Homework 1 (hand in by 21st Oct 2019)

1. Quickies. (3 points)
   (a) Explain the cosmological principle.
   (b) What is the meaning of the critical density and how is it defined.
   (c) Use the first Friedmann equation to derive the expansion law for an Einstein-
dee-Sitter universe (hint: \( H(a) = \dot{a}/a \)).

2. Flat rotation curve. (2 points)
   We know that the rotation curve of the Milky Way is flat, \( v(r) \approx \text{const} \).
   Assume a spherically-symmetric density distribution \( \rho(r) \). Determine the functional form of
   \( \rho(r) \) which yields a flat rotation curve?

3. Age of the Universe. (3 points)
   Based on the Hubble law \( v = H_0 D \), we can get a simple first estimate of the
   age of the Universe.
   (a) Consider a galaxy at distance \( D \) whose radial velocity is given by this Hubble
   Law, and assume that this velocity was the same throughout cosmic time. In
   this case, at some time in the past the separation was zero, and we can identify
   that instant as the Big Bang. Under these assumptions, calculate the current
   age of the Universe.
   (b) Does it depend on the choice of the galaxy, i.e., the current distance \( D \)? Compare
   your result with the age of the oldest stars found in our Galaxy, which is about
   \( 12 \times 10^9 \) yr.
   (c) Since no signal can propagate faster than the speed of light \( c \), the age of the
   Universe times the speed of light is often called the ‘size of the visible Universe’.
   How large is that?
4. De Sitter spacetime and Quintessence. (6 points)

(a) Consider a Universe without matter and radiation, $\Omega_m = \Omega_r = 0$, only with vacuum energy. Obtain the general solution of the expansion equation for $a(t)$ in this case [hint: try (a combination of) exponential functions]. Find the solution which satisfies $a(t = 0) = 0$. Can such a solution be found for all $\Omega_\Lambda$? What is the age of the Universe today in this world model?

(b) From the Friedmann equation and the ‘first law of thermodynamics’, derive the expansion equation [i.e. $H(a)$] for a Universe containing matter, radiation and an additional substance with an equation-of-state $p = w c^2 \rho$, where $w$ is a constant (say, $w \in [-1, 0]$).

5. Age of the Universe at its milestones. (2 points)

Use Eq. (4.26) to find the age of the Universe:

(a) when matter and radiation had the same energy density
(b) when recombination occurred at a redshift $z = 1100$

For this use the parameters $h = 0.72$ and $\Omega_m = 0.3$.

You can find the exercise sheets, class details, literature, links, etc. on our web page:

http://www.astro.uni-bonn.de/~Cosmo