Homework 3 (hand in by 4th Nov 2019)

1. Quickies. (4 points)
   
   (a) Write down the epochs of the thermal history of the universe in chronological order along with their redshifts or temperature!
   
   (b) Find the temperature of the Universe 10 minutes after the Big Bang.
   
   (c) How would this temperature change if there were 5 families of neutrinos instead of 3 (provided they are massless and weakly interacting)?
   
   (d) What would be the temperature of the neutrino-background today if there were 5 families of neutrinos instead of 3?

2. Stellar Nucleosynthesis. (3 points)

   Assume that the baryonic matter of mass $M$ of a galaxy, such as the Milky Way, consisted purely of hydrogen when it was formed. In this case, all heavier elements must have formed from nuclear fusion in the interior of its stellar population. Assume further that the total luminosity $L$ of the galaxy is caused by burning hydrogen into helium, and let this luminosity be constant over the total lifetime of the galaxy, here assumed to be $10^{10}$ yr, with a correspondingly constant baryonic mass-to-light ratio of $M/L = 3M_\odot/L_\odot$. What is the mass fraction in helium that would be generated by the nuclear fusion process? Would this fraction be large enough to explain the observed helium abundance of $\sim 27\%$?

3. Big Bang Nucleosynthesis. (4 points)

   (a) Why does the abundance of helium from Big Bang nucleosynthesis not depend on $\Omega_m$ and $\Omega_\Lambda$? Why does it depend on the combination $\Omega_b h^2$?
   
   (b) Calculate the baryon density at the epoch of nucleosynthesis. How does it compare with the density in the central regions of stars where nuclear burning takes place?
   
   (c) It takes the Sun some 10 billion years to convert $\sim 10\%$ of its hydrogen into helium, whereas in BBN, all helium is formed on a time-scale of a minute. Can you speculate about the reasons for this difference?
   
   (d) During BBN, energy is released from the fusion process. Obtain an estimate of this fusion energy generated per unit volume, and compare it to the energy density of the photons at the epoch of BBN. Does BBN cause a substantial heating of the Universe?
4. Basic statistical physics. (2 points)

Show that the Bose–Einstein distribution function $f(p)d^3p$ for photons yields the Planck spectrum (i.e. spectral energy density $f(p)Ed^3p$).