

# **The Globular Cluster Luminosity Function**

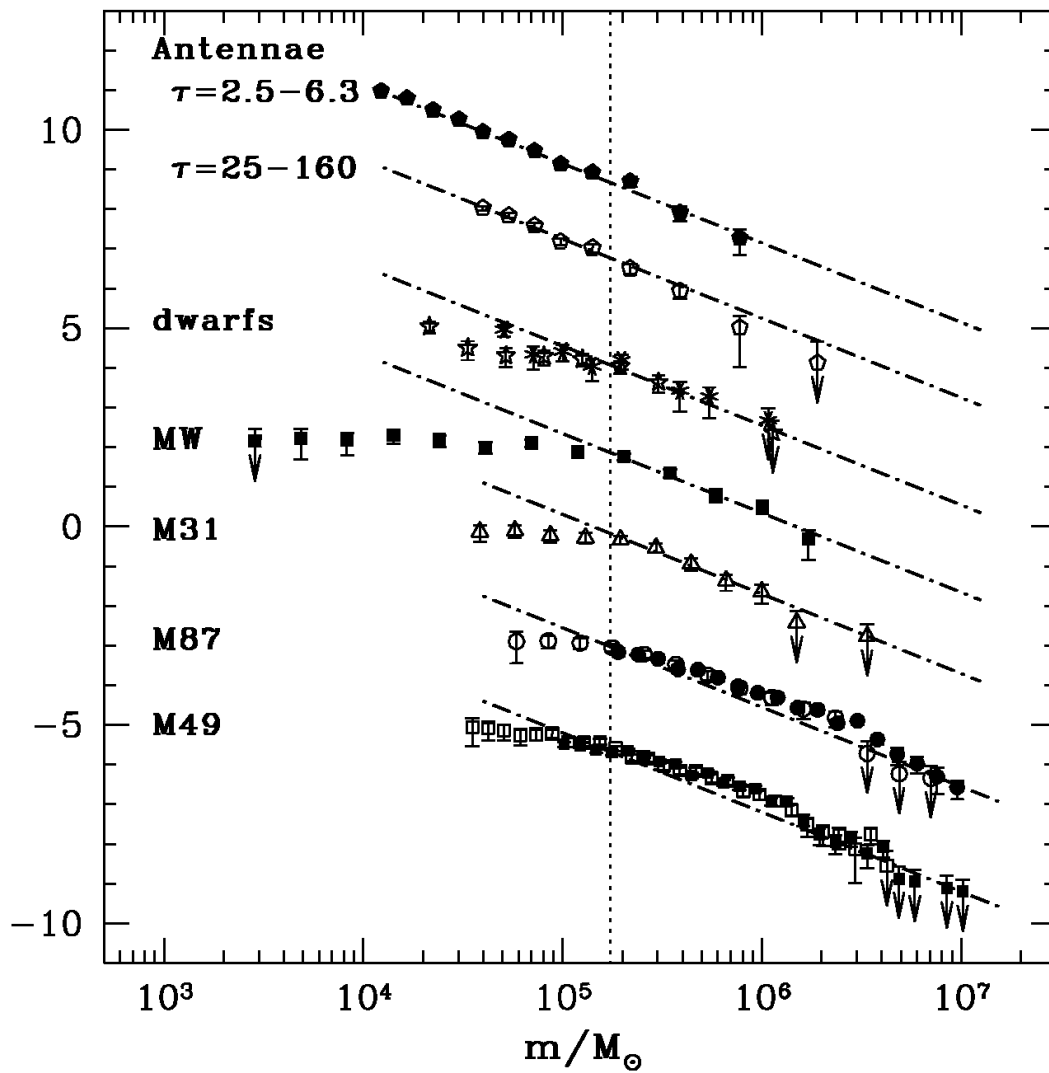
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Keele University

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D. E. McLaughlin & S. M. Fall 2007, ApJ, submitted

- ◆  $dN/d(\log M) = \text{GC mass function (GCMF)}$ :  
not universal
  - ◆ peak or turnover  $M_{\text{TO}}$  :  
increases with cluster half-mass density in  
the Milky Way
  - ◆ width:  
decreases with increasing  $\rho_h$
  - ◆ must be accounted for in any theory of GCMF
  - ◆ signature of cluster evaporation driven by  
internal two-body relaxation
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$$dN/dM \propto M^{-\beta} \Rightarrow$$

$$dN/d(\log M) \propto M^{1-\beta} \Rightarrow$$

observable  $dN/d(\log L)$

$\beta > 1$  : GCMF rises to low M

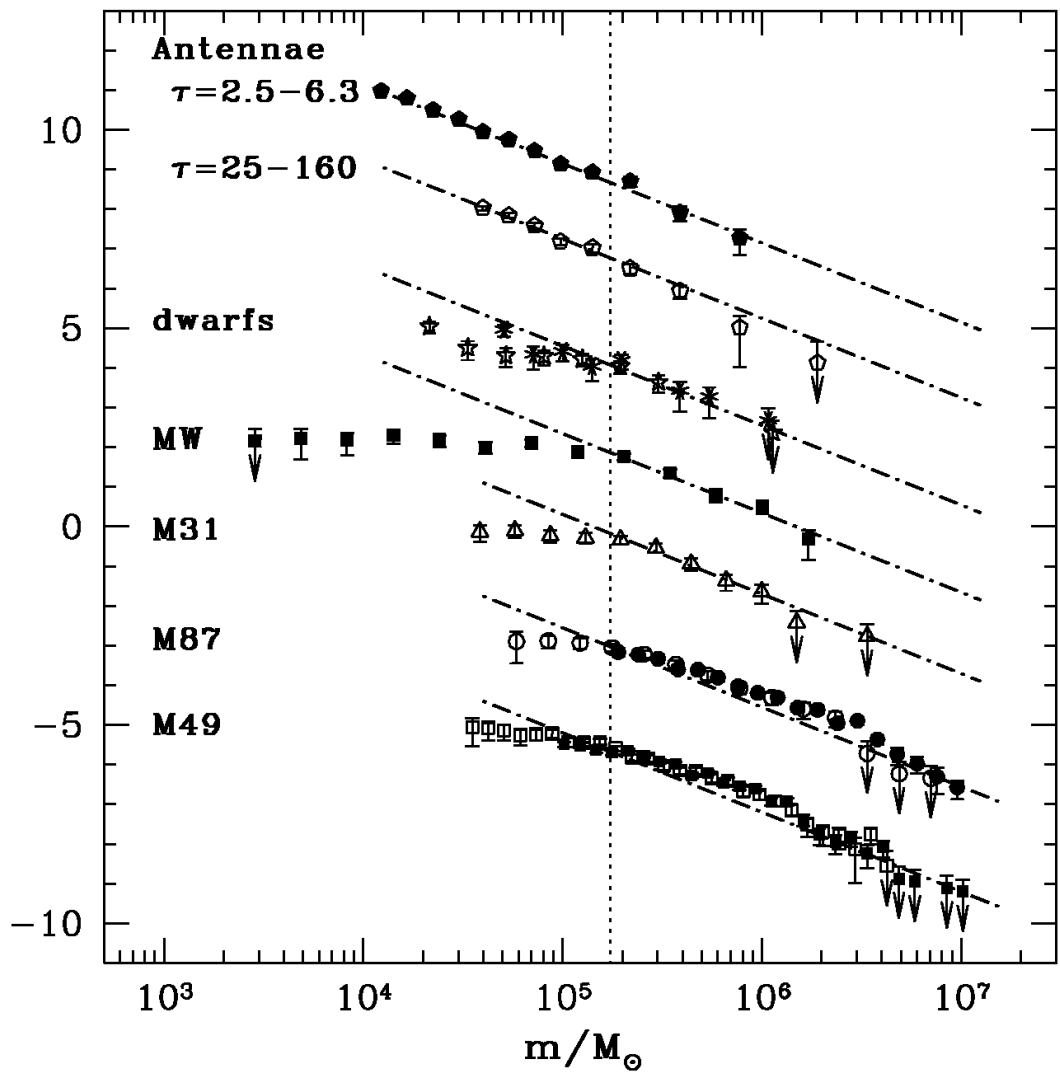
$\beta < 1$  : GCMF falls to low M

$\beta > 1 \leftrightarrow \beta < 1$  : peak  $M_{TO}$

young clusters:  $\beta \approx 2$  for  $10^6 - 10^4 M_{\odot}$

old GCs:  $\beta \sim 2$  at  $M > M_{TO}$  (with curvature!)

$M_{TO} \approx 1 - 2 \times 10^5 M_{\odot}$



**$M < M_{TO} : \beta \rightarrow 0$**

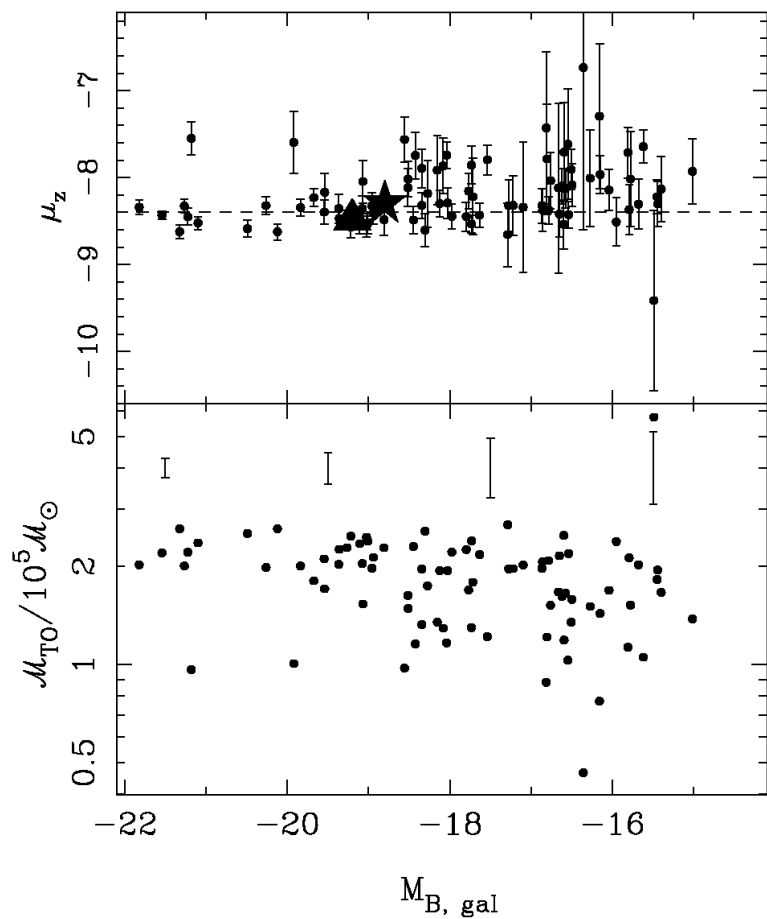
$dN/dM \rightarrow \text{const.}$   
 $dN/d(\log M) \rightarrow M^{+1}$

generic?

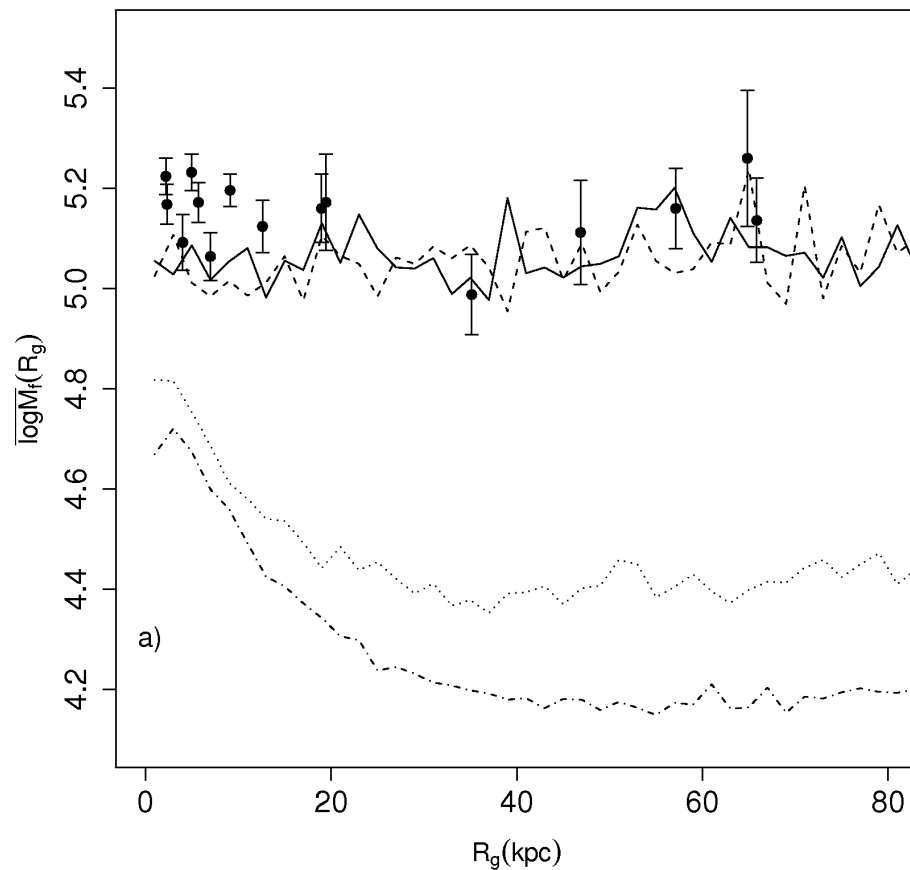
Milky Way  
 M87  
 Sombrero  
 89 Virgo galaxies

Fall & Zhang (2001)  
 Waters et al. (2006)  
 Spitler et al. (2006)  
 Jordán et al (2007)

# GCMF turnover $\approx$ constant between and within galaxies



$M_{TO}/(10^5 M_\odot)$  vs  $L_{gal}$   
(Jordán et al. 2006, 2007)



$M_{TO}$  vs  $R_{gc}$  in M87  
(Vesperini et al. 2003)

“universality” of  $M_{\text{TO}} \approx 1-2 \times 10^5 M_{\odot}$  and  $\beta \rightarrow 0$  for  $M < M_{\text{TO}}$

(near-)initial conditions?

e.g., Kroupa & Boily (2002); Vesperini & Zepf (2003);  
Parmentier & Gilmore (2007)

long-term evolution from initial  $\beta \simeq 2$  power law at low  $M$ ?  
low-mass clusters preferentially destroyed over  
 $\simeq 13$  Gyr of evolution in galaxy tidal fields  
(gravitational shocks, two-body relaxation)

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star formation and “infant mortality” inevitably affect the  
“initial” GCMF at  $\sim 10^8-10^9$  yr

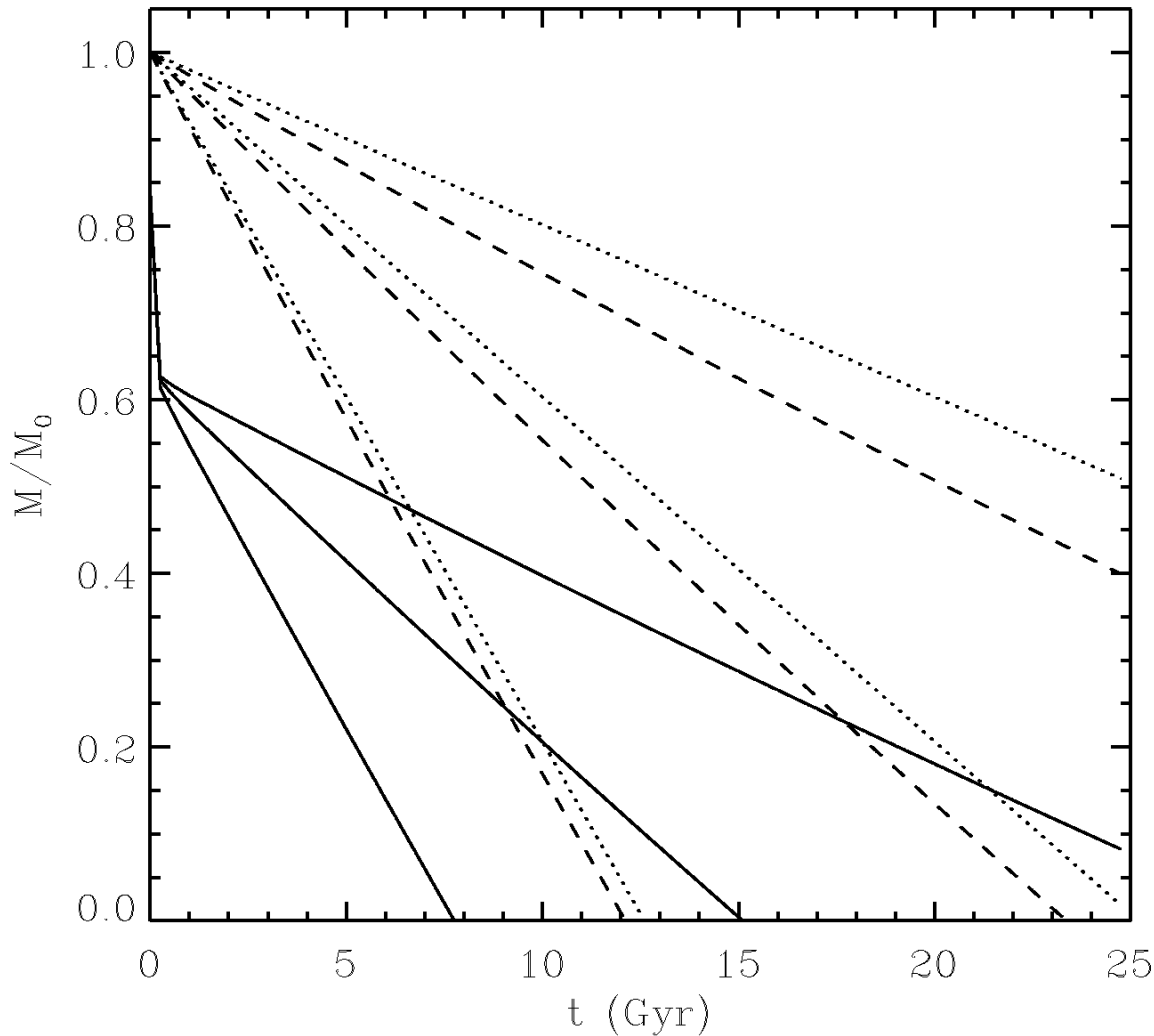
subsequent dynamical evolution inevitably affects the mass  
distribution of old clusters as observed

what is **necessary** vs. what is **possible**?

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# Cluster mass-loss rates (Fall & Zhang 2001, ApJ)



Two-body relaxation:  
most important

Gravitational Shocks:  
secondary  
(Gnedin et al. 1999 ;  
Prieto & Gnedin 2006)

Stellar evolution:  
no effect on MF

**NOTE! relaxation-dominated evolution  $\Rightarrow$  **M linear in time****

(also: Vesperini & Heggie 1997; Gnedin et al. 1999; Giersz 2001;  
Baumgardt & Makino 2003; Trenti et al. 2007; ...)



Linear decrease  $dM/dt \simeq \text{constant}$  is crucial  
(Fall & Zhang 2001)

$\mathbf{M(t) = M_0 - \mu_{ev}t} \Rightarrow$  if initial  $dN/d(\log M_0)$  rose to low masses, peak develops at  
 $\mathbf{M_{TO} \sim \mu_{ev}t}$  (roughly speaking)

more generally:

current masses  $M \ll \mu_{ev}t$  are  
remnants of initial  $M_0 \approx \mu_{ev}t$

number at all  $M \ll \mu_{ev}t$  is  
 $\approx$  initial number at  $M_0 \approx \mu_{ev}t$

i.e., low-mass  $\beta \rightarrow \mathbf{0} : dN/d(\log M) \rightarrow \mathbf{M^{+1}}$  *always*

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## Relaxation-driven mass loss

Average mass-loss rate  $\mu_{\text{ev}} \equiv M_0/t_{\text{dis}}$  ;  $M(t) \simeq M_0 - \mu_{\text{ev}}t$



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(Hénon; Spitzer)

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N-body:  $t_{\text{dis}} \propto t_{\text{rh},0}^{3/4} t_{\text{cross},0}^{1/4} \Rightarrow \mu_{\text{ev}} \propto \sum_{\text{h},0}^{3/4}$

(Baumgardt & Makino)

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# Relaxation-driven mass-loss rate increases with cluster half-mass density

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***then*** infer strong radial-orbit bias for GC system: to reproduce near-constant  $M_{\text{T0}} \sim \rho_{\text{h}}^{1/2} \sim r_{\text{p}}^{-1}$  vs. current  $r_{\text{gc}}$

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- ◆ abandon long-term dynamical evolution for peak and low-mass shape of GCMF?  
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GC  $\rho_h$  vs.  $r_p$  and  $r_{gc}$  that may be ***unrealistic***

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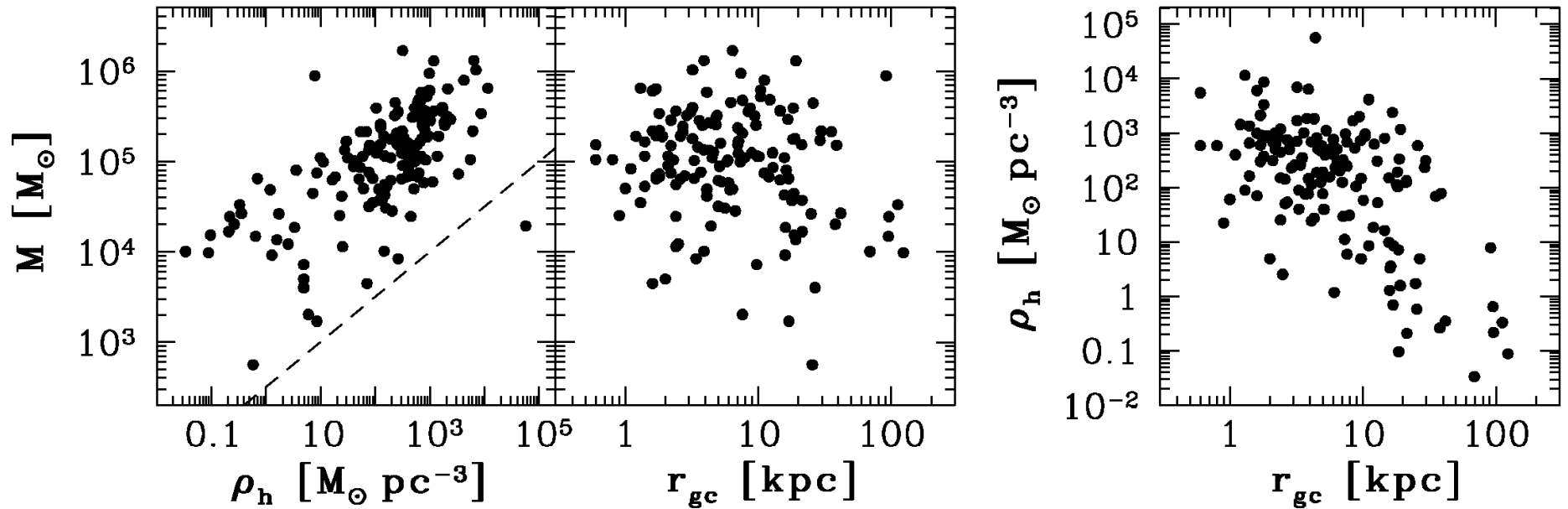
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relaxation-dominated cluster mass loss  $\Rightarrow$   
prediction of GC  $dN/d(\log M)$  dependence on cluster density  $\rho_h$  (or  $\Sigma_h$ ) : ***check!***

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## McLaughlin & Fall (2007)

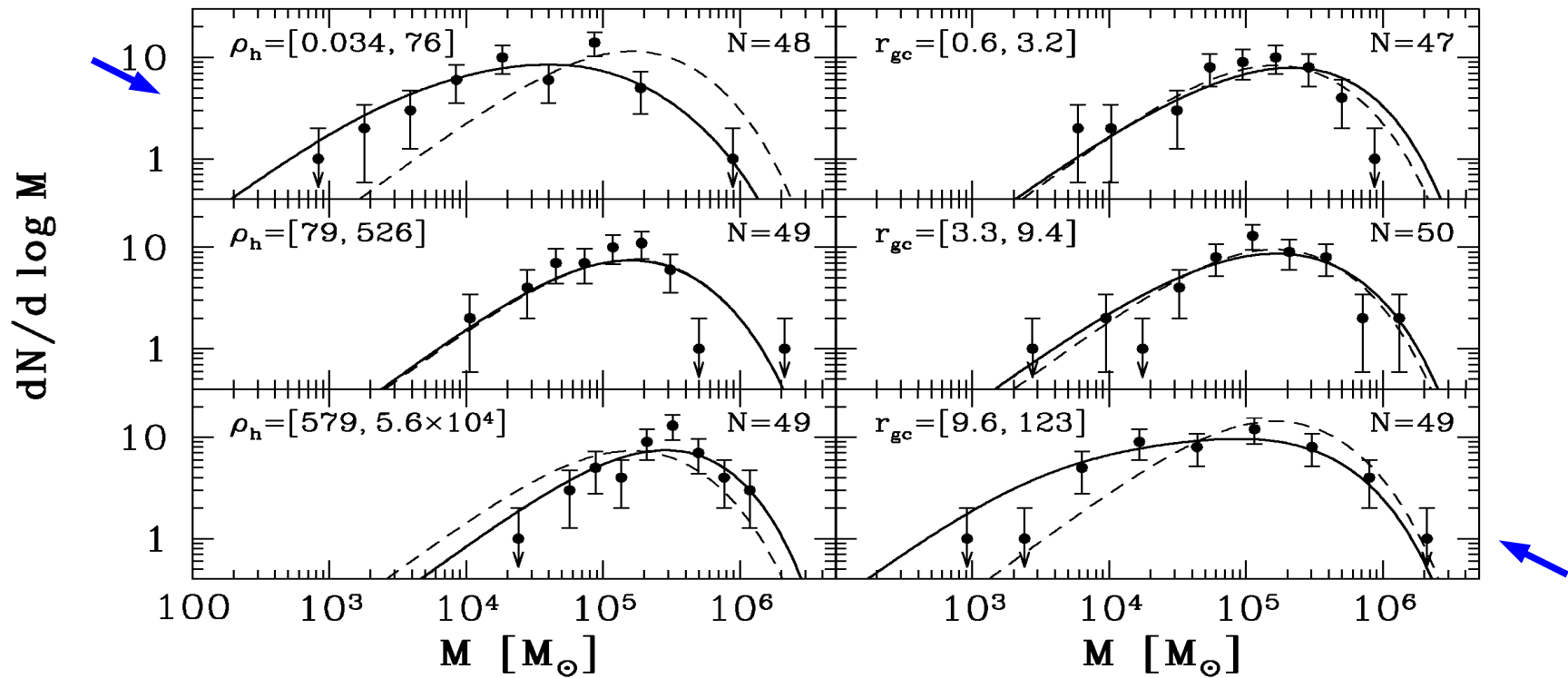


$M$  vs  $\rho_h$  :  $M \uparrow$  and "scatter"  $\downarrow$  as  $\rho_h \uparrow$   
lower envelope  $M \propto \rho_h^{1/2}$  (constant  $t_{\text{dis}}$ )

$M$  vs  $r_{gc}$  : no strong trend (broader at large  $r_{gc}$ )

$\rho_h$  vs  $r_{gc}$  : large scatter, no *unique* fit like  $\rho_h \propto r_{gc}^{-2}$   
convolving with  $M(\rho_h)$  erases GCMF variations

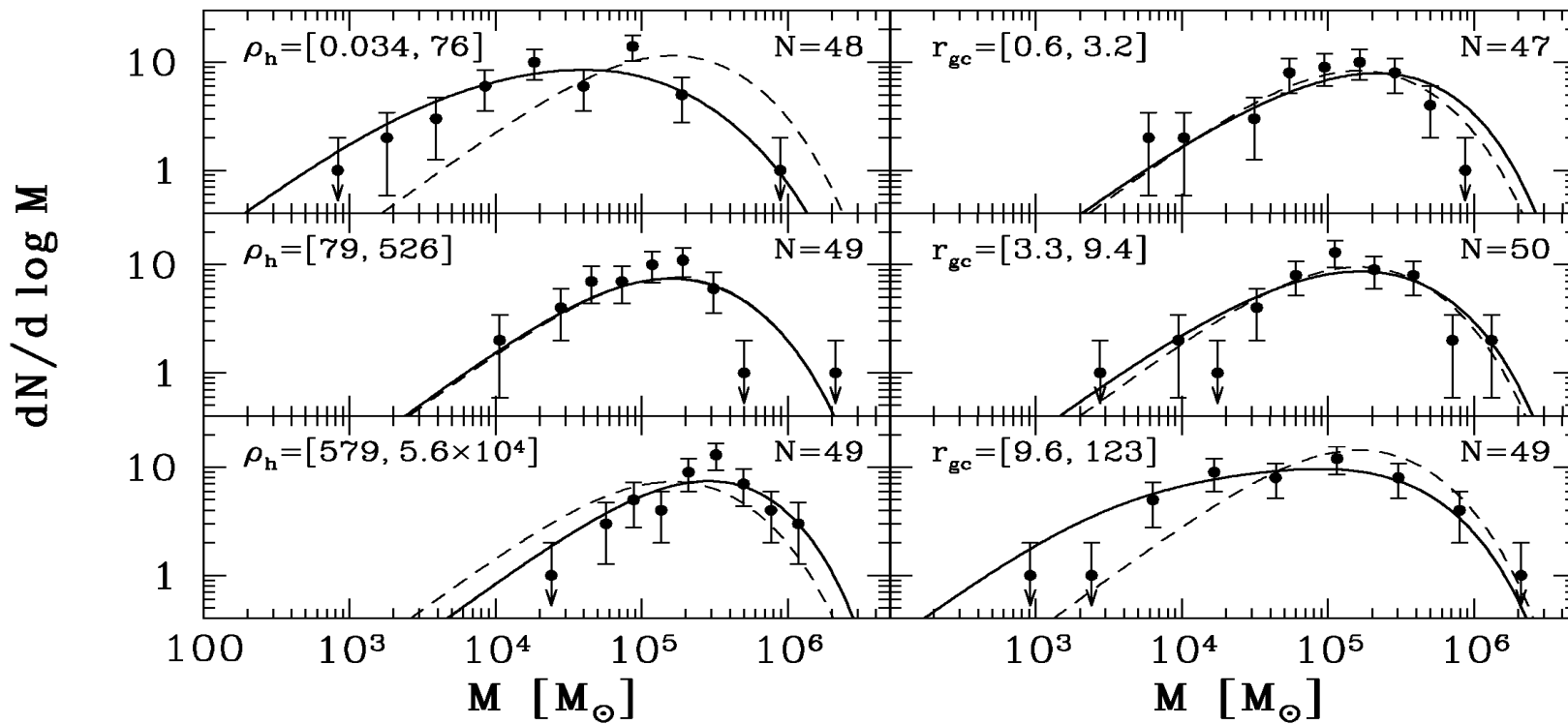
# McLaughlin & Fall (2007)



$M_{TO}$  increases with  $\rho_h$  : not universal  
*expected for  $\mu_{ev} \uparrow$  as  $\rho_h \uparrow$*

broader distribution at low  $\rho_h$  : less evolved

broad GCMF at large  $r_{gc}$  : very broad range of  $\rho_h$



*initial* GCMF:  $dN/d(\log M_0) \propto M_0^{-1} \exp(-M_0/M_c)$  [ $\beta=2$  at low  $M$ ]

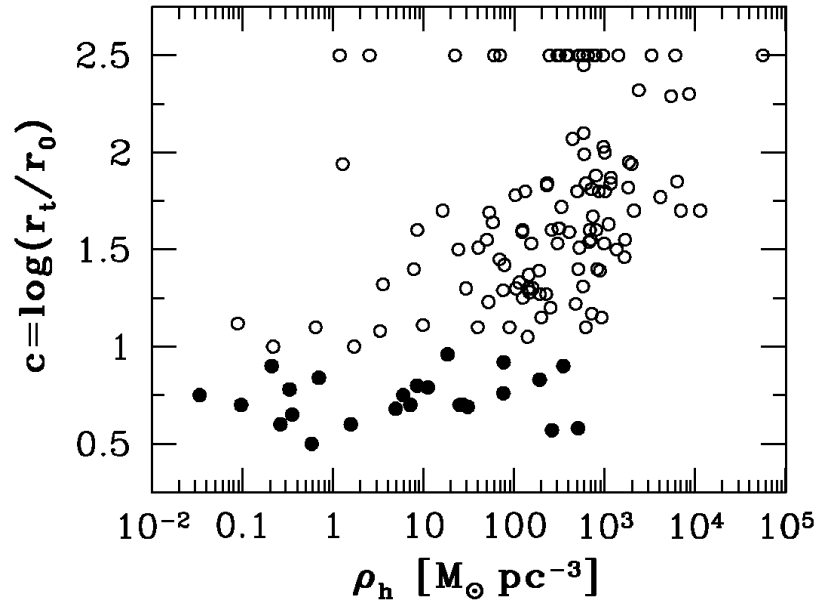
$$\frac{dN}{d \log M} = \sum_{i=1}^N A_i \frac{M}{(M + \Delta_i)^2} \exp\left(-\frac{M + \Delta_i}{M_c}\right) \quad (M_c \simeq 10^6 M_\odot)$$

$$\Delta_i = 1.45 \times 10^4 M_\odot (\rho_{h,i} / M_\odot \text{ pc}^{-3})^{1/2}$$

$$\mu_{ev,i} = \Delta_i / 13 \text{ Gyr} = 1.1 \times 10^3 M_\odot \text{ Gyr}^{-1} (\rho_{h,i} / M_\odot \text{ pc}^{-3})^{1/2}$$

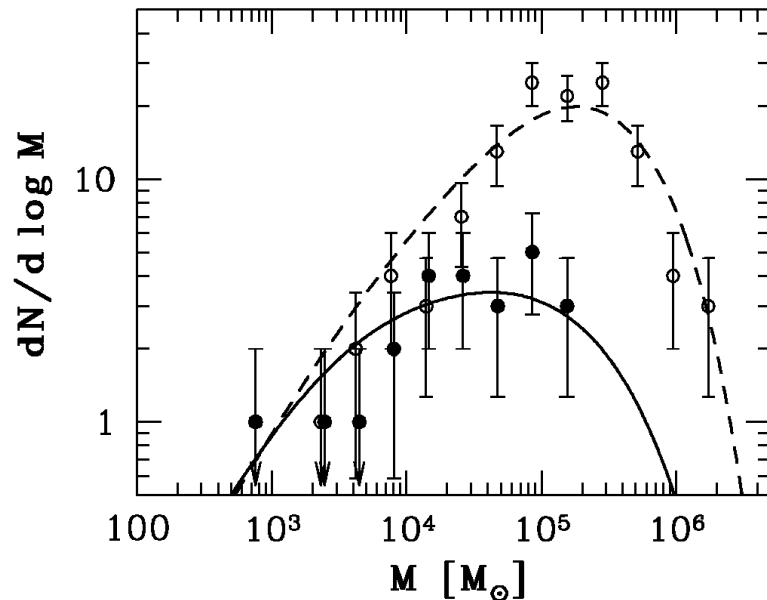


# McLaughlin & Fall (2007)



Galactic GCMF vs cluster concentration

Smith & Burkert (2002):  
difference between  $c < 0.99$   
and  $c \geq 0.99$



result of different  $\rho_h$  distributions  
in different  $c$  ranges

same  $\Delta \propto \rho_h^{1/2}$  as before

Single  $\mu_{\text{ev}} \propto \rho_{\text{h}}^{1/2}$  as in relaxation-dominated evaporation  
explains GCMF as function of  $\rho_{\text{h}}$   
as (non-)function of  $r_{\text{gc}}$   
in different concentration ranges  
etc....

weak/null radial variation of GCMF inside galaxy not  
an argument against long-term dynamical evolution

strong radial orbit bias results from too-simple attempt  
to connect  $\rho_{\text{h}}$  to  $r_{\text{gc}}$  : Galaxy is not spherical and static  
over a Hubble time

derive  $\rho_{\text{h}}(r_{\text{gc}})$  distribution from detailed simulations in  
hierarchical cosmology (e.g., Prieto & Gnedin 2006)

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from “fitted” mass-loss rate  $\mu_{\text{ev}} \propto \rho_{\text{h}}^{1/2}$  :

$$\text{GCMF } t_{\text{dis}} = M_0 / \mu_{\text{ev}} \approx 10 t_{\text{rh},0}$$

standard relaxation theory:  $t_{\text{dis}} \sim 20\text{--}30 t_{\text{rh},0}$

GC lifetimes estimated from GCMF within factor of  $\sim 2$  of independent theory

various possible resolutions: shocks? stellar IMF?

initial low-mass power law  $\beta < 2$  ?

current  $\rho_{\text{h}} \neq \rho_{\text{h},0}$

or relaxation theory itself:  $t_{\text{dis}} \sim 6\text{--}7 t_{\text{rh},0}$  at  $10^5 M_{\odot}$

in Baumgardt & Makino (2003)

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Baumgardt & Makino (2003):

$$t_{\text{dis}} \propto t_{\text{rh},0}^{3/4} t_{\text{cross},0}^{1/4} \Rightarrow \mu_{\text{ev}} \propto \Sigma_{\text{h},0}^{3/4}$$

replacing  $\rho_{\text{h}}^{1/2}$  with  $\Sigma_{\text{h}}^{3/4}$  in preceding:  
no change to basic results

key point: near-linear  $M(t)$  and  $\mu_{\text{ev}}$  increasing with  
*some* measure of half-mass density

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mass loss rate  $-\mu_{ev} \propto \sqrt{\rho} = \text{constant} \Rightarrow$

$$M(t) = M_0 - \mu_{ev} t = M_0 - \Delta$$

$$\frac{dN}{dM}(t) = \frac{dN}{dM_0} \left| \frac{\partial M_0}{\partial M} \right| = \frac{dN}{dM_0}$$

$$\frac{dN}{dM_0} = M_0^{-2} \exp(-M_0/M_c) \Rightarrow$$

$$\frac{dN}{dM} = \frac{1}{(M + \Delta)^2} \exp\left(\frac{-M + \Delta}{M_c}\right)$$

## Evolved Schechter Function

**$M_c$  sets scale**       **$\Delta/M_c$  fixes shape**

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