CMB photon number far dominates the baryon number, the entropy of the universe is practically equal to the entropy of the CMB photons!

$$\boxed{baryon\ to\ photon\ ratio : \eta = \frac{n_B}{n_R} \approx \frac{0.22\,m^{-3}}{4.1\times 10^8\,m^{-3}} \approx 5\times 10^{-10}}$$

>the CMB number density dominates over that of baryons by ~10^9. (valid for all times).

Thermalization of the CMB

Process that changes photon energy, not number:

Compton scattering: $e + \gamma = e + \gamma$

Processes that creates photons:

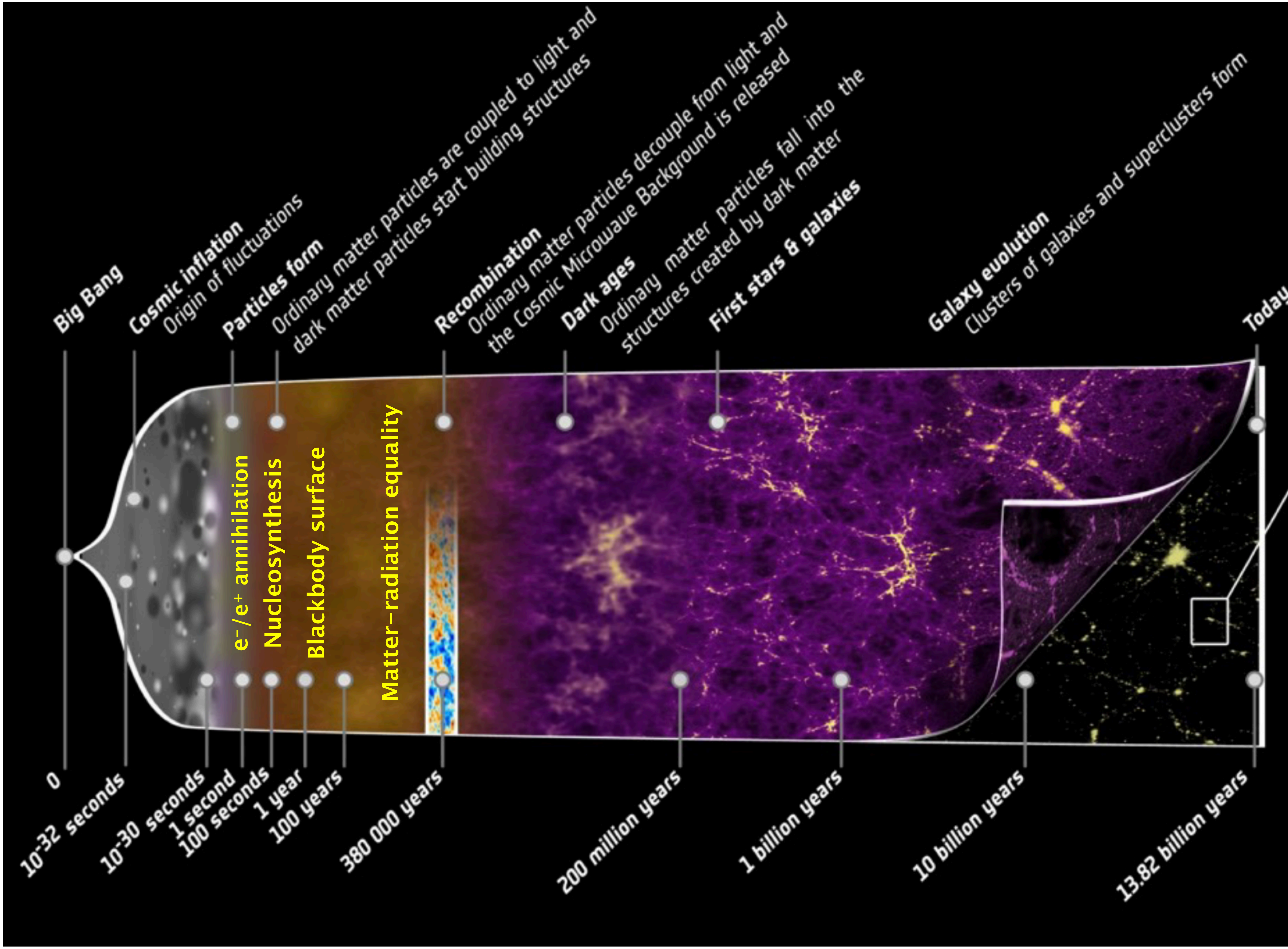
Radiative (double) Compton scattering: $e + \gamma = e + \gamma + \gamma$

Bremsstrahlung: $e + Z = e + Z + \gamma$

At an early enough epoch, timescale of the radiative Compton scattering rate Is much shorter than the expansion timescale. They are equal at **$z \sim 2 \times 10^6$** , or roughly two months after the big bang.

The universe reaches thermal equilibrium by this time through scattering and photon-generating processes. Thermal equilibrium generates a blackbody radiation field. Any energy injection *before* this time cannot leave any spectral signature on the CMB blackbody.

For pure adiabatic expansion of the universe *afterwards*, a blackbody spectrum — once established — should be maintained.



See also: https://www.physicsoftheuniverse.com/topics_bigbang_timeline.html

Events in the history of the CMB spectrum

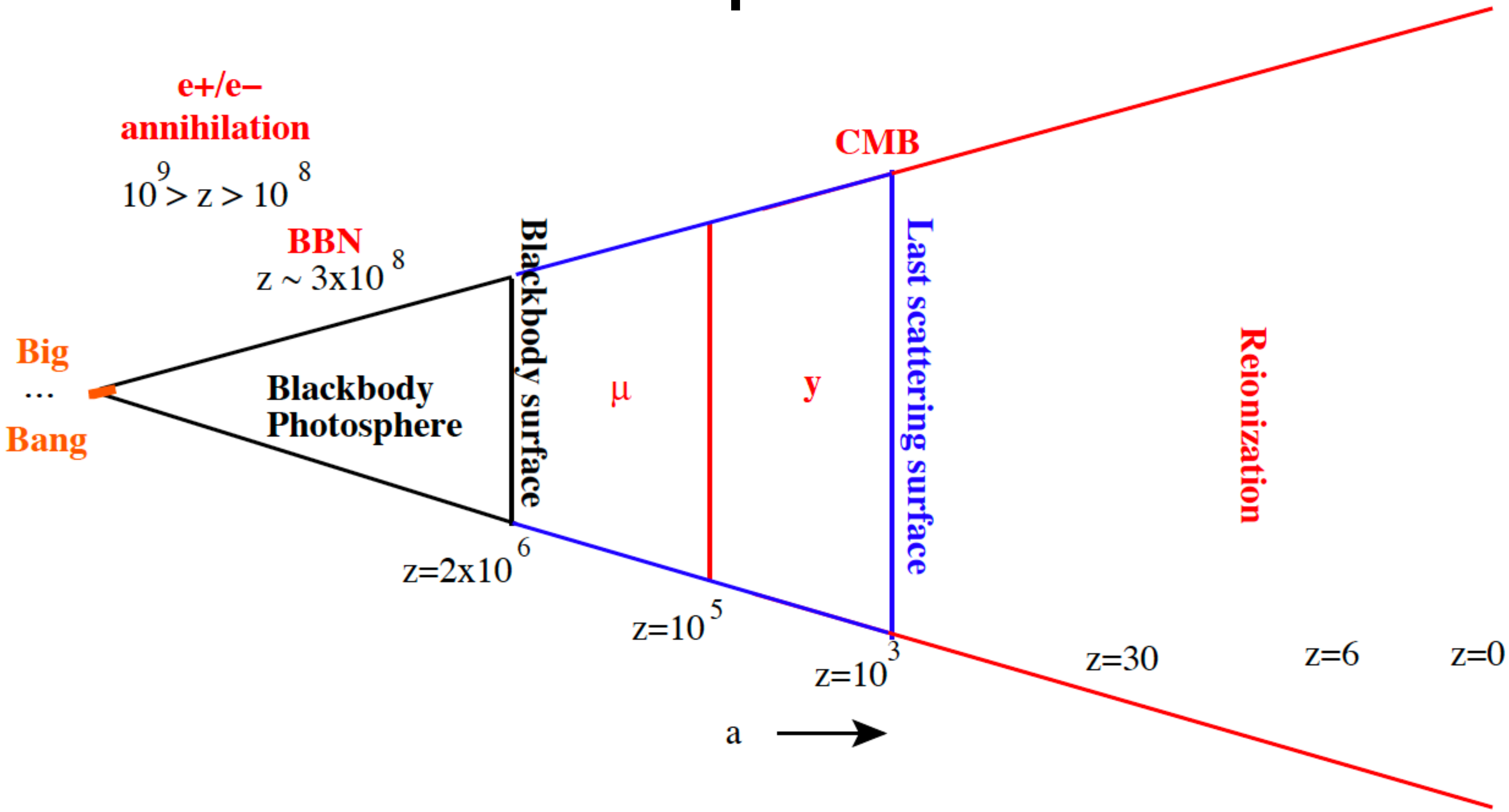
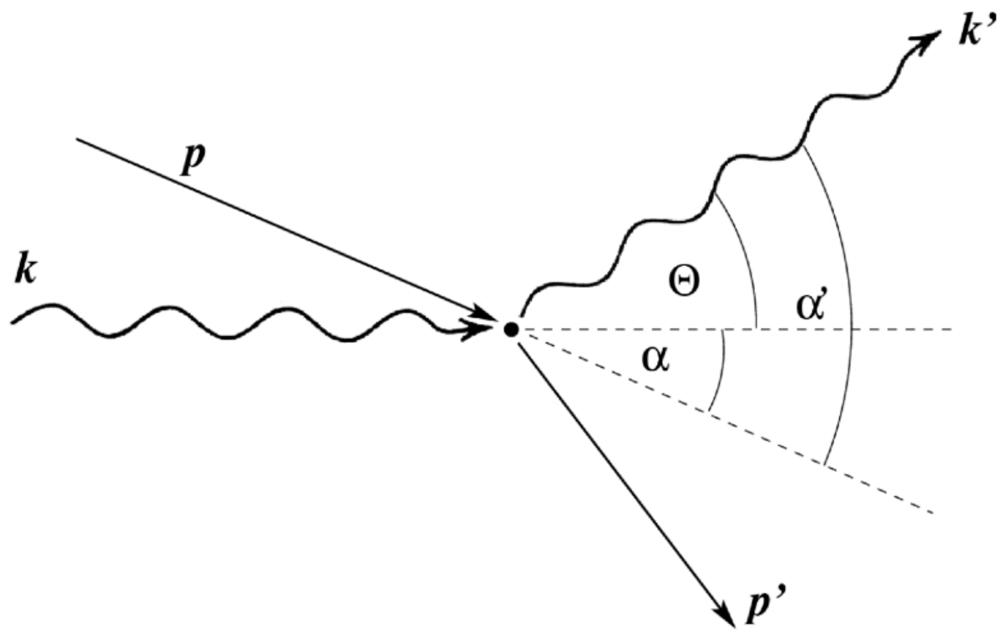
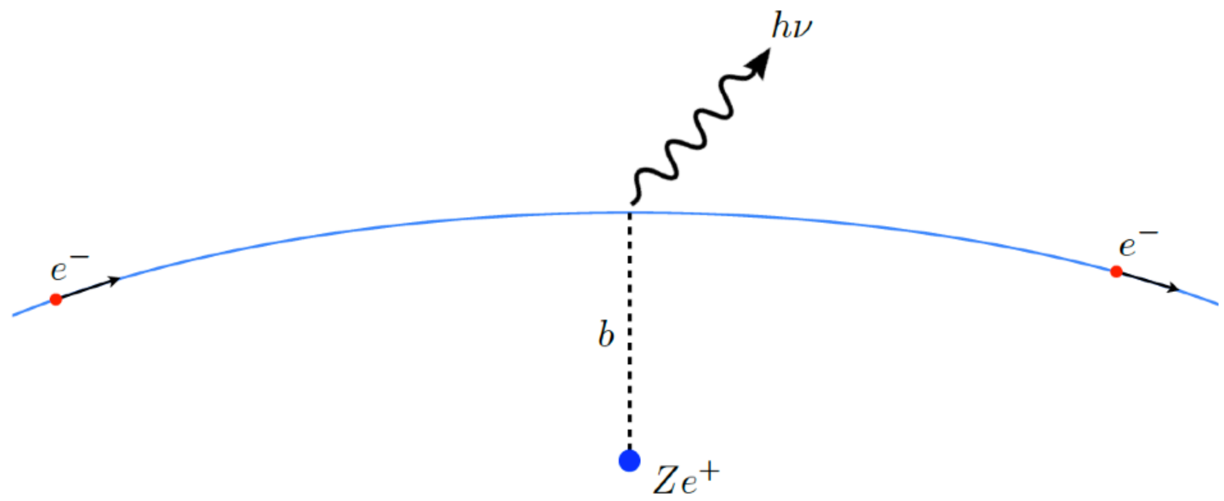


Figure 1. Important events in the history of the CMB spectrum and anisotropy formation in big bang cosmology. Redshift range ($2 \times 10^6 \gtrsim z \gtrsim 10^5$), where the energy injection would give rise to a Bose-Einstein spectrum (μ-type distortion), is marked as μ. At much smaller redshifts ($z \lesssim 10^4$), any heating of CMB through Compton scattering would create a y-type distortion. The spectrum in the intermediate redshift range would not be a pure μ or y type but in between the two types.

From Khatri & Sunyaev (2012)

Compton scattering and bremsstrahlung





One would expect the bremsstrahlung process to be the main culprit for producing photons, but due to the **high photon-to-baryon ratio** of our universe, the double Compton scattering (or radiative Compton scattering) plays a dominant role in the early phase.

After the photon creation is done (which is more efficient at low frequencies), the redistribution of photon energies is done by the standard Compton scattering. However, this process may not be fully efficient. We can define two epochs: that of 'efficient' Comptonization (or the era of μ-type distortion, before z~ 5×10^4), and the era of 'inefficient' Comptonization (or $\mathbf{y}$-type distortion) below that redshift.

The change in photon energy via Compton scattering is given by the well-known equation:

$$\frac{\nu'}{\nu} = \frac{1-\beta\mu}{1-\beta\mu' + \frac{h\nu}{\gamma m_e c^2}(1-\mu_{sc})},$$

where $\beta = v/c$, $\gamma = 1/\sqrt{1-\beta^2}$, $\mu = \cos\alpha$, $\mu' = \cos\alpha'$ and $\mu_{sc} = \cos\Theta$

Time scales vs expansion rate

To understand the relative importance of different scattering process, we need to compare their characteristic time-scales with the expansion time-scale of the universe.

Whenever an interaction rate $\boldsymbol{\Gamma}$ drops below the expansion rate of the universe

$$\boldsymbol{\Gamma} < H$$

we can consider the corresponding reaction "frozen". It becomes negligible.

We will also see this later for recombination and decoupling: When the recombination rate drops below the expansion rate, recombination freezes and the ionization fraction remains constant. When the Thomson scattering rate of photons on electrons falls below the expansion rate, photons become free to propagate without further scattering.

Time scales vs expansion rate

To understand the relative importance of different scattering process, we need to compare their characteristic time-scales with the expansion time-scale of the universe.

The typical expansion time-scale is given by the inverse of Hubble expansion rate, *H(z)*

$$t_{\rm exp} = H^{-1} \simeq \begin{cases} 4.8 \times 10^{19}\,(1+z)^{-2}\ \ \text{sec} & \text{(radiation domination)} \\ 8.4 \times 10^{17}\,(1+z)^{-3/2}\ \ \text{sec} & \text{(matter domination),} \end{cases}$$

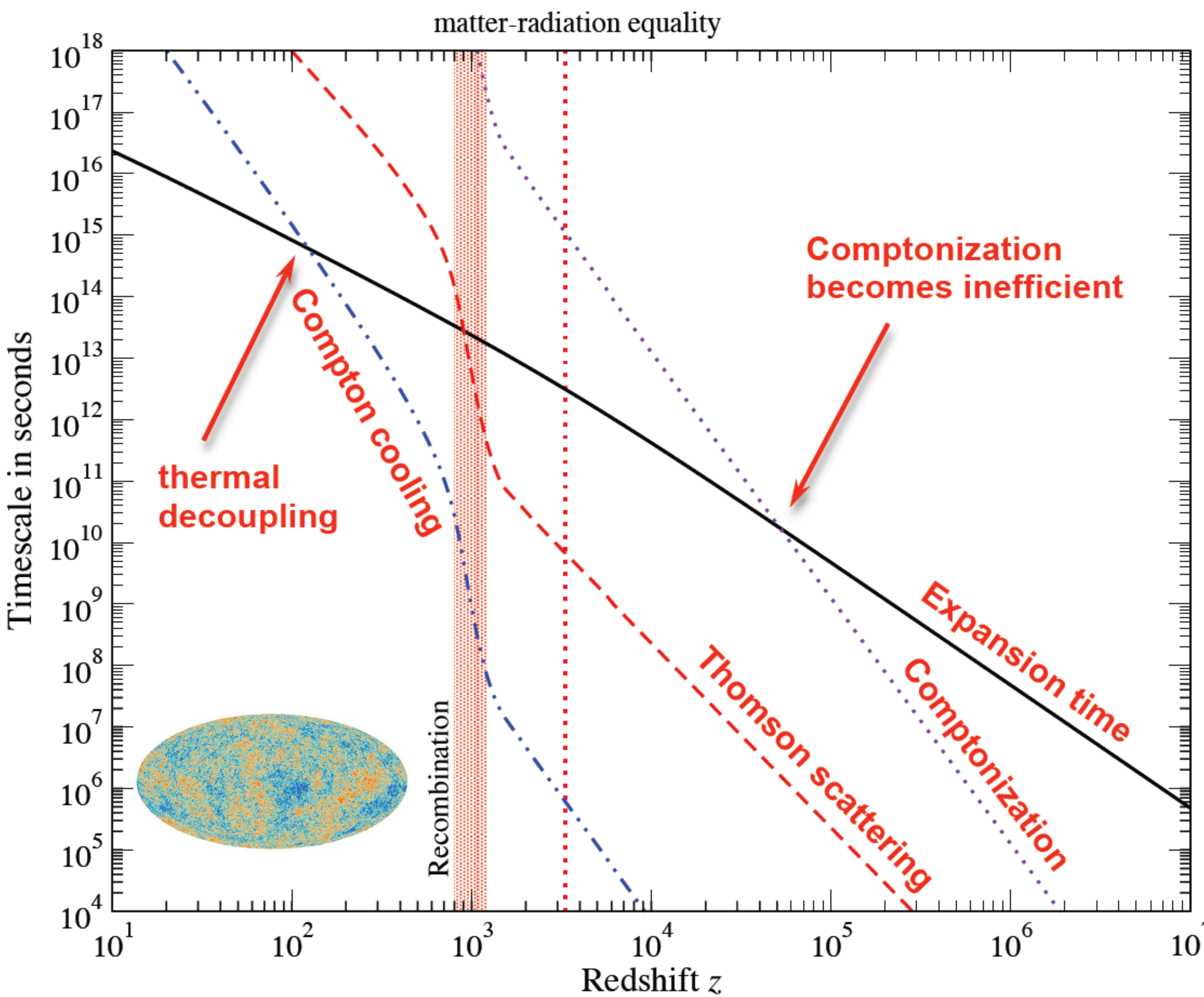
Comparing it with the Thomson scattering time-scale, we can find when the universe becomes transparent to the CMB photons. The Thomson scattering time-scale is:

$$t_{\rm T} = (\sigma_{\rm T} N_{\rm e} c)^{-1} \simeq 2.7 \times 10^{20}\, X_{\rm e}^{-1} (1+z)^{-3}\ \ \text{sec} \simeq 4.0 \times 10^{4} \left[\frac{X_{\rm e}}{0.16}\right]^{-1} \left[\frac{1+z}{1100}\right]^{-3}\ \ \text{years,}$$

Comparing it with the Compton time-scale for transfer of energy from the electrons to the photons, we find that the *Comptonization process* becomes inefficient at roughly z ~ $5{\times}10^4$

$$t_{\rm e\gamma} = \frac{t_{\rm T}}{4\theta_{\rm e}} \simeq 4.9 \times 10^{5}\, t_{\rm T} \left[\frac{1+z}{1100}\right]^{-1} \simeq 1.2 \times 10^{29} (1+z)^{-4}\ \ \text{sec}\,.$$

Compton scattering efficiency



From Chluba (2018), arXiv:1806.02915

The Bose–Einstein spectrum

When many scatterings occur, the photon spectrum is driven towards an equilibrium via the Comptonization process, and the result is a Bose-Einstein (BE) spectrum:

$$n_{\mathrm{BE}} = \frac{1}{\mathrm{e}^{x_{\mathrm{e}}+\mu_0} - 1},$$

Note that the statement "photons have no rest mass and hence their chemical potential is zero" is only true if there is full equilibrium, i.e. we have a blackbody at the temperature of the medium! More generally, for a fixed photon number, the chemical potential can be non-zero.

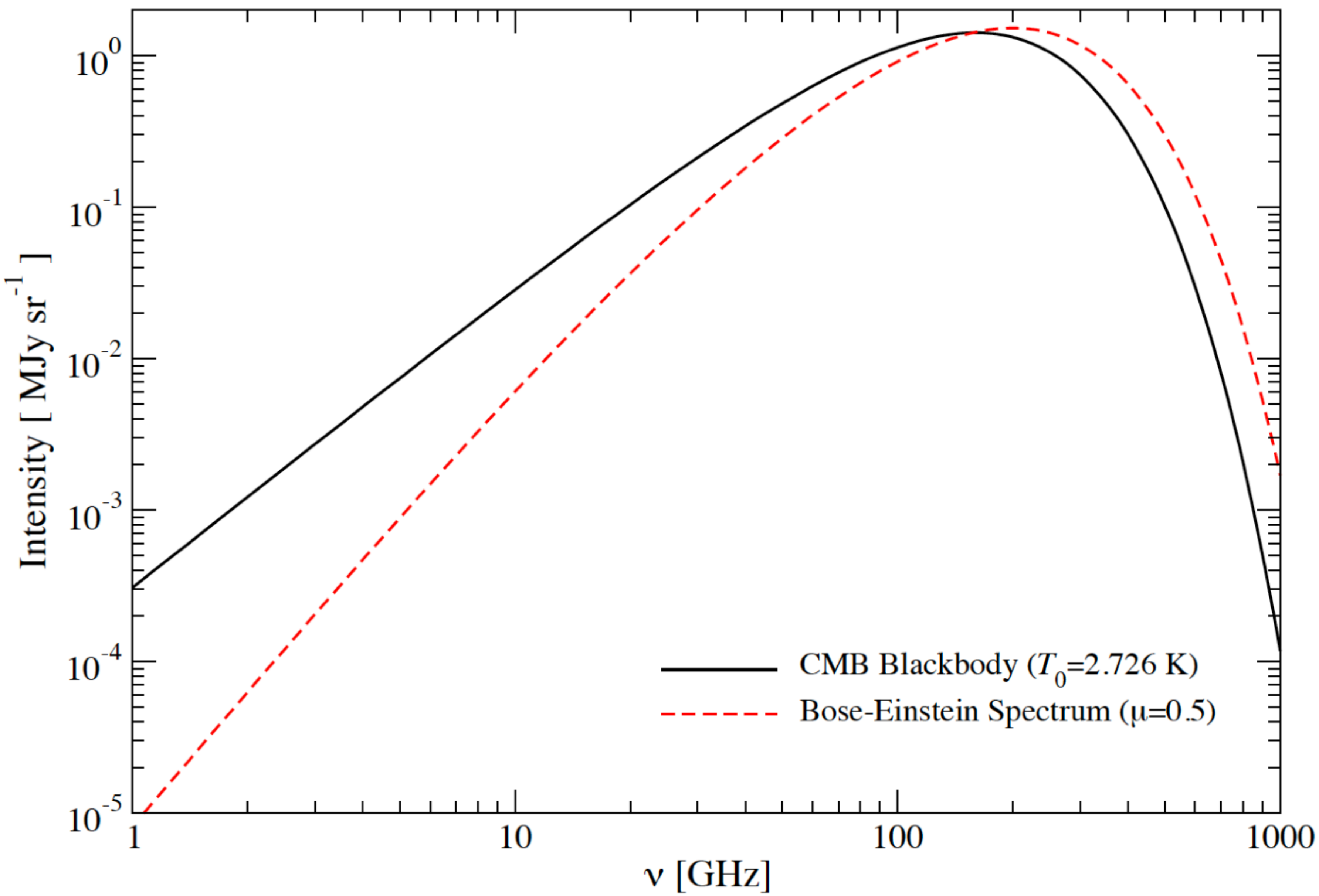
- $\mu_0 > 0$: *fewer* photons than a in blackbody at T_{e} $\rightarrow$ *energy release / photon destruction*
- $\mu_0 \equiv 0$: blackbody at temperature T_{e} $\rightarrow$ *full equilibrium*
- $\mu_0 < 0$: *more* photons than a in blackbody at T_{e} $\rightarrow$ *energy extraction / photon injection*

The statement thermal equilibrium is established at the high redshift ($z > 2\times10^6$) is equivalent to say that μ is driven essentially to zero by that redshift.

When energy is added to the CMB radiation field *after* redshift of $\sim2\times10^6$, there may still be time to reintroduce kinetic equilibrium, but not full thermal equilibrium. In that case, *the spectrum would be a Bose-Einstein spectrum.* This phase is generally termed as the epoch of μ–distortion ($\sim2\times10^6 < z < \sim10^5$). Here Comptonization is efficient in driving the photon energy distribution to an equilibrium.

The Bose–Einstein spectrum

Bose-Einstein spectrum for large chemical potential $\mu = 0.5$ and $T_i = T_0 = 2.726\,\mathrm{K}$.



If we expand the BE spectrum in powers of the chemical potential, we can see that the maximum departure from a blackbody spectrum is expected at low frequencies ($x_e << 1$)

$$f_{\rm BE} = \frac{1}{e^{x_e+\mu_0}-1} \approx \frac{1}{e^{x_e}-1} - \frac{G(x_e)}{x_e}\mu_0 + O(\mu_0^2)$$

μ– and y–type distortions

A µ-type distortion can not be generated at recent epochs (Comptonization is very inefficient) and thus directly probe the physics of the pre-recombination era.

At later times, leading up to the epoch of recombination, and then during reionization, any energy input will generally create a y-type distortion. Here Comptonization is inefficient ($\Gamma > H^{-1}$), but Compton scatterings still take place, and we get y << 1. This can be created at any redshift after z_{rec} , down to z = 0 (e.g. in hot clusters, SZ effect).

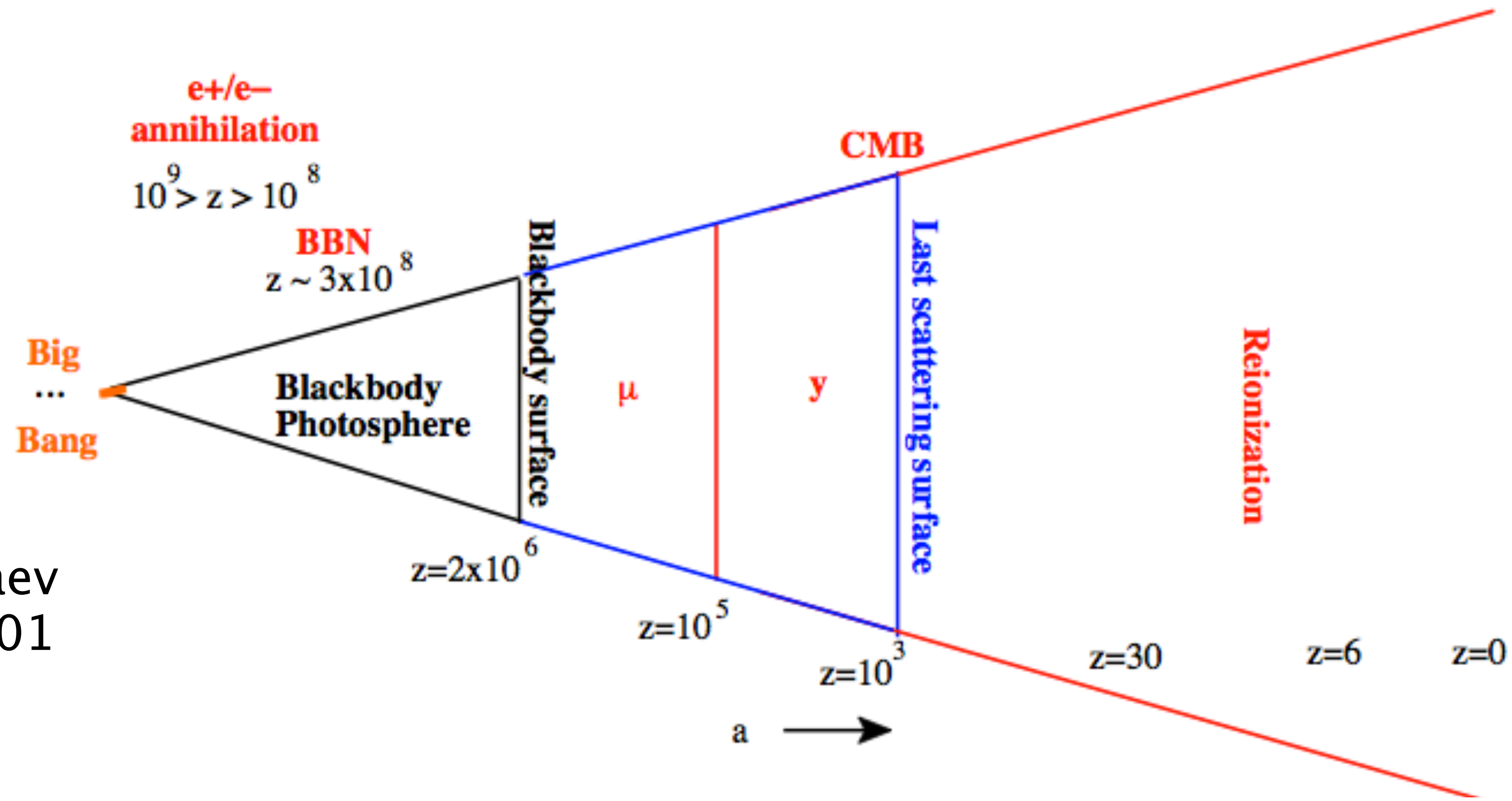


Fig. ref:
Khatri & Sunyaev
arXiv:1203.2601

The Compton-y parameter depends on the number of scattering (related to τ) and the net energy exchange per scattering.

$$y = \int_0^{\tau} \frac{k(T_e - T_\gamma)}{m_e c^2}\, d\tau' = \int_0^{t} \frac{k(T_e - T_\gamma)}{m_e c^2} \sigma_T N_e c\, dt'.$$

Compton-y distortion

Compton-y distortion is created when scattering between electrons and photons are inefficient in causing an energy exchange. This is typically the case when the electron and photon temperatures are vastly different, so that electrons practically don't change energy after scattering. The energy exchange is parametrized by the Compton *y*-parameter and in this typical case we have *y*<<1.

The Comptonization of photons, i.e. repeated scattering of photons by electrons in an isotropic medium, can be described by the *Kompaneets equation*:

Kompaneets, A. (1956), Sov.Phys. 31, 876

$$\frac{\partial n}{\partial \tau} \equiv \frac{\theta_e}{x^2}\frac{\partial}{\partial x} x^4 \left[\frac{\partial}{\partial x} n + \frac{T_\gamma}{T_e} n(1+n)\right],$$

$\theta_e = kT_e/m_e c^2$ (dimensionless electron temperature)

This photon evolution equation in presence of electron-photon collisions can be solved analytically only in limiting cases. Starting with an initial blackbody distribution function, n_{BB}, one gets after a very short time interval (i.e., $\Delta\tau << 1$) the following expression:

$$\begin{aligned}\Delta n &\approx \frac{\Delta\tau\,\theta_e}{x^2}\frac{\partial}{\partial x} x^4 \left[\frac{\partial}{\partial x} n_{bb} + \frac{T_\gamma}{T_e} n_{bb}(1+n_{bb})\right] \approx \frac{\Delta\tau\,(\theta_\gamma-\theta_e)}{x^2}\frac{\partial}{\partial x} x^4 n_{bb}(1+n_{bb}) \\ &\approx \Delta\tau\,(\theta_\gamma-\theta_e)\left[4x n_{bb}(1+n_{bb}) - x^2 n_{bb}(1+n_{bb})(1+2n_{bb})\right] \\ &\approx \Delta\tau\,(\theta_e-\theta_\gamma)\,G(x)\left[x\frac{e^x+1}{e^x-1} - 4\right] \equiv \Delta\tau\,(\theta_e-\theta_\gamma)\,Y_{SZ}(x),\end{aligned}$$

where we used $\partial_x n_{bb} = -n_{bb}(1+n_{bb}) = -e^x/(e^x-1)^2 = -G(x)/x$

Compton–y distortion

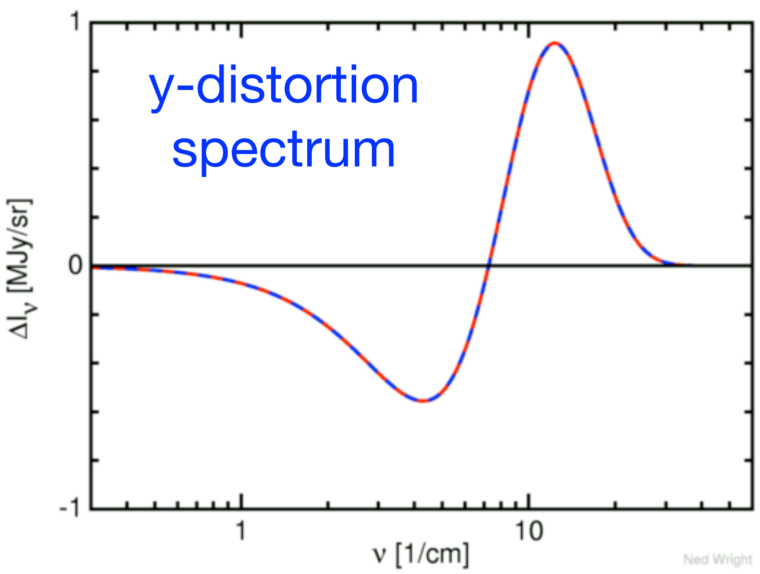
Compton-y distortion is created when scattering between electrons and photons are inefficient in causing an energy exchange. This is typically the case when the electron and photon temperatures are vastly different, so that electrons practically don't change energy after scattering. The energy exchange is parametrized by the Compton *y*-parameter and in this typical case we have *y*<<1.

$$\Delta n \approx \frac{\Delta\tau\,\theta_{\rm e}}{x^2}\frac{\partial}{\partial x}x^4\left[\frac{\partial}{\partial x}n_{\rm bb}+\frac{T_\gamma}{T_{\rm e}}n_{\rm bb}(1+n_{\rm bb})\right]\approx\frac{\Delta\tau\,(\theta_\gamma-\theta_{\rm e})}{x^2}\frac{\partial}{\partial x}x^4 n_{\rm bb}(1+n_{\rm bb})$$

$$\approx \Delta\tau\,(\theta_\gamma-\theta_{\rm e})\left[4x n_{\rm bb}(1+n_{\rm bb})-x^2 n_{\rm bb}(1+n_{\rm bb})(1+2n_{\rm bb})\right]$$

$$\approx \Delta\tau\,(\theta_{\rm e}-\theta_\gamma)\,G(x)\left[x\,\frac{{\rm e}^x+1}{{\rm e}^x-1}-4\right]\equiv\Delta\tau\,(\theta_{\rm e}-\theta_\gamma)\,Y_{\rm SZ}(x),$$

$$\boxed{Y_{\rm SZ}(x)=G(x)\left[x\,\frac{{\rm e}^x+1}{{\rm e}^x-1}-4\right]\approx\begin{cases}-\frac{2}{x} & \text{for} \quad x\ll 1\\ x(x-4){\rm e}^{-x} & \text{for}\quad x\gg 1.\end{cases}}$$

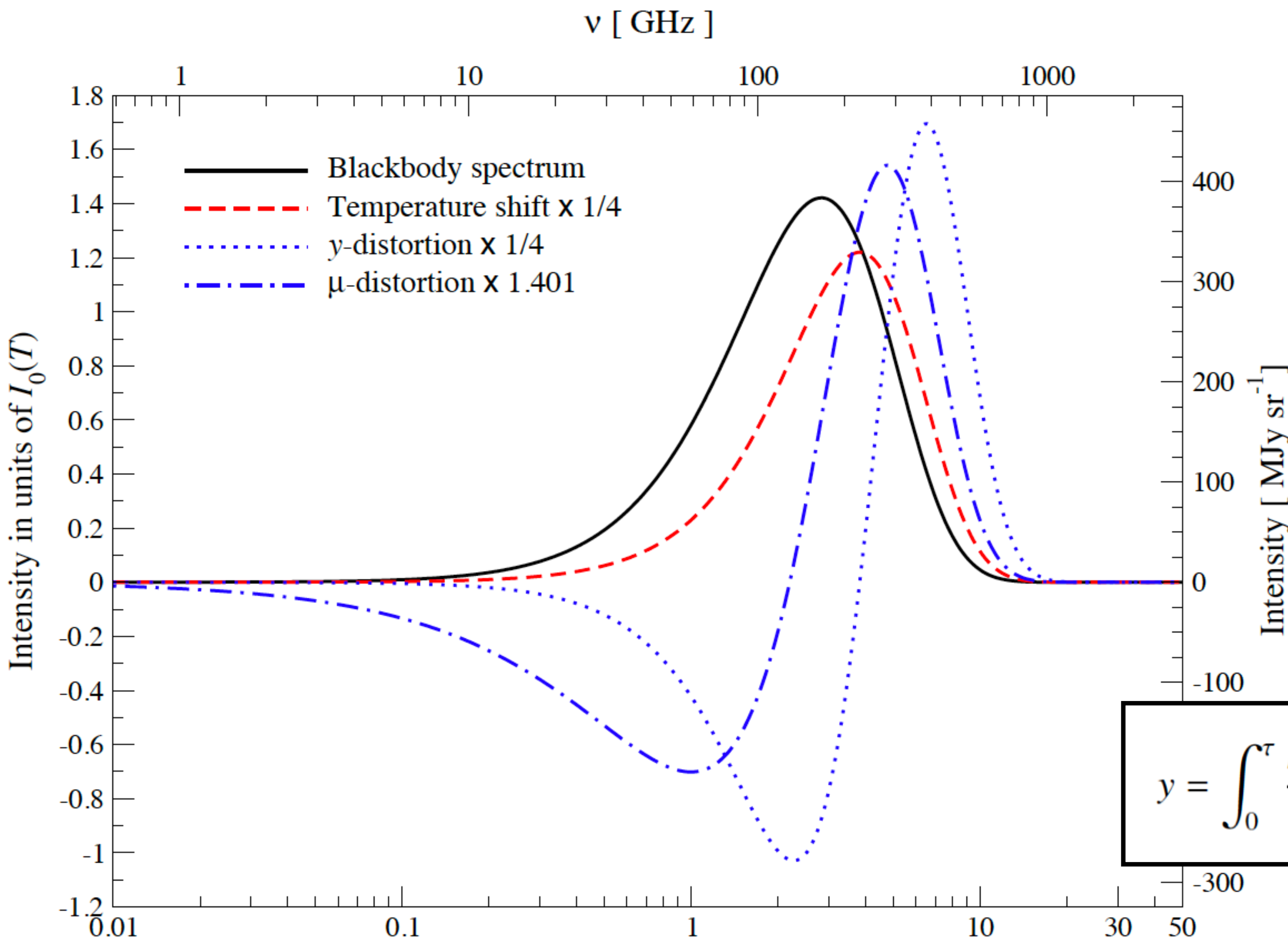


This is the definition of the Compton-y distortion, first studied by *Sunyaev & Zeldovich in 1968* for the case of CMB scattering by hot plasma inside galaxy clusters. The important parameter is the Compton-y parameter (below), which depends on the number of scattering and the net energy exchange, $\Delta\nu/\nu \simeq 4(\theta_{\rm e}-\theta_\gamma)$, per scattering.

$$y=\int_0^\tau \frac{k(T_{\rm e}-T_\gamma)}{m_{\rm e}c^2}\,{\rm d}\tau' = \int_0^t \frac{k(T_{\rm e}-T_\gamma)}{m_{\rm e}c^2}\sigma_{\rm T}N_{\rm e}c\,{\rm d}t',$$

Compton–y distortion

Compton-y distortion is created when scattering between electrons and photons are inefficient in causing an energy exchange. This is typically the case when the electron and photon temperatures are vastly different, so that electrons practically don't change energy after scattering. The energy exchange is parametrized by the Compton *y*-parameter and in this typical case we have *y*<<1.



The y-distortion spectrum zero crossing frequency:

$$\nu \simeq 217\text{GHz} \, (\equiv x \simeq 3.830)$$

The BE spectrum zero crossing frequency:

$$\nu_{\rm cr} \approx 124\,\text{GHz}\,(1 - 0.304\,\mu \ln \mu)$$

$$y = \int_0^\tau \frac{k(T_e - T_\gamma)}{m_e c^2}\,d\tau' = \int_0^t \frac{k(T_e - T_\gamma)}{m_e c^2}\sigma_T N_e c\,dt',$$

- $y > 0$: overall energy is transferred from the electrons to the photons → *Comptonization*
- $y < 0$: energy flows from the photons to the electrons → *Compton cooling*

y-distortion from Sum of Blackbodies

Here we show the important result that when you mix two (or more) blackbodies, the result is not another blackbody at an intermediate temperature. Rather, it leads to y-type spectral distortion!

We need to use the derivative of blackbody intensity spectrum at a fixed temperature:

$$\boxed{\frac{\Delta I}{I} = \frac{d\ln I}{d\ln T}\frac{\Delta T}{T} = \frac{xe^x}{e^x-1}\frac{\Delta T}{T}}$$

Let's assume we have two blackbodies, $B_\nu(T)$ and $B_\nu(T')$, where the temperature difference is $\Delta = (T' - T) / T$. Then from Taylor expansion we get the following:

$$\begin{aligned}\frac{\Delta I}{I} &= \frac{I'-I}{I} = \frac{e^x-1}{e^{x(1+\Delta)}-1} \\ &\approx \frac{xe^x}{e^x-1}\left[\Delta + g(x)\frac{\Delta^2}{2}\right] + \mathcal{O}(\Delta^3)\end{aligned}$$

where $g(x) = x\frac{e^x+1}{e^x-1} - 2.$

By recasting to relative temperature difference: $\frac{\Delta T}{T} = \Delta + g(x)\frac{\Delta^2}{2} + \mathcal{O}(\Delta^3)$

➜ so should there be a global y-distortion from the CMB temperature anisotropies?

μ and y distortions: Summary

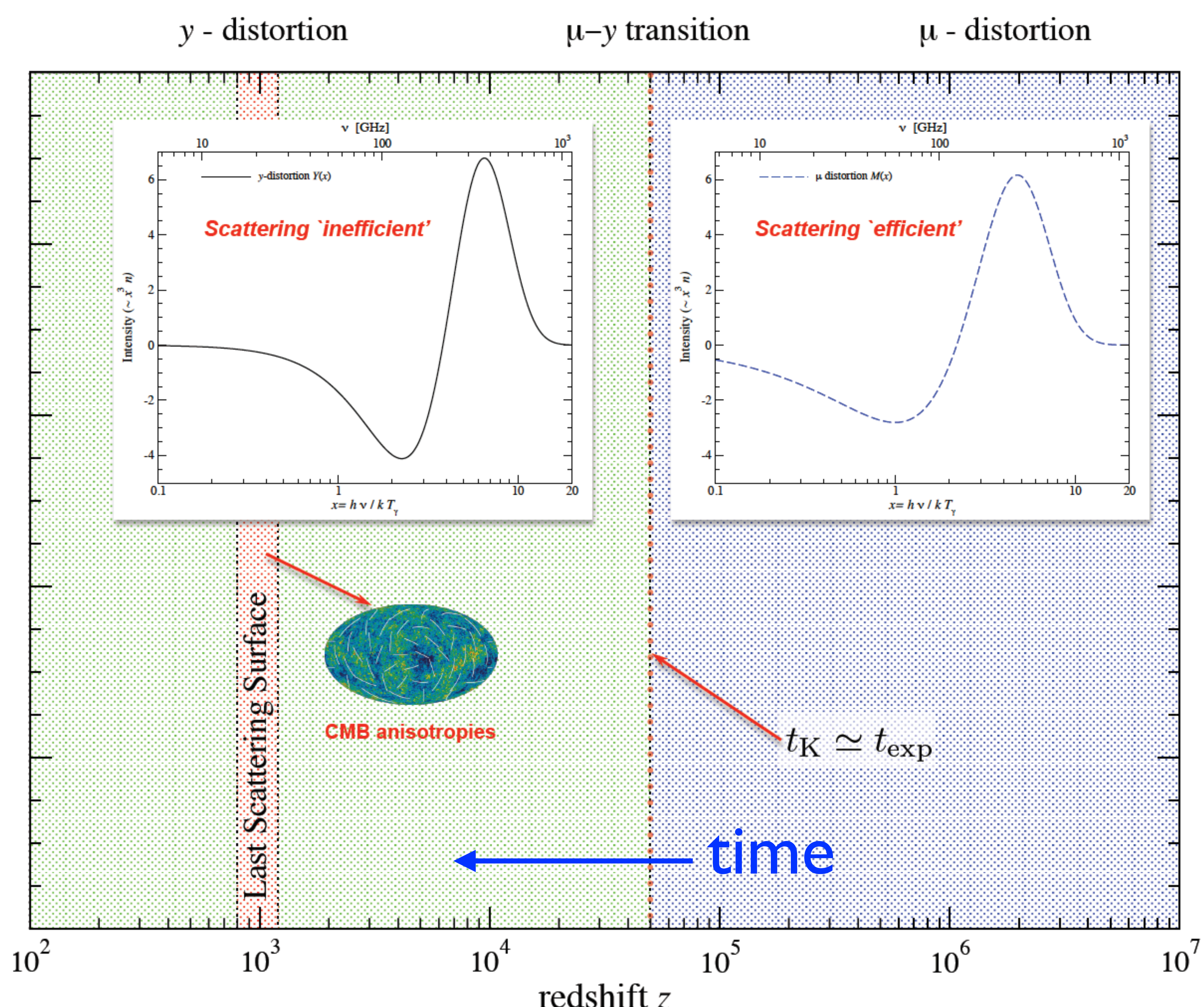
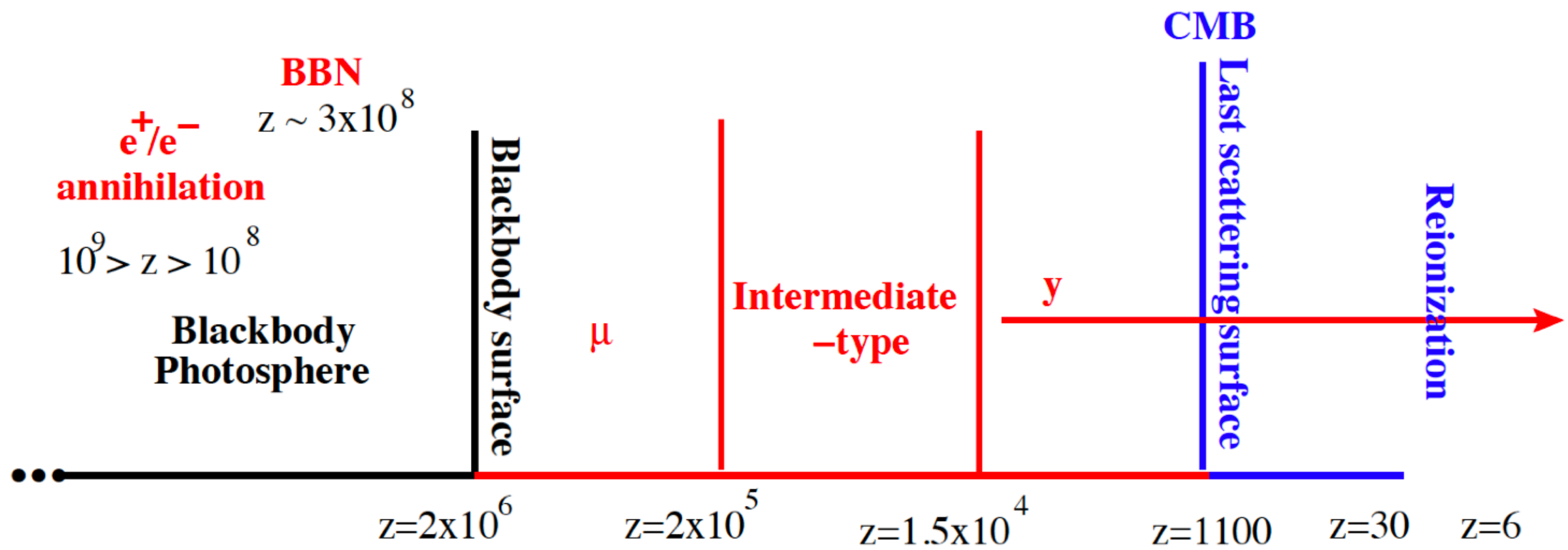


Figure credit: Jens Chluba

μ and y distortions: Summary



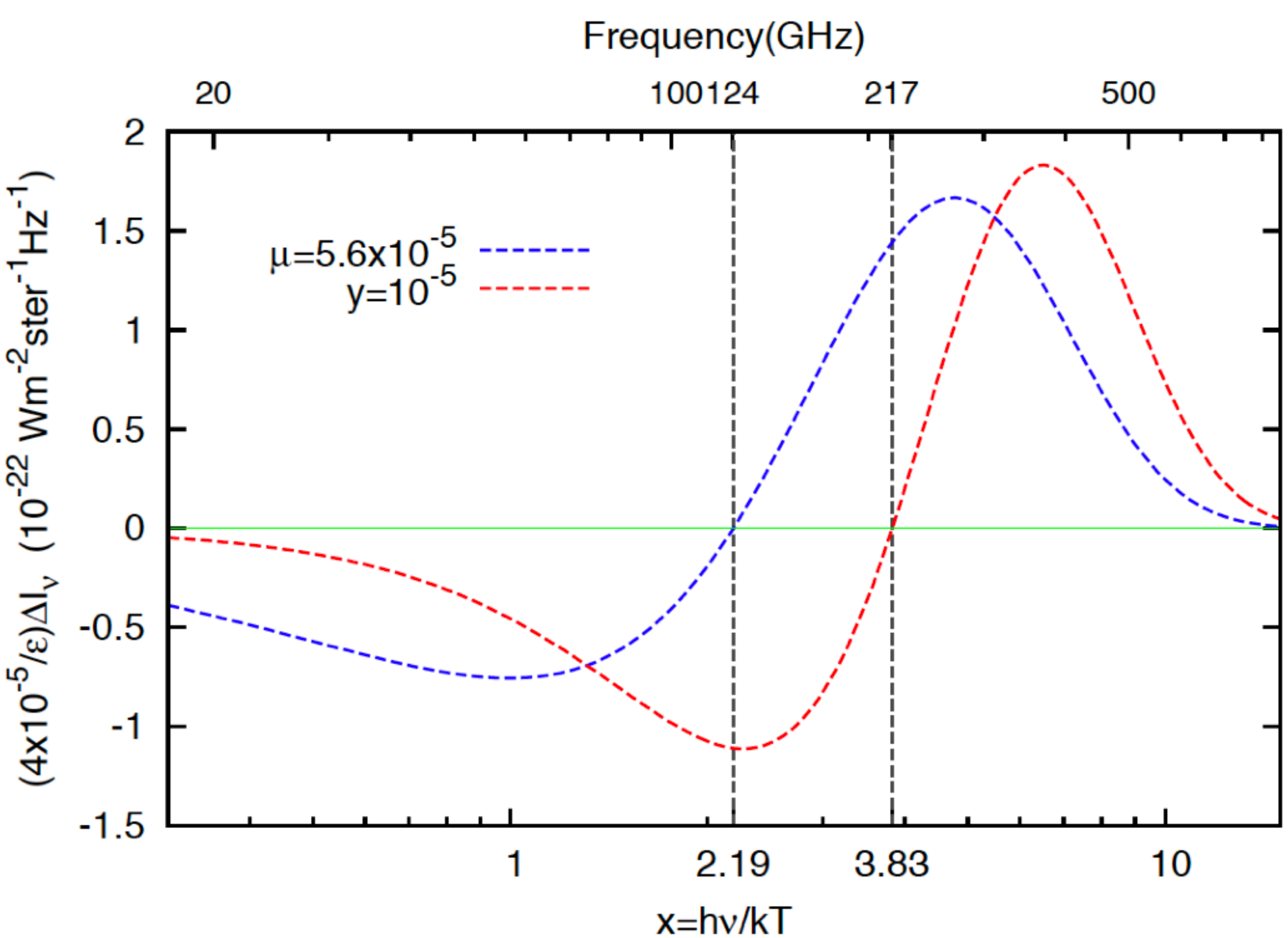


Fig. left: μ- and y-type distortions for a fixed addition of energy $\varepsilon=\Delta E/E=4\text{x}10^{-5}$ to a blackbody at temperature T.

Shown here is the intensity difference w.r.t the reference CMB blackbody.

These values are roughly at the level of current observational limits, obtained roughly 30 years ago!

(From Khatri & Sunyaev 2012)

Current limits on Spectral Distortions

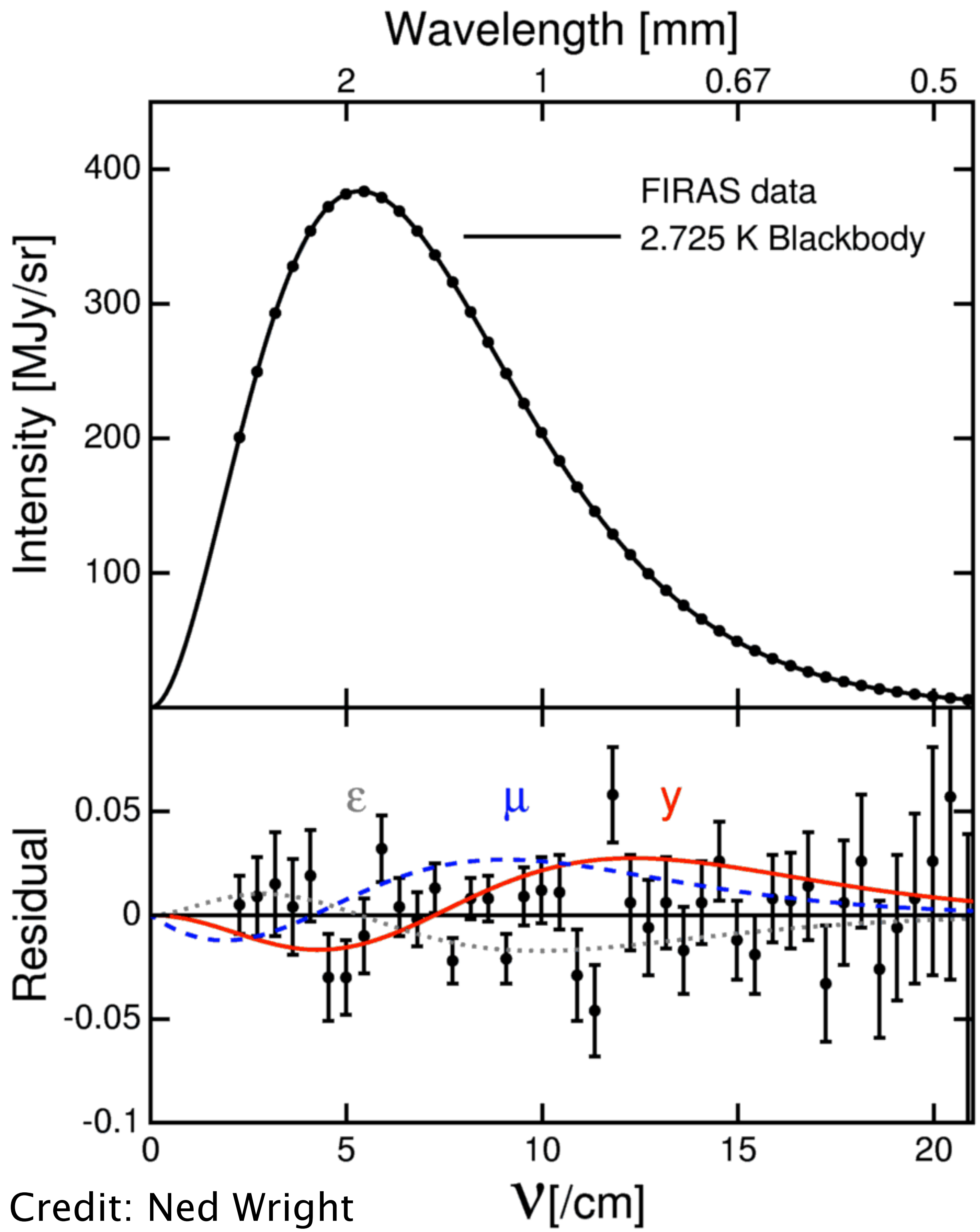
(from COBE/FIRAS, early 90-s)

- Energy added after $z \sim 2 \times 10^6$ will show up as spectral distortions. Departure from a Planck spectrum at fixed T is known as "μ distortion" (B-E distribution). μ distortion is easier to detect at wavelengths $\lambda > 10$ cm.
  COBE measurement: $|\mu| < 9 \times 10^{-5}$ (95% CL)

- The amount of inverse Compton scattering at later epochs ($z < 10^5$) show up as "y distortion", where $y \sim \sigma_T n_e kT_e$ (e.g. the Sunyaev-Zel'dovich effect). This rules out a uniform intergalactic plasma as the source for X-ray background.
  COBE measurement: $y < 1.2 \times 10^{-5}$ (95% CL)

- Energy injection at much later epochs ($z << 10^5$), e.g. free-free distortions, are also tightly constrained.
  COBE measurement: $Y_{ff} < 1.9 \times 10^{-5}$ (95% CL)

COBE/FIRAS Measurements



Credit: Ned Wright

Fundamental FIRAS measurement is the plot at the bottom: the difference between the CMB and the best-fitting blackbody. The top plot shows this residual added to the theoretical blackbody spectrum at the best fitting cold load temperature.

The three curves in the lower panel represents three likely non-blackbody spectra:

Red and blue curves show effect of hot electrons adding energy before and after recombination (roughly), the grey curve shows effect of a non-perfect blackbody as calibrator (less than 10^{-4})

$\mu-$ and $y-$type distortion candidates

Table 1. Census of μ distortions in standard cosmology. The adiabatic cooling of matter results in negative distortions shown in red. Table is taken from Ref. 63.

Process	μ
electron-positron annihilation	10^{-178}
BBN tritium decay	2×10^{-15}
BBN ^{7}Be decay	10^{-16}
WIMP dark matter annihilation	$3 \times 10^{-9} f_\gamma \frac{10\mathrm{GeV}}{m_{\mathrm{WIMP}}}$
Silk damping	$10^{-8} - 10^{-9}$
Adiabatic cooling of matter and Bose-Einstein condensation	-2.7×10^{-9}

Table 2. Census of y-type distortions in standard cosmology. y-type distortion from the mixing of blackbodies in our CMB sky[31] are also shown. Adiabatic cooling of matter creates negative distortions shown in red. Reionization/WHIM contributions after recombination dominate. Table is taken from Ref. 63.

Process	y
WIMP dark matter annihilation	$6 \times 10^{-10} f_\gamma \frac{10\mathrm{GeV}}{m_{\mathrm{WIMP}}}$
Silk damping	$10^{-8} - 10^{-9}$
Adiabatic cooling of matter and Bose-Einstein condensation	-6×10^{-10}
Reionization/WHIM	$10^{-6} - 10^{-7}$
Mixing of blackbodies: CMB $\ell \geq 2$ multipoles	8×10^{-10}

Ref: Khatri & Sunyaev, arXiv:1203.2601

μ– and y–type distortion candidates

➡ **WIMP dark matter annihilation:** WIMPs are a popular candidate of dark matter that can be thermally produced in the early universe. Even after the DM reactions freeze out in the very early stage, a small number of residual annihilation can keep on happening throughout the history of the universe, and release energy.

➡ **BBN Be^7 and tritium decay:** Even though BBN happens before the blackbody photosphere, Be^7 and tritium (H^3) survives until late times, whose decay can cause a μ-type spectral distortion.

➡ **Silk damping:** The acoustic waves generated in the photon-baryon fluid decay on the small scales due to viscous drag and thermal conduction. This effect is known as Silk damping (see lecture on CMB angular power spectrum later!) and it releases energy.

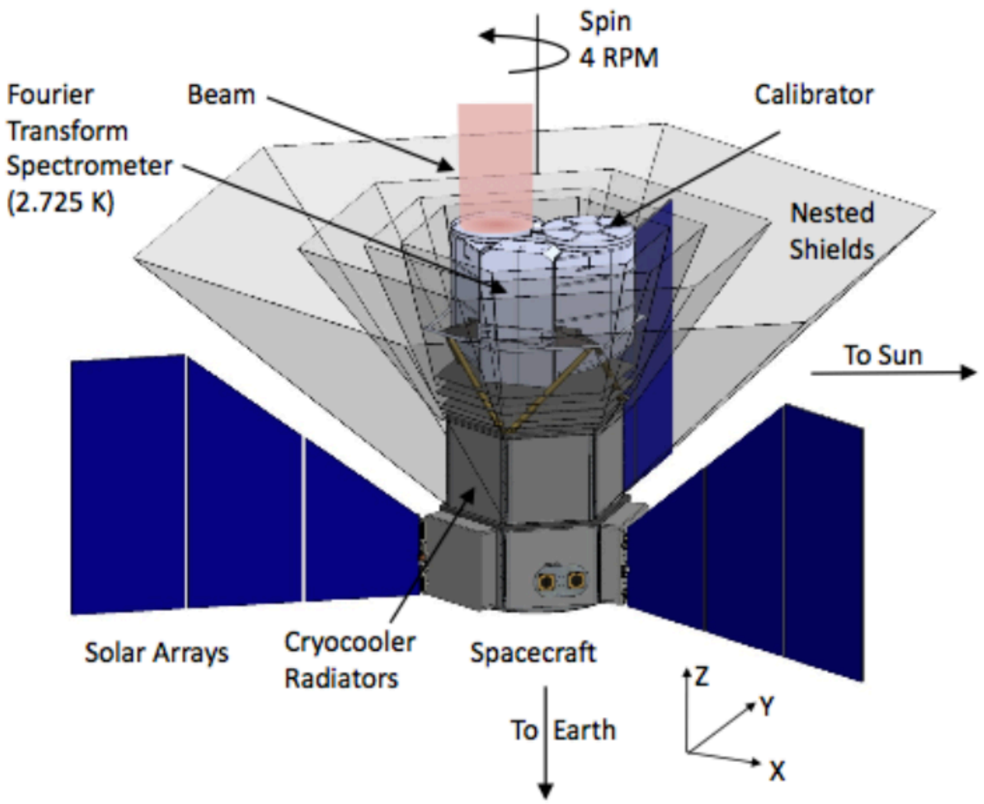
➡ **Bose-Einstein condensate:** The cooling of electron/baryons and the photons have different time (or scale factor) dependence, but Comptonization keep them at equal temperature in the early universe. This leads to a BE-condensate for the photons where energy is transferred from radiation to the matter. The resulting μ- and y-distortions have opposite sign compared to energy release.

➡ **Reionization / WHIM:** Star formation and shock-heating in the cosmic filaments ionize the neutral hydrogen, creating y-distortion on a global scale.

For more details and references, see Khatri & Sunyaev (2012)

Factor ~10^3 better with current technology

Sensitivity of the proposed Pixie satellite
Kogut et al. (2011), arXiv:1105.2044



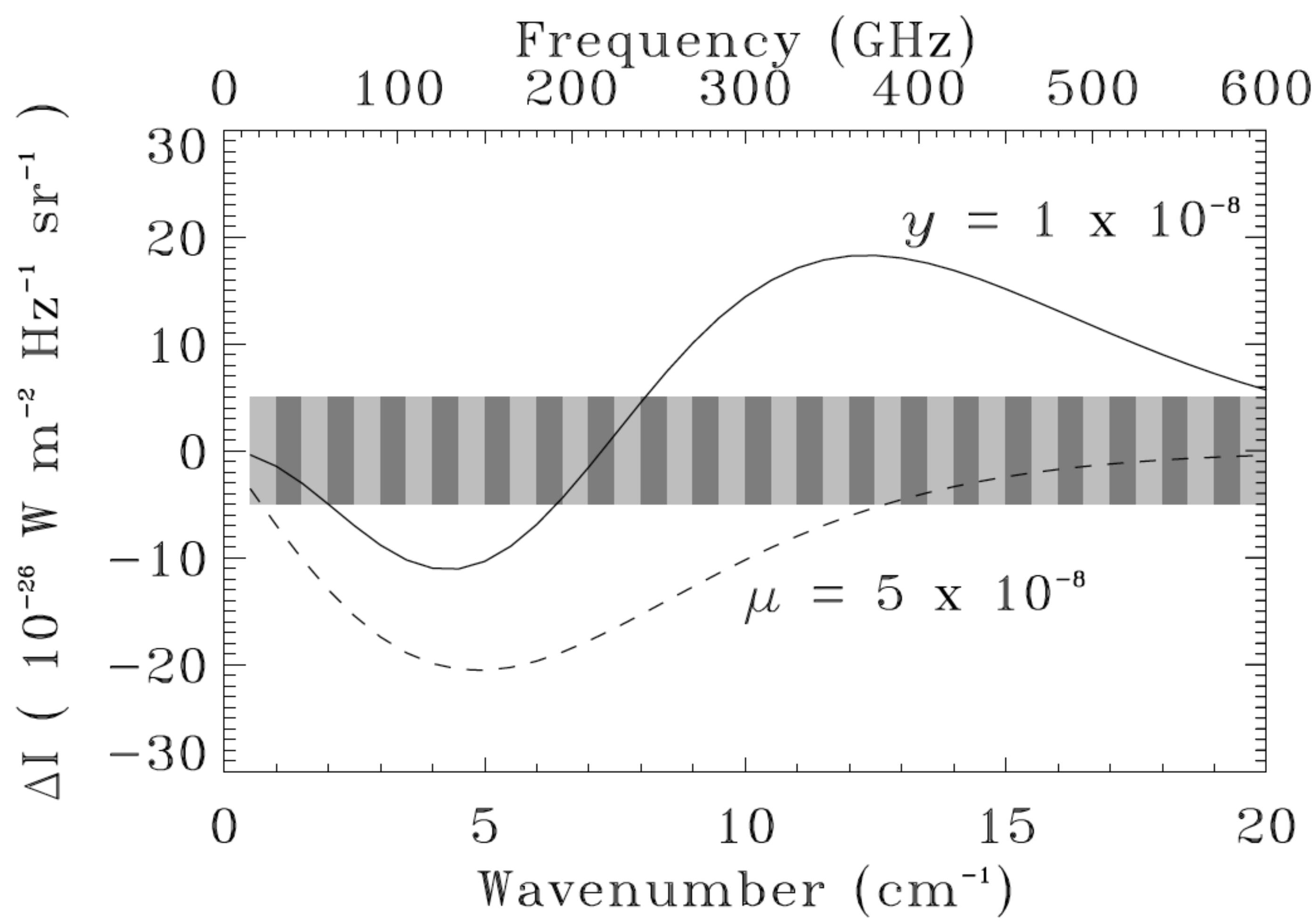
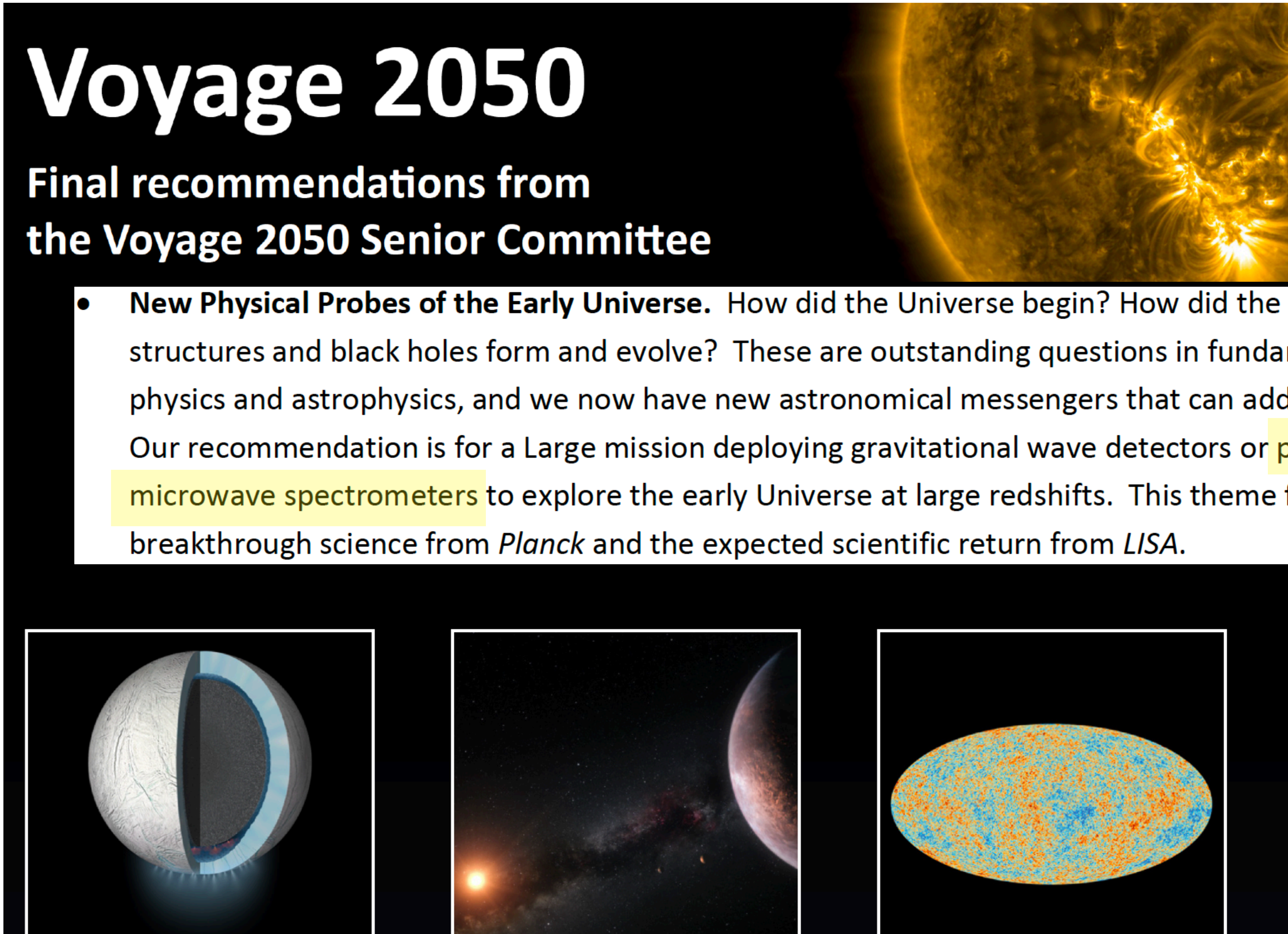


Figure 12. Distortions to the CMB blackbody spectrum compared to the PIXIE instrument noise in each synthesized frequency channel. The curves show 5σ detections of Compton (y) and chemical potential (μ) distortions. PIXIE measurements of the y distortion determine the temperature of the intergalactic medium at reionization, while the μ distortion probes early energy release from dark matter annihilation or Silk damping of primordial density perturbations.

Successor to PIXIE: SuperPIXIE and the Voyage2050 concept (see, e.g., arXiv:1909.01593)

Spectral distortion science in focus

https://www.esa.int/Science_Exploration/Space_Science/Voyage_2050_sets_sail_ESA_chooses_future_science_mission_themes



- **New Physical Probes of the Early Universe.** How did the Universe begin? How did the first cosmic structures and black holes form and evolve? These are outstanding questions in fundamental physics and astrophysics, and we now have new astronomical messengers that can address them. Our recommendation is for a Large mission deploying gravitational wave detectors or precision microwave spectrometers to explore the early Universe at large redshifts. This theme follows the breakthrough science from *Planck* and the expected scientific return from *LISA*.

Instrumentation for Voyage2050 CMB probe

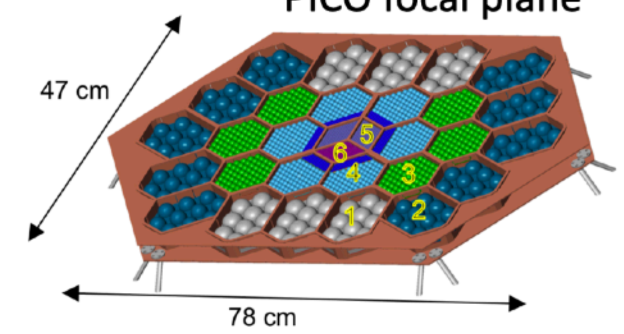
Microwave Imaging and Spectroscopy Telescope

1. A broad-band, multi-frequency polarised imager
   - Reference model: PICO instrument at the focus of 3.5m cold telescope
   - 21 overlapping bands from 20 to 800 GHz, $\Delta\nu/\nu \approx 0.3$
   - Angular resolution 1' at 300 GHz, 25" at 800 GHz
   - Full-sky sensitivity > 5000 Planck missions

2. A sensitive filterbank spectrometer with $R \approx 300$
   - Reference model: Extended Deshima at the focus of the same telescope
   - Frequency range 100-1000 GHz (goal 50-2000 GHz)
   - Angular resolution 1' at 300 GHz

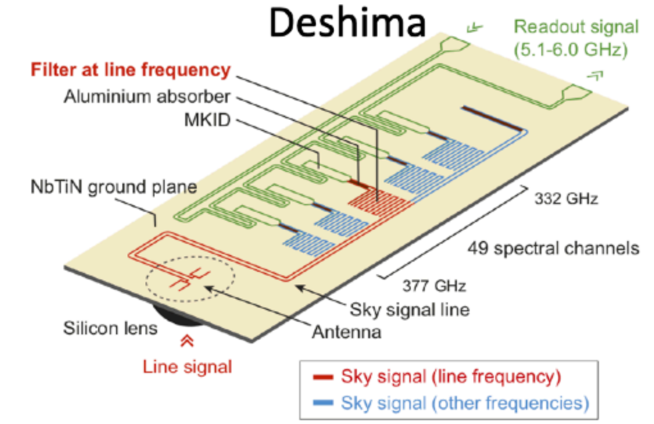
3. An absolutely calibrated Fourier Transform Spectrometer
   - Reference model: a three-module version of PIXIE / PRISTINE
   - Frequency range 10-2000 GHz
   - Degree-scale angular resolution
   - Sensitivity 10^4 to 10^5 times better than COBE/FIRAS

PICO focal plane

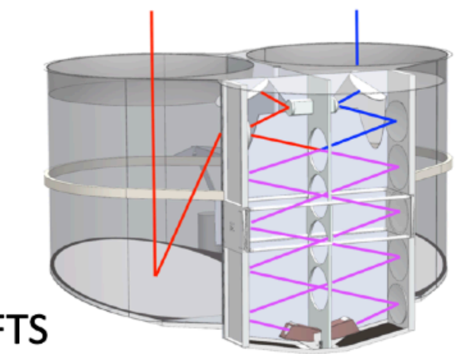


(S. Hanany et al. 2019)

Deshima



(A. Endo et al. 2019)



PIXIE FTS

(A. Kogut et al. 2019)

Questions?

Feel free to email me or ask questions in our eCampus Forum