

# A New SEP Test and its Theoretical Implications

Preliminary results from timing the triple system

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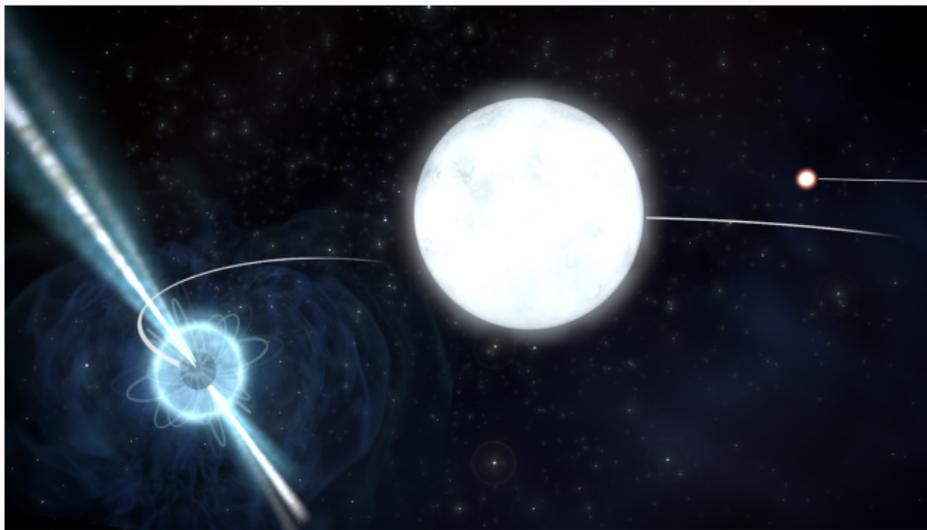
**Anne Archibald**, Nina Gusinskaia, Jason Hessels, Scott Ransom, Ingrid Stairs,  
Ryan Lynch, and Duncan Lorimer

2017 December 11



# The Triple System

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- Discovered as part of the 2007 GBT drift scan survey
- Consists of a hierarchical triple:
  - $1.4M_{\odot}$  radio pulsar with a period of 2.73 ms
  - $0.2M_{\odot}$  inner white dwarf in a 1.6-day orbit
  - $0.4M_{\odot}$  outer white dwarf in a 327-day orbit

# **The Strong Equivalence Principle**

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# The Strong Equivalence Principle

The Strong Equivalence Principle (SEP) states:

- All experiments, *including gravitational ones*, give the same result regardless of which inertial frame they are carried out in

And in particular:

- gravitational binding energy falls the same way as other mass, or
- $M_G = M_I$  even for objects with strong gravity.

Most alternatives to GR violate the SEP at some level.



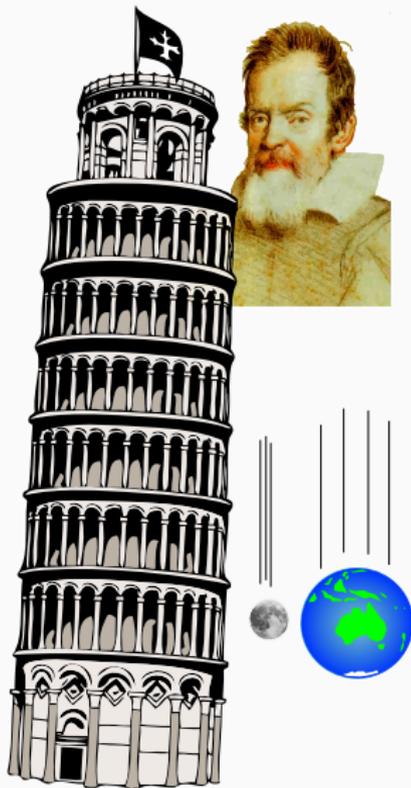
# Effects of an SEP violation

**Key idea:** test whether two bodies fall the same way in the gravitational field of a third

**Need:** binary falling in an external gravitational field

- Earth and Moon falling in Sun's gravity (LLR)
- Pulsar-WD binary falling in Galactic potential (e.g. Gonzalez et al.)
- Triple system: pulsar and inner WD falling in gravity of outer WD

Fractional difference in acceleration ( $\Delta$ ) gives rise to **excess eccentricity** in the direction of the external acceleration



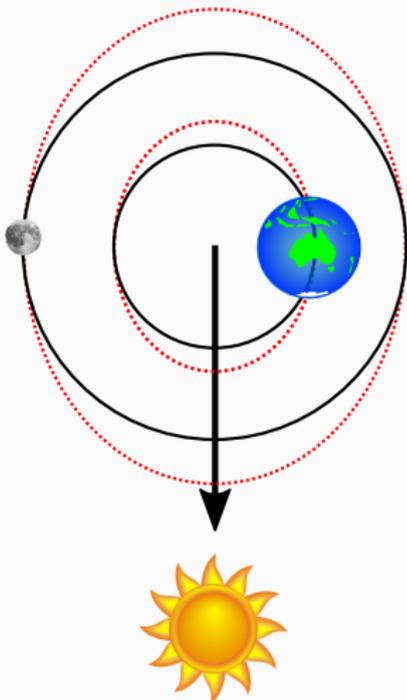
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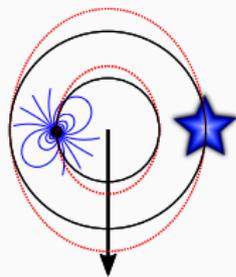
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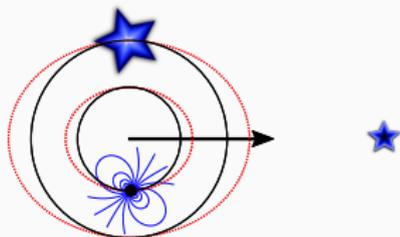
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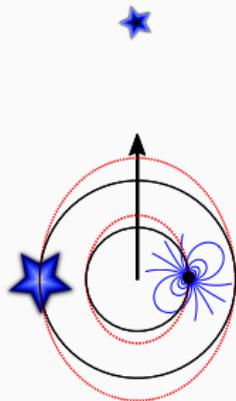
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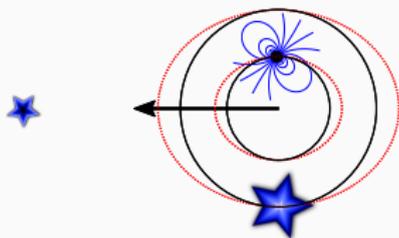
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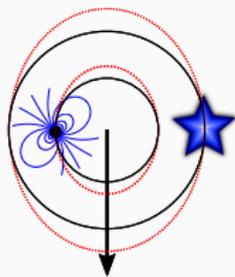
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# Timing the Triple System

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# Observations

Tel.	Band	Num.	Hours	Date range
AO	1400	92	58.9	2012 Mar – 2017 Mar
GBT	1400	172	236.0	2012 Feb – 2017 May
WSRT	1400	439	836.7	2012 Jan – 2013 Jul
AO	430	36	12.9	2012 May – 2017 Mar
WSRT	350	20	17.3	2012 Feb – 2013 Jul



Arecibo Observatory (AO)



Green Bank Telescope (GBT)



Westerbork Synthesis Radio Telescope  
(WSRT)

# Relativistic timing model

- Nordtvedt (1985) derives a “point particle” Lagrangian
  - Taylor expansion around the Newtonian Lagrangian
  - Lorentz invariance and symmetry used to eliminate terms
  - Bodies may contain strong fields but internal structure is frozen
  - Fields *away from bodies* approximated to first post-Newtonian order
- Computer algebra straightforwardly yields equations of motion
  - Direct integration simulates orbits

$$\begin{aligned} L_{PPN} = & - \sum_i M_{i,G} \left( 1 - \frac{v_i^2}{2} - \frac{v_i^4}{8} \right) \\ & + \frac{1}{2} \sum_{i,j} \frac{M_{i,G} M_{j,G}}{r_{ij}} \left( 1 + \frac{v_i^2 + v_j^2}{2} - \frac{3\mathbf{v}_i \cdot \mathbf{v}_j}{2} - \frac{(\mathbf{v}_i \cdot \hat{\mathbf{r}}_{ij})(\mathbf{v}_j \cdot \hat{\mathbf{r}}_{ij})}{2} \right) \\ & + \frac{\gamma}{2} \sum_{i,j} \frac{M_{i,G} M_{j,G}}{r_{ij}} (\mathbf{v}_i - \mathbf{v}_j)^2 + \left( \frac{1}{2} - \beta \right) \sum_{i,j,k} \frac{M_{i,G} M_{j,G} M_{k,G}}{r_{ij} r_{ik}} \end{aligned}$$

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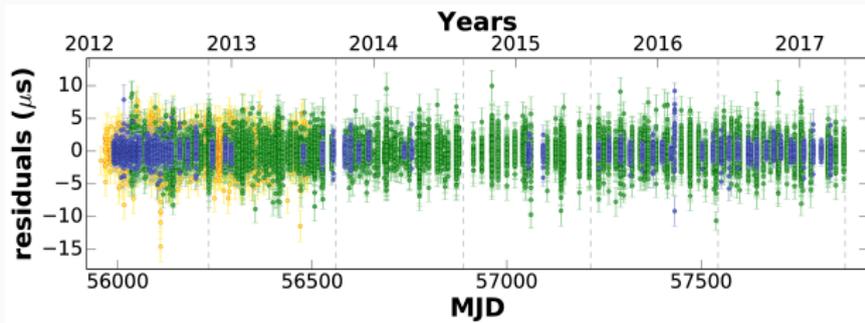
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# Testing the SEP



In principle we simply:

- include  $\Delta$  in the timing model,
- fit timing model to TOAs, and
- determine best-fit values and uncertainties.

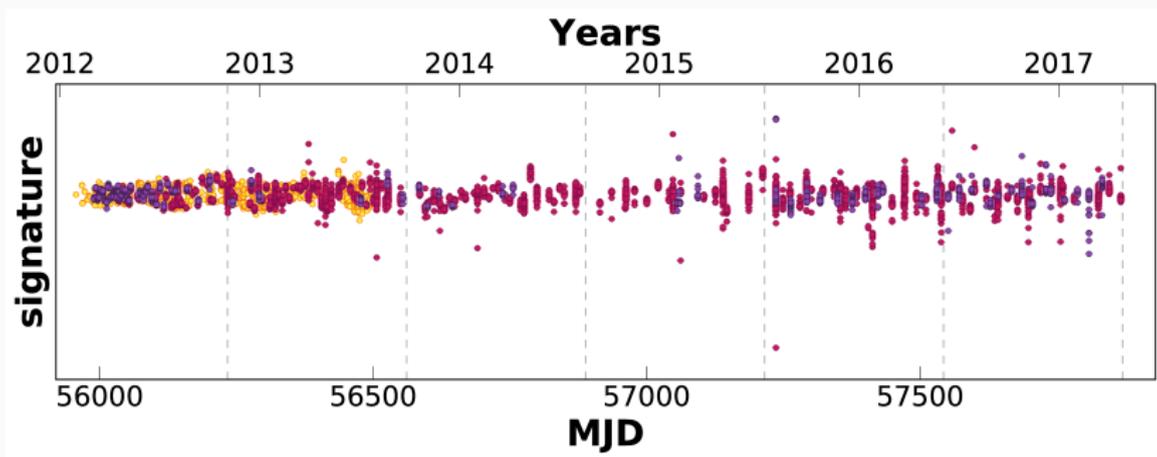
Ideally, the value of  $\Delta$  and its uncertainty would determine how well we constrain SEP violation and whether GR is violated.

**But:** only correct once we've accounted for all systematics, and formally the effects of  $\Delta$  are constrained **at the 7 ns level**.

# The signature of an SEP violation

**Key idea:** look for structure in the residuals that *looks like* SEP violations.

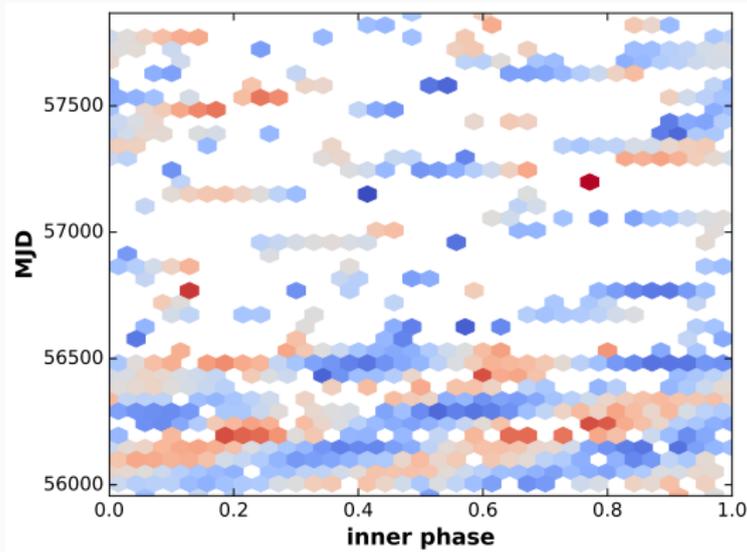
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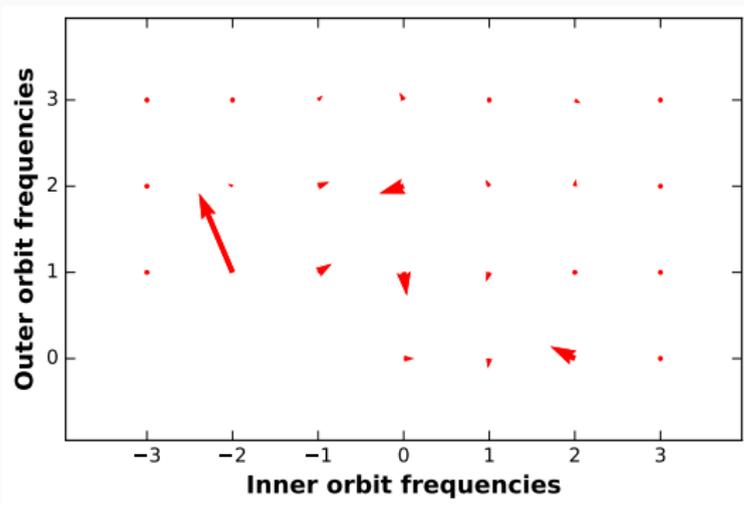
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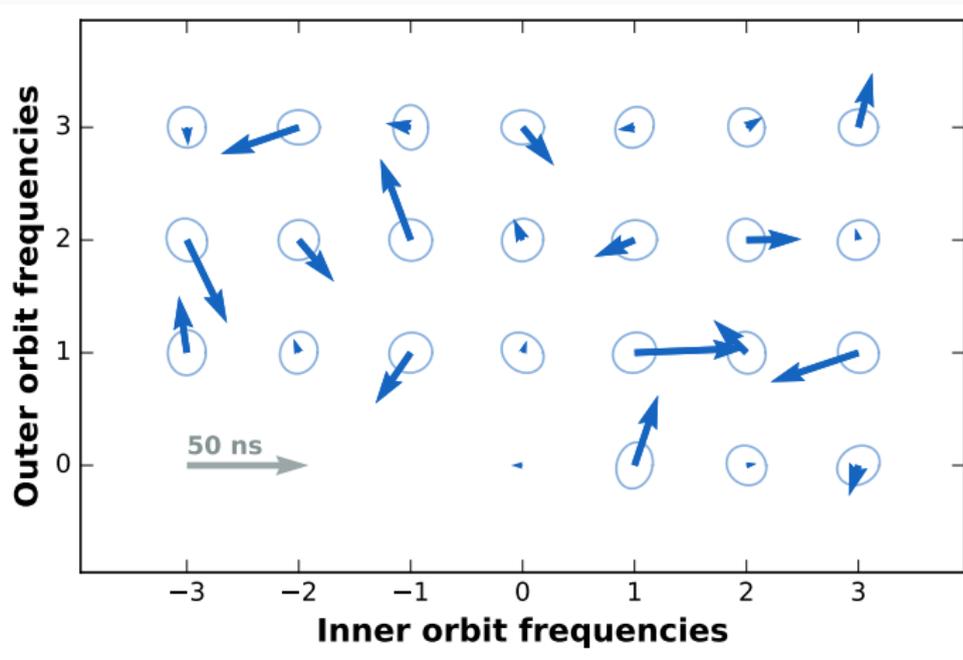
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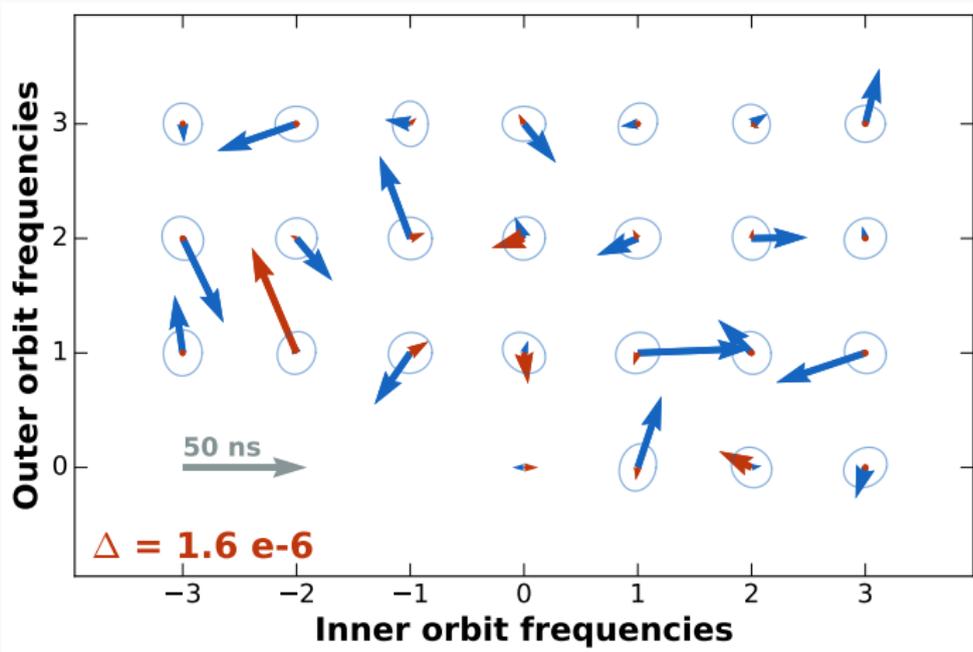
# Wiggles in our residuals

Look at sinusoids with frequency  $kf_{\text{outer}} + lf_{\text{inner}}$ :



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## **An Upper Limit on SEP Violation**

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## An upper limit on SEP violation

Our data sets a **preliminary** upper limit on wiggles like the SEP violation signature of 50 ns amplitude. This corresponds to constraint that for a  $1.4378M_{\odot}$  neutron star:

$$|\Delta| < 1.6 \times 10^{-6}$$

(Triple system)

For comparison, wide pulsar-white-dwarf binaries falling in the Galactic potential give:

$$|\Delta| \lesssim 4.6 \times 10^{-3} \quad \text{(WB)}$$

**But:** how do we compare this to lunar laser ranging or dipole gravitational wave tests?

## Weak- versus strong-field tests

Within the PPN framework, there's a simple relation,

$$\Delta = \eta E_B, \quad (1)$$

where  $E_B$  is the fractional binding energy of the test mass. For the earth,  $E_B = 4.6 \times 10^{-10}$  and lunar laser ranging can constrain  $|\eta| \lesssim 10^{-3}$ .

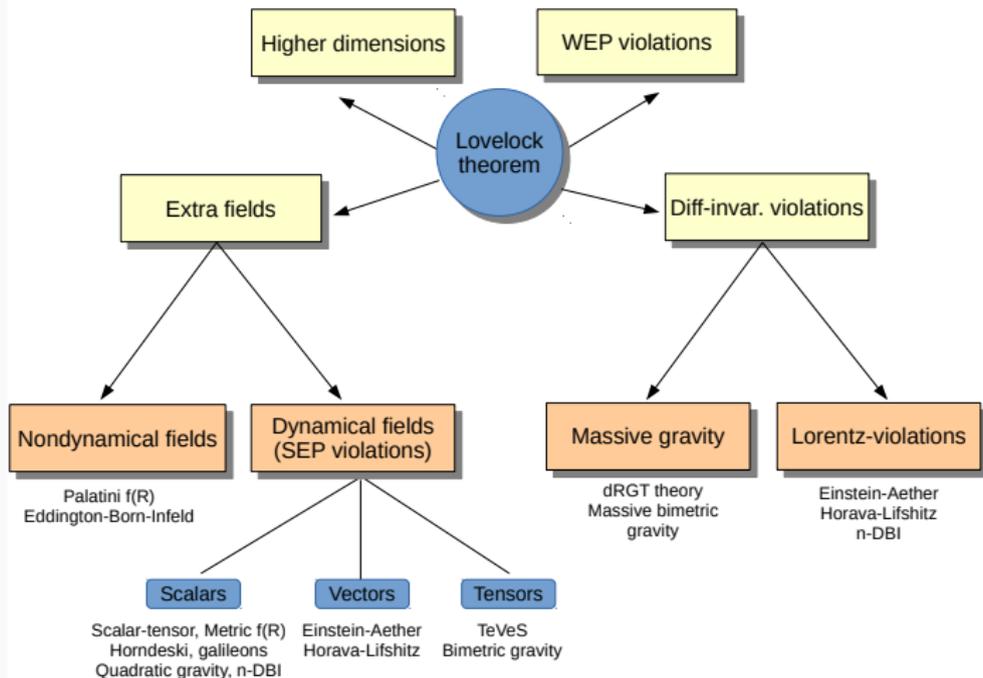
In general, though,

$$\Delta = \eta E_B + \eta_2 E_B^2 + \dots, \quad (2)$$

and our pulsar has an  $E_B$  of 0.1–0.15, so we can't obtain a clean constraint on  $\eta$ .

**We must use strong-field theories to compare different tests.**

# Alternative theories of gravity



## Quasi-Brans-Dicke scalar-tensor theories

These theories include a scalar field  $\phi$  in addition to the metric that mediates gravity. Matter responds to a modified version of the metric:

$$\tilde{g}_{\mu\nu} = e^{2(\alpha_0\phi + \beta_0\phi^2/2)} g_{\mu\nu}^*$$

The scalar field is sourced in matter:

$$\square\phi = -\frac{4\pi G^*}{c^4}(\alpha_0 + \beta_0\phi)T_*$$

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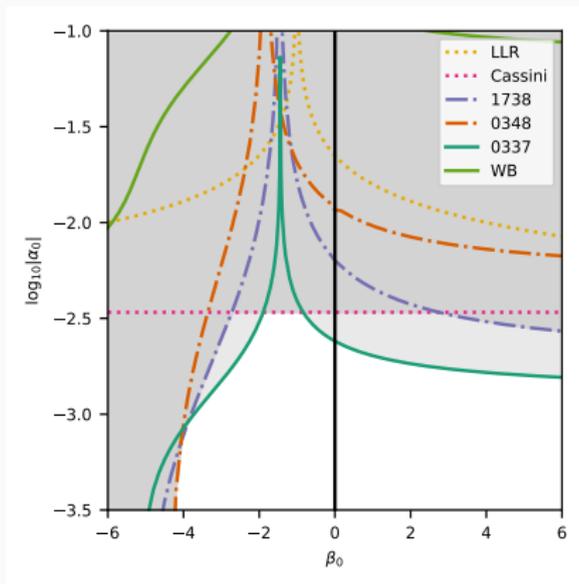
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# Our constraint on quasi-Brans-Dicke theories

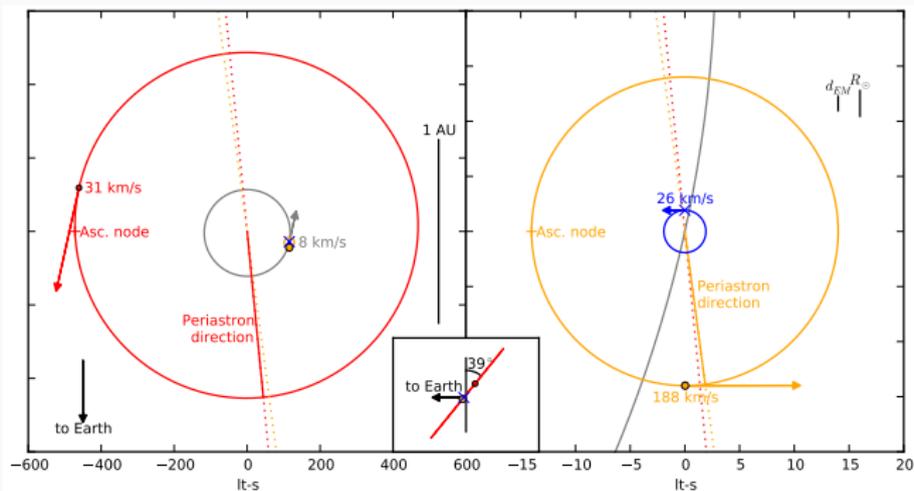


Our **preliminary** constraint  $|\Delta| \lesssim 1.6 \times 10^{-6}$  rules out the light-gray area.

# Appendix

Slides that follow are in case of questions.

# Basic Timing



- Microsecond-level timing allows measurement of the system
- Orbits are computed by direct integration
- Three-body interactions break the usual degeneracies without reference to relativistic effects, for example:
  - System inclination is  $39.1^{\circ}$  and the orbits are nearly coplanar
  - Pulsar mass is  $1.4378(13)M_{\odot}$

# The Weak Equivalence Principle



Dave Scott of Apollo 15 dropping a hammer and a feather. Painting by Alan Bean.

The Weak Equivalence Principle states:

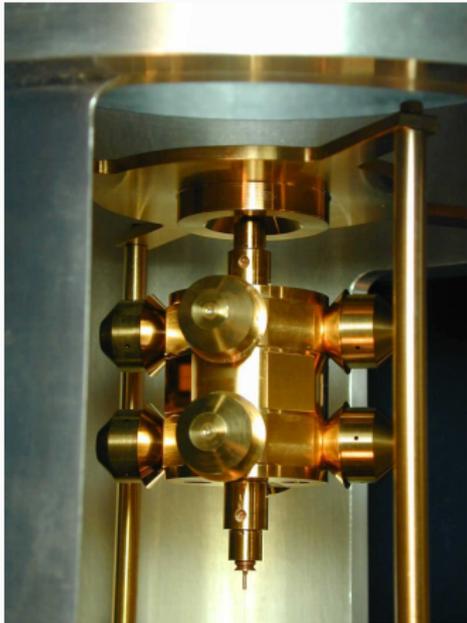
- All *non-gravitational* experiments give the same result regardless of which inertial frame they are carried out in

And in particular:

- The following fall identically: proton rest mass, nuclear binding energy, magnetic fields..., or
- gravitational mass equals inertial mass regardless of composition.

This has been tested to exquisite accuracy in laboratory experiments

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Torsion pendulum for WEP tests; from Wagner et al. 2012

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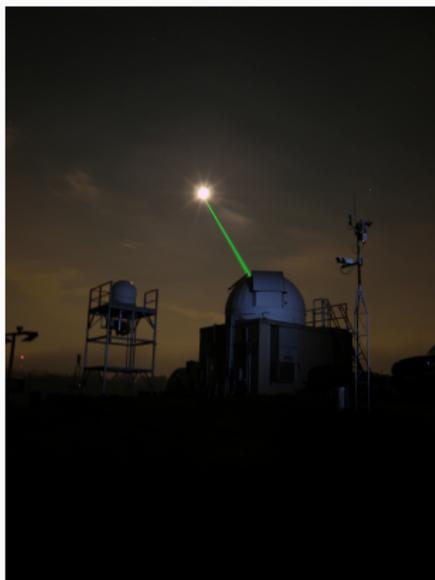
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Lunar Laser Ranging ground station in operation.  
Photo courtesy of NASA.

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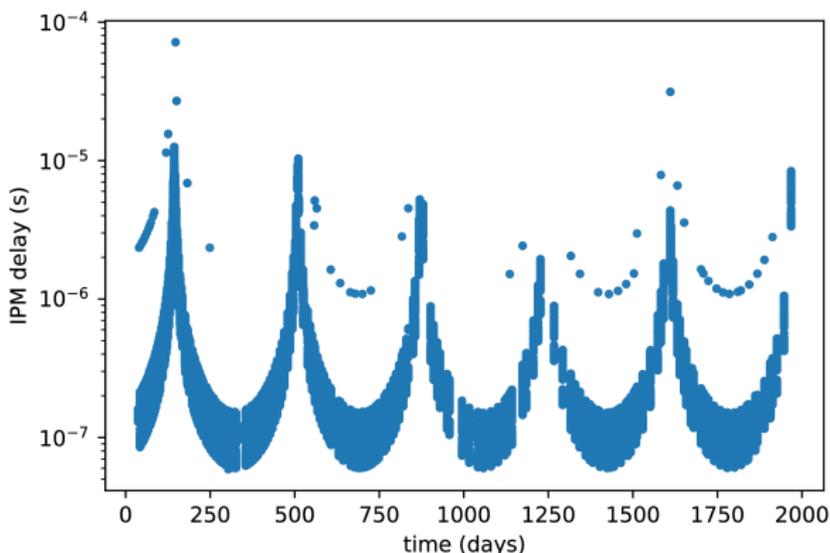
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This requires astrophysical experiments.

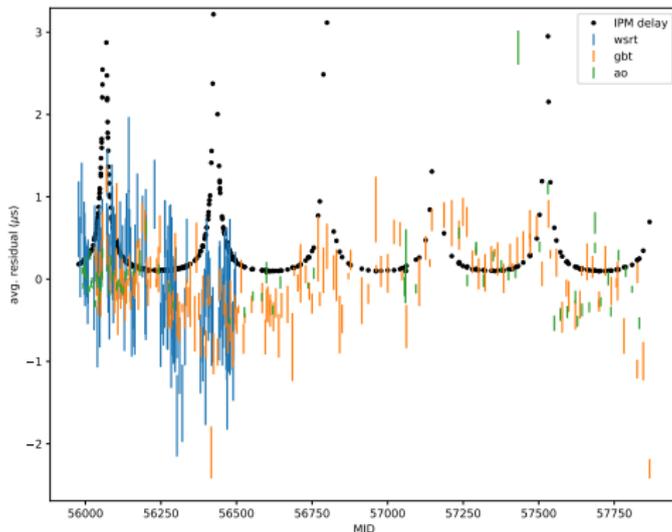
## Effects of the interplanetary medium

The ecliptic latitude of our source is only 2.1 degrees, so our line of sight passes close to the Sun every March. Using a simple model of the IPM, and assuming a density of 10 electrons per cubic centimeter at 1 AU, we obtain:



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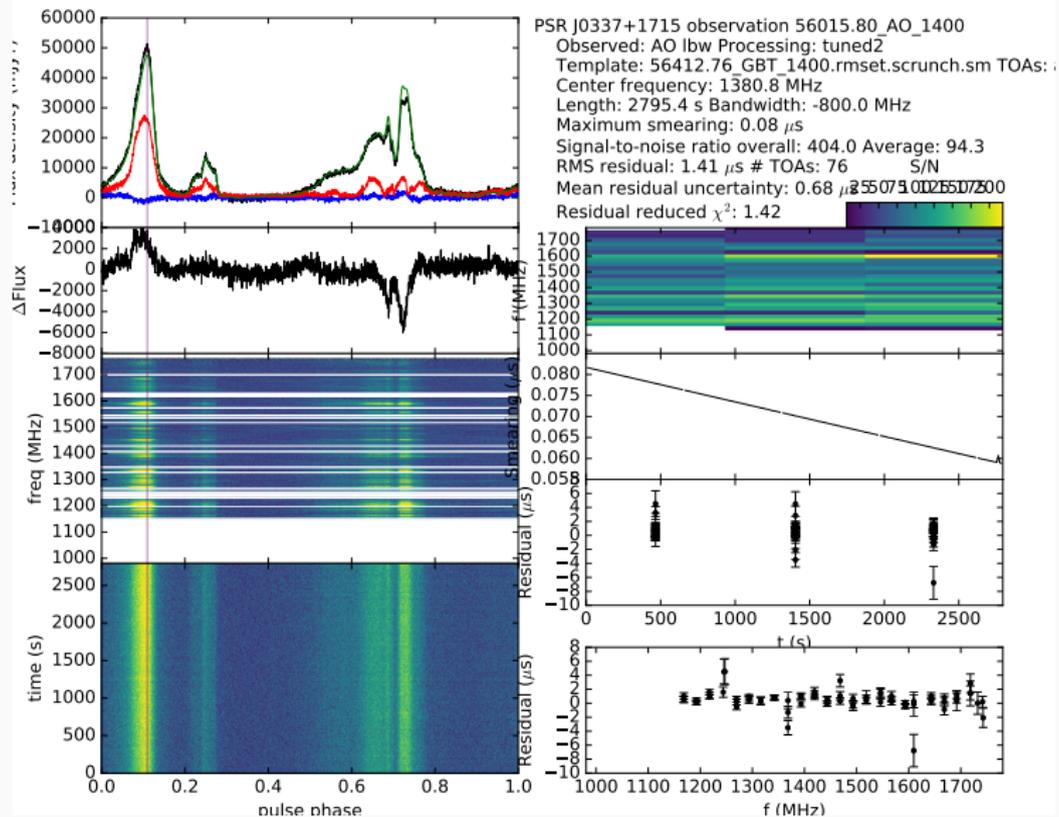
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# Data processing

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- Follows NANOGrav “how we do it” paper except:
  - Include WSRT
  - Realign with short-term ephemeris
  - Matrix template matching
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  - Summary plot per observation

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  - Summary plot per observation
- TOAs every 20 minutes  $\times$  20 MHz at 1400 MHz
  - 25153 TOAs currently in use
  - 1.2  $\mu$ s median uncertainty

## Timing model

No adequate formula is known for directly describing the orbit, so we use direct integration of equations of motion:

$$F_j = M_j a_j, \quad (3)$$

and

$$F_j = - \sum_k \frac{GM_j M_k}{r_{jk}^2} \hat{r}_{jk} \quad (4)$$

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## Known systematics

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Cause	Remedy
Profile variation with frequency	TOAs no more than 20 MHz
Telescope polarization variations	Matrix template matching
Intrinsic profile variations	?
Interstellar DM variations	Variable DM fitting
Interplanetary medium effects	IPM fitting
Tidal effects in inner WD	Too small
GW losses	Too small
Red noise	Too small at freq. of interest
Uncertainty in DE430 ephemeris	Position fitting
Kopeikin and inverse parallax	Too small
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We need to estimate the impact of unknown or poorly modeled systematics.

# Measuring SEP violations



Lunar Laser Ranging ground station in operation.  
Photo courtesy of NASA.

Theoretical framework (weak field):

- Measured quantity: fractional difference in accelerations  $\Delta$ 
  - $M_G = (1 + \Delta)M_I$
- Theory (Nordtvedt) parameter:  $\eta$
- For mass  $M$  and gravitational binding energy  $E_g$ ,

$$\Delta = \eta \frac{E_g}{M}$$

Lunar Laser Ranging:

- $|\Delta| \lesssim 2 \times 10^{-13}$
- Earth  $E_g/M \sim 4.6 \times 10^{-10}$
- $|\eta| \lesssim 10^{-3}$

Neutron star  $E_g/M$  is 0.1–0.15!