astro8405

An Introduction to the Cosmic Microwave Background

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eCampus | Lernplattform der Universität Bonn

astro8405: The Cosmic Microwave Background

Aktionen

This course intends to give you a modern and up-to-date introduction to the science and experimental techniques relating to the Cosmic Microwave Background. No prior knowledge of cosmology is necessary, your prerequisite are a basic understanding of electrodynamics and thermal physics and some familiarity with Python programming.

Lecture 2:

Part 1: Detectors and Experiments

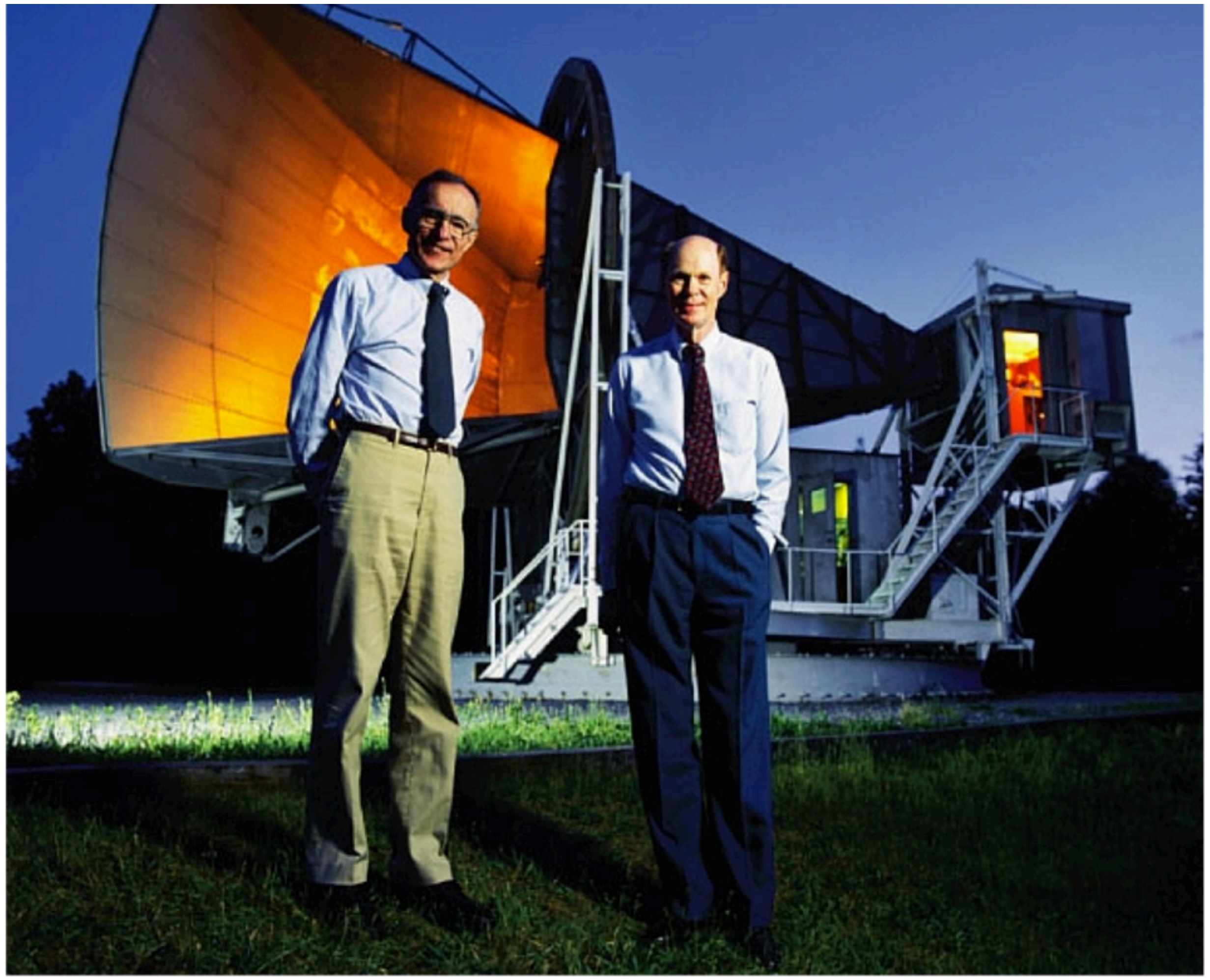
Part 2: Radiation Fundamentals

Arno Penzias & Robert Wilson, 1965

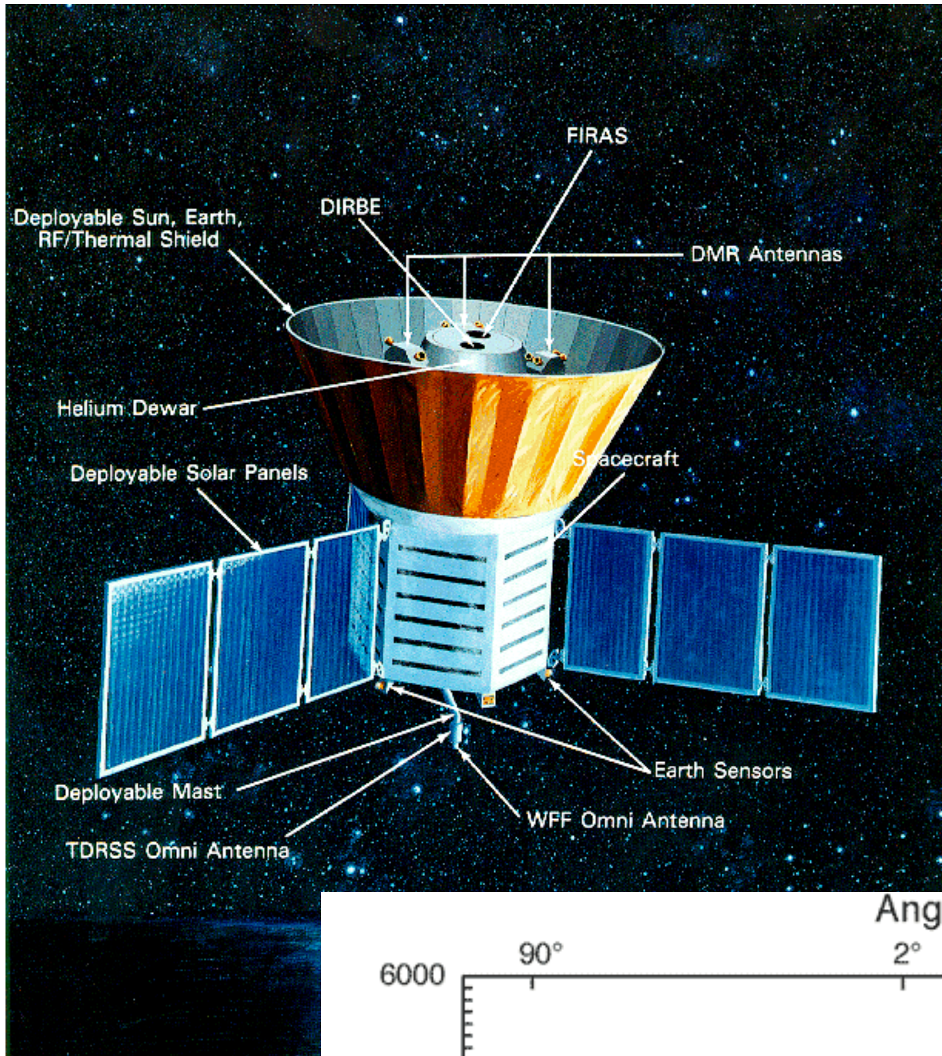
A MEASUREMENT OF EXCESS ANTENNA TEMPERATURE
AT 4080 Mc/s

Measurements of the effective zenith noise temperature of the 20-foot horn-reflector antenna (Crawford, Hogg, and Hunt 1961) at the Crawford Hill Laboratory, Holmdel, New Jersey, at 4080 Mc/s have yielded a value about 3.5° K higher than expected. This excess temperature is, within the limits of our observations, isotropic, unpolarized, and free from seasonal variations (July, 1964–April, 1965). A possible explanation for the observed excess noise temperature is the one given by Dicke, Peebles, Roll, and Wilkinson (1965) in a companion letter in this issue.

May 13, 1965
BELL TELEPHONE LABORATORIES, INC
CRAWFORD HILL, HOLMDEL, NEW JERSEY



COBE (1990) & WMAP (2001–2010)



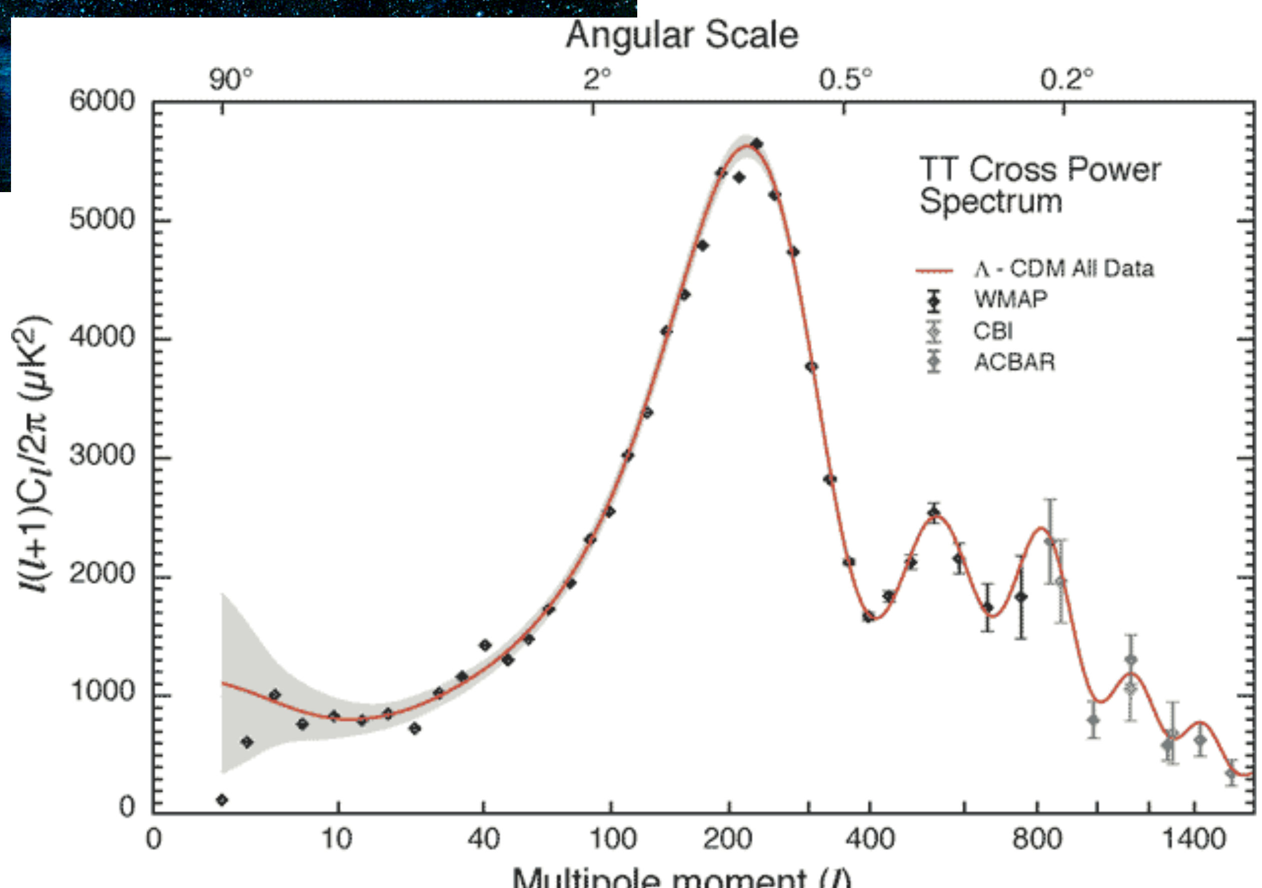
Credit: NASA

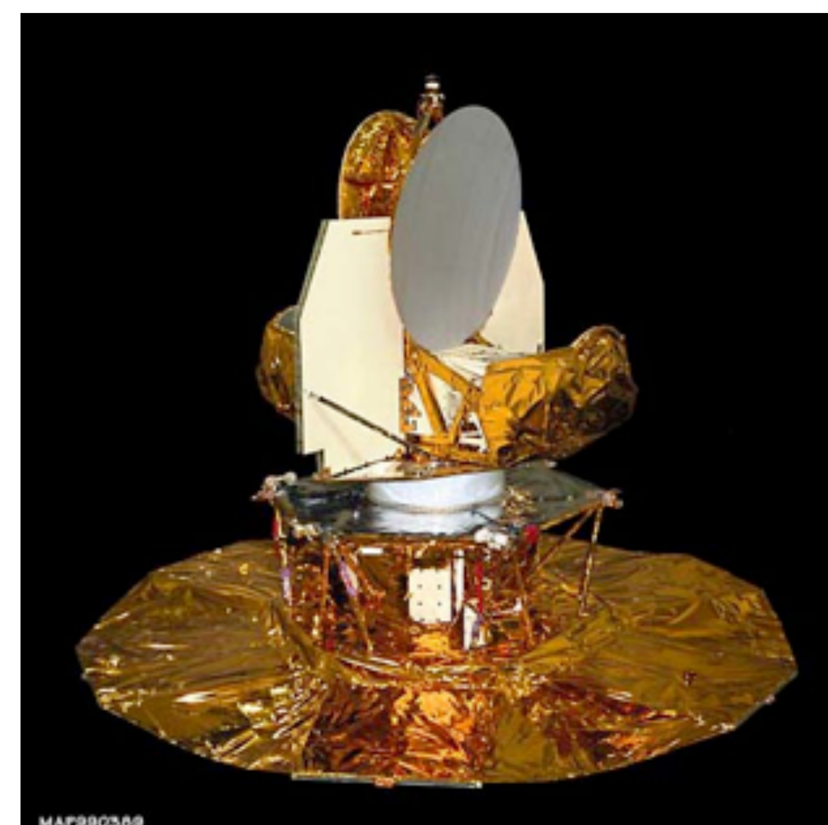
COBE made the first precise measurement of the thermal spectrum, as well as the first measurement of temperature anisotropies.

FIRAS: Measured the spectrum of the CMB, finding it to be an almost perfect blackbody with $T_0 = 2.725 \pm 0.002$ K

DMR: Found "anisotropies" in the CMB for the first time, at a level of 1 part in 10^5

WMAP made the first precise measurement of the angular power spectrum over a wide angular scale, putting tight constraints on cosmological models.

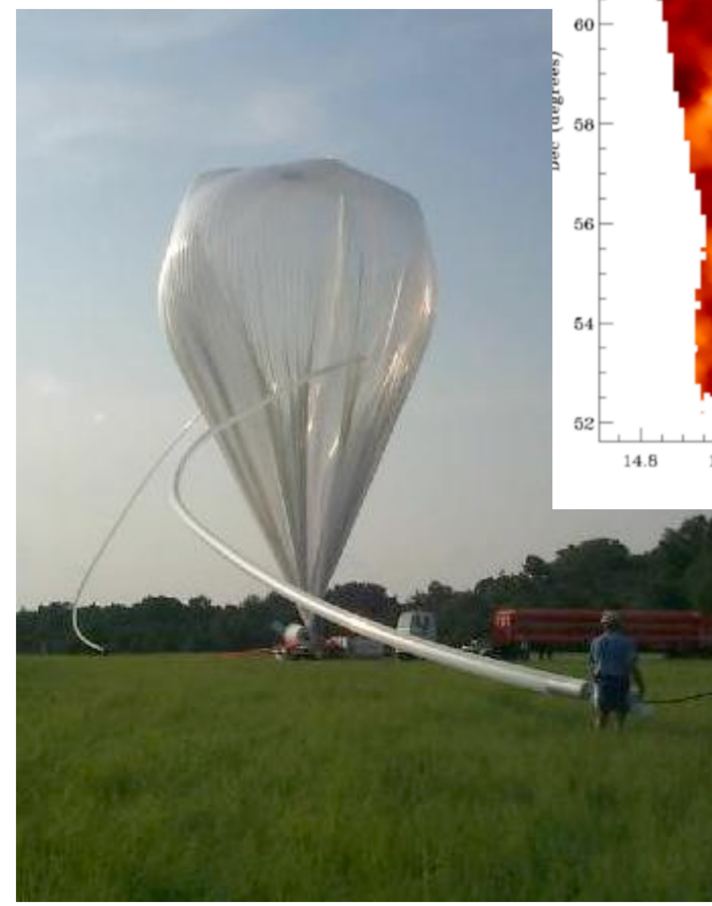


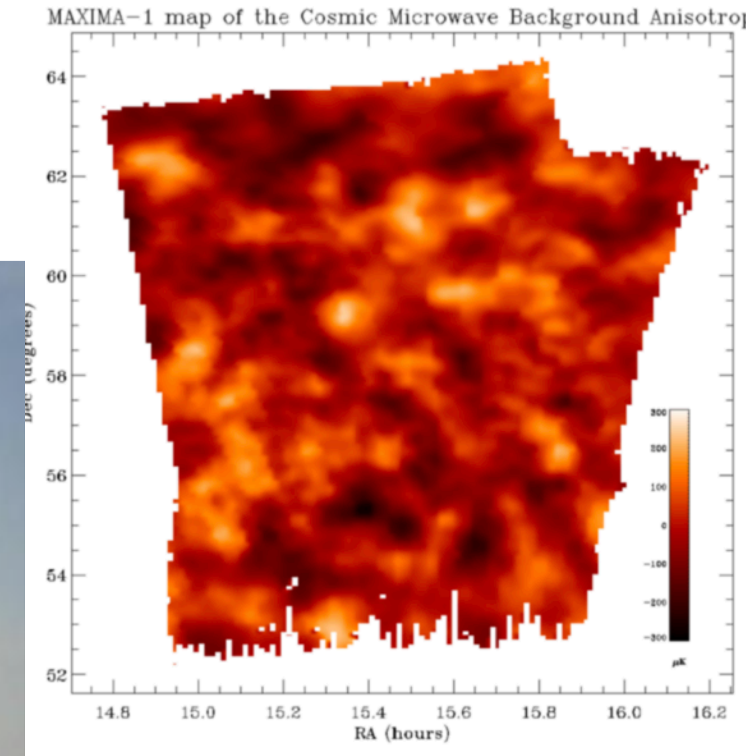


Planck satellite (2009–2013)

Credit: ESA

BOOMERanG and MAXIMA





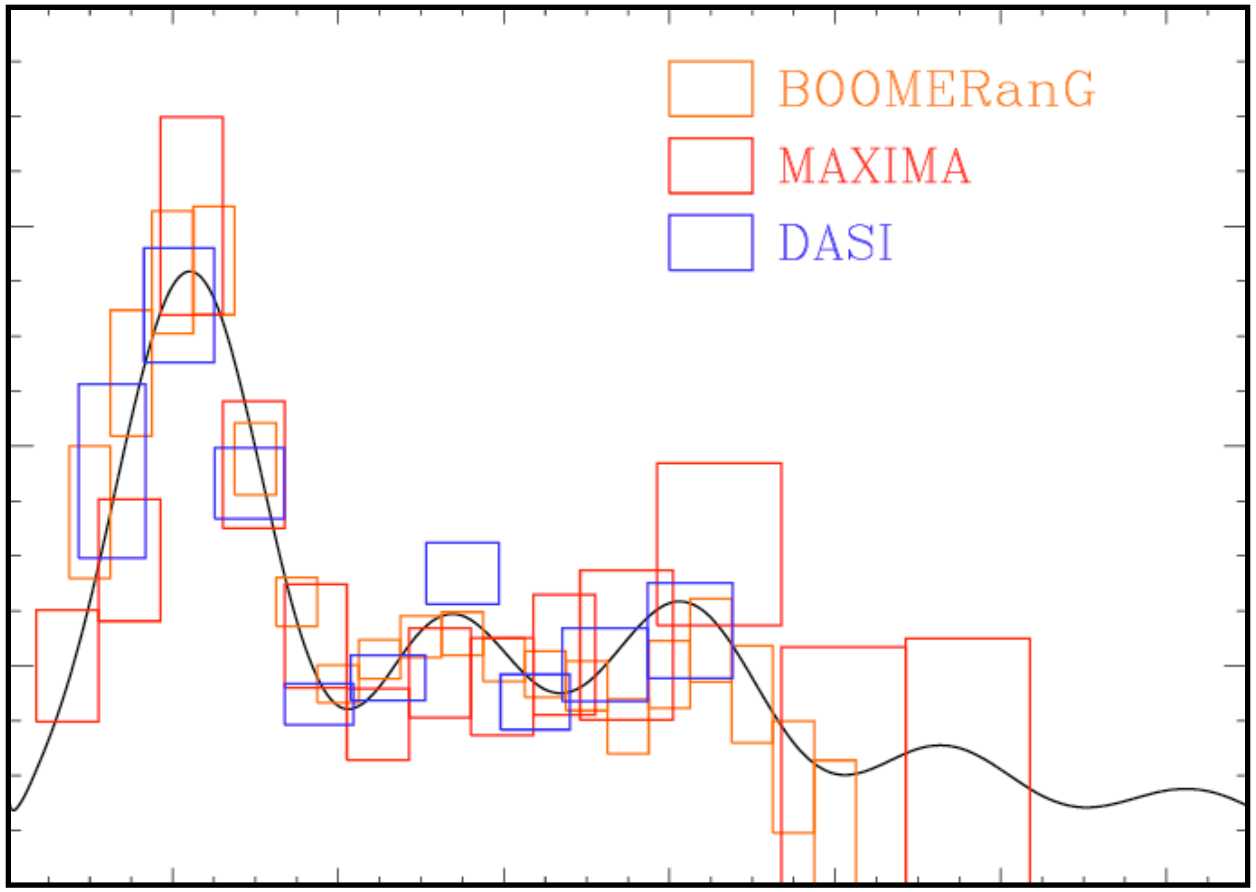
Boomerang launch Dec 1998

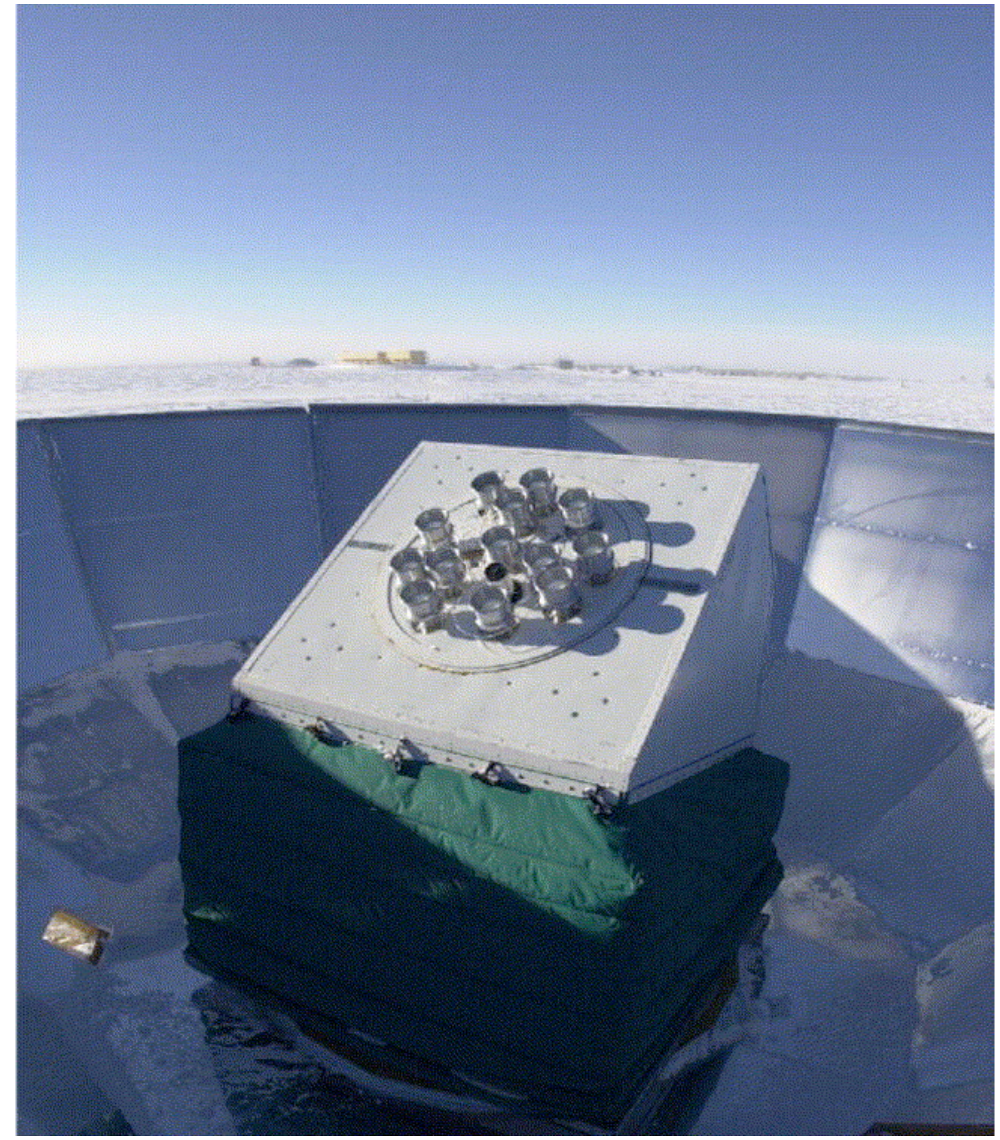
(Balloon Observations Of Millimetric Extragalactic Radiation ANd Geophysics)

Maxima launch Aug 98, Jun 99

Launched from Texas

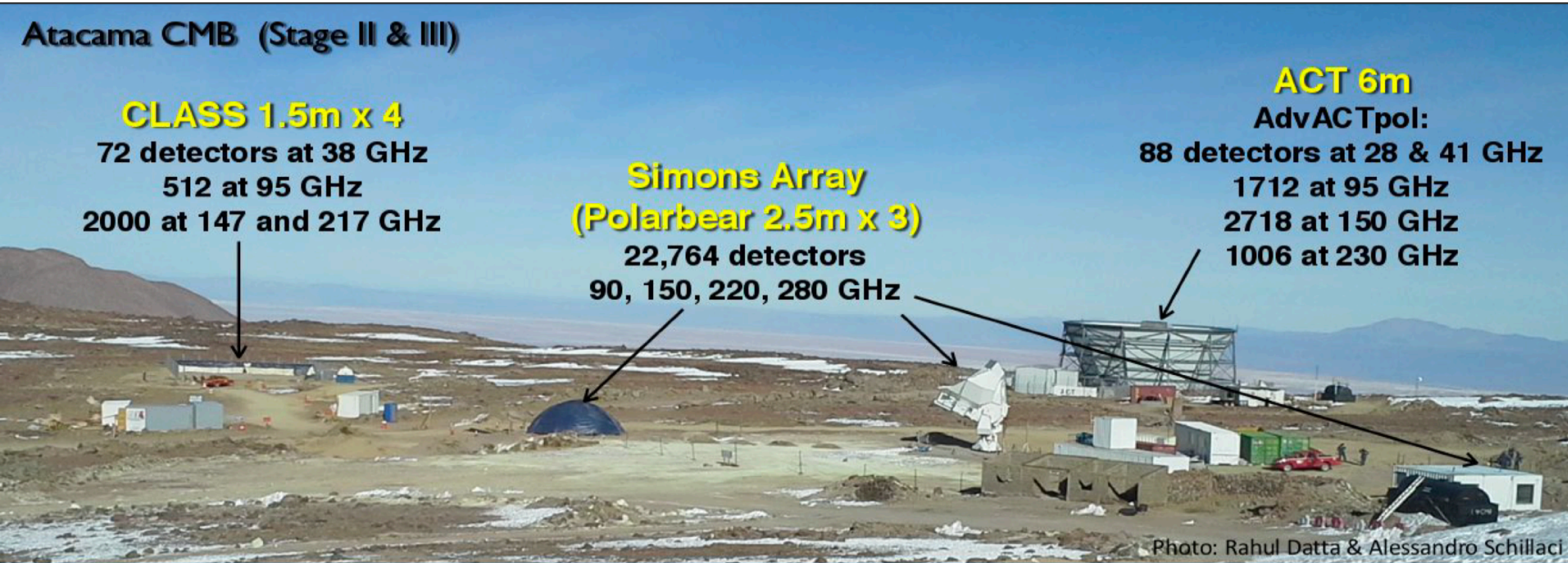
DASI from South Pole



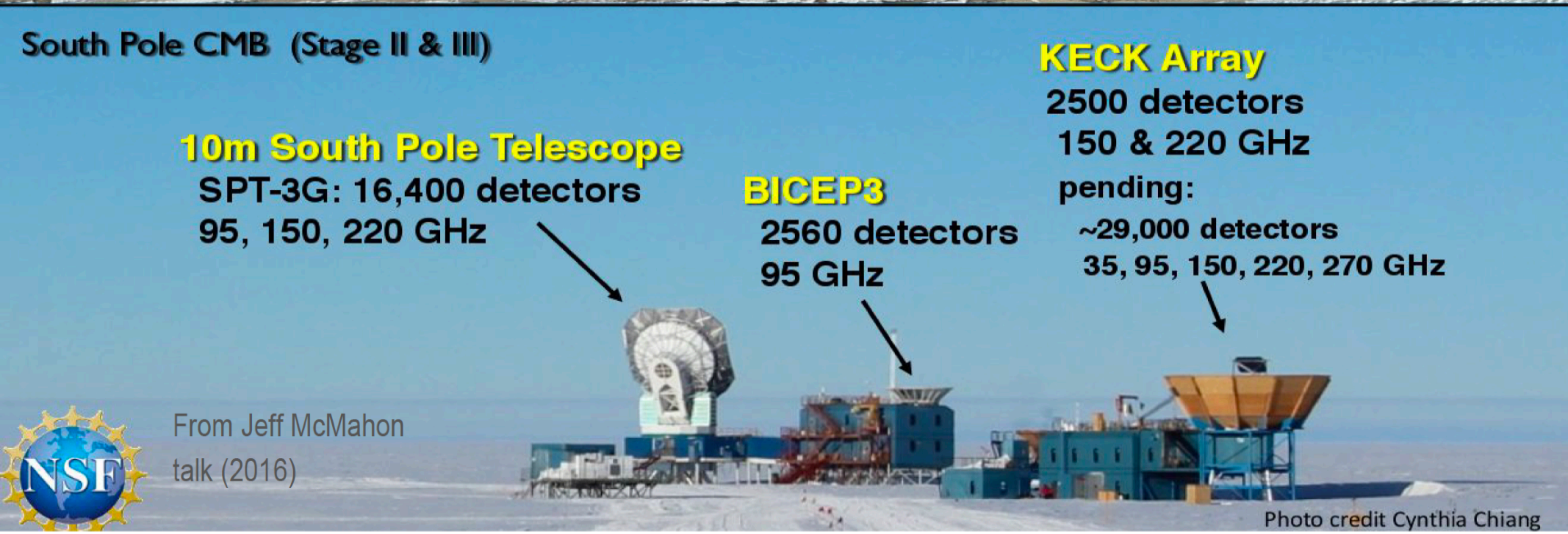


DASI (Degree Angular Scale Interferometer) was a 13-element **interferometer** operating from South Pole, and in 2002 reported the first detection of polarization anisotropies (E-mode).

DASI was replaced by the QUaD and then Keck Array, both bolometer instruments.

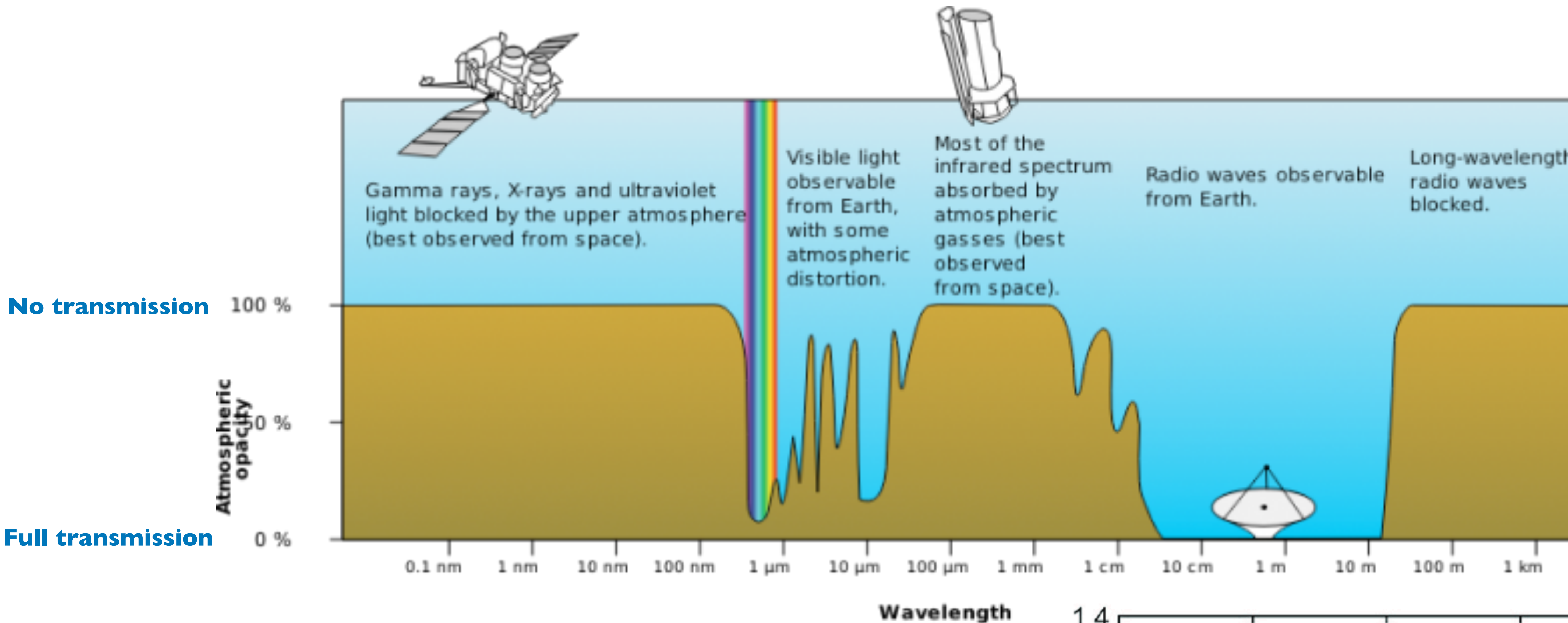


Atacama CMB (Stage II & III)
CLASS 1.5m x 4
72 detectors at 38 GHz
512 at 95 GHz
2000 at 147 and 217 GHz
Simons Array
(Polarbear 2.5m x 3)
22,764 detectors
90, 150, 220, 280 GHz
ACT 6m
AdvACTpol:
88 detectors at 28 & 41 GHz
1712 at 95 GHz
2718 at 150 GHz
1006 at 230 GHz
Photo: Rahul Datta & Alessandro Schillaci



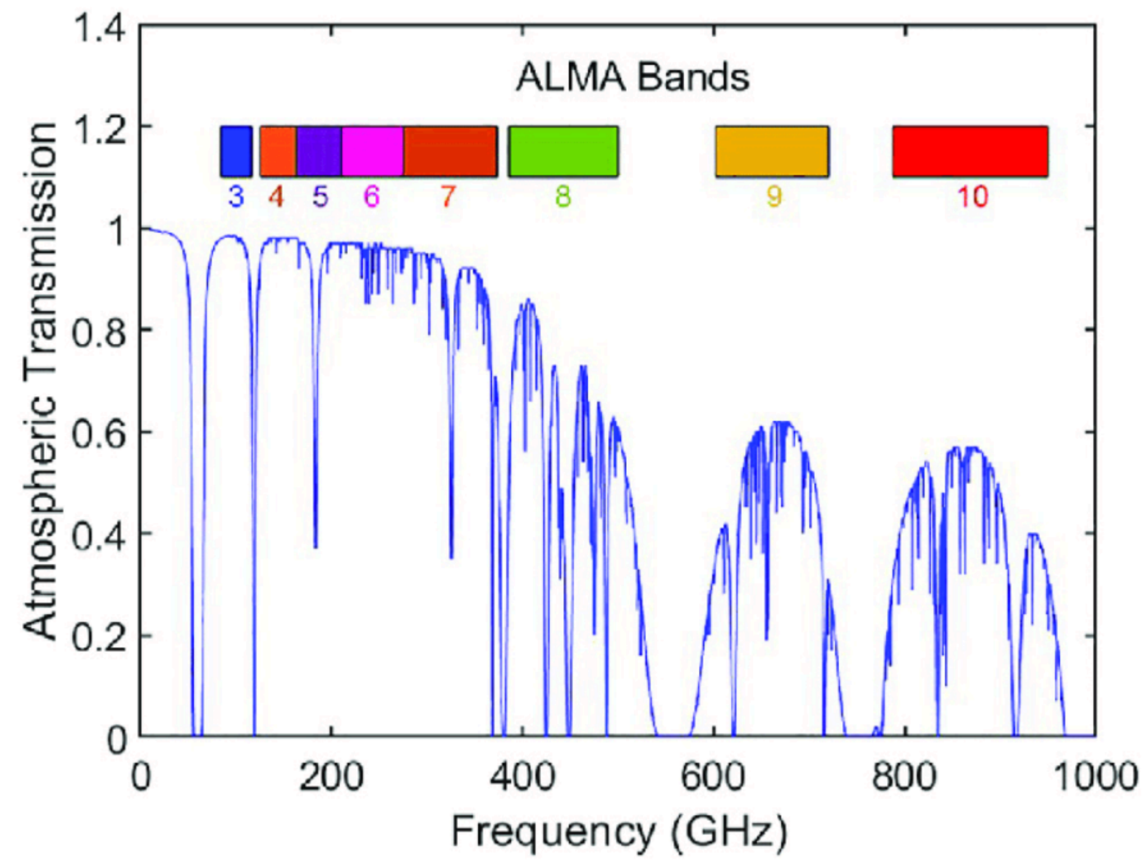
South Pole CMB (Stage II & III)
10m South Pole Telescope
SPT-3G: 16,400 detectors
95, 150, 220 GHz
BICEP3
2560 detectors
95 GHz
KECK Array
2500 detectors
150 & 220 GHz
pending:
~29,000 detectors
35, 95, 150, 220, 270 GHz
From Jeff McMahon
talk (2016)
Photo credit Cynthia Chiang

Atmospheric transmission: High & Dry



To get suitable atmospheric conditions for mm/submm observations (roughly above 90 GHz, λ ~ 3mm), one needs to get to extreme high-and-dry locations, with very little moisture in the air.

Apart from high microwave transmission, these locations also allow excellent stability for long-duration observations.



Ground- and space-based detector load comparison

From Delabrouille et al., CORE mission paper

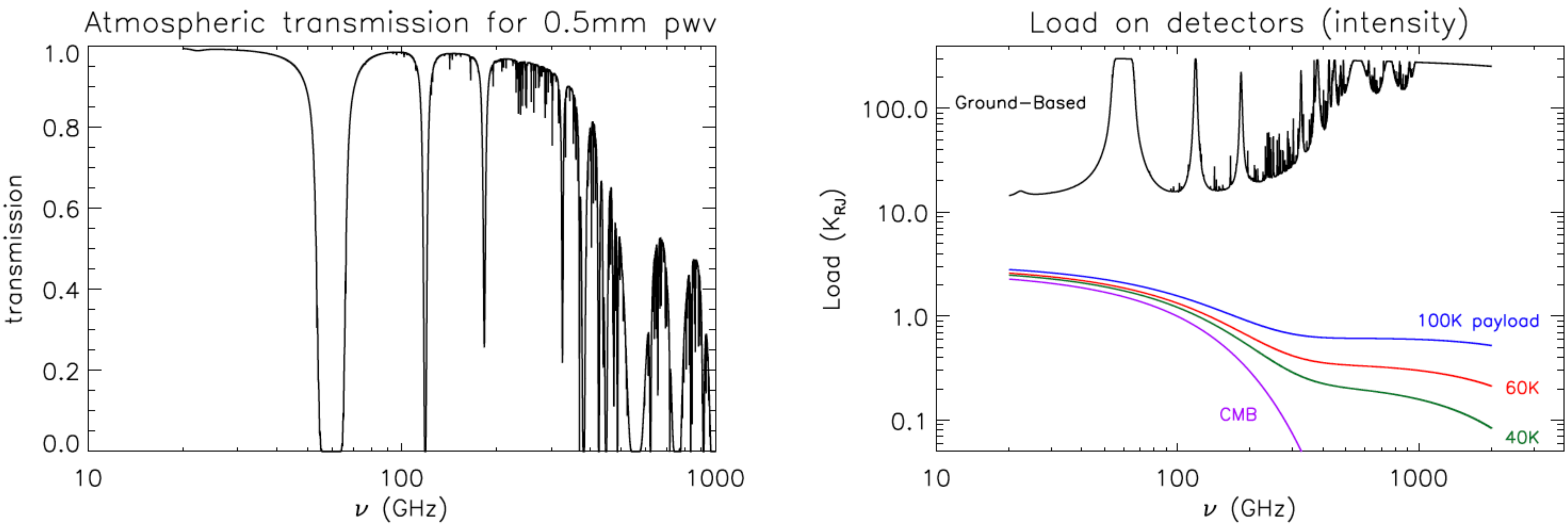
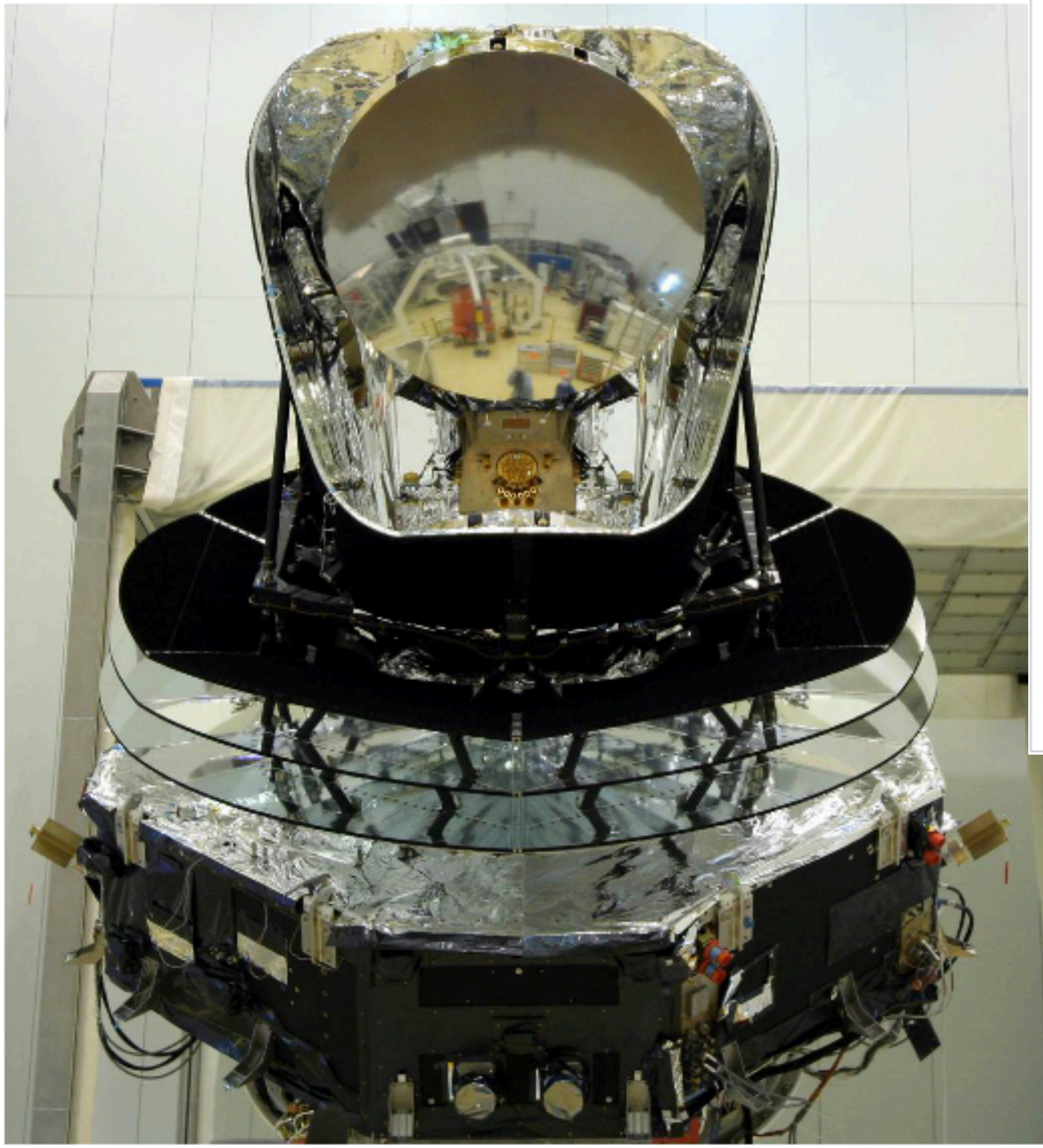


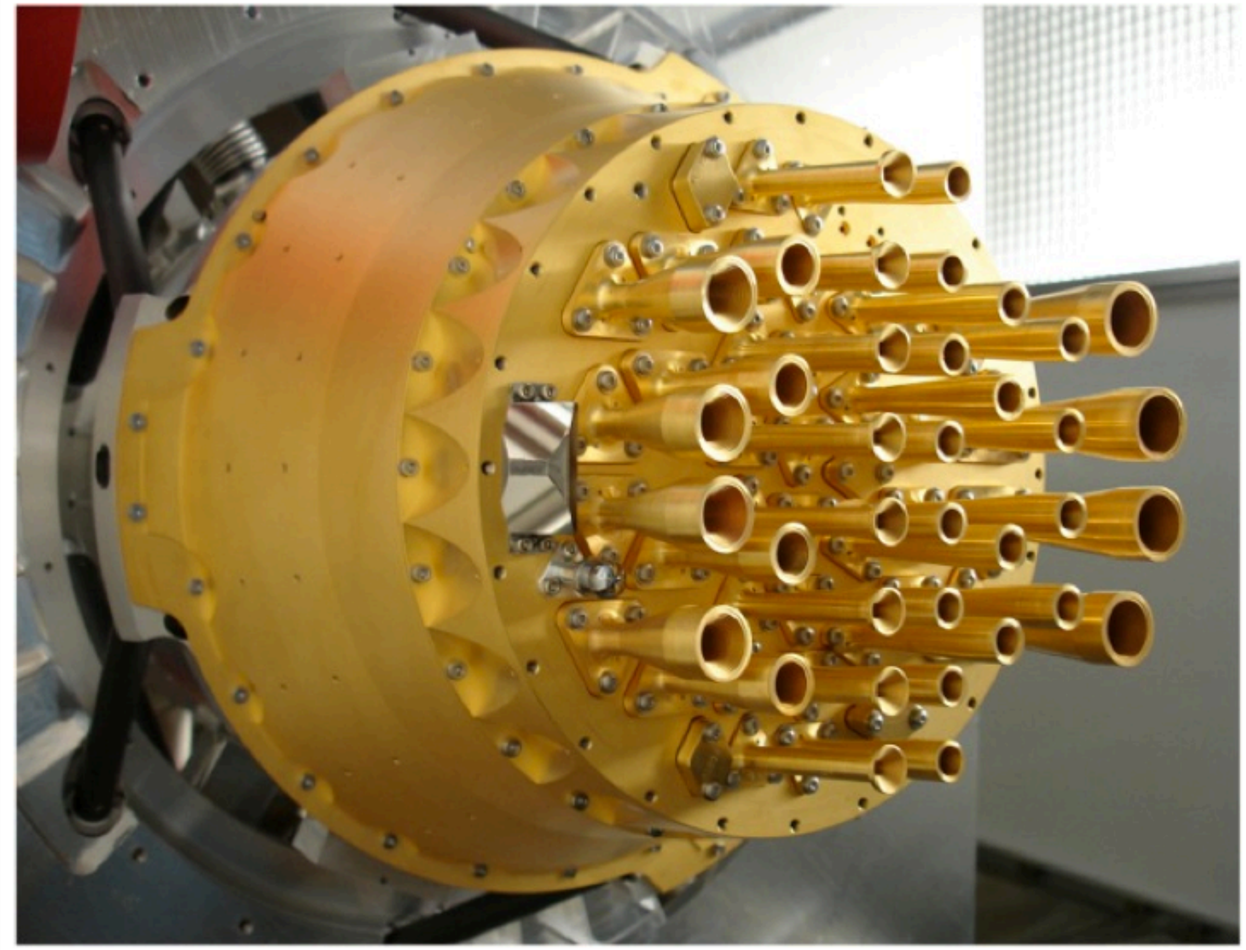
Figure 11. *Top left*: Typical atmospheric transmission from the Atacama plateau at 60° elevation, for an average of half a millimetre of integrated precipitable water vapour. *Top right*: Load on a detector for a ground-based instrument (black) and for a space-borne instrument with various payload temperatures.

A single space-borne detector can reach a sensitivity equivalent to 100-1000 ground-based detectors (depending on frequency).

Planck satellite detector array



The fully assembled Planck satellite a few days before integration into the Ariane 5 rocket. Herschel is visible by reflection on the primary reflector.



The HFI focal plane optics and 4K thermo-mechanical stage

Planck

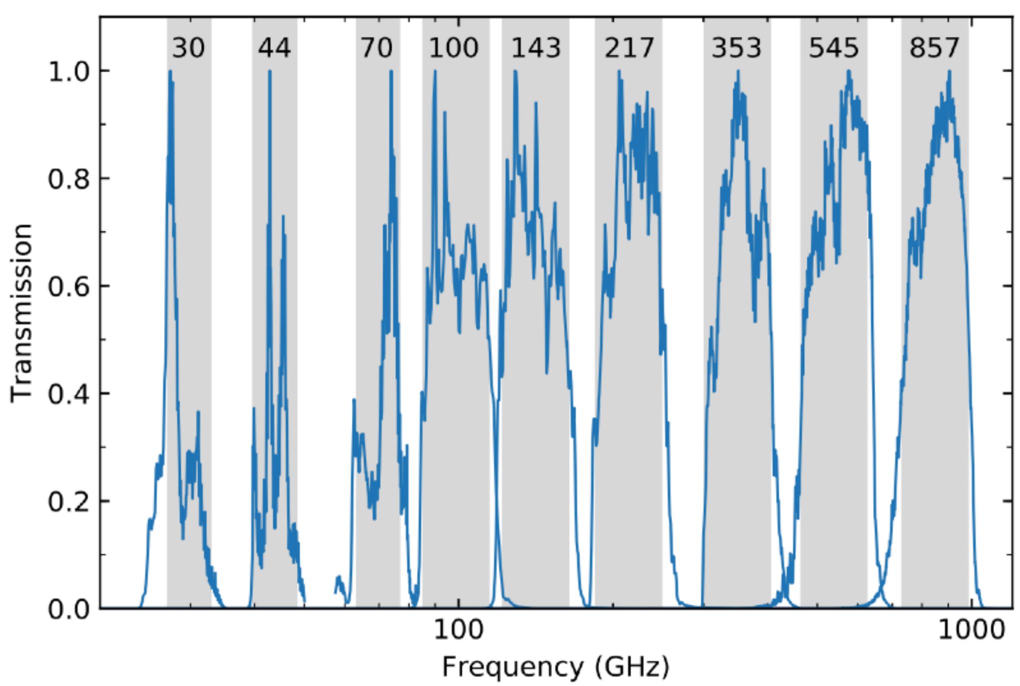
- 20K HEMT amplifiers at 30, 45, 70 GHz
  - ~ 20 amplifiers
- 100 mK bolometers at 100 -> 850 GHz
  - ~ 50 bolometers

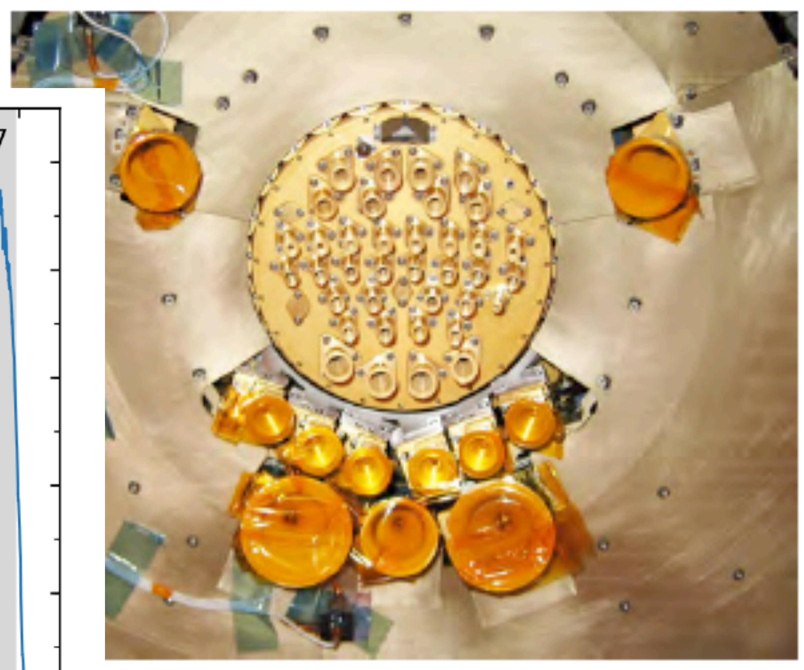
Planck detectors

INSTRUMENT CHARACTERISTIC	LFI			HFI					
Detector Technology	HEMT arrays			Bolometer arrays					
Center Frequency [GHz]	30	44	70	100	143	217	353	545	857
Bandwidth ($\Delta\nu/\nu$)	0.2	0.2	0.2	0.33	0.33	0.33	0.33	0.33	0.33
Angular Resolution (arcmin)	33	24	14	10	7.1	5.0	5.0	5.0	5.0
$\Delta T/T$ per pixel (Stokes I)[a]	2.0	2.7	4.7	2.5	2.2	4.8	14.7	147	6700
$\Delta T/T$ per pixel (Stokes Q & U)[a]	2.8	3.9	6.7	4.0	4.2	9.8	29.8	...	...

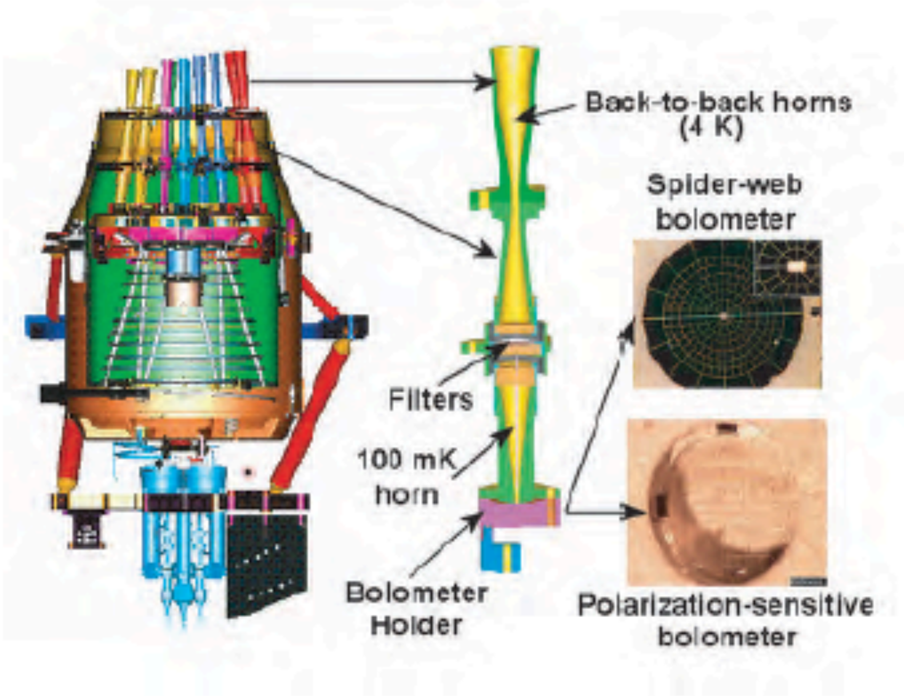
[a] Goal (μK/K, 1σ), 14 months integration, square pixels whose sides are given in the row "Angular Resolution".

Planck Bluebook





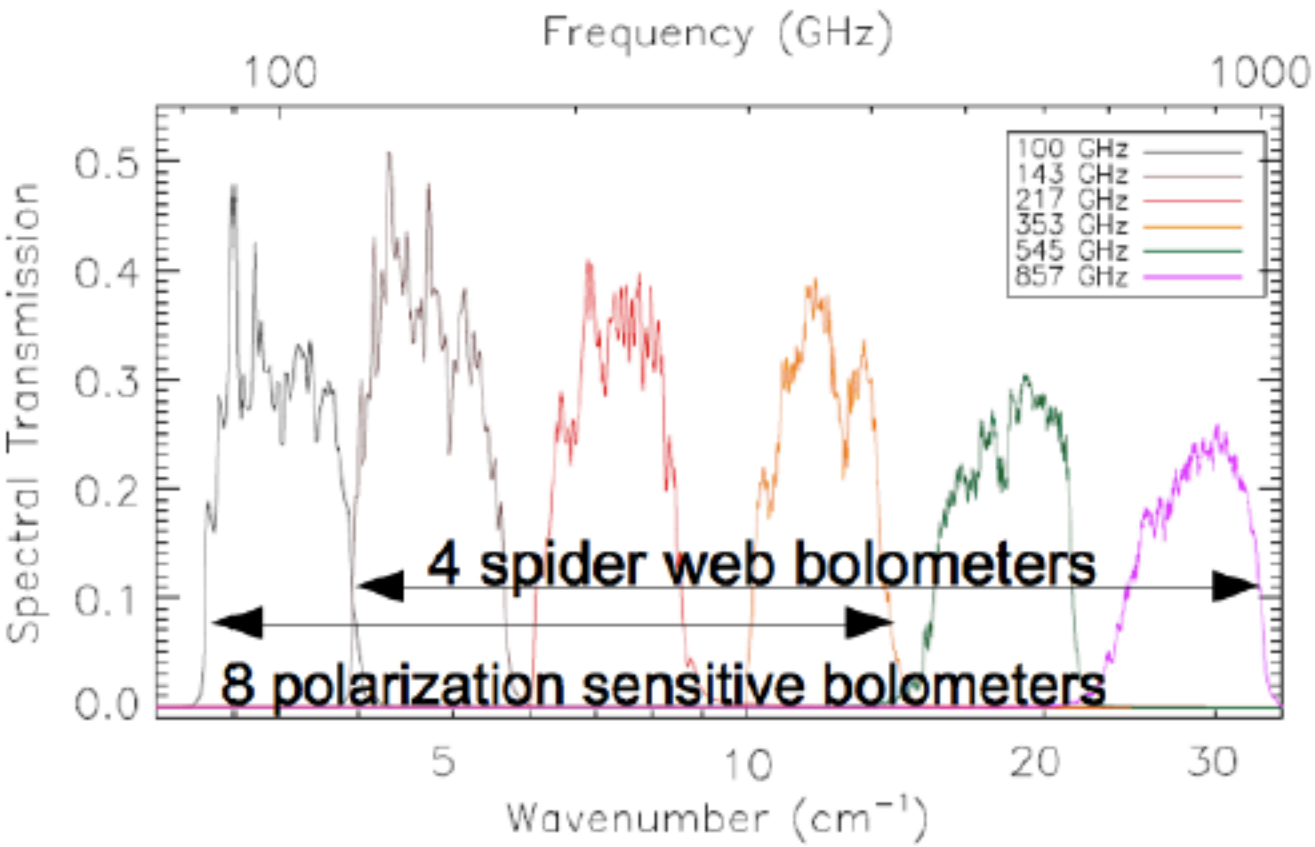
Thales/Alenia Space+ESA



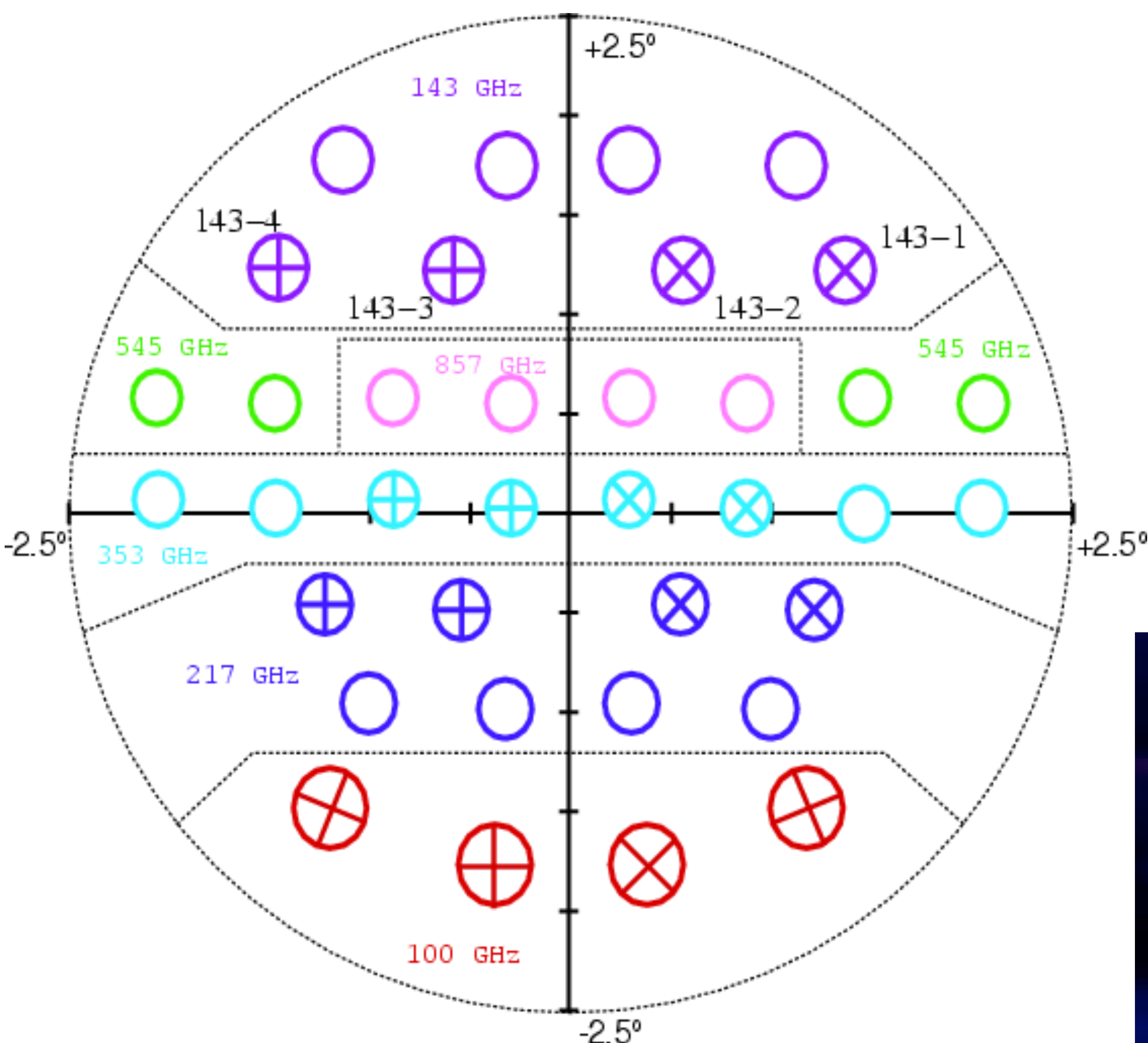
Planck Bluebook

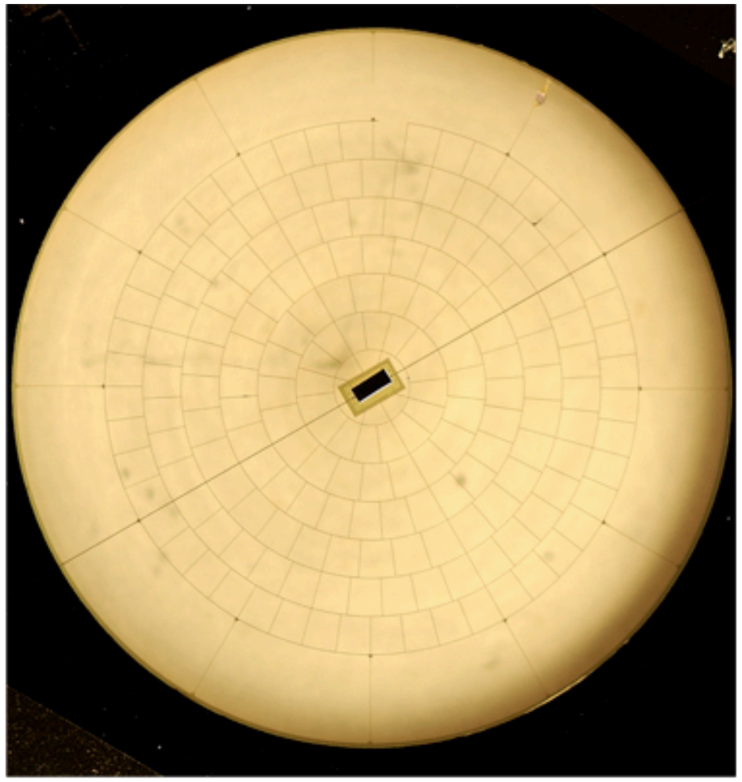
Planck HFI (bolometer detectors)

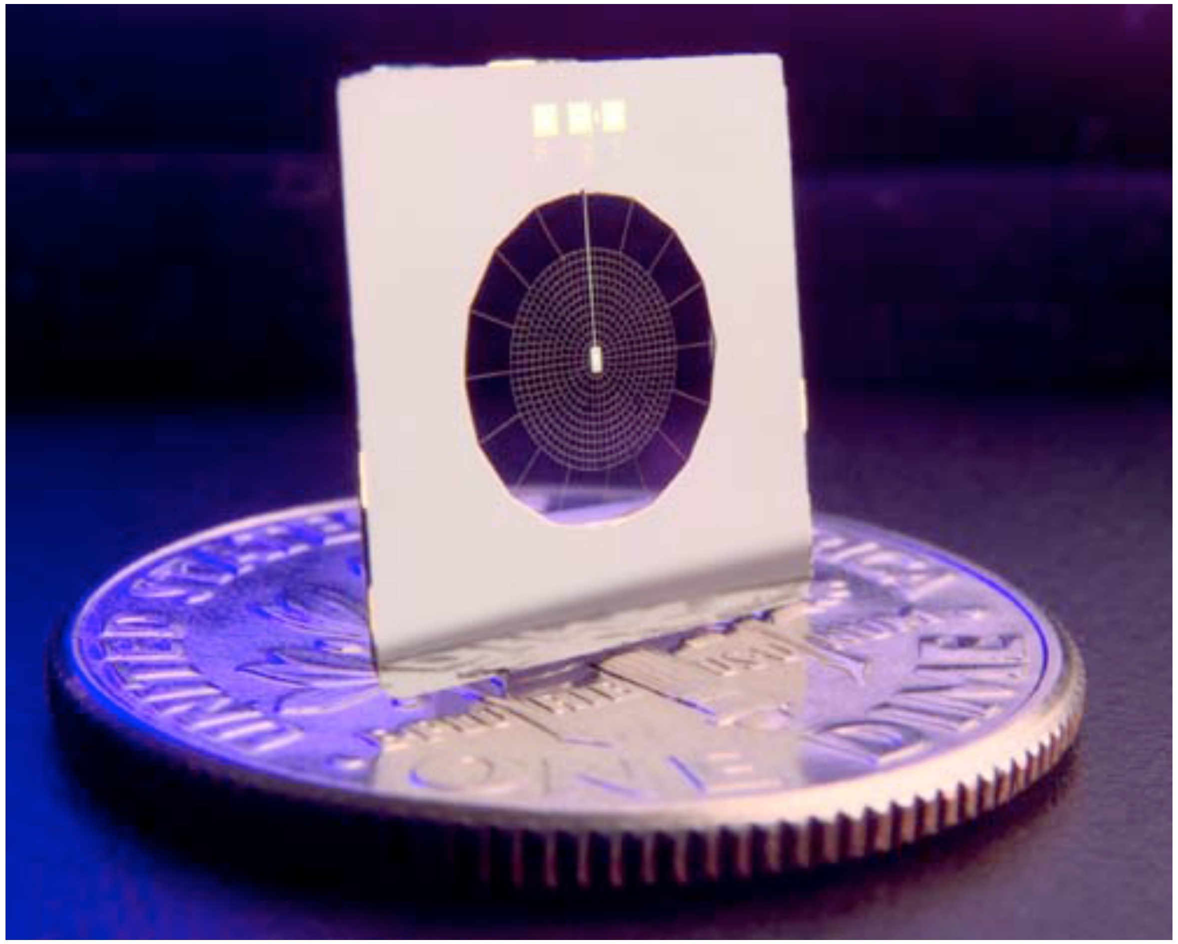
Center Frequency (GHz)	100	143	217	353	545	857
N Detectors	8	11	12	12	3	4
Resolution (arcmin)	9.5	7.1	4.7	4.5	4.7	4.4
Noise in maps μK_{CMB} deg	1.6	0.9	1.4	5.0	70	1180
Array NET (μK s)	22.6	14.5	20.6	77.3	4.9 (RJ)	2.1 (RJ)



Planck: polarization sensitivity

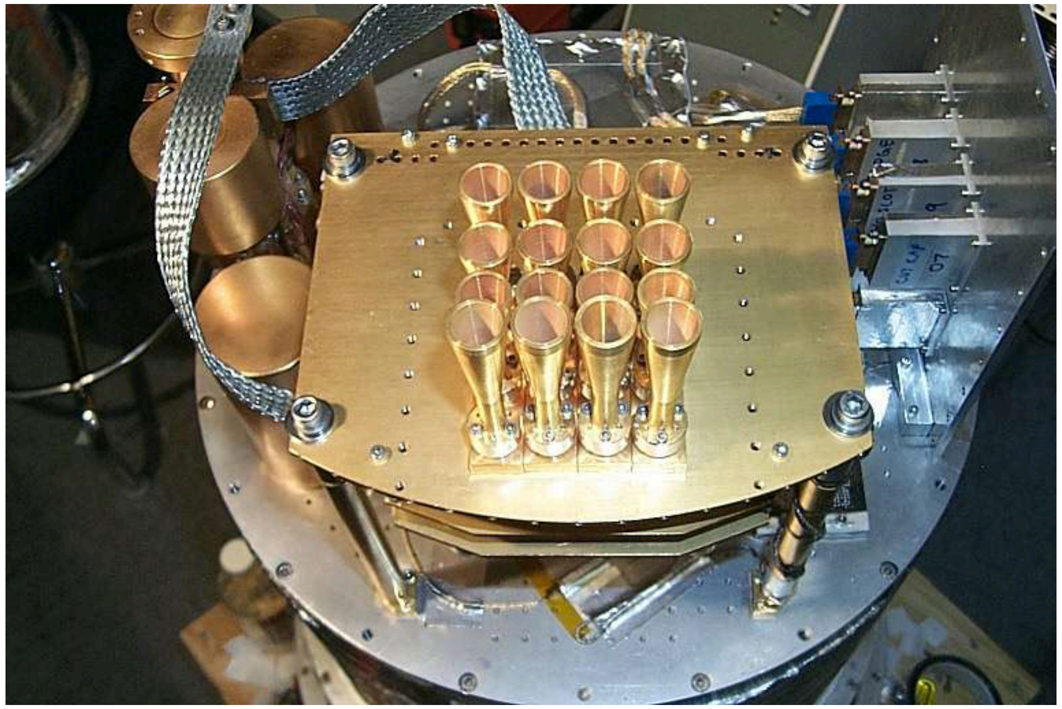




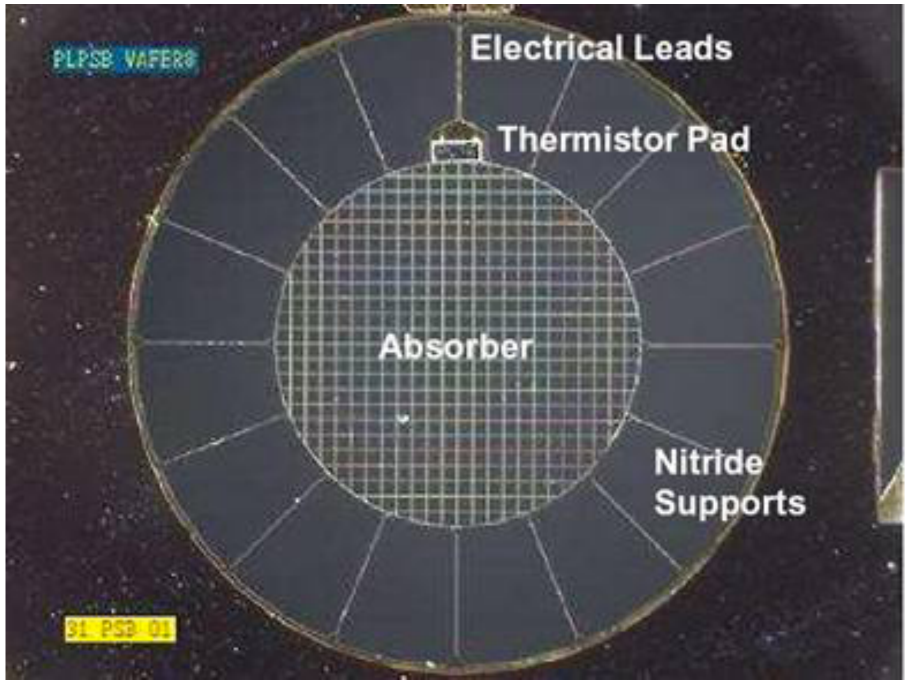


Planck Focal Plane Unit with polarization sensitive bolometers (spiderweb bolometers). Here one has two bolometers back-to-back with orthogonal grids.

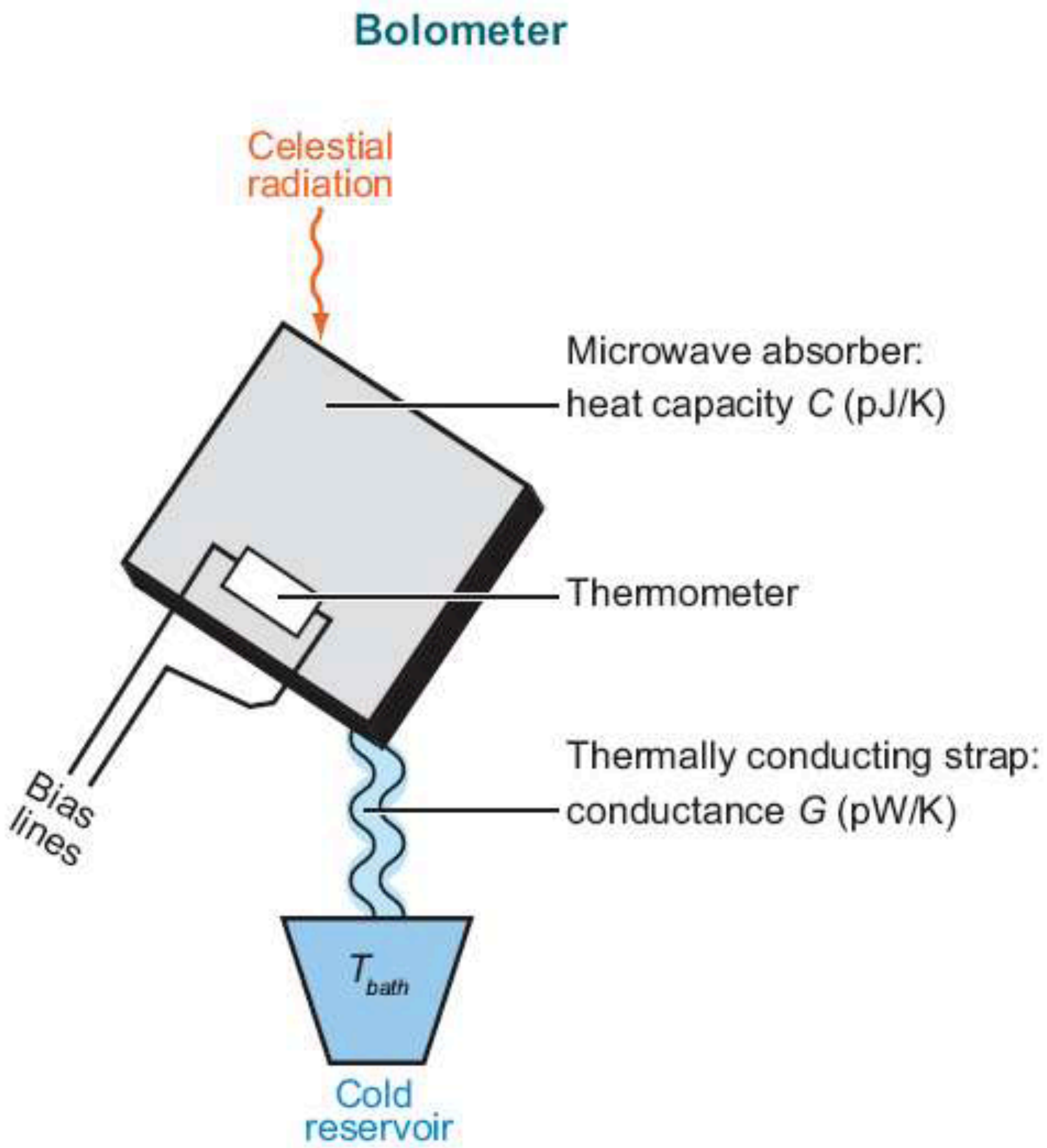
Bolometers (heat detectors): Workhorse of all CMB experiments



ACBAR



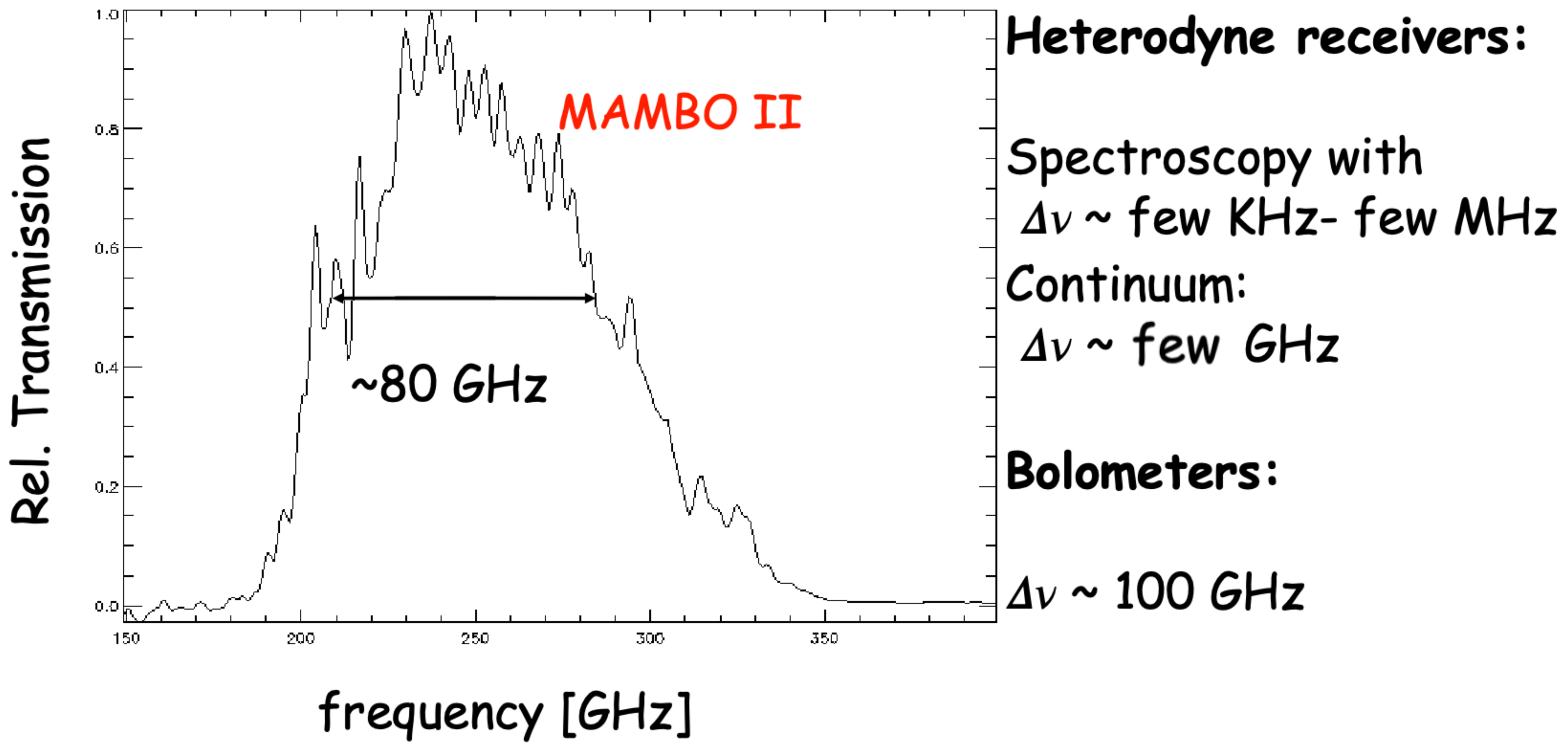
Boomerang



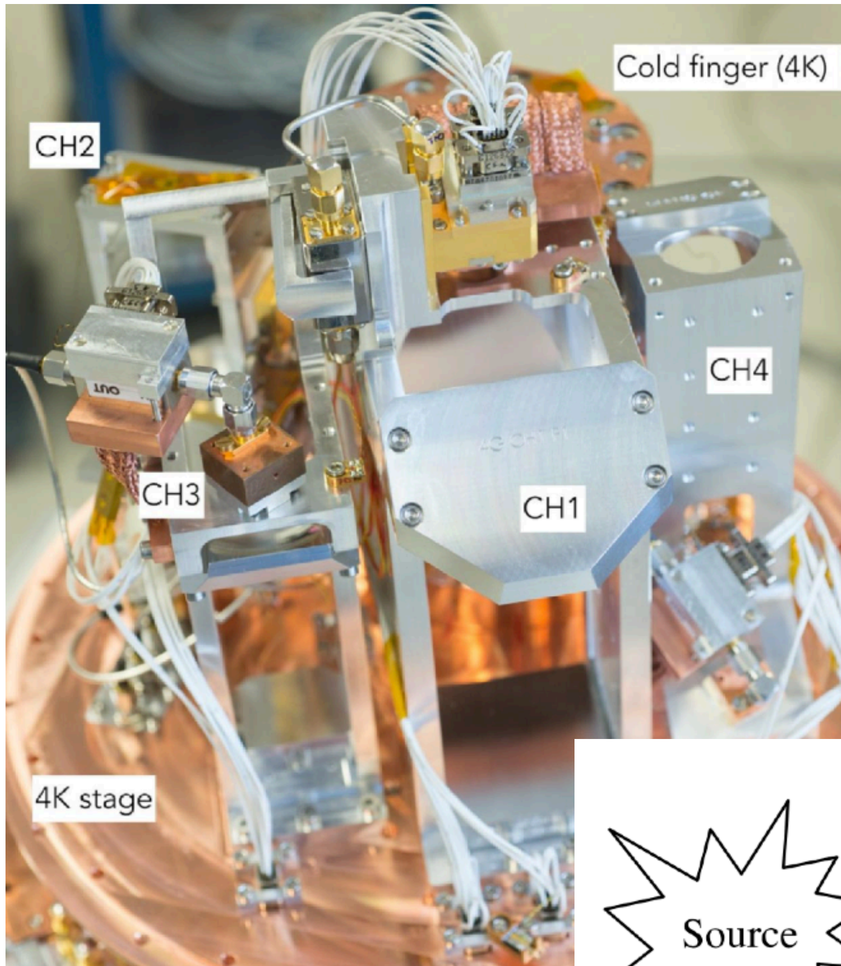
Sensitivity of bolometers

Bolometers collect radiation in a wide frequency band, which makes them much more sensitive than narrow-band heterodyne receivers. Consequently, bolometers are suitable only for continuum measurements (i.e. signals that change slowly with frequency).

No phase information is preserved, so signals from different bolometers cannot be added coherently, to use in an interferometer. But incoherent adding of time-averaged signals is no problem, and current bolometer arrays employ 10s of thousands of individual elements.



CMB receiver types



4GREAT receiver

Coherent receivers:

Phase-preserving amplification
(suitable for interferometers)
Correlation of different polarization

- Heterodyne receivers (e.g. DASI, CBI)

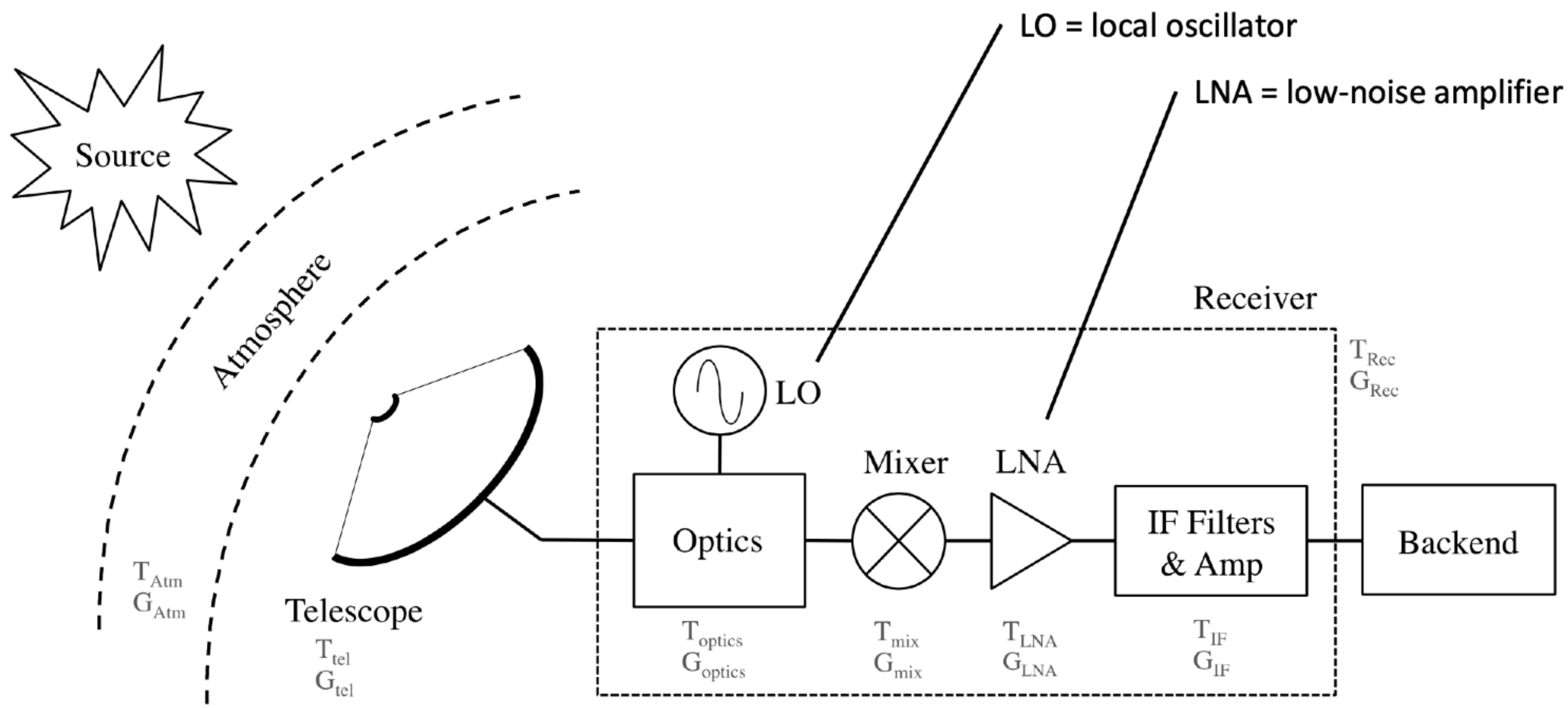
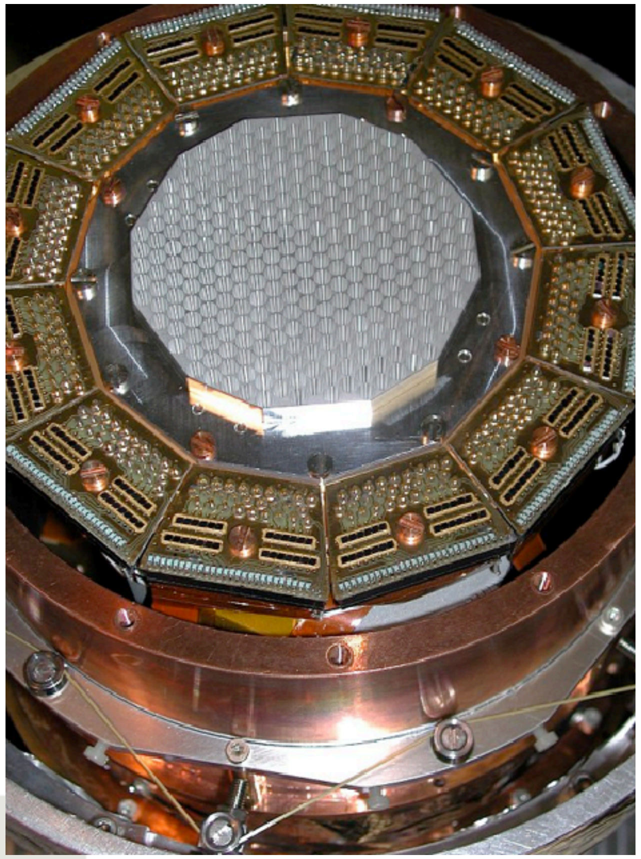
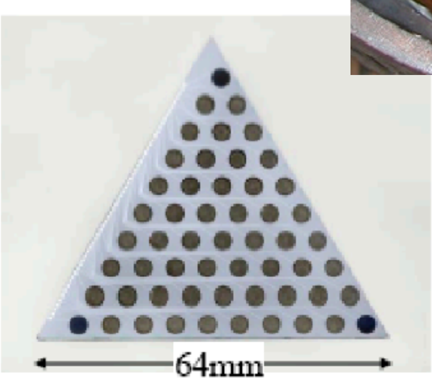


Image credit: Oliver Ricken

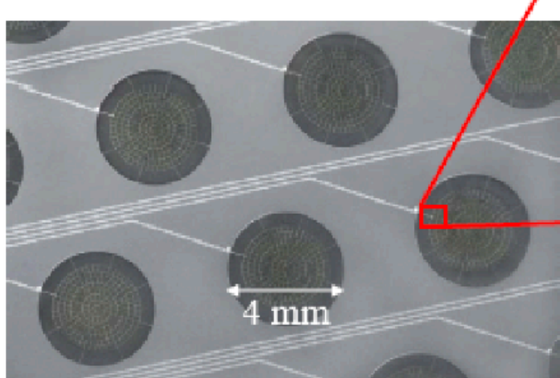
CMB receiver types

APEX-SZ
receiver

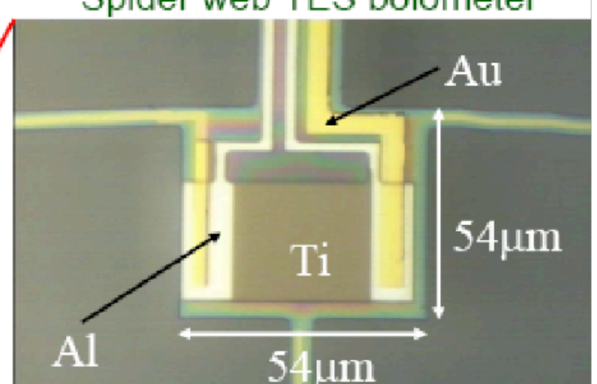




Prototype 55 element wedge



Spider web TES bolometer



Coherent receivers:

Phase-preserving amplification
(suitable for interferometers)
Correlation of different polarization

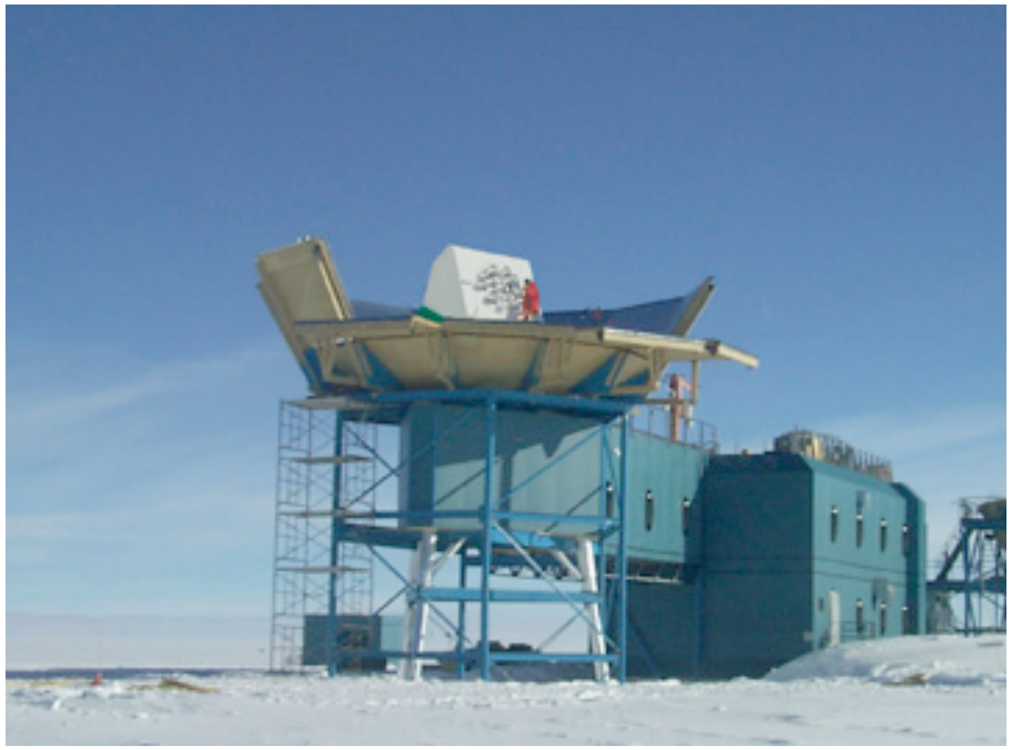
- Heterodyne receivers (e.g. DASI, CBI)

Incoherent receivers (bolometers):

Direct detection of radiation,
no phase information kept
Multiplexing possible → *Large arrays!*

- Bolometers (e.g. ACBAR, Boomerang, BICEP, Clover, Planck, ACT, South Pole Telescope, Maxima, BICEP array, Polarbear, Simons Observatory, CCAT, CMB-S4)

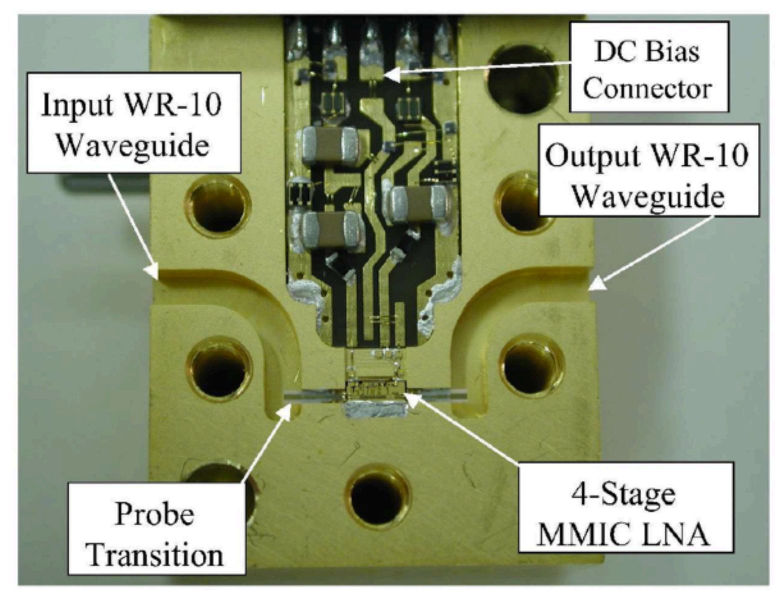
Coherent receivers: Interferometers for CMB



DASI in South Pole

CBI in Atacama desert

Coherent receivers can be configured so that the output is the correlation of two input signals.
HEMT (High Electron Mobility Transistor), commonly called Heterodynes, are a class of coherent receivers that allow coherent amplification with low noise and high gain.

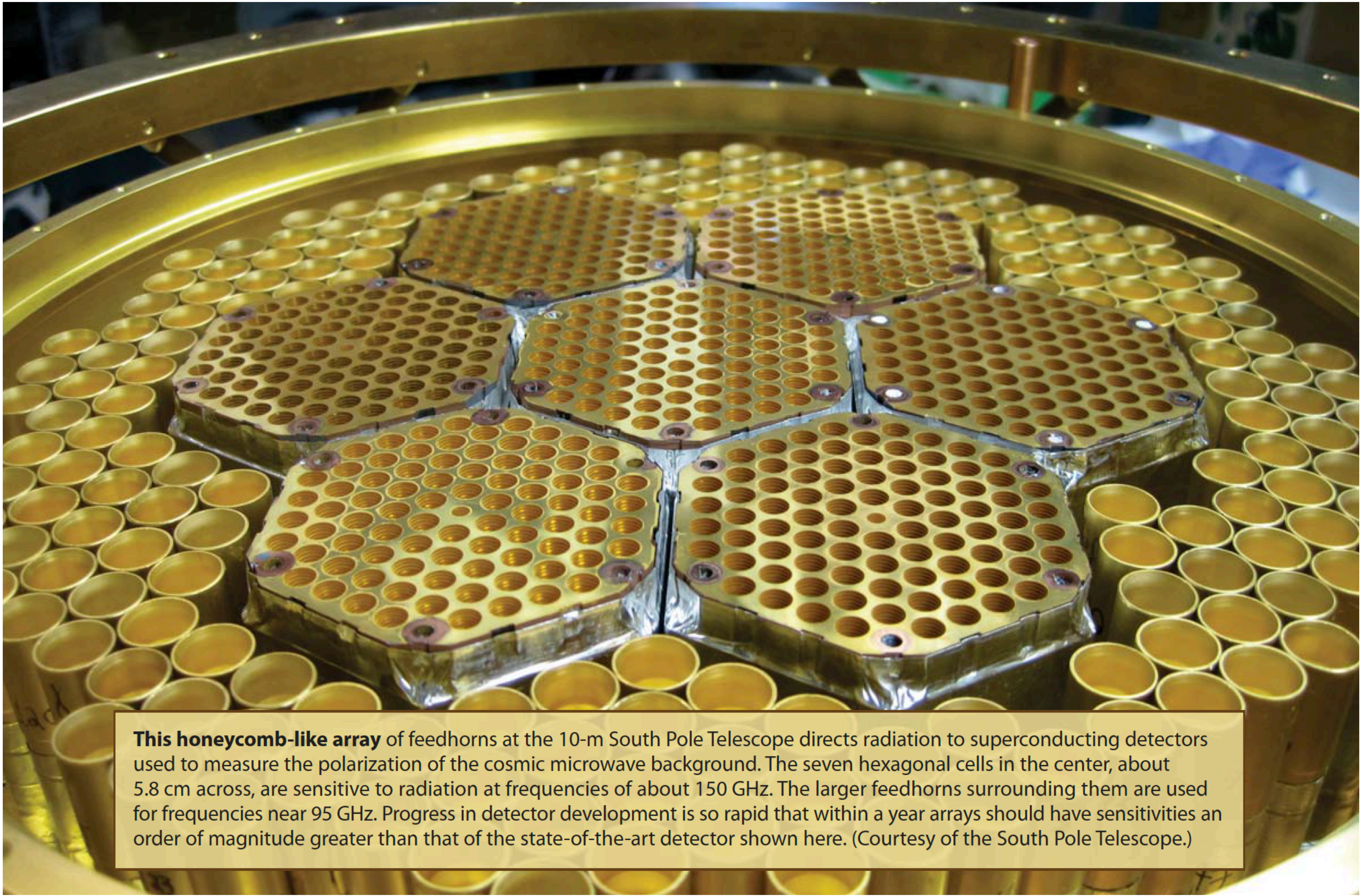


Bolometer and HEMT sensitivities

	Bolometers	MMIC HEMTs
• Sensitivity	great	good @ < 100 GHz
• Response time	~ msec -> sub-msec	fast
• Frequency Coverage	comprehensive	limited
• Cooling Requirements	little P at low T	large P at higher T
• Linearity	adequate	excellent
• Gain Stability	excellent	poor
• Offset Stability	excellent	good
• Focal Plane Density	better	feedhorn limited
• Polarization Sensitivity	good	good
• EMI / RFI / B-field / microphonic susceptibility	adequate	better
• Array Uniformity	good	???

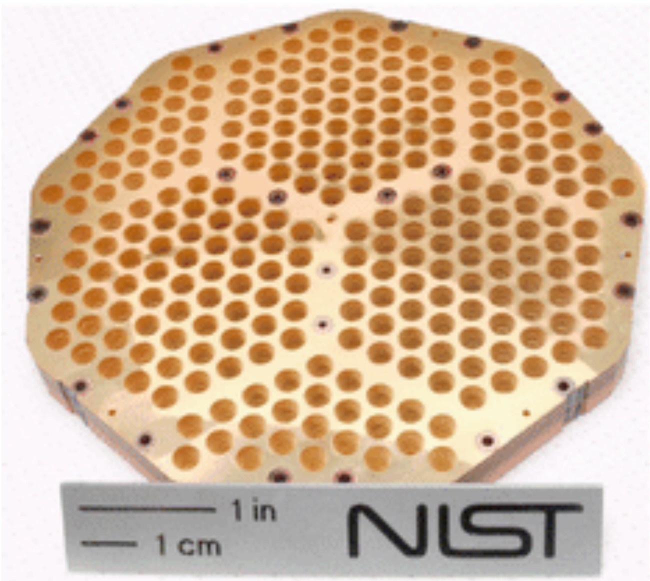
(Slide from Andrew Lange)

A modern bolometer feedhorn array

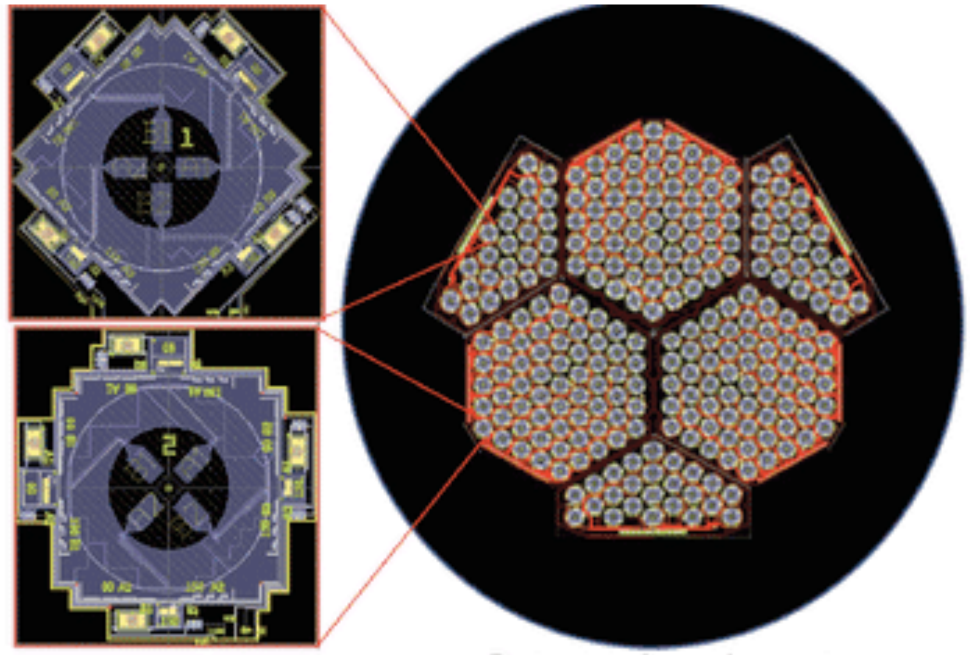


This honeycomb-like array of feedhorns at the 10-m South Pole Telescope directs radiation to superconducting detectors used to measure the polarization of the cosmic microwave background. The seven hexagonal cells in the center, about 5.8 cm across, are sensitive to radiation at frequencies of about 150 GHz. The larger feedhorns surrounding them are used for frequencies near 95 GHz. Progress in detector development is so rapid that within a year arrays should have sensitivities an order of magnitude greater than that of the state-of-the-art detector shown here. (Courtesy of the South Pole Telescope.)

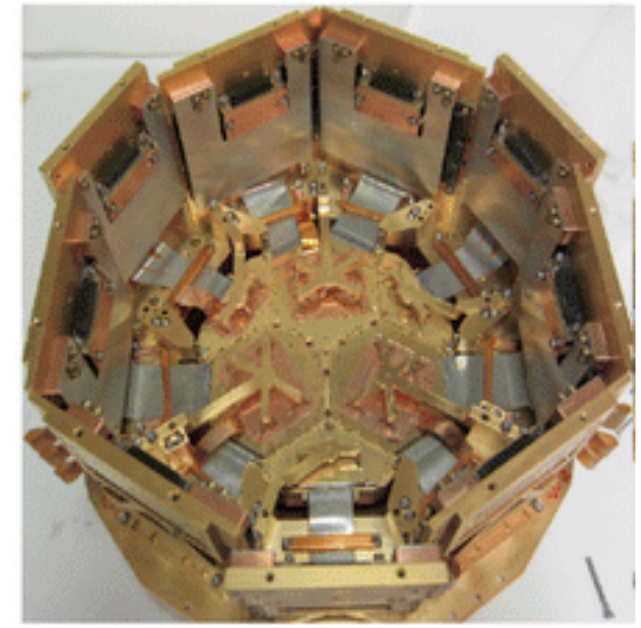
More bolometer array examples



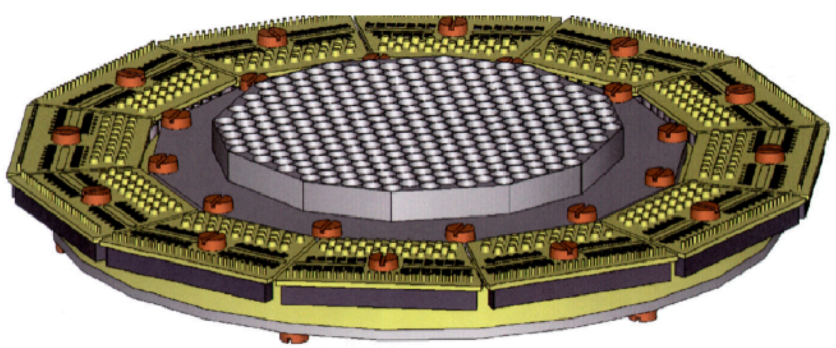
Platelet Feedhorn Array

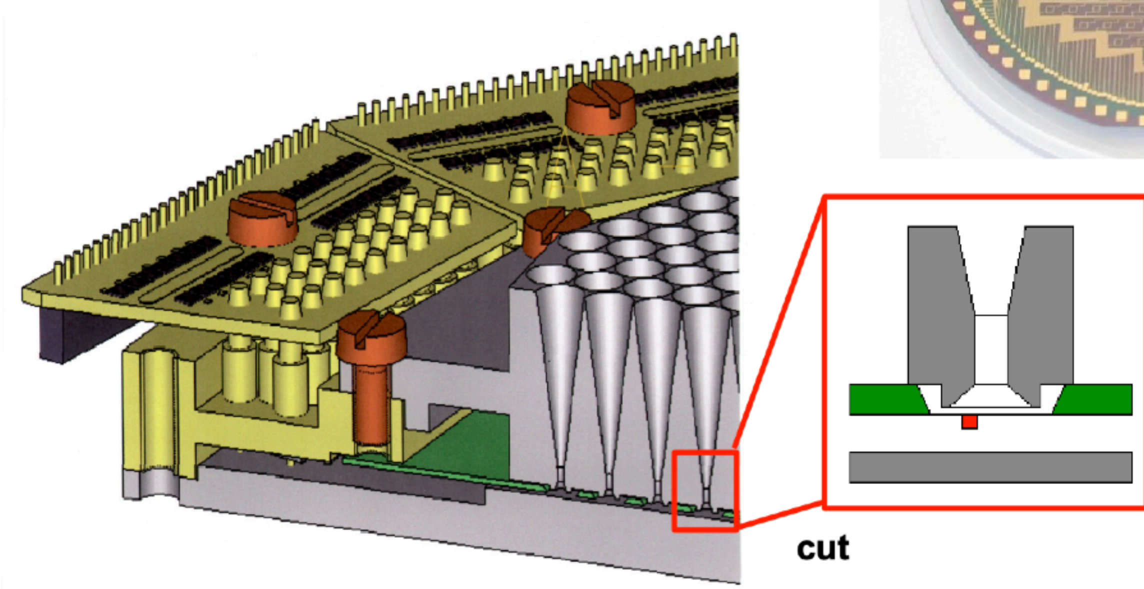


Detector Array Layout

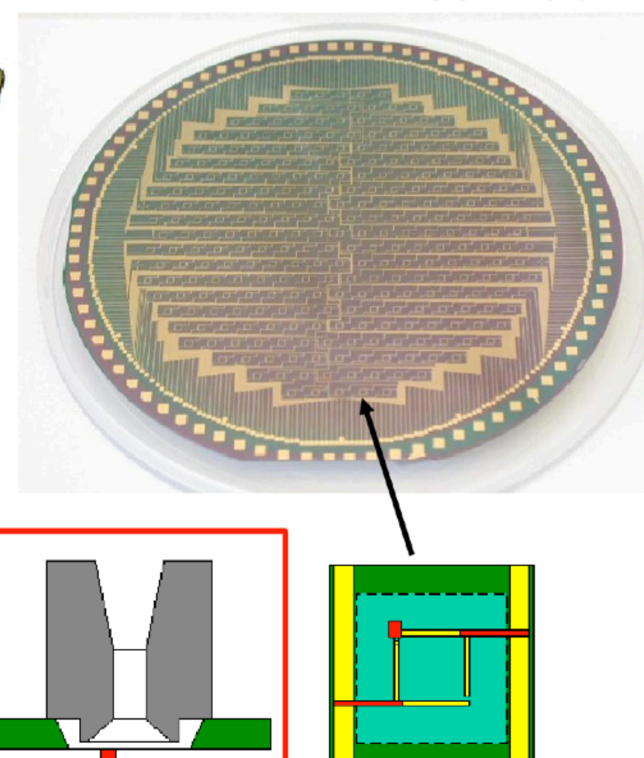


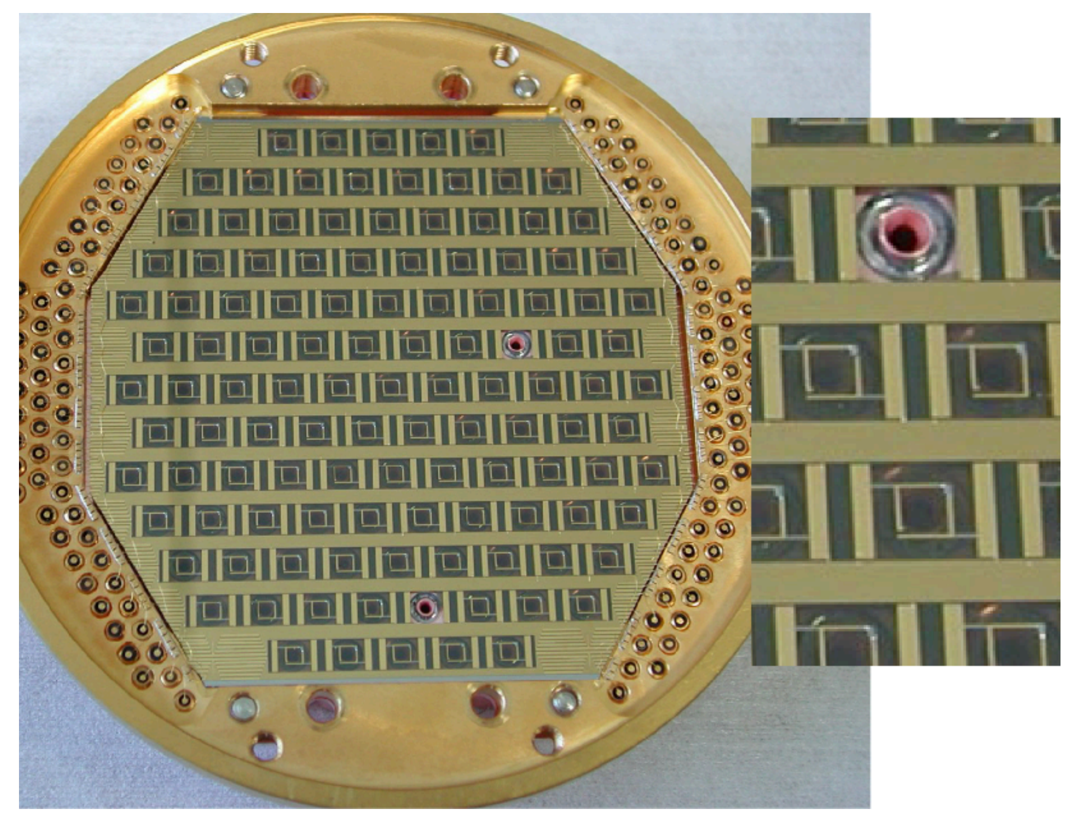
Assembled Detector Array Package





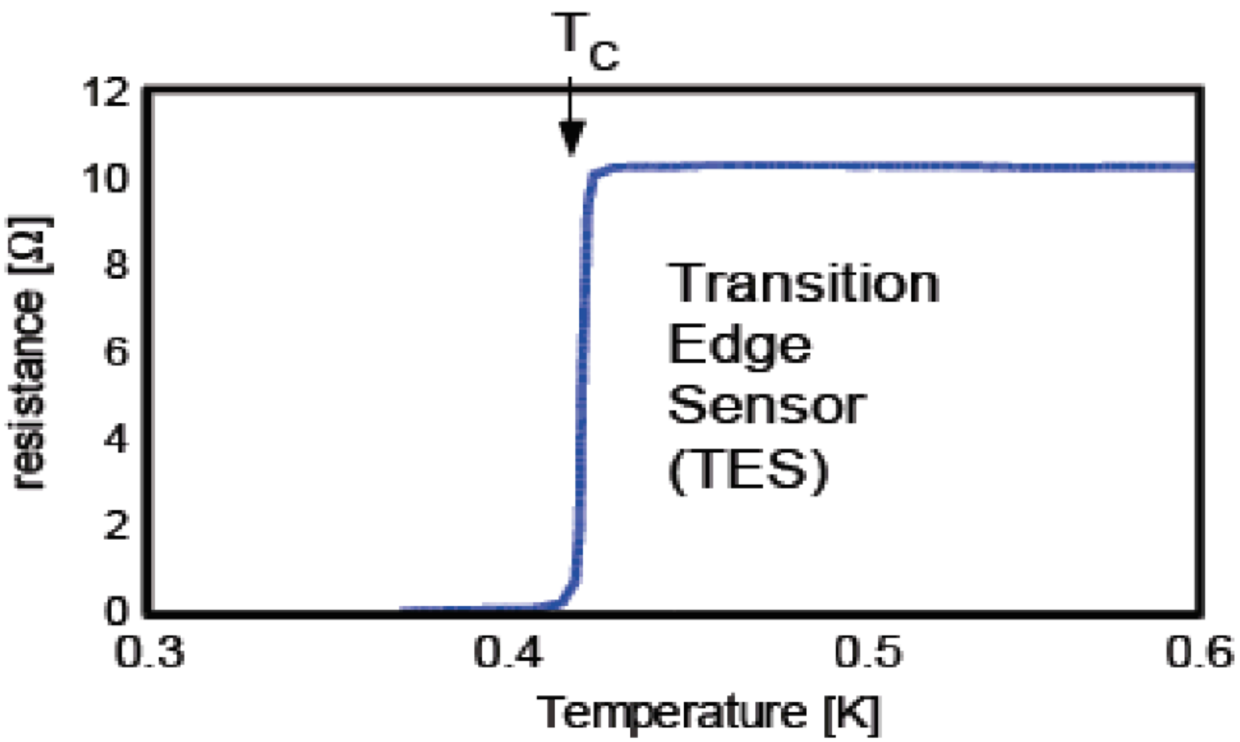
LABOCA Wafer





Bolometer technology

TES bolometers

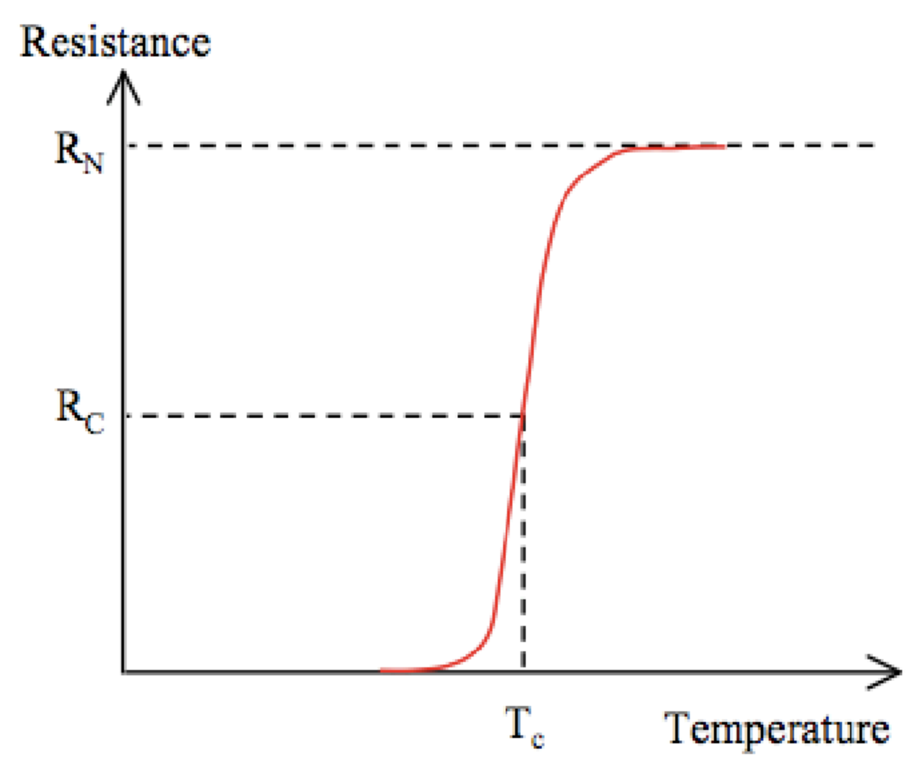


- Film held at constant V-bias on normal/ superconducting transition - change in R leads to change in current I
- Resistance has very steep (500-1000) dependence on T in transition range
- low-noise, low-power (~1 nW) SQUID A-meter readout

MKID bolometers

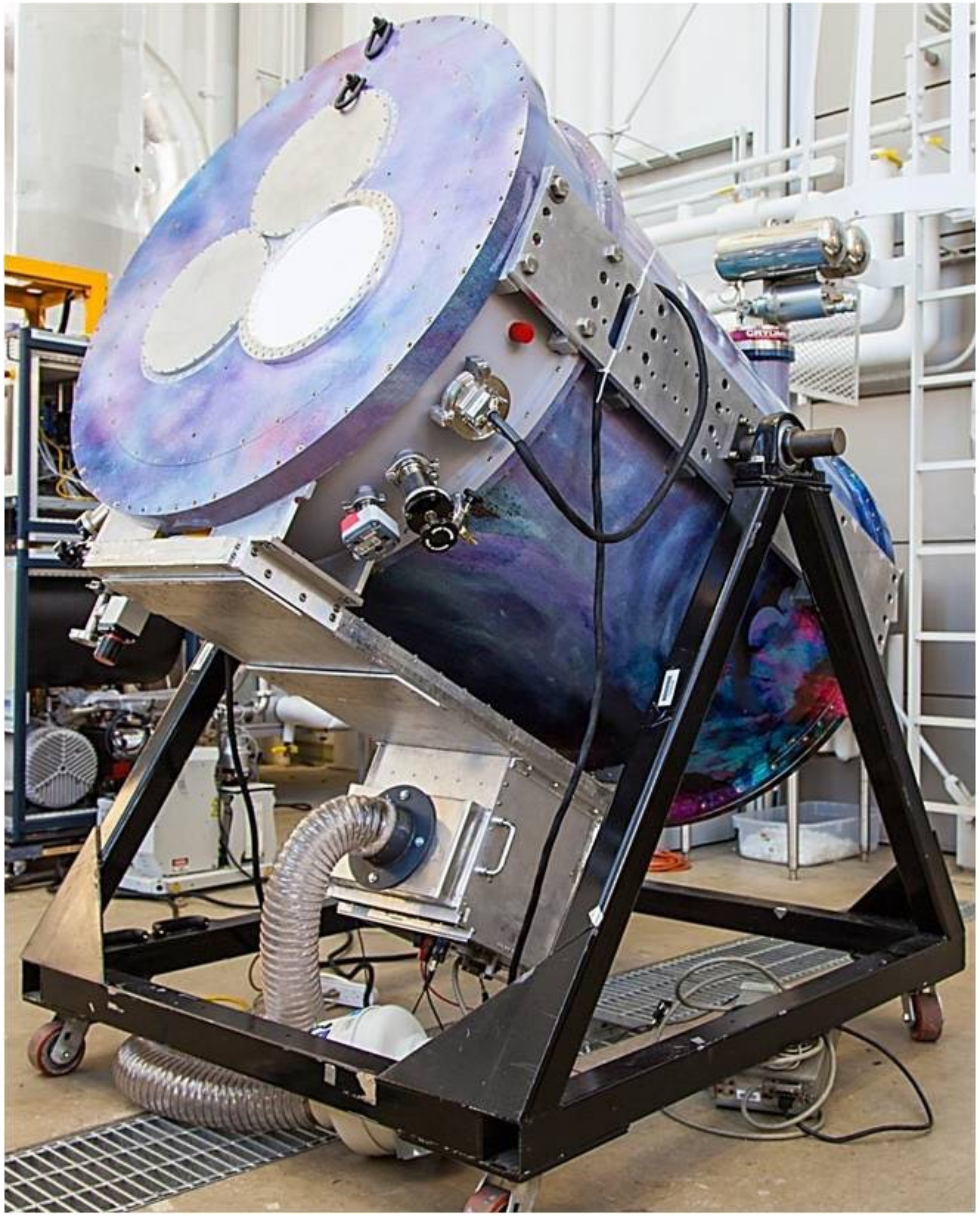
MKID = Microwave Kinetic Inductance Detector

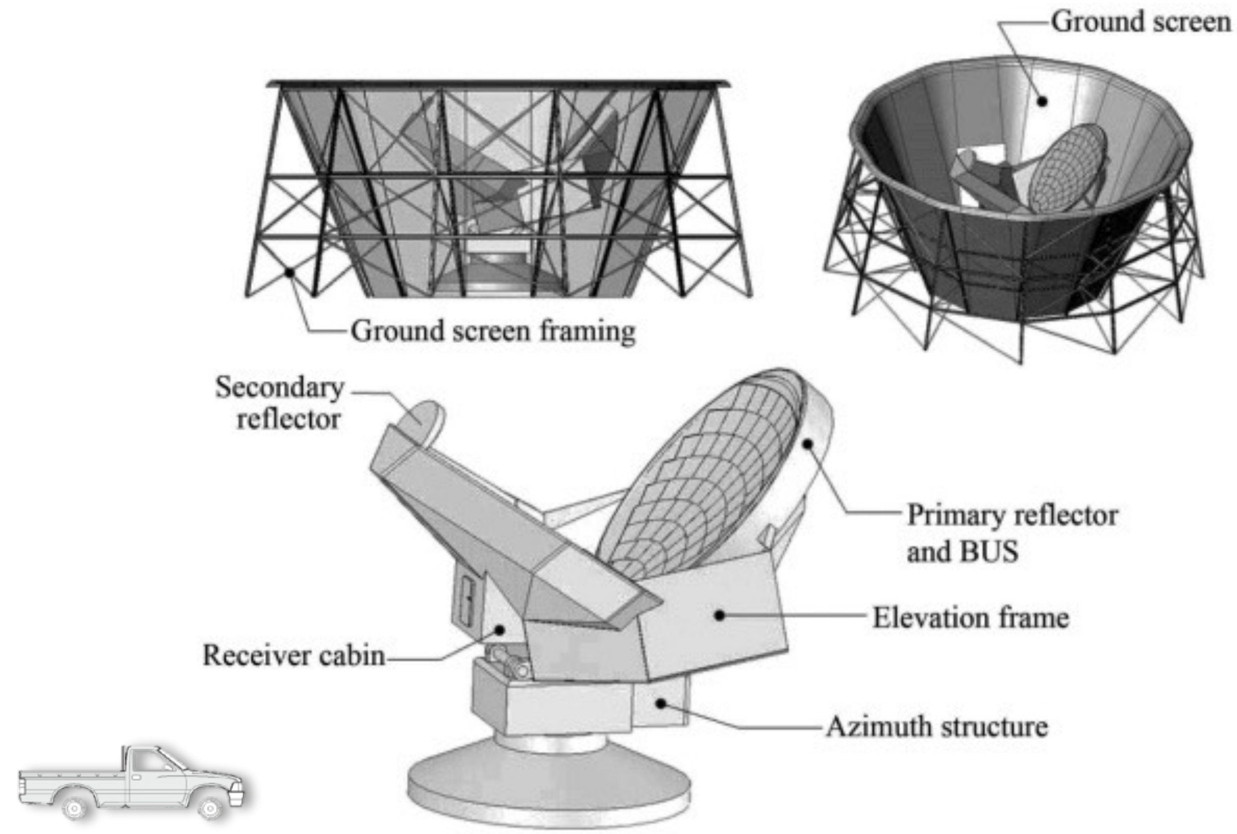
An MKID uses the change in the microwave surface impedance of a superconducting thin-film microresonator to detect photons. Absorption of photons in the superconductor breaks Cooper pairs into quasiparticles, changing the complex surface impedance. Quasiparticles recombine on timescale micro- to milliseconds.



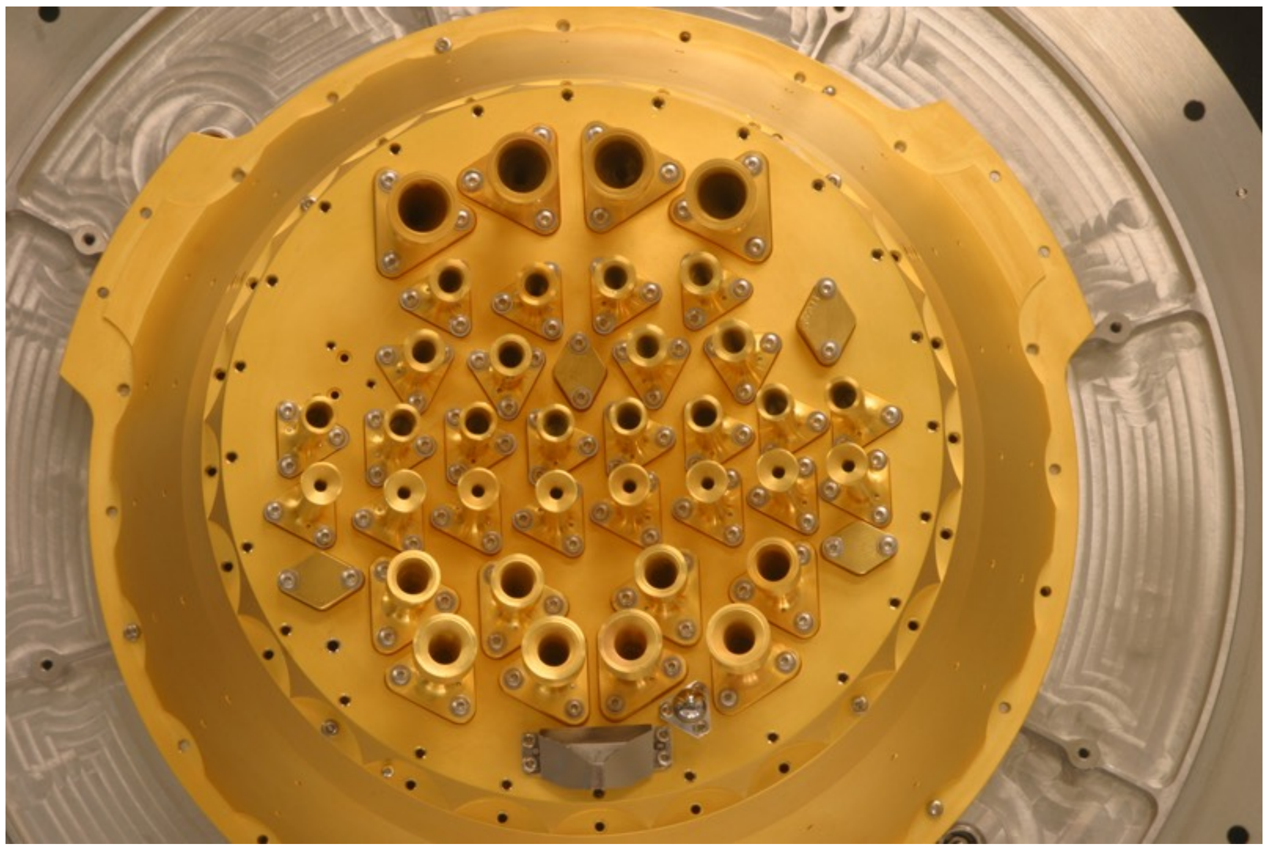
Credit: Frank Bertoldi

Bolometer cryostat for the ACT

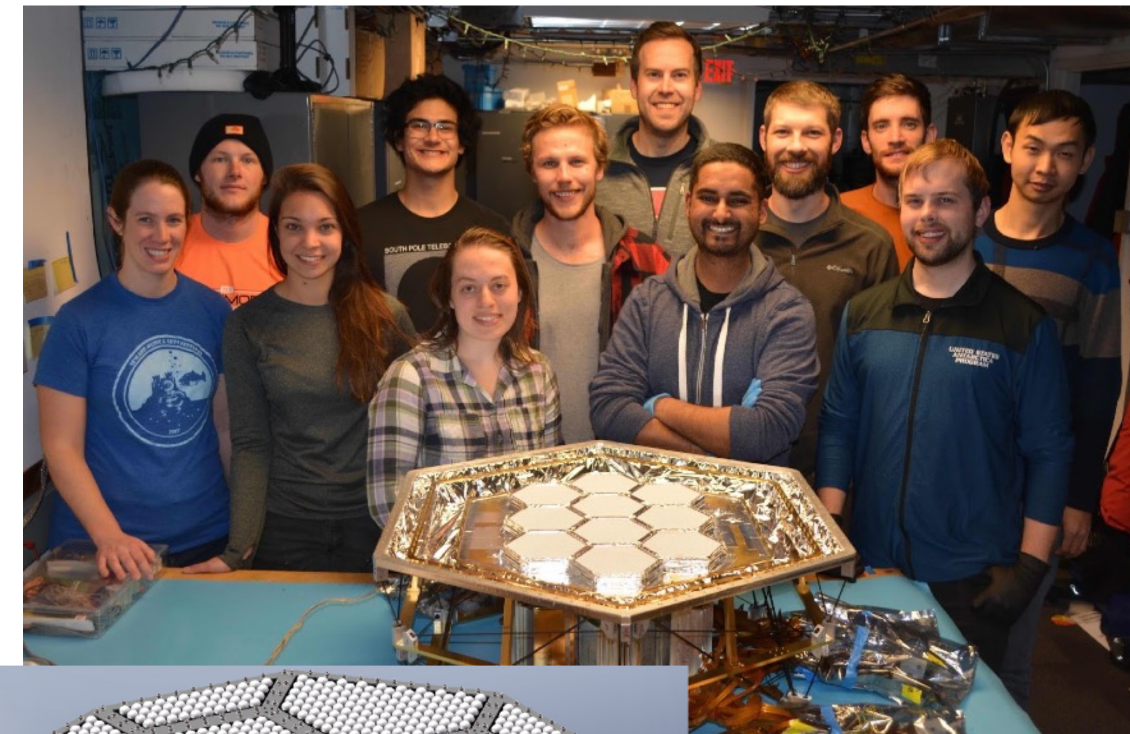




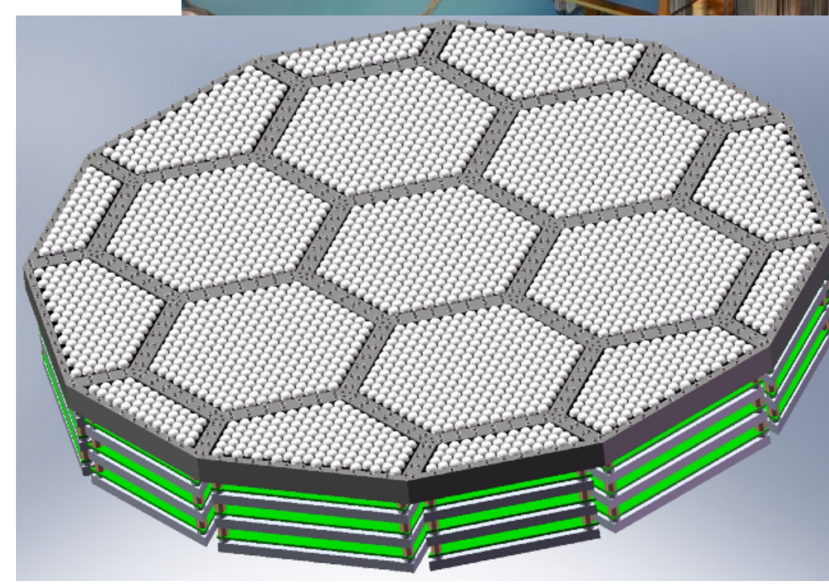
Ground- and space-based Bolometer detector arrays



Planck HFI focal plane, showing the feed horns for 32 bolometer detectors



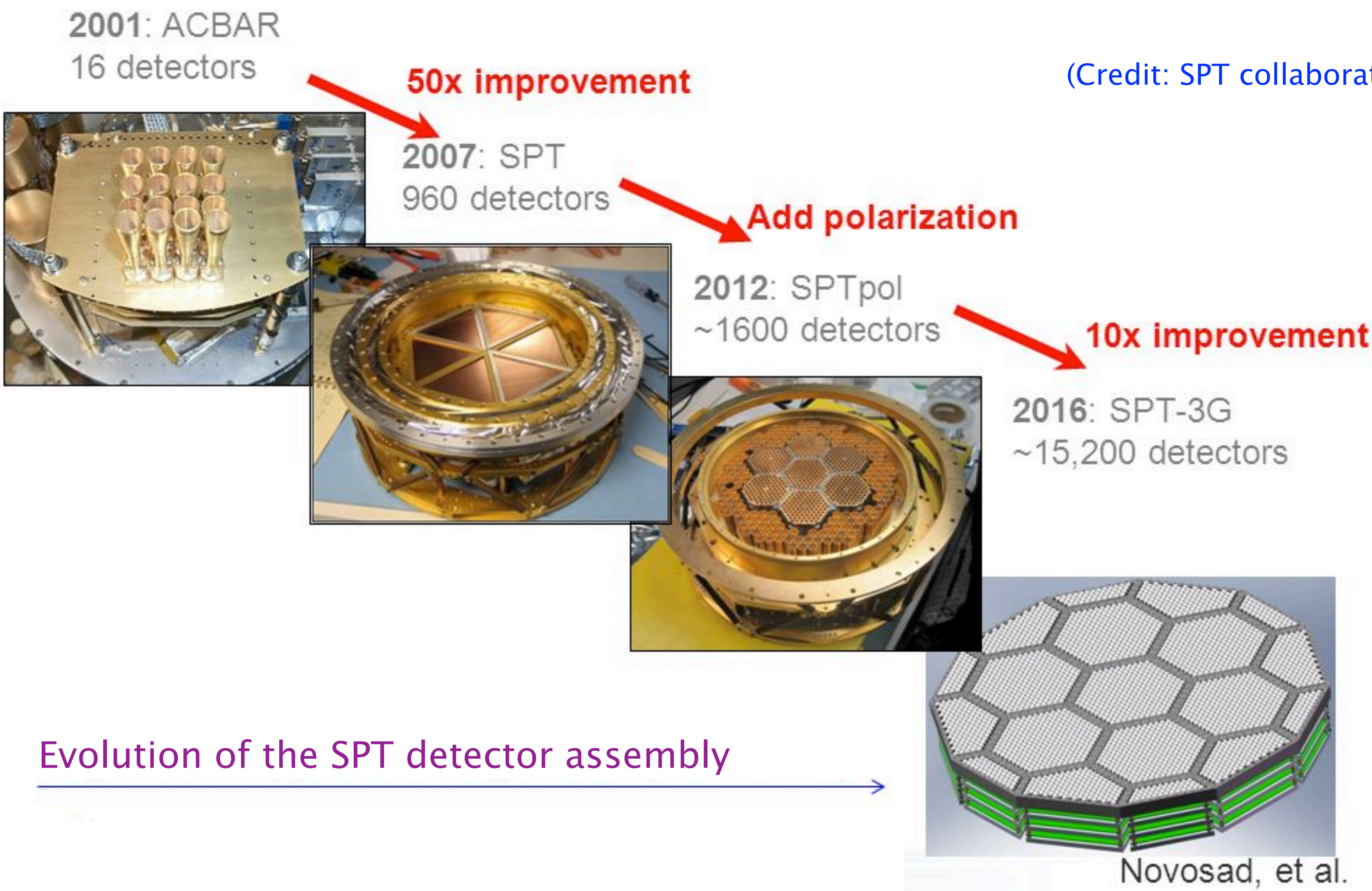
SPT-3G collaboration
Dec 2017



SPT-3G focal plane, with over 15 000 detectors (0.5 m diameter)

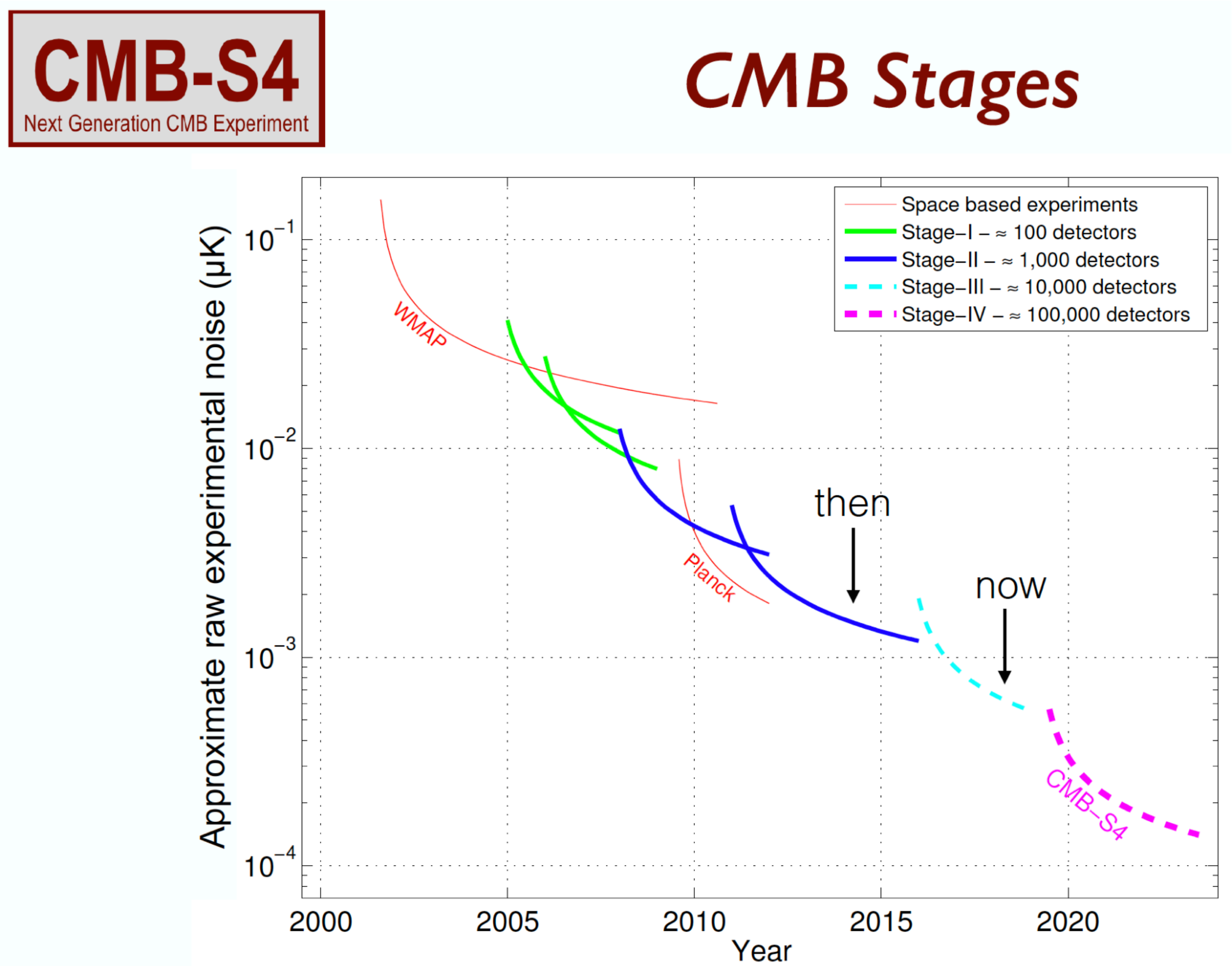
Detectors for the ground-based telescopes

(Credit: SPT collaboration)

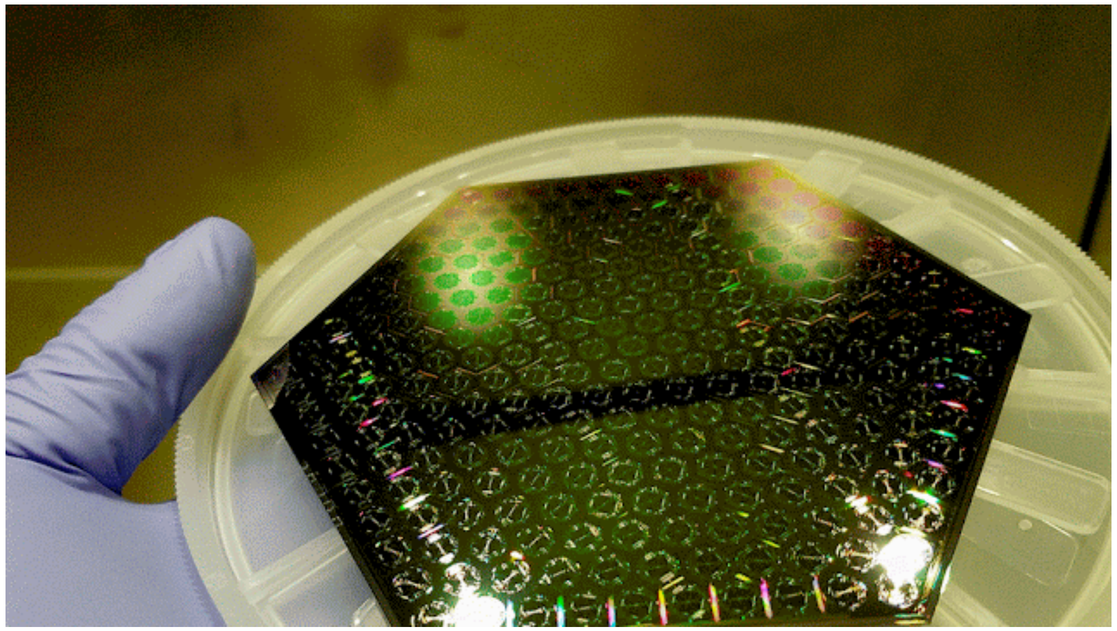


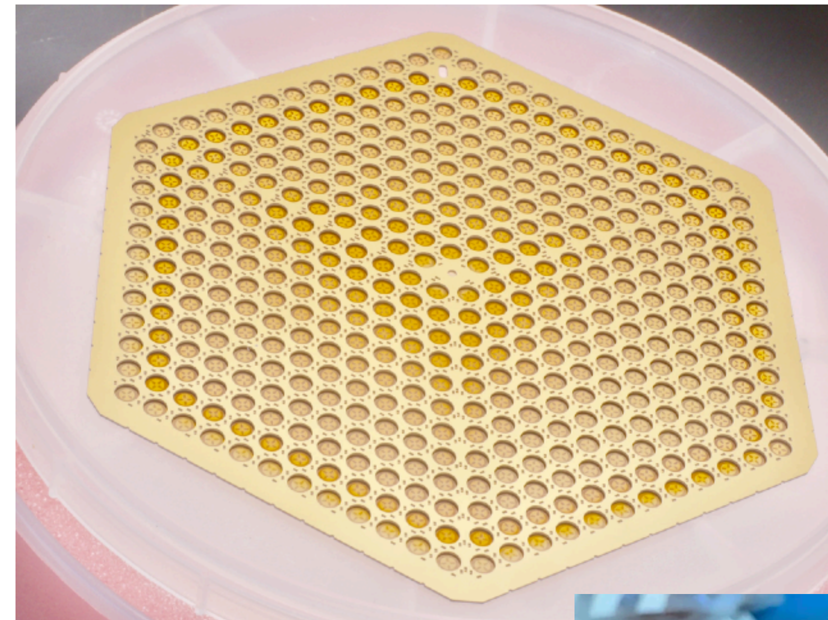
Evolution of the SPT detector assembly

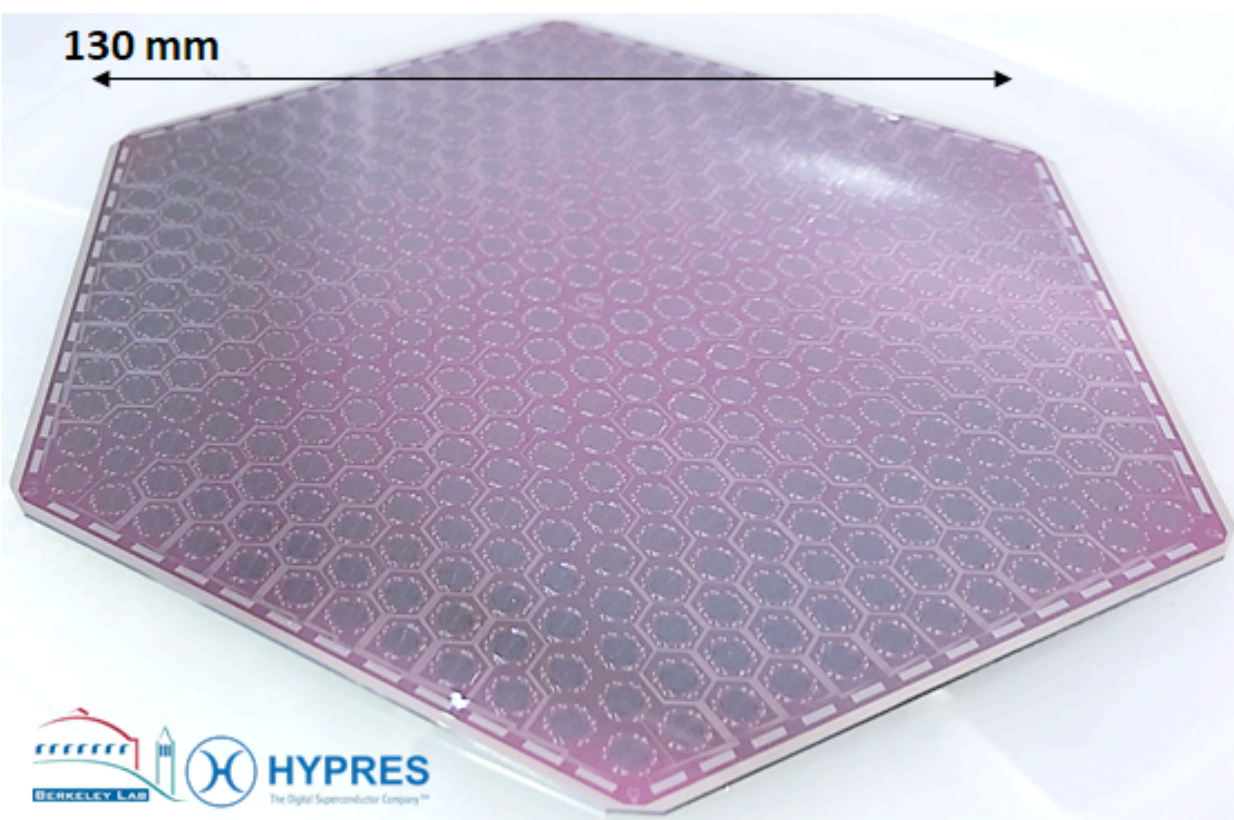
Moore's law for CMB detectors

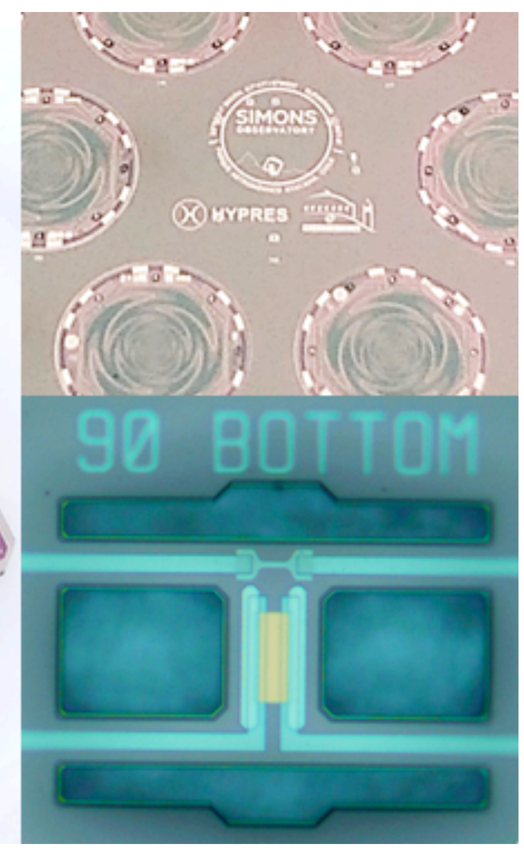


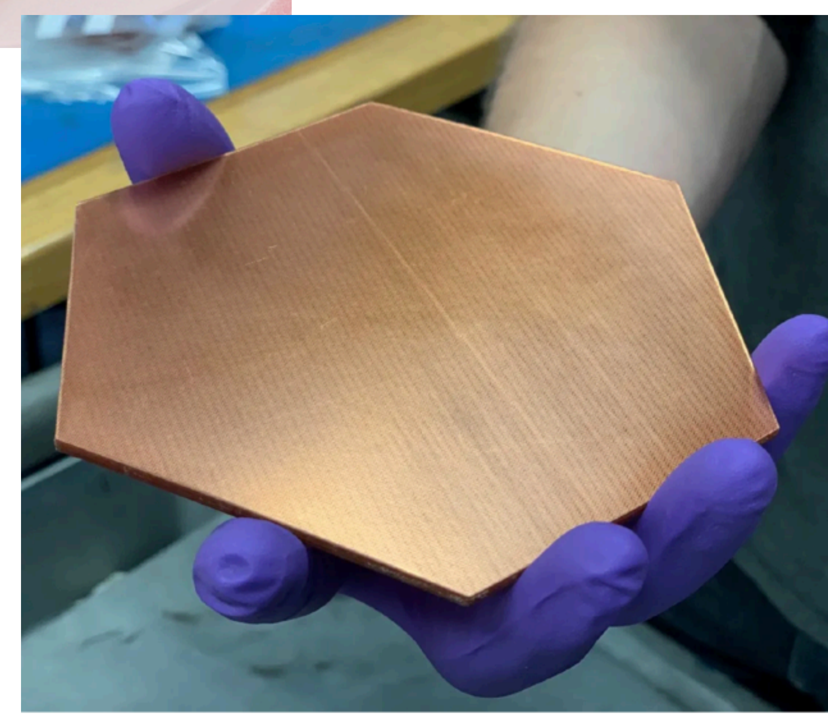
Detector development for Simons Observatory



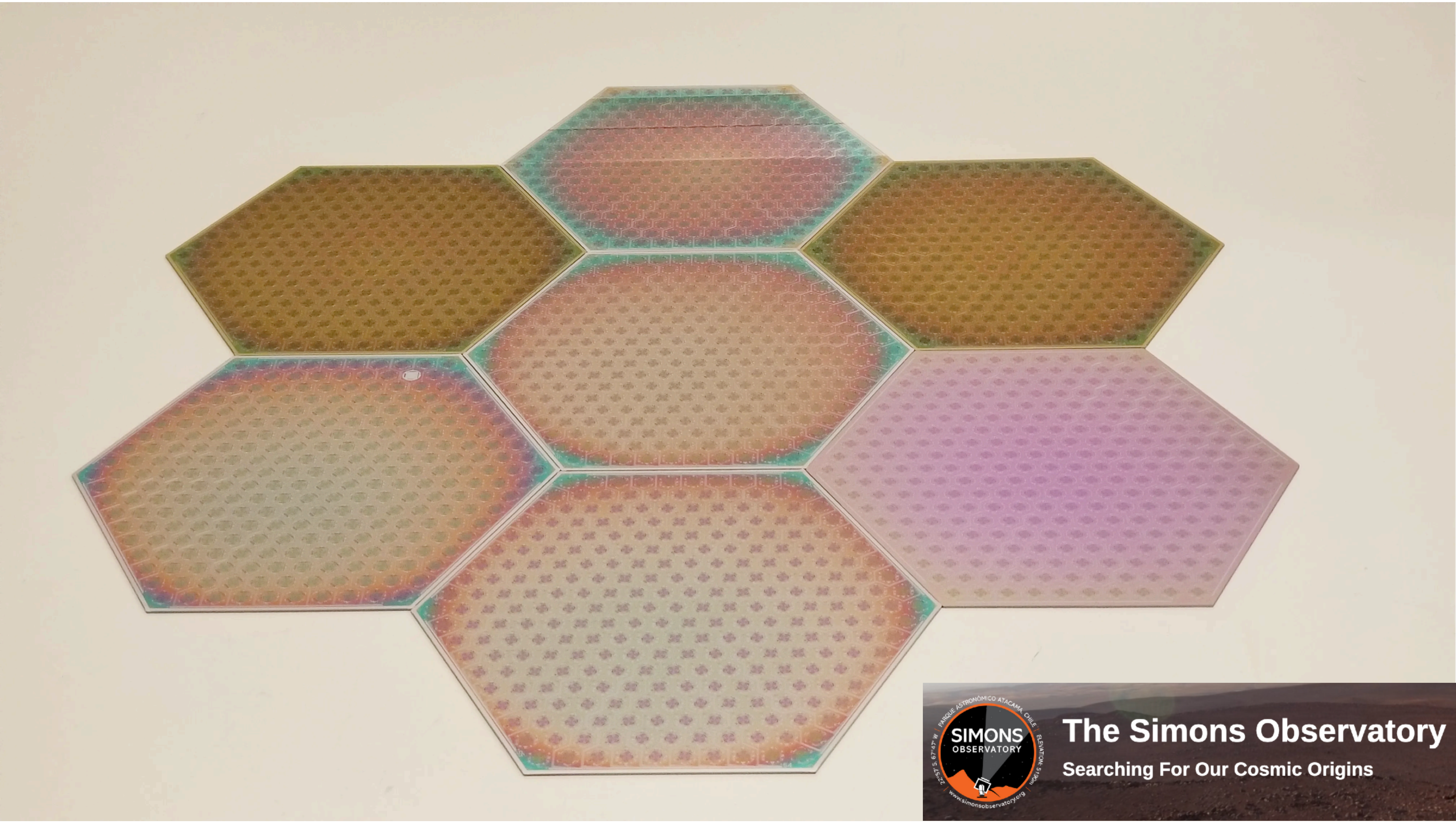








Detector development for Simons Observatory



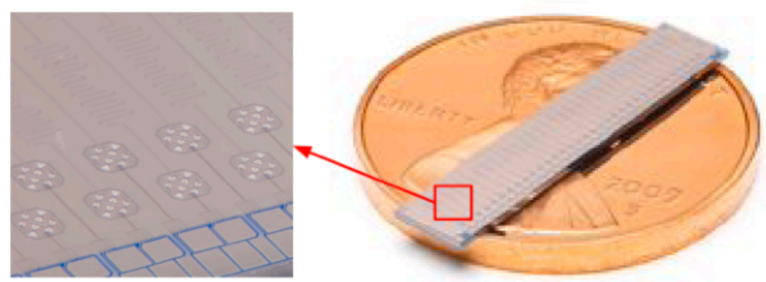
Simons Observatory plans

Credit: Adrian Lee

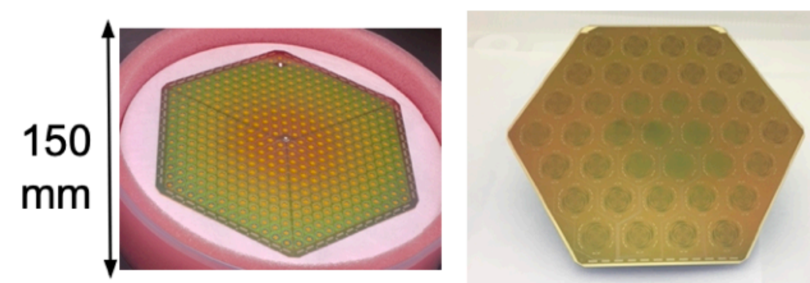
SO construction at Chile (mid-2023)

Readout: SO is using microwave SQUID multiplexing (umux) readout with a 1000x multiplexing factor in collaboration with SLAC (warm electronics) and NIST (cold).

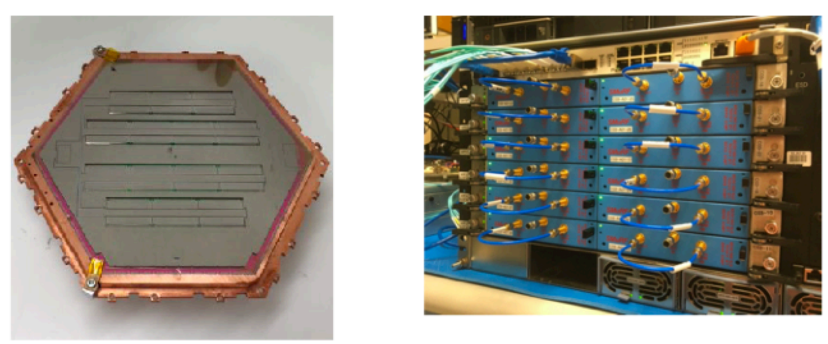
Detectors: SO will use dual-polarization, dichroic TES bolometer detectors spanning 27 - 270 GHz. Each mid-frequency (MF) and high-frequency (HF) array contains ~1700 detectors, with >60,000 detectors total.



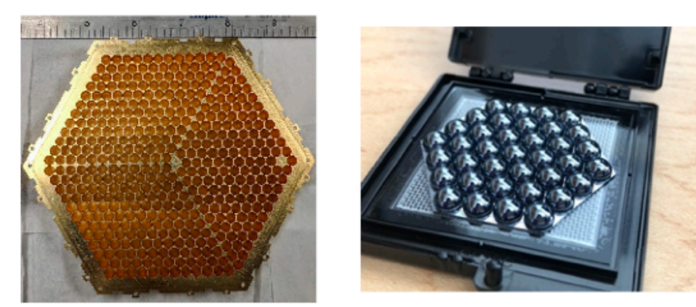
uMux readout channels (left) and NIST uMUX chip with 66 channels (right)



SO MF detector array (left) and LF array (right)

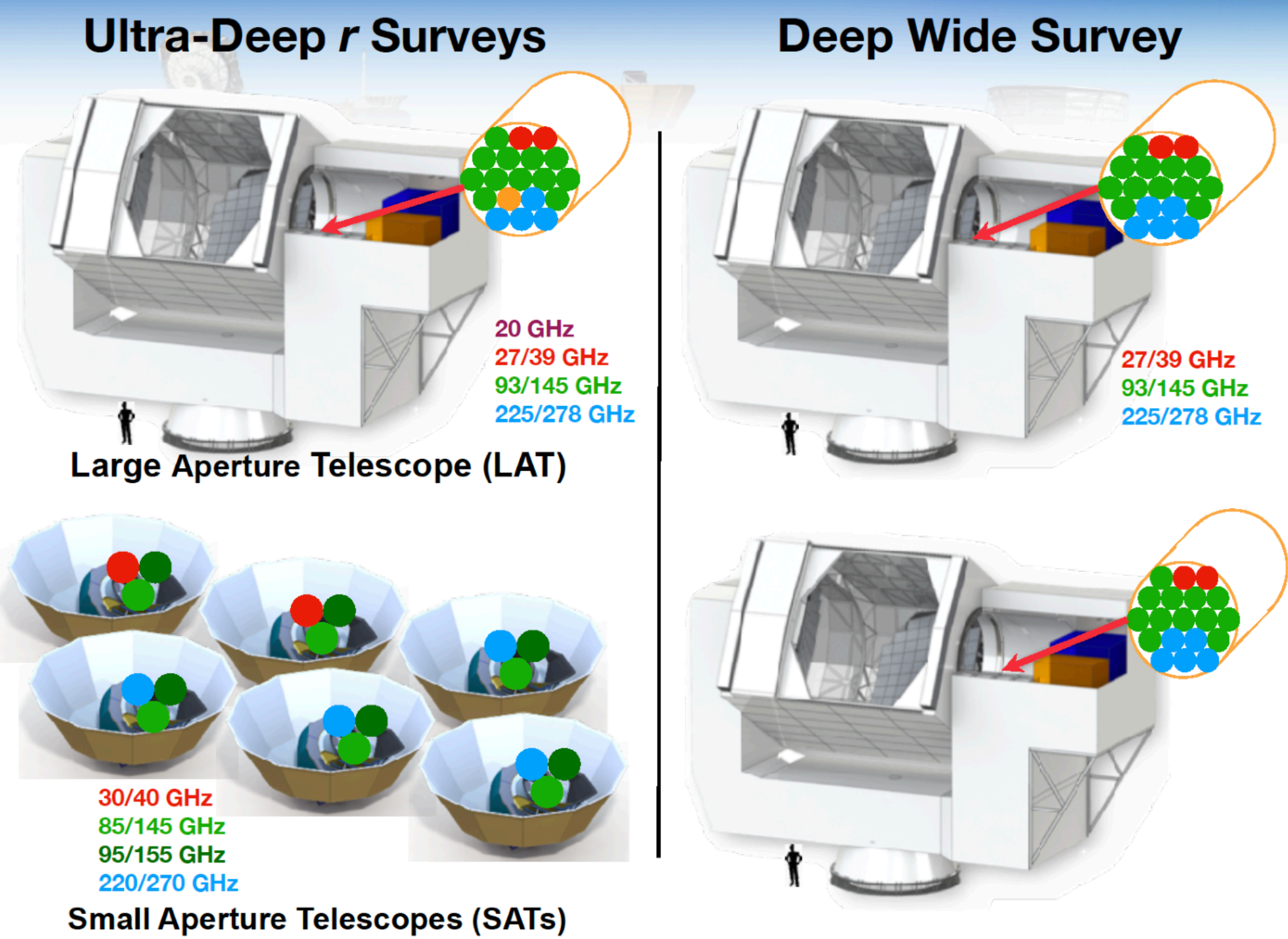


Prototype SO cold readout module with 1848 readout channels (left). SMuRF warm electronics with 12,000 tones (right).



Horn array (left) and lenslet (right) optical coupling for the MF and UFM detector arrays and LF detector array, respectively.

CMB–S4 plans



- Uses field proven technology, including multiplexed TES detectors
- CMB-S4 requires 500 science-grade 150 mm detector wafers
- CMB-S4 has 57 LAT optics tubes and 18 SAT tubes, **500,000** detectors
- SPO has 1 LAT and 5 SAT tubes, 52,000 detectors (CMB-S4 compatible LAT proposed)
- SO nominal has 7 LAT optics tubes plus 3 SAT tubes, 60,000 detectors (CMB-S4 compatible LAT)

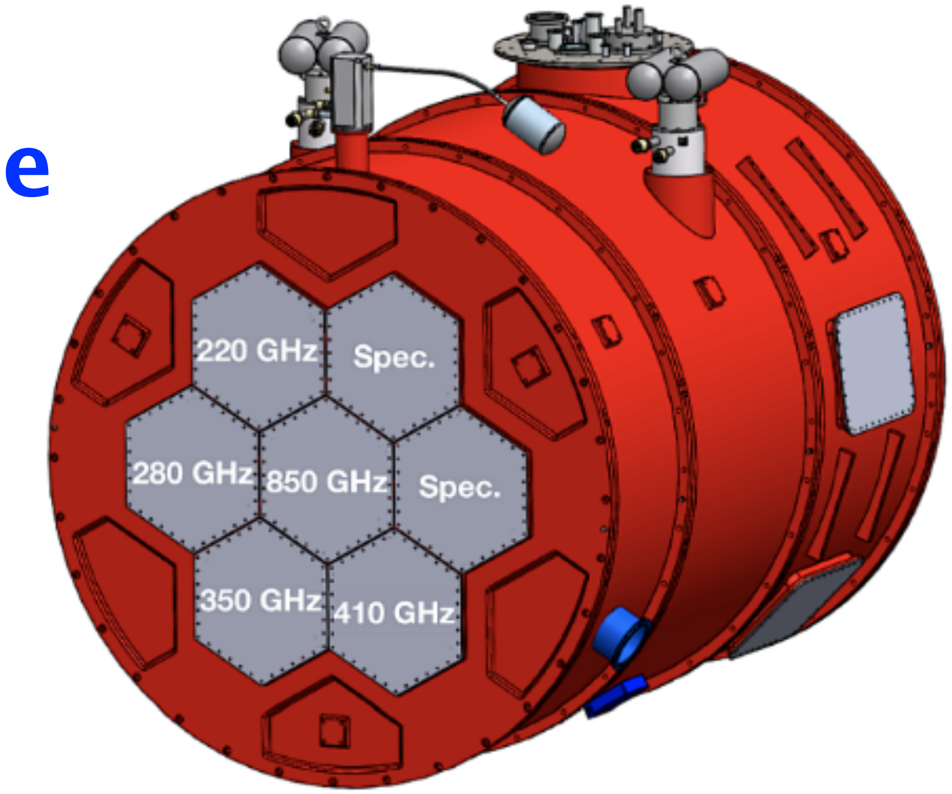
Credit: CMB-S4 collaboration

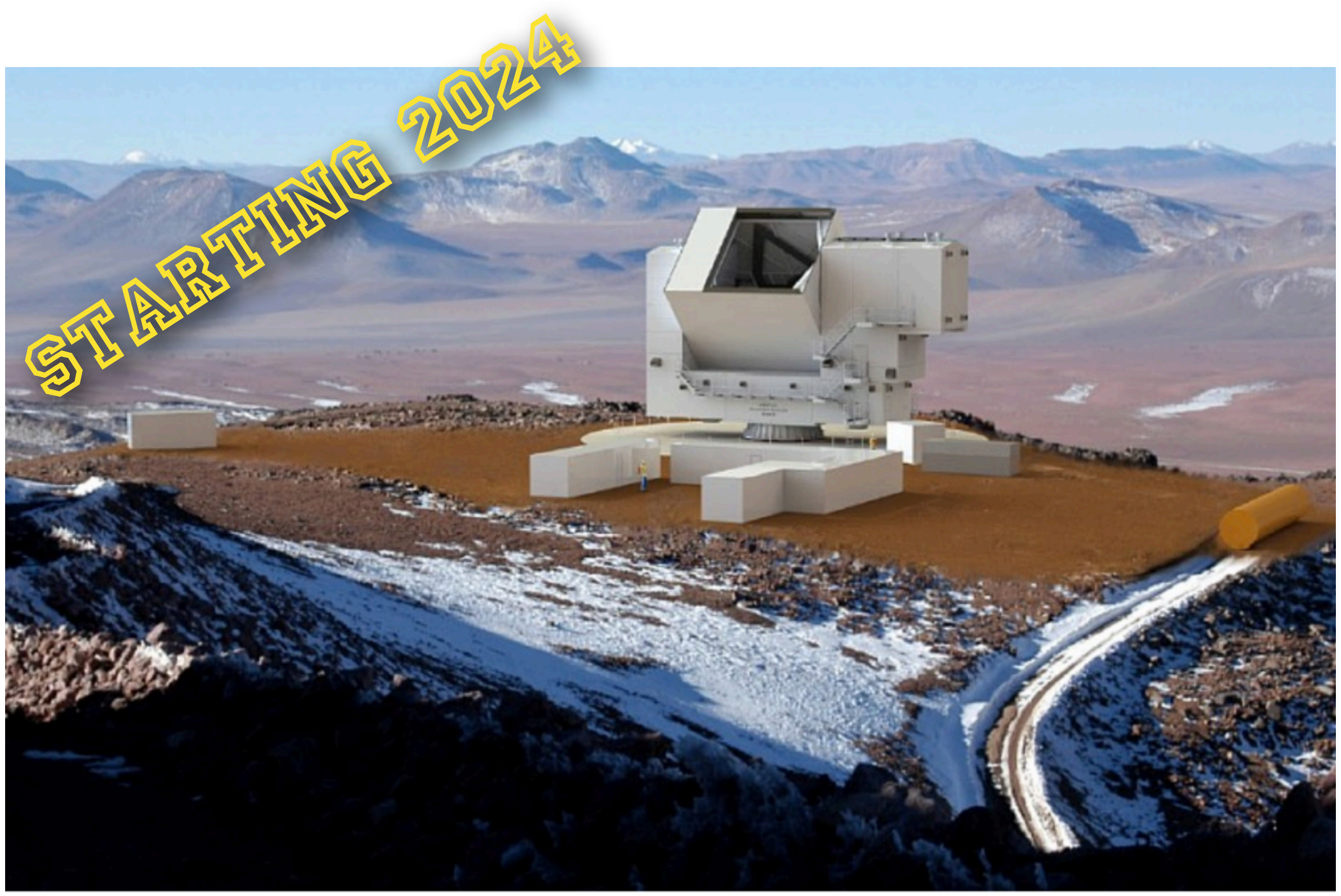
Fred Young Submillimeter Telescope (FYST)

The experiment is called CCAT-prime

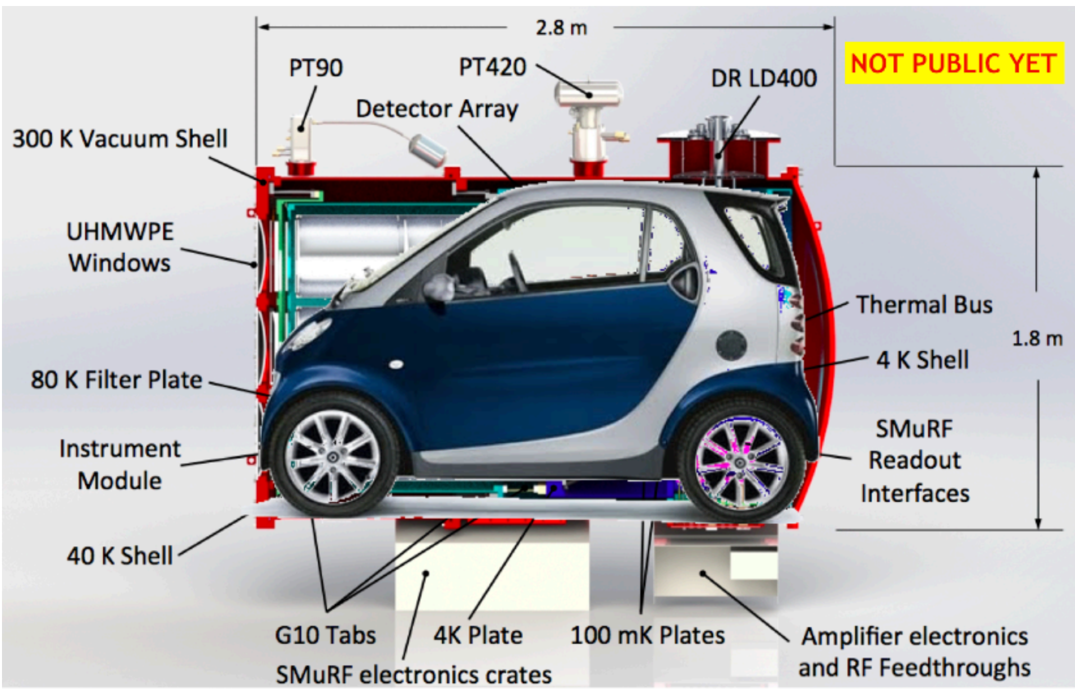
6 meter aperture and *extremely large* field-of-view sub-millimeter telescope on the Cerro Chajnantor (at 5600m) Chile

Partners: Cornell, **Bonn**-Cologne-Munich, Canadian universities

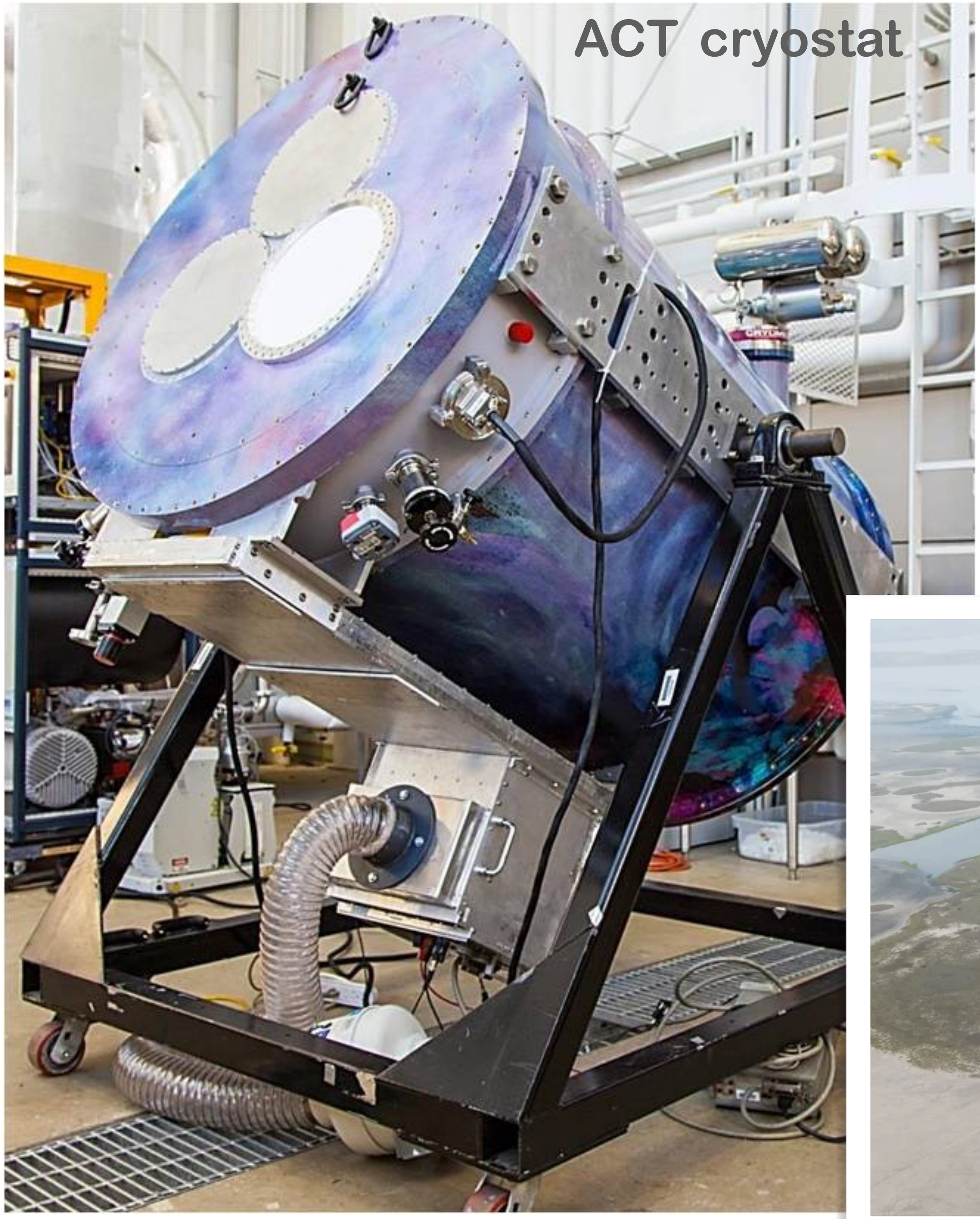


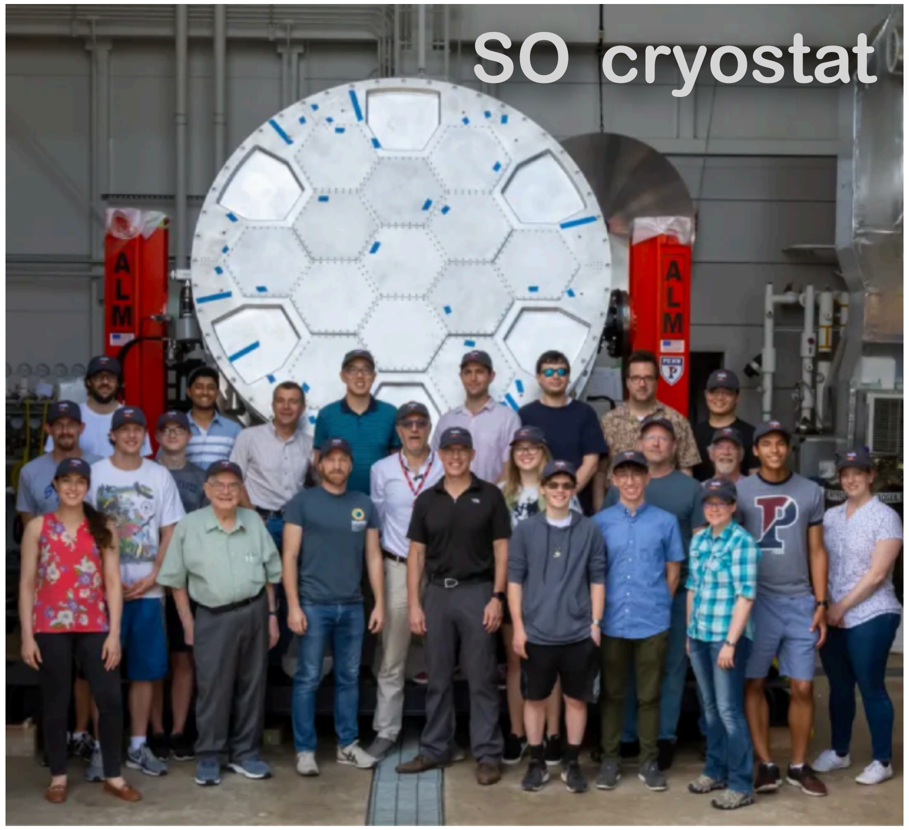


2.5 m diameter bolometer receiver



When will these fly to space?





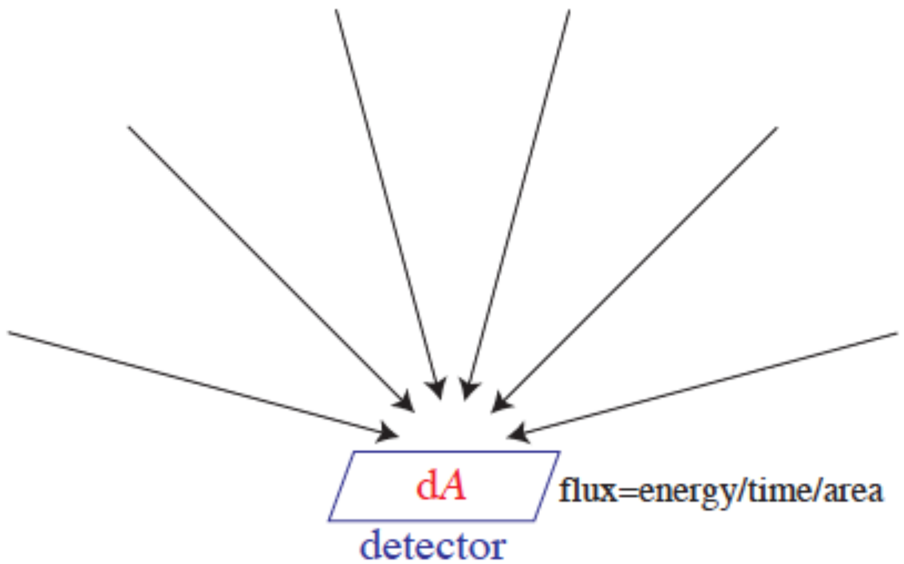
Radiation Basics:

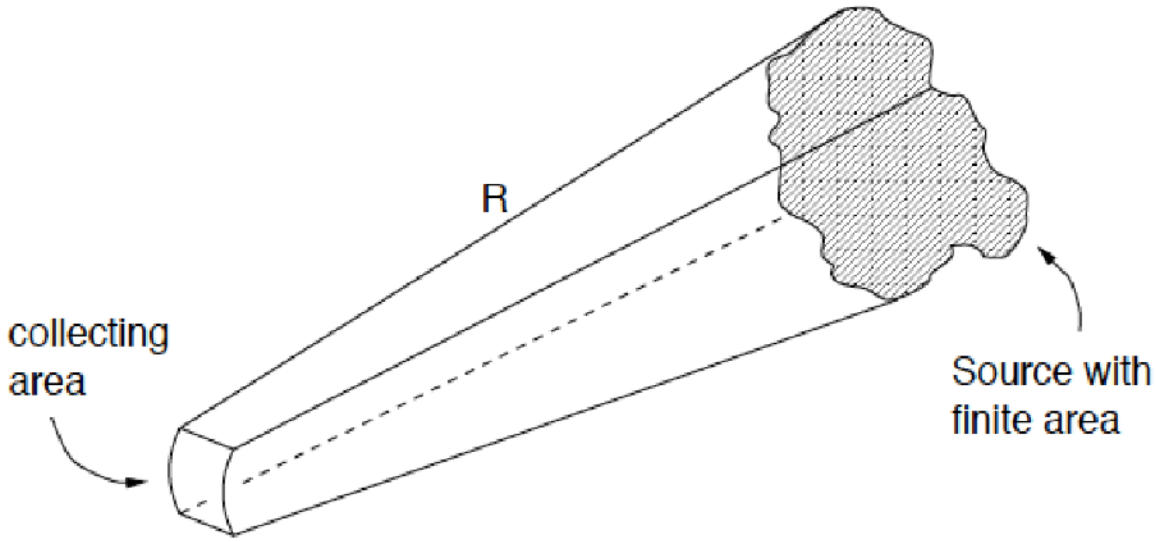
Intensity, Temperatures, & Radiative Transfer

Radiation observables

For an astronomer, there are 5 main observables for radiation:

- Energy flux
- Direction of flux
- Frequency
- Time interval
- Polarization



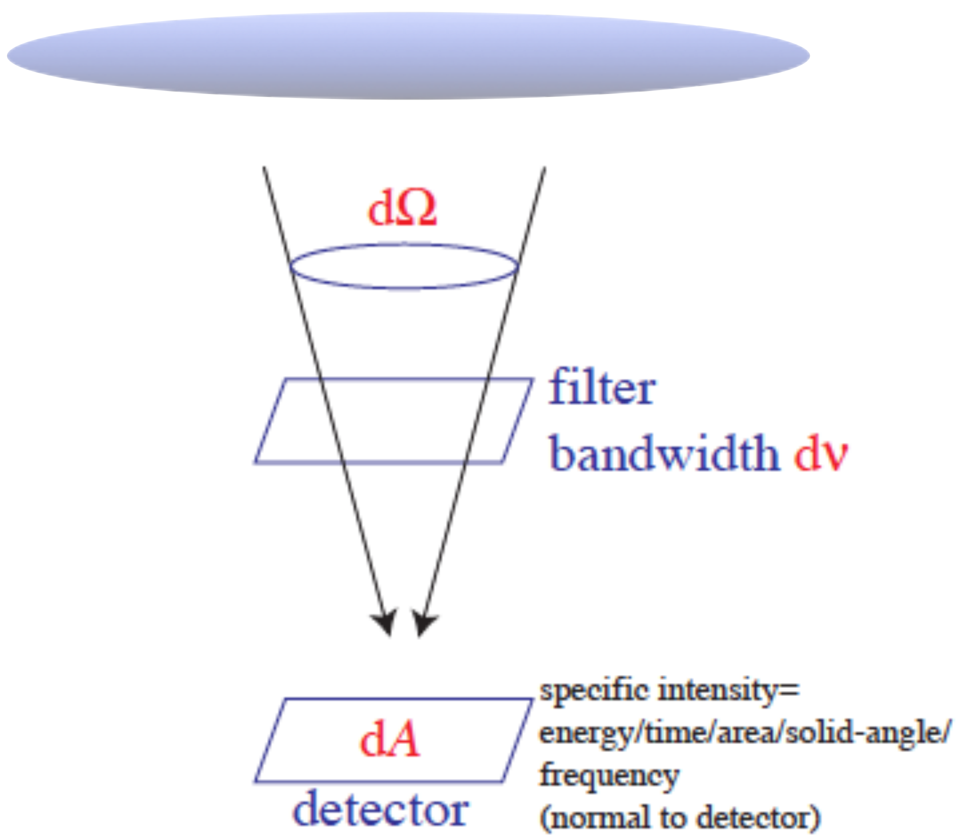


To start, we can think of extended object that is well resolved by our telescope. Here we can talk about the flux received per unit solid angle of the source. This one dimensional quantity is the **specific intensity**.

Specific intensity

Specific intensity (or spectral intensity or spectral brightness) is a line-of-sight quantity, defined as the power received per unit surface area (e.g. of a detector) orthogonal to it, per unit solid angle, and at a specific frequency (or wavelength).

In practice, we can talk about this quantity only for well-resolved sources, i.e. where we can define the source solid angle.



$$dI_\nu = \frac{dE}{(\cos\theta dA)\; d\nu\; d\Omega\; dt}$$

Unit: erg s^{-1} cm^{-2} sr^{-1} Hz^{-1}

Corresponding unit would be: **Jy sr^{-1}**
(this sr^{-1} is extremely important!)

Jansky ➔ 1 Jy = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}

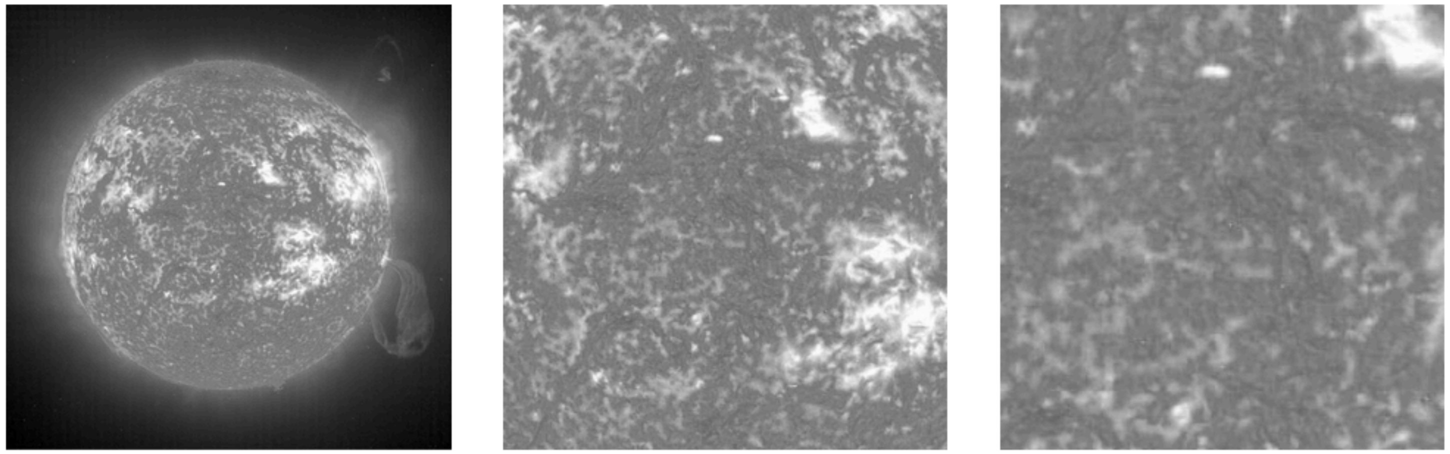
$$I_\lambda \equiv \frac{dP}{(\cos\theta\, d\sigma)\; d\lambda\, d\Omega}$$

$$|I_\nu d\nu| = |I_\lambda d\lambda|\,.$$

$$\boxed{\frac{I_\lambda}{I_\nu} = \left|\frac{d\nu}{d\lambda}\right| = \frac{c}{\lambda^2} = \frac{\nu^2}{c}\,.}$$

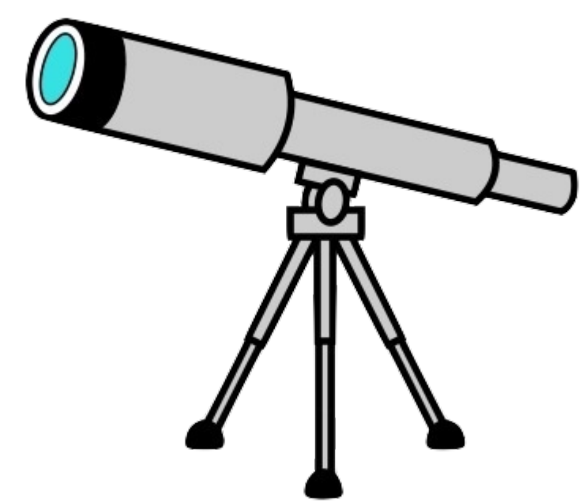
Total intensity: $I \equiv \int_0^\infty I_\nu(\nu)\, d\nu = \int_0^\infty I_\lambda(\lambda)\, d\lambda$

Conservation of Specific Intensity



The Sun as it would appear in three photos taken with the same camera from long (left), medium (center), and short distances (right). The images would have constant brightness but increasing angular size.

There is some confusion over specific intensity and the *contrast* of an object in an image. A telescope cannot change the brightness (specific intensity) of an object, but a photographic plate can collect more photons with a long exposure, thereby increasing the contrast and giving us the *appearance* of a brighter source.



Flux density and total flux

Flux density (or more accurately, spectral flux density, at a specific $\boldsymbol{\nu}$) is defined for discrete sources, i.e. when the source has a well-defined solid angle.

For point-like sources this is the only measurable quantity.

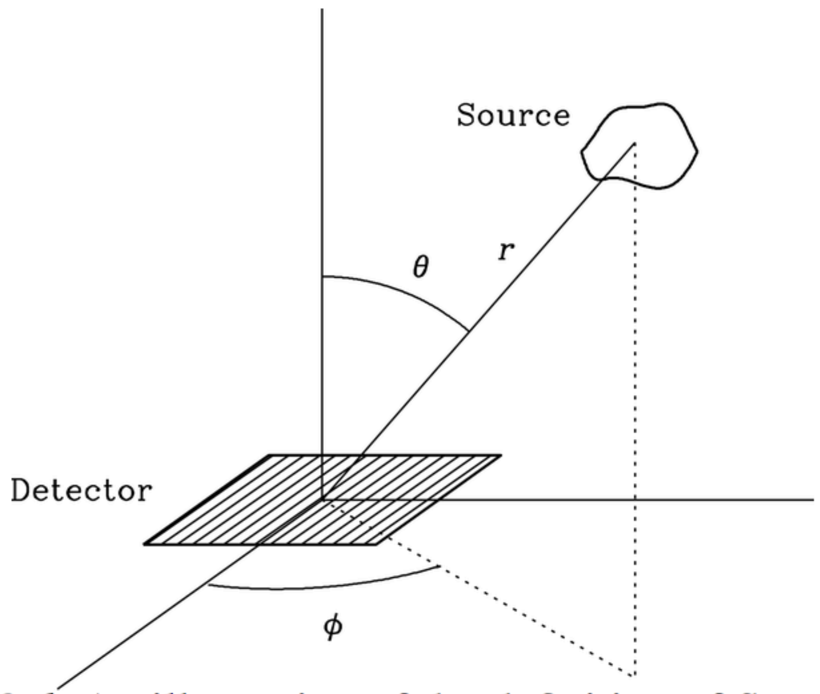


Figure 2.6: An illustration of the definition of flux density.

$$\boxed{S_\nu \equiv \int_{\text{source}} I_\nu(\theta,\phi)\cos\theta \, d\,\Omega\,.}$$

For a source with very small angular extent (≪1 rad), cosθ ≈ 1, and the definition is simpler

$$\boxed{S_\nu \approx \int_{\text{source}} I_\nu(\theta,\phi) d\,\Omega\,.}$$

The cgs/mks unit is too large for radio astronomers, so we use Jy (or mJy, or μJy)

$$1 \ \text{jansky} = 1 \ \text{Jy} \equiv 10^{-26} \ \text{W m}^{-2} \ \text{Hz}^{-1},$$

The total flux is simply $S \equiv \int_0^\infty S_\nu \, d\nu\,.$

Spectral and bolometric luminosities

The spectral luminosity is defined as the total power radiated by the source per unit bandwidth at frequency ν

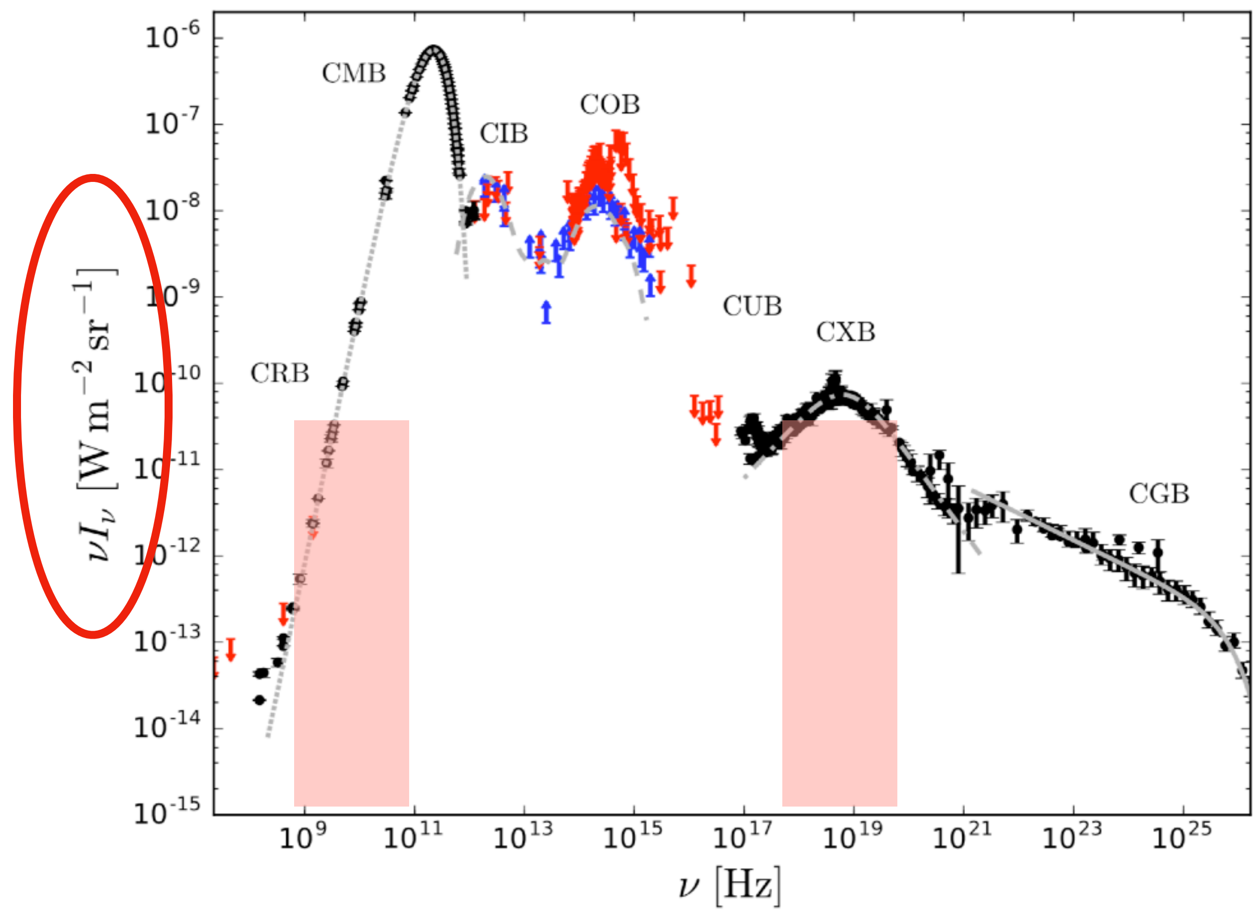
$$\boxed{L_\nu = 4\pi d^2 S_\nu,}$$

The integral over all frequencies is the total luminosity, which radio astronomers also call *bolometric luminosity*, because of the large-bandwidth bolometer detectors they use.

$$L \equiv \int_0^\infty L_\nu \, d\nu \, .$$

Side note: In many spectral plots the Y-axis representing the specific intensity or flux is often multiplied by the frequency ν. This is just a simple "mental assist" to help us estimate the *total energy content* by estimating the areas under the curves, when the X-axis has log units.

$$I_\nu d\nu = \nu I_\nu d(\ln\nu)$$



When to use I_ν and when S_ν

(or, when to use the definition of a temperature)

Clearly, we would be using the definition of flux density only for relatively compact sources. For unresolved sources, it is the only quantity that can be measured.

For extended objects, flux density per unit solid angle can be measured, which is nothing but the specific intensity. But intuitively, we can guess that **temperature** would already be a suitable variable for that. However, temperature only makes sense if the emitting matter is in thermodynamic (local or global) equilibrium.

Irrespective of this limitation, radio astronomers define a new quantity, called the **brightness temperature** (also called Rayleigh-Jeans temperature), to describe this flux density per unit solid angle in terms of an equivalent blackbody temperature.

$$\boxed{I_\nu = \frac{2\,\nu^2\,k}{c^2} \cdot T_b}$$

Recap: Blackbody radiation

$$\boxed{B_\nu(T) = \frac{2h}{c^2}\frac{\nu^3}{e^{h\nu/kT}-1}.}$$

$[B_\nu(T)] = \text{ergs sec}^{-1}\,\text{cm}^{-2}\text{Hz}^{-1}\text{sr}^{-1} = 10^{17}\,\text{MJy sr}^{-1}.$

Jansky ➔ 1 Jy = 10^{-26} W m^{-2} Hz^{-1} = 10^{-23} erg s^{-1} cm^{-2} Hz^{-1}

Planck's law determines the intensity of photons per unit frequency.

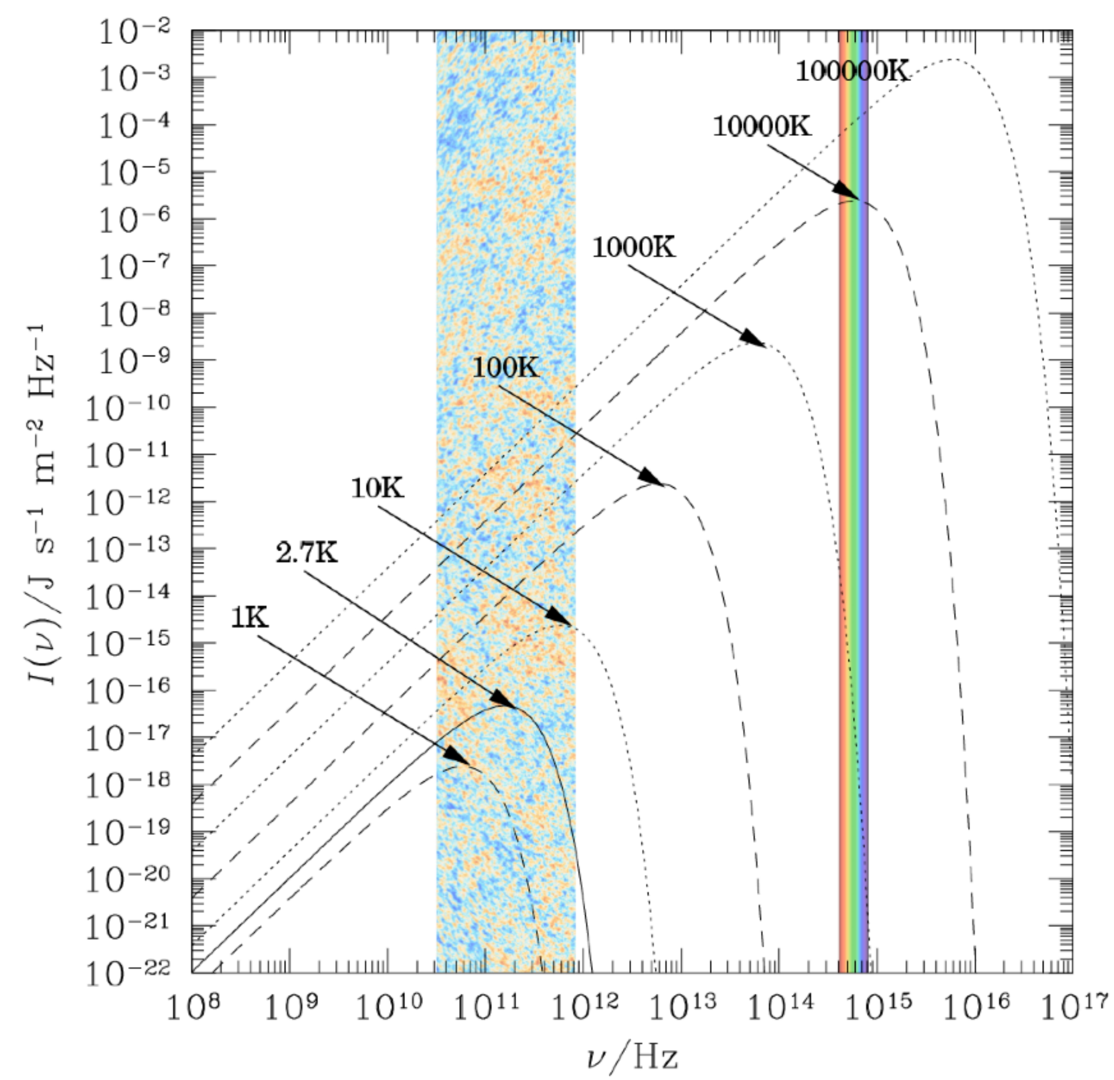
At low frequencies, we have the Rayleigh-Jeans limit

Radio Astronomy

$$\boxed{B_\nu(T) \overset{h\nu\ll kT}{\approx} \frac{2\nu^2}{c^2}kT \propto \nu^2 T,}$$

At the high-frequency limit we have the Wien law

$$B_\nu(T) \overset{h\nu\gg kT}{\approx} \frac{2h\nu^3}{c^2}e^{-h\nu/kT}.$$



Brightness temperature

Brightness temperature is a convenient quantity, defined as the temperature of a blackbody (which is defined in the Rayleigh-Jeans limit) that will produce the observed specific intensity

$$\boxed{I_\nu = \frac{2k\nu^2 T_B}{c^2}}$$

Correspondingly, the flux density is : $S_\nu = \int_{\Omega_s} I_\nu d\Omega = \frac{2k\nu^2}{c^2} \int T_B d\Omega$

A very useful approximation:

$$\boxed{(\Delta T_{\mathrm{RJ}}/\mu\mathrm{K}) = 340\,(\Delta S/\mu\mathrm{Jy\ beam}^{-1})(\nu/\mathrm{GHz})^{-2}(h_{\mathrm{a}}/\mathrm{arcmin})^{-1}(h_{\mathrm{b}}/\mathrm{arcmin})^{-1}}$$

It doesn't matter if the emitting source is thermal or non-thermal.
T_B is just a definition, in terms of an equivalent blackbody radiation.

Applying this definition blindly can then lead to very unphysical temperatures for astronomical sources, e.g., 10^{12} K for radio AGN lobes! This simply means that these sources are not thermal emitters, and there is no meaningful temperature values to speak of.

Antenna temperature

The actual measurement done by a telescope is often expressed in terms of *antenna temperature*, T_A. This is the "beam convolved" brightness temperature, with additional factor of telescope efficiency.

The effective area of an antenna is defined as the constant of proportionality between the source flux density and the monochromatic power received by the antenna (the factor 1/2 is for each mode of polarization):

$$P_\nu = \frac{1}{2} A_{eff} \cdot S_\nu$$

Now imagine this power is being *radiated* by the antenna, there is no source! But instead, the thermal load of resistor, kept at a certain temperature, outputs that equivalent amount of power. This gives the definition of *antenna temperature*.

This conversion is done using the definition of Johnson–Nyquist noise, where a resistor at temperature T produces a noise power kT dν per unit bandwidth.

$$P_\nu \, d\nu = kT \, d\nu$$

System temperature

A telescope/receiver system itself generates noise, which usually drowns the antenna temperature T_A by factor of several. *This is called the system temperature, in terms of which telescope sensitivities are defined.*

$$P_{out} = P_A + P_{sys} \quad \Rightarrow \quad T_{out} = T_A + T_{sys}$$

The *system temperature*, T_{sys}, represents noise added by the system:

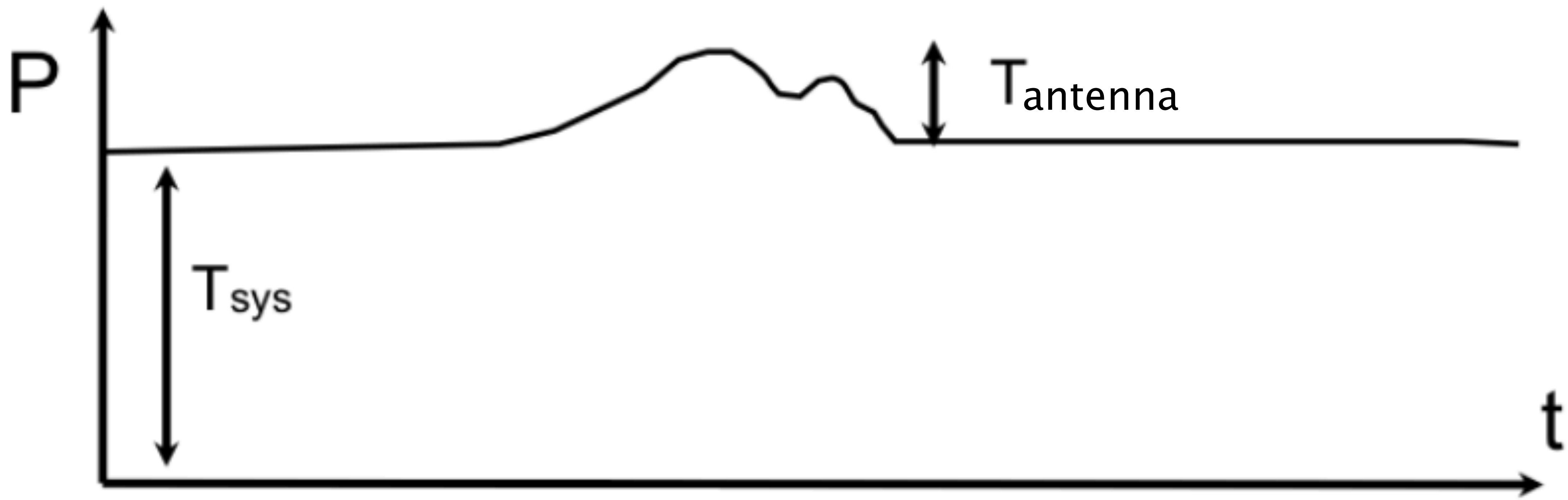
$$T_{sys} = T_{bg} + T_{sky} + T_{spill} + T_{loss} + T_{cal} + T_{rx}$$

T_{bg} = microwave and galactic background (3K, except below 1GHz)
T_{sky} = atmospheric emission (increases with frequency--dominant in mm)
T_{spill} = ground radiation (via sidelobes) (telescope design)
T_{loss} = losses in the feed and signal transmission system (design)
T_{cal} = injected calibrator signal (usually small)
T_{rx} = receiver system (often dominates at cm — a design challenge)

detector noise

Note that T_{bg}, T_{sky}, and T_{spill} vary with sky position and T_{sky} is time variable

Radio telescope measurements

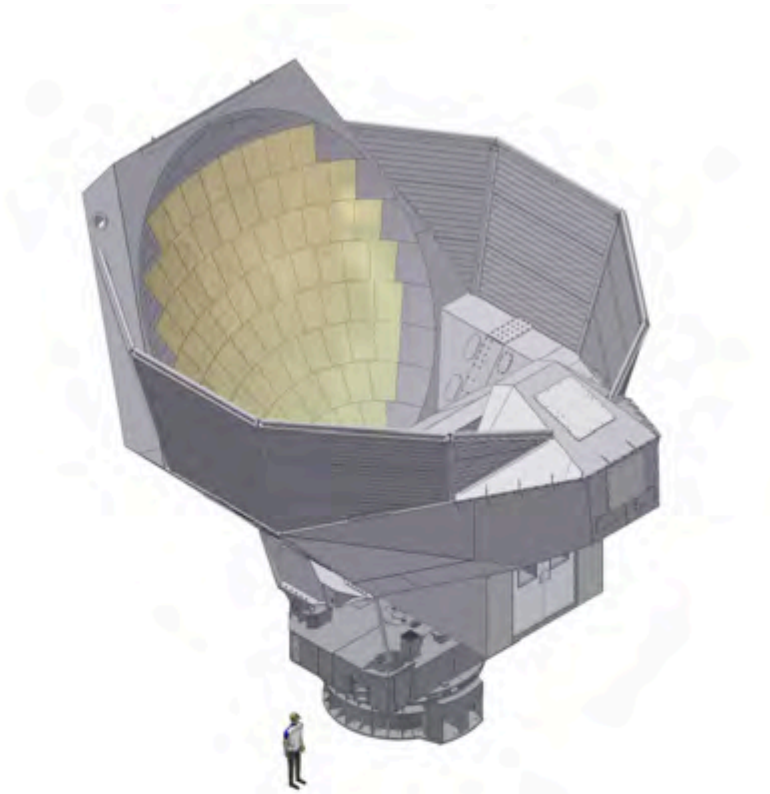


•**System temperature**: temperature of blackbody producing same power as telescope + instrumentation without a source

• **Brightness temperature:** Flux density per unit solid angle of a source measured in units of equivalent blackbody temperature

• **Antenna temperature:** The flux density transferred to the receiver by the antenna. Some of the incoming power is lost, represented by the aperture efficiency

Noise in ground-based experiments



Atacama Cosmology Telescope

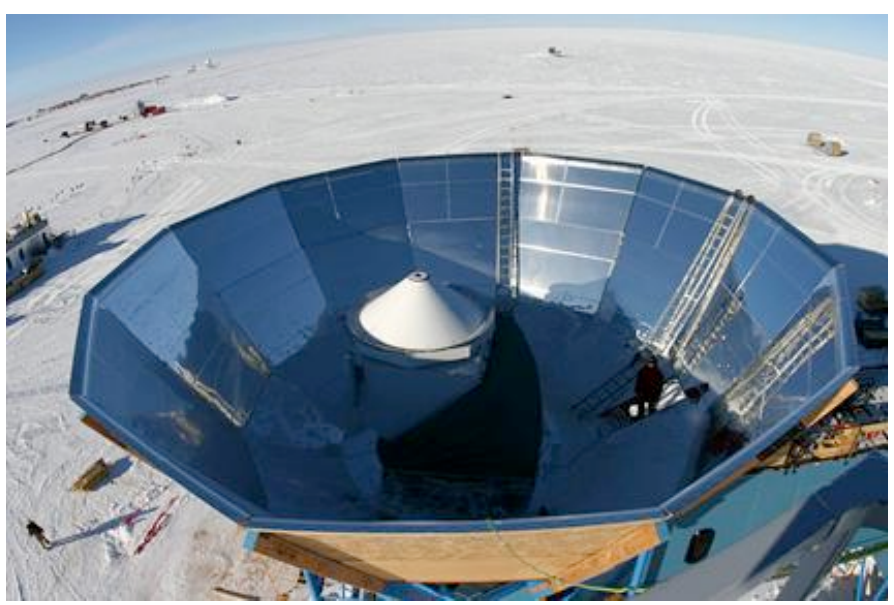
Since 3 K << 300 K, CMB measurements are sensitive to thermal emission from their environments.

CMB telescopes are specially designed to be very directional, but ~300 K in the sidelobes is always a worry. **Hence most telescopes use ground shields.**

The atmospheric noise totally dominates scales larger than ~few tens of arcminutes, making low multipole CMB measurements from the ground extremely challenging.

A receiver measures system temperatre, T_{sys}

$$T_{sys} = T_{detector} + T_{CMB} + T_{atmosphere} + T_{ground} + \ldots$$



QUaD at south pole

The radiometer equation gives an estimation of the noise:

antenna efficiency

$$\delta T = \frac{bT_{sys}}{\sqrt{\Delta\nu\ \tau}}$$

bandwidth

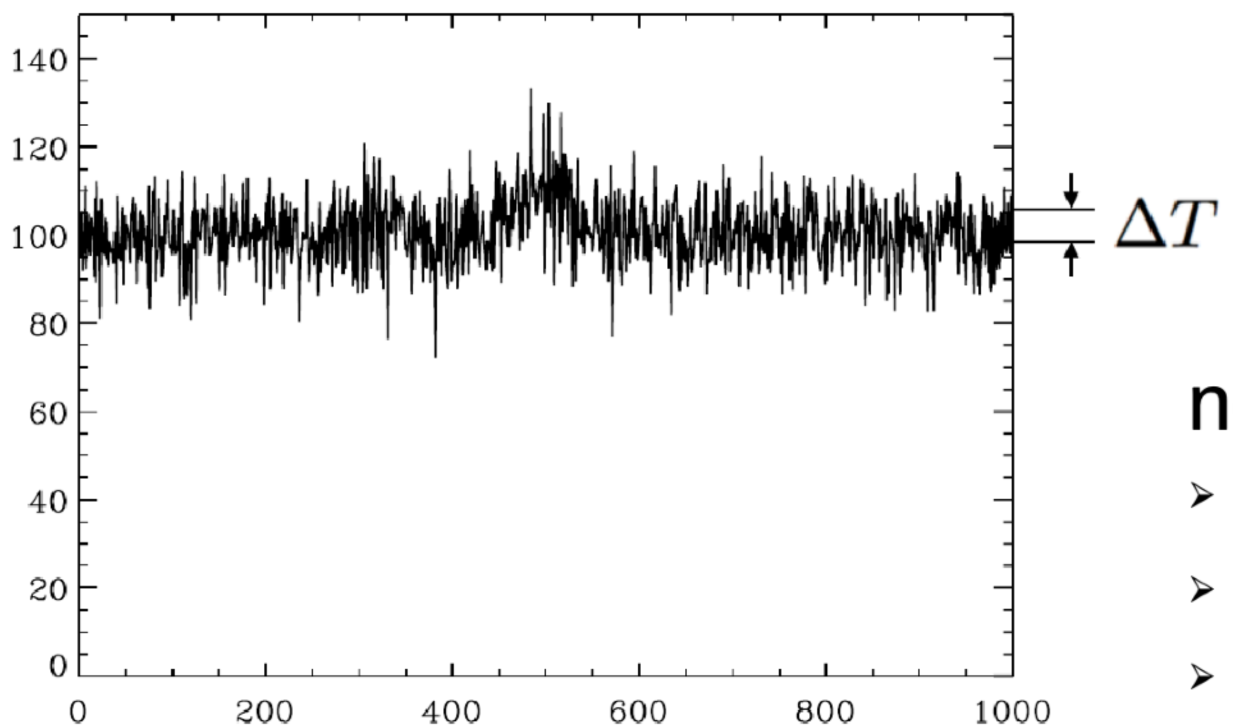
time

Radiometer equation

A radio receiver used to measure the average power of the noise coming from a radio telescope in a well-defined frequency range is called a radiometer.

The **radiometer equation** expresses the ideal noise amplitude in terms of the bandwidth and averaging time. The noise amplitude follows a Gaussian distribution with zero mean and fluctuations on very short timescales.

$$\boxed{\Delta T = \frac{k \cdot T_{sys}}{\sqrt{\Delta\nu \cdot \tau}}}$$



noise depends on

- system temperature T_{sys}
- bandwidth of the receiver $\Delta\nu$
- integration time τ

Signal-to-noise ratio:
SNR = $T/\Delta T$

Telescope and Beams

Primary mirror converts incoming plane wave into a converging spherical wave

Power received by the primary mirror:

$$P = \frac{1}{2} \cdot A_{eff} \cdot S_\nu \cdot \Delta\nu$$

effective collecting area (telescope) — flux density (source) — bandwidth (receiver)

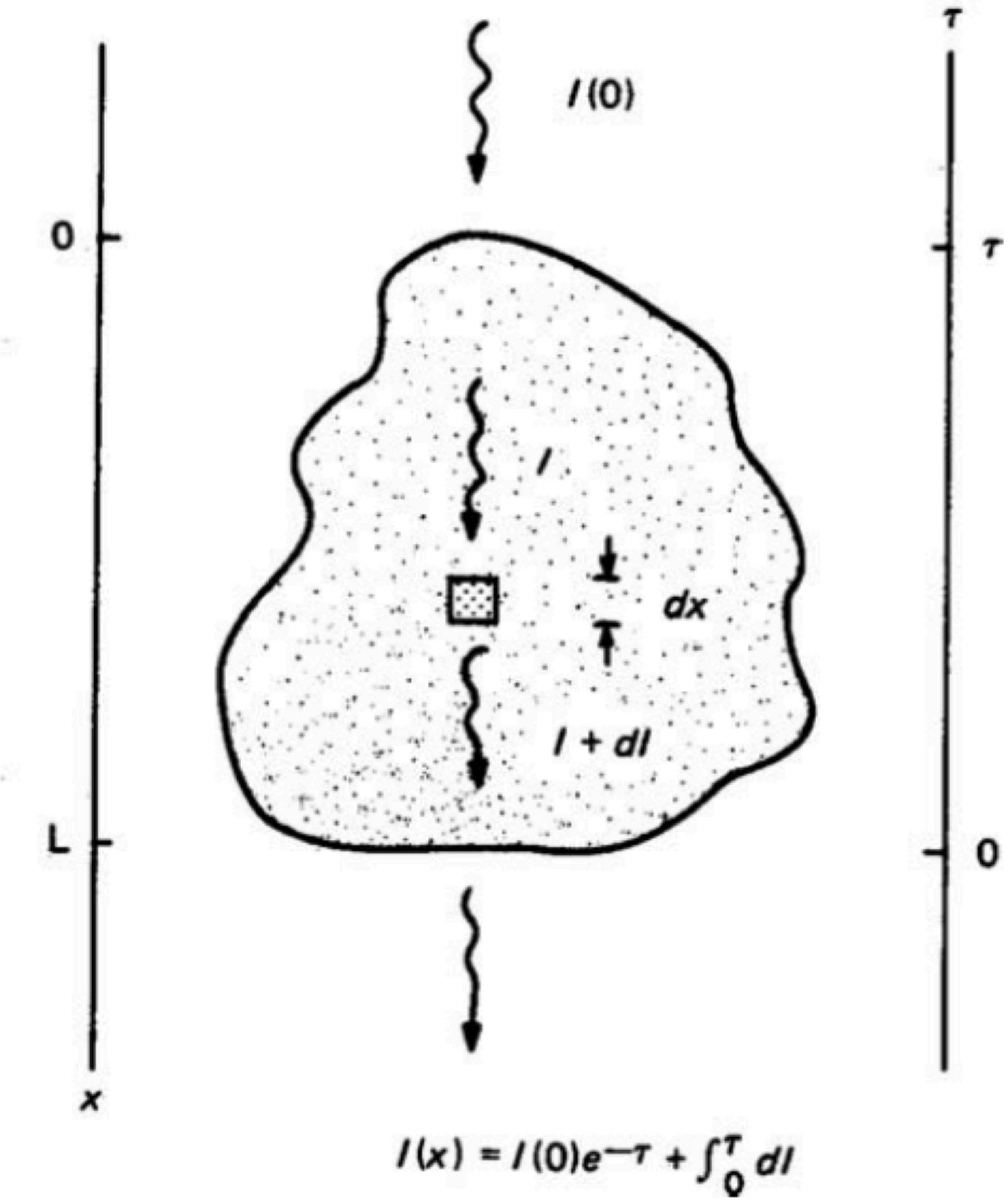
Geometric collecting area: $A_{geo} = \frac{\pi}{4} D^2$

$A_{\rm eff} = \eta_A \cdot A_{\rm geo}$ with $\eta_A \sim 0.5$

Angular resolution: (→ *Fourier Optics*)

Half Power Beam Width (HPBW) without taper $HPBW = 1.02 \cdot \frac{\lambda}{D}\ \mathrm{rad} = 58\overset{\circ}{.}4 \cdot \frac{\lambda}{D}$

Radiative transfer in its entirety



Radiation transfer describes the changes in the *one-dimensional radiation beam* as it passes through a medium in a **steady state** (i.e. $dI_\nu/dt = 0$).

In its most common form, effect of scattering is neglected (we discuss this later), but phenomena like stimulated emission are accounted for.

$$\boxed{dI_\nu = -I_\nu \kappa_\nu ds + j_\nu ds}$$

The general solution will be

$$\boxed{I_\nu(s) = I_\nu(0)\,\mathrm{e}^{-\tau_\nu(s)} + B_\nu(T)\left(1 - \mathrm{e}^{-\tau_\nu(s)}\right)}$$

Note that, as $\tau_\nu(0) \to \infty$, we get $I_\nu = B_\nu(T)$.

Absorption coefficient, optical depth

$$\boxed{\kappa \equiv \frac{dP}{ds}}$$

is the quantity that measures the probability *dP* of a photon to be absorbed along the slab-with of thickness *ds*. This is the linear absorption coefficient and has dimension of inverse length.

Note the distinction between the two definitions: absorption coefficient, $\boldsymbol{\kappa}_\nu$, and mass absorption coefficient, $\boldsymbol{\alpha}_\nu$ (also called opacity) These are related by $\boldsymbol{\alpha}_\nu = \rho \boldsymbol{\kappa}_\nu$. Hence, one has unit cm^{-1} and the other is $gm^{-1}\,cm^2$.

The optical depth (or opacity) is the dimensionless quantity:

$$\boxed{\tau \equiv -\int_{s_{\mathrm{out}}}^{s_{\mathrm{in}}} \kappa(s')\ ds'}$$

Hence $d\tau = -\kappa\, ds$. The negative sign makes sure that optical depth increases as one looks “deeper” into the material.

Hence, for absorption only,

$$\boxed{\frac{I_\nu(s_{\mathrm{out}})}{I_\nu(s_{\mathrm{in}})} = \exp(-\tau)}$$

$$I_\nu = I_\nu(0)\, e^{-\tau_\nu}$$

Optical depth is therefore the number of e-foldings of change in intensity.

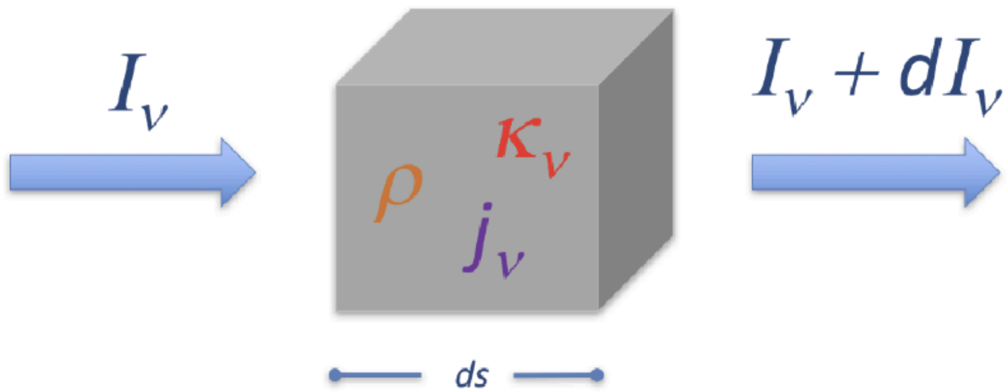
Emission coefficient, radiative transfer

The intervening medium can also emit photons, which will be proportional to the volume along the path length *ds* (assuming isotropic emission). The emission coefficient is defined as (when there is no absorption):

$$\boxed{j_\nu \equiv \frac{dI_\nu}{ds}}$$

(Note the dimension of j_ν : it is W m^{-3} Hz^{-1} sr^{-1}, which is the power per unit volume)

$$I_\nu(s) = I_\nu(0) + \int ds\, j_\nu$$



Now we have the full radiative transfer equation combining both the effects of absorption and emission:

$$\boxed{\frac{dI_\nu}{ds} = -\kappa I_\nu + j_\nu\,.}$$

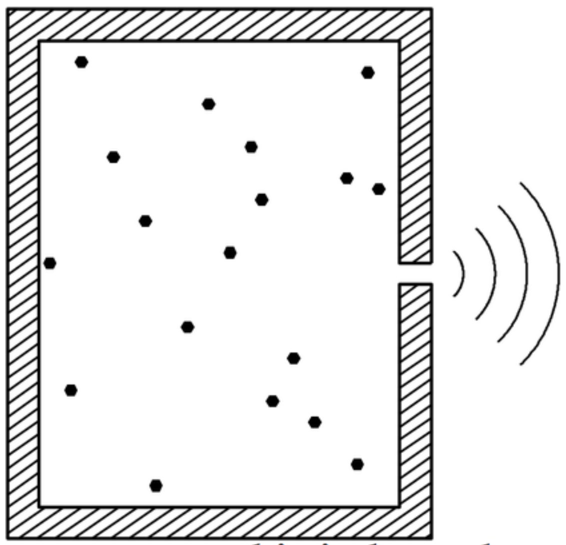
In terms of the optical depth,

$$\boxed{\frac{dI_\nu}{d\tau_\nu} = -I_\nu + S_\nu}$$

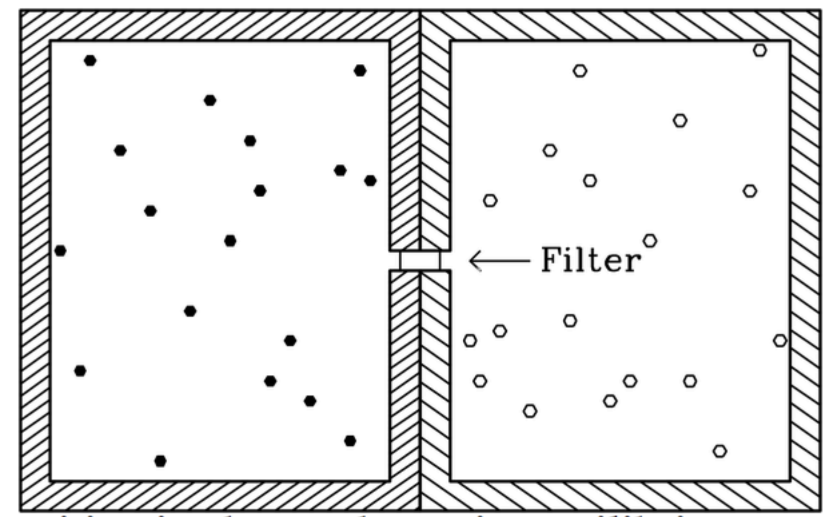
S_ν is called the ***source function***, defined as the ratio between the emission coefficient and absorption coefficient in equilibrium. (Don't confuse this with the flux density!)

Kirchhoff's law & Blackbody radiation

To understand the source function, consider the famous thought-experiment by Kirchhoff.



In **thermodynamic equilibrium (TE)**, the cavity radiation from an opaque container will be that of a blackbody. Blackbody radiation is defined by only one parameter, its temperature *T*.



When two containers are connected and are in equilibrium, there is no net transfer of power (otherwise it will violate the 2nd law of thermodynamics). This establishes a relation between the absorption and emission coefficients.

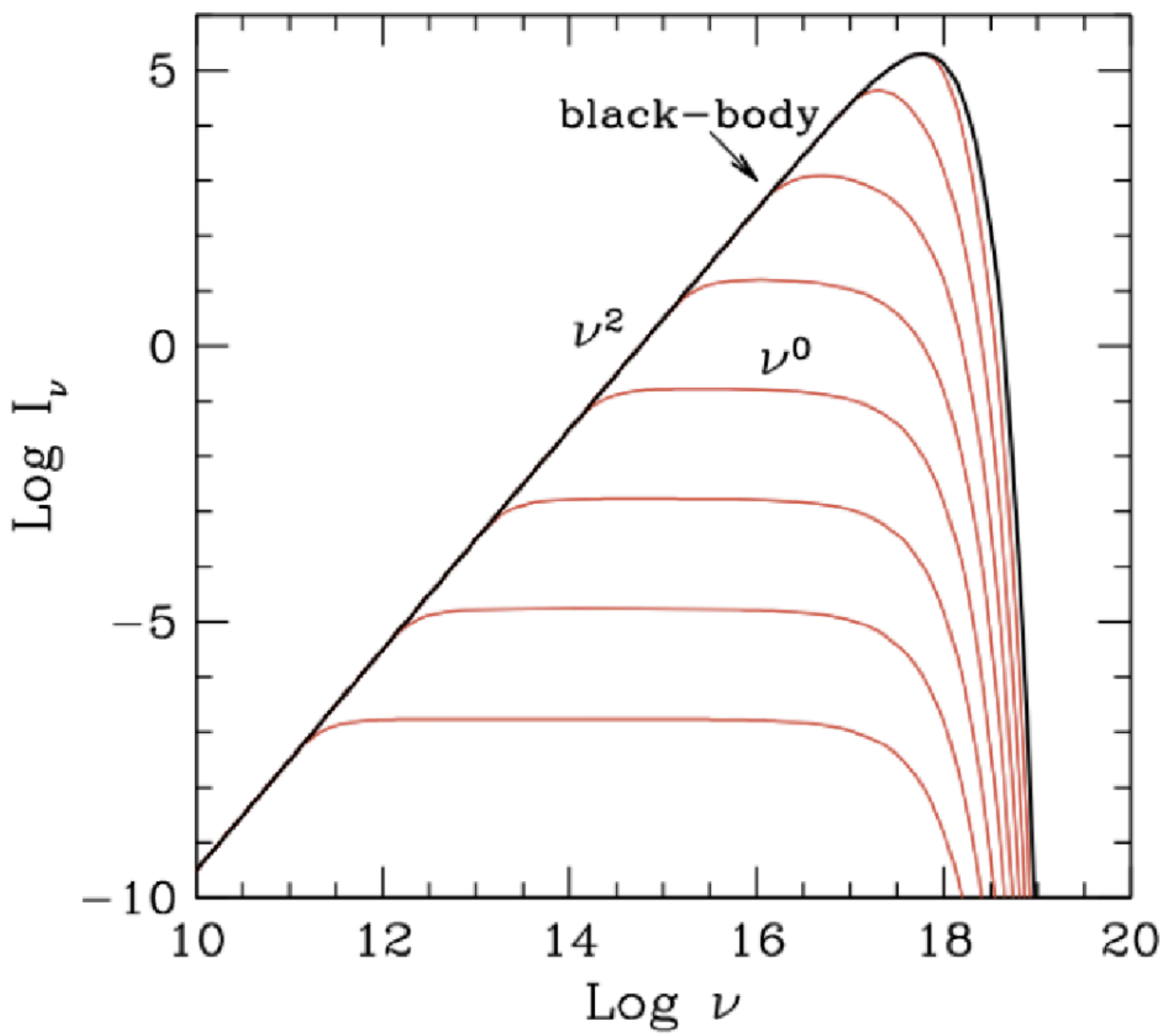
In **thermodynamic equilibrium** at temperature *T*, $\frac{dI_\nu}{ds} = 0$ and $I_\nu = B_\nu(T)$,

Hence, from $\boxed{\frac{dI_\nu}{ds} = -\kappa I_\nu + j_\nu}$ we get the **Kirchoff's law**: $\boxed{\frac{j_\nu(T)}{\kappa(T)} = B_\nu(T),}$

Kirchhoff's law also applies whenever the radiating/absorbing material is in thermal equilibrium, in any radiation field (applies in TE and LTE).

Thermal radiation & Blackbody radiation

Thermal Radiation & Black Body Radiation: Thermal radiation is the continuum emission arising from particles colliding, which causes acceleration of charges (atoms typically have electric or magnetic dipole moments, and colliding those results in the emission of photons). This thermal radiation tries to establish **thermal equilibrium** with the matter that produces it via photon-matter interactions. If thermal equilibrium is established (locally), then the **source function** $S_\nu \equiv j_\nu/\alpha_\nu = B_\nu(T)$ (**Kirchoff's law**).



This figure shows the so-called "blackbody limit" of thermal radiation, as particles are added to an optically thin gas, holding the volume and temperature constant. The thermal bremsstrahlung curve rises upwards (emission increases with density) and reaches the blackbody curve, but first only at the low-frequencies. Here the emission becomes optically thick at a break-point, known as the low-frequency cutoff. Eventually, the entire cloud becomes optically thick at all frequencies and the radiation becomes a blackbody.

Effect of scattering

We have considered the cases for photon destruction (absorption) and creation (emission). But photons can also be scattered, which changes their line of propagation. We consider inelastic scattering only (e.g. Thomson scattering) when there is no change of photon energy.

The reduction in intensity along a ray's path is simply proportional to the number density of absorbers, n, each with a cross-section area σ_ν

$$dI_\nu = -n\sigma_\nu I_\nu ds$$

Assuming the scattering to be isotropic, in the steady state the power absorbed will be equal to the power emitted

$$j_\nu = n\sigma_\nu S_\nu = n\sigma \int \frac{1}{4\pi} I_\nu d\Omega$$

Equating the two, we get the transfer equation for coherent, isotropic scattering, which is very similar to the original radiative transfer equation.

$$\frac{dI_\nu}{ds} = -n\sigma_\nu \left(I_\nu - S_\nu\right)$$

$$\frac{dI_\nu}{d\tau} = S_\nu - I_\nu$$

More definitions

Optical depth:
(for scattering)

We see that the scattering optical depth is nothing but the line-of-sight integral of the total cross section

$$\tau_\nu = \int n\sigma_\nu ds = \int n\sigma_\nu c dt$$

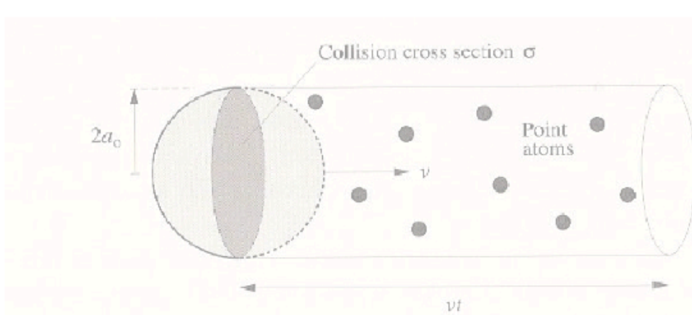
$$\boxed{\alpha_\nu = n\sigma_\nu}$$

Mean free path:

The mean free path is the reciprocal of the absorption coefficient, α_ν, in a homogeneous material

$$l \sim 1/n\sigma_\nu = 1/\alpha_\nu$$

Consider a single atom of radius $2a_0$ moving with speed v through a collection of stationary points that represent the centres of other atoms:



$$V = \pi(2a_0)^2 v\ t = \sigma\ v\ t$$

$$\sigma \equiv \pi(2a_0)^2$$

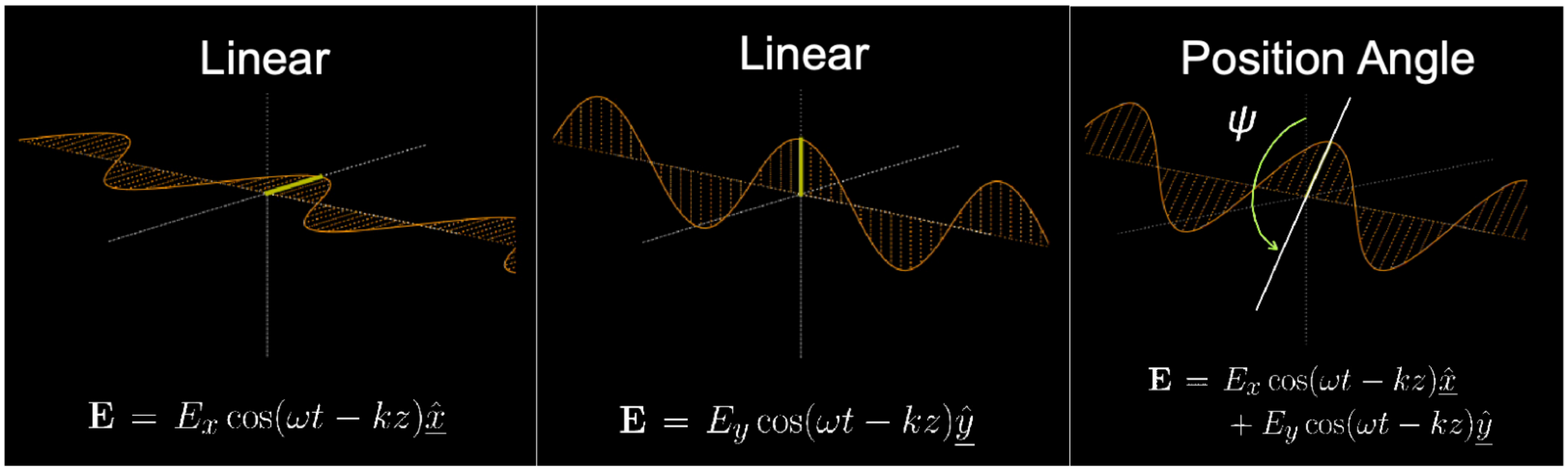
collision cross-section of the atom

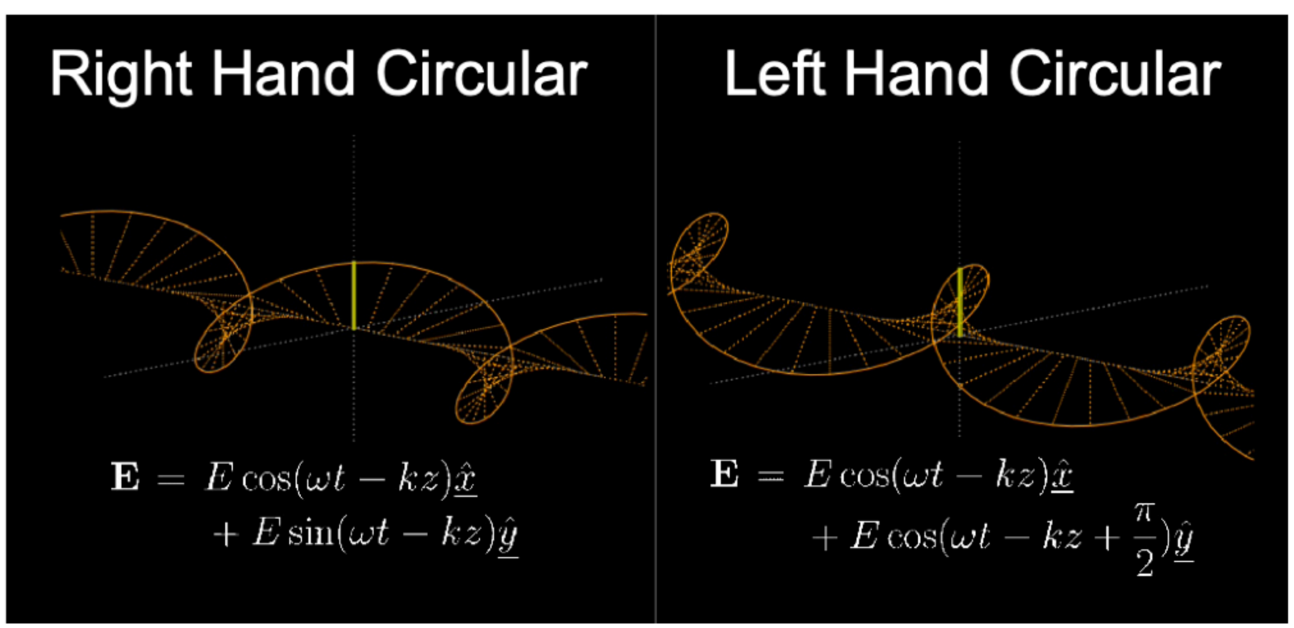
within volume, V, are $nV = n\sigma vt$ point-like atoms with which the moving atom has collided, thus the average distance between collisions:

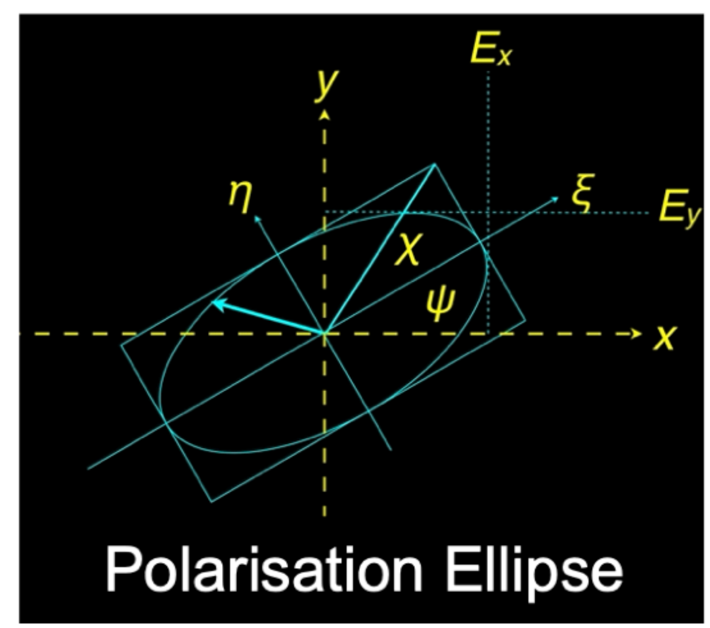
$$l = \frac{v\ t}{n\ \sigma\ v\ t} = \frac{1}{n\ \sigma}$$

Polarization of radiation

Polarization is the behaviour of electric fields with time. Astrophysical sources, like synchrotron emission, can emit partially polarized emission (but never 100% polarized). Intervening material, like the ISM, can change the state of polarization of background radiation, even depolarize it. The state of radiation polarization is fully characterized by the **four Stokes parameters**.







Quantifying polarization

Stokes polarization parameters are one way of describing polarization (but not the only way!). ***Stokes I*** describes the total intensity of radiation, it is the sum of any two orthogonal component of polarization. ***Stokes Q and U*** completely describes the state of linear polarization. ***Stokes V*** uniquely describes the circular polarization.

Stokes vector

$$\mathbf{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix}$$

$$S_0 = I = E_x^2 + E_y^2$$
$$S_1 = Q = E_x^2 - E_y^2$$
$$S_2 = U = 2E_xE_y \cos\delta$$
$$S_3 = V = 2E_xE_y \sin\delta$$

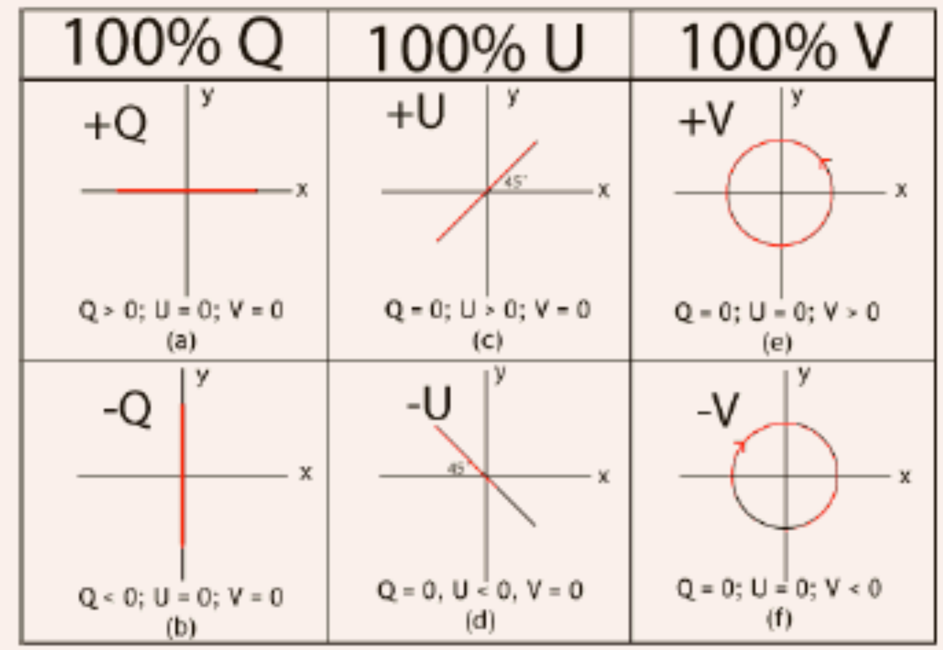
Polarization fraction

$$p = \frac{\sqrt{S_Q^2 + S_U^2 + S_V^2}}{S_0}$$

Polarization angle (linear only!)

$$\Theta = \frac{1}{2}\tan^{-1}\left(\frac{U}{Q}\right)$$

STOKES PARAMETERS FORMALISM



$$\left\{ \begin{matrix} I \\ Q \\ U \\ V \end{matrix} \right\}$$

- ★ *I, intensity*
- ★ *Q, U, linear polarization*
- ★ *V, circular polarization*

★*in the case of the CMB,* $V = 0$

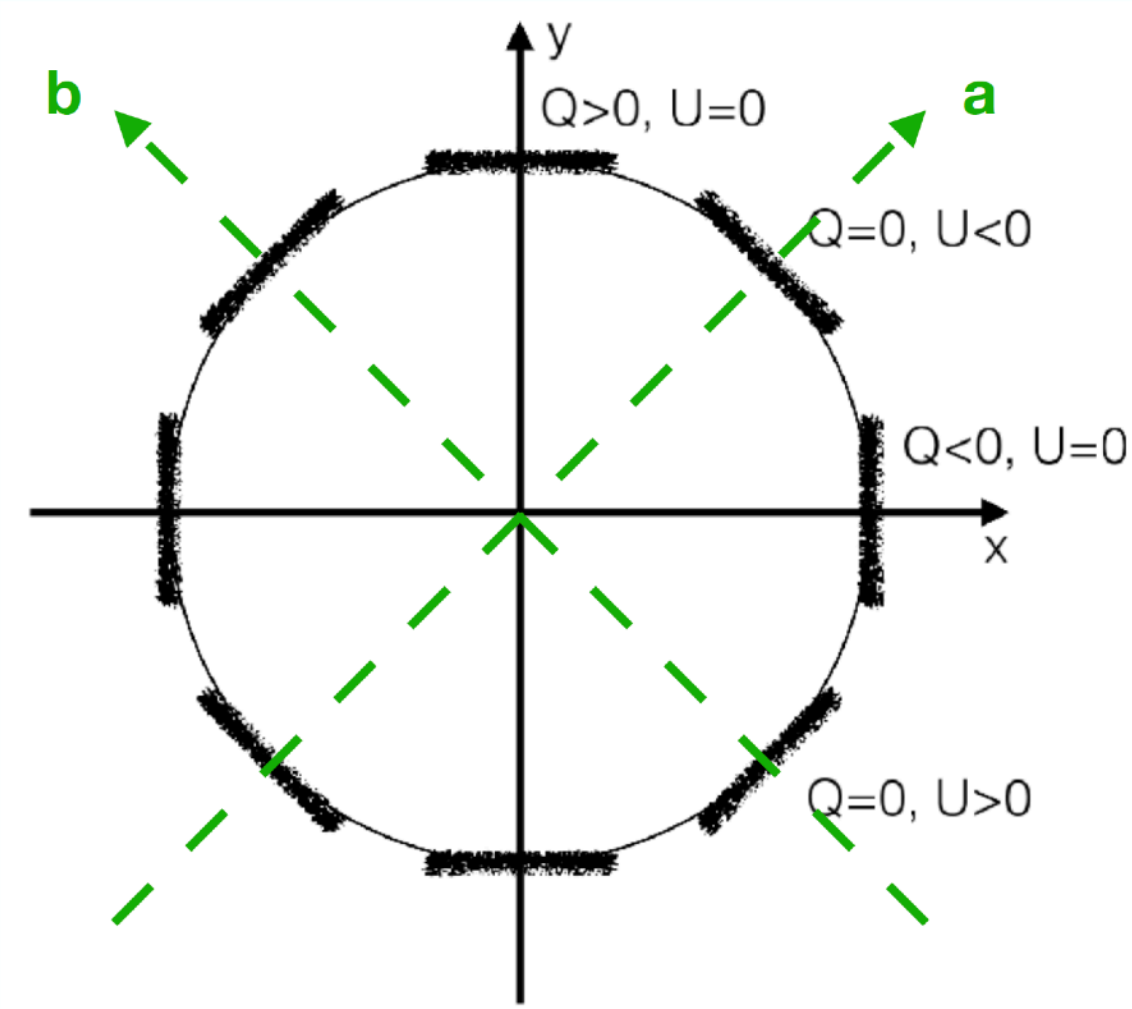
Coordinate-independent description

$$\boxed{\begin{aligned} Q &\propto E_x^2 - E_y^2 \\ U &\propto E_a^2 - E_b^2 \end{aligned}}$$

Under (x,y) -> (x',y'):

$$Q \longrightarrow \tilde{Q}$$
$$U \longrightarrow \tilde{U}$$

$$\boxed{\begin{pmatrix} \tilde{Q} \\ \tilde{U} \end{pmatrix} = \begin{pmatrix} \cos 2\varphi & \sin 2\varphi \\ -\sin 2\varphi & \cos 2\varphi \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix}}$$



For CMB, we need to define the polarization vectors in a coordinate-independent way.

$$\tilde{Q} + i\tilde{U} = \exp(-2i\varphi)(Q + iU)$$

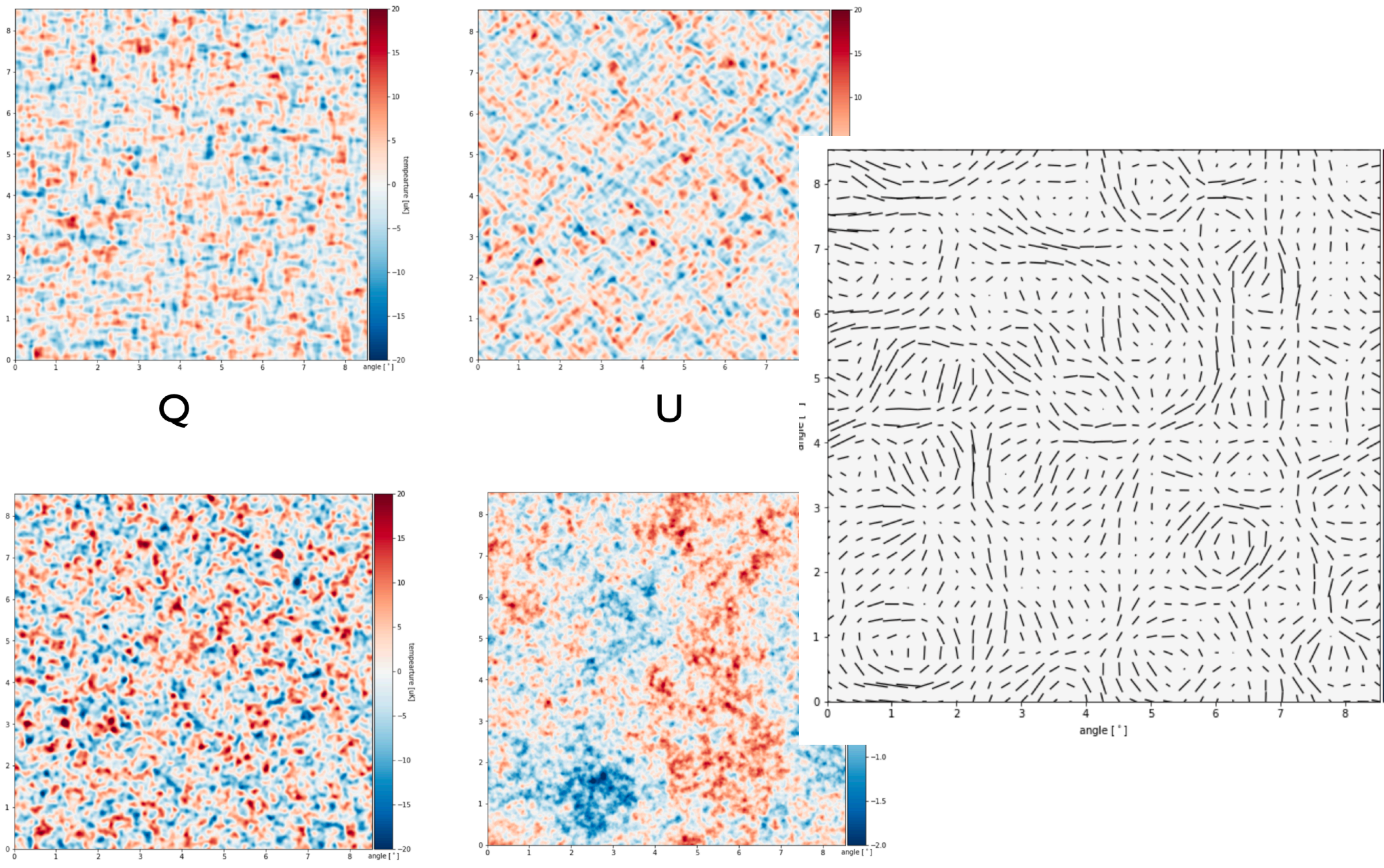
To do that, we write the polarization as a complex vector, which behaves as a spin-2 field. We then choose special spin-2 spherical harmonic coefficients which has the rotation term in-built, which cancels out the rotation of $Q + iU$ vector.

$$Q(\boldsymbol{\theta}) + iU(\boldsymbol{\theta}) = \int \frac{d^2\ell}{(2\pi)^2}\, a_{\boldsymbol{\ell}} \exp(i\boldsymbol{\ell}\cdot\boldsymbol{\theta})$$

where we write the coefficients as(*)

$$a_{\boldsymbol{\ell}} = -{}_2a_{\boldsymbol{\ell}} \exp(2i\phi_\ell)$$

E/B and Q/U for CMB polarization



Questions?

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