Numerical modeling of star clusters and their evolution

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Outline

1) Models of evolution

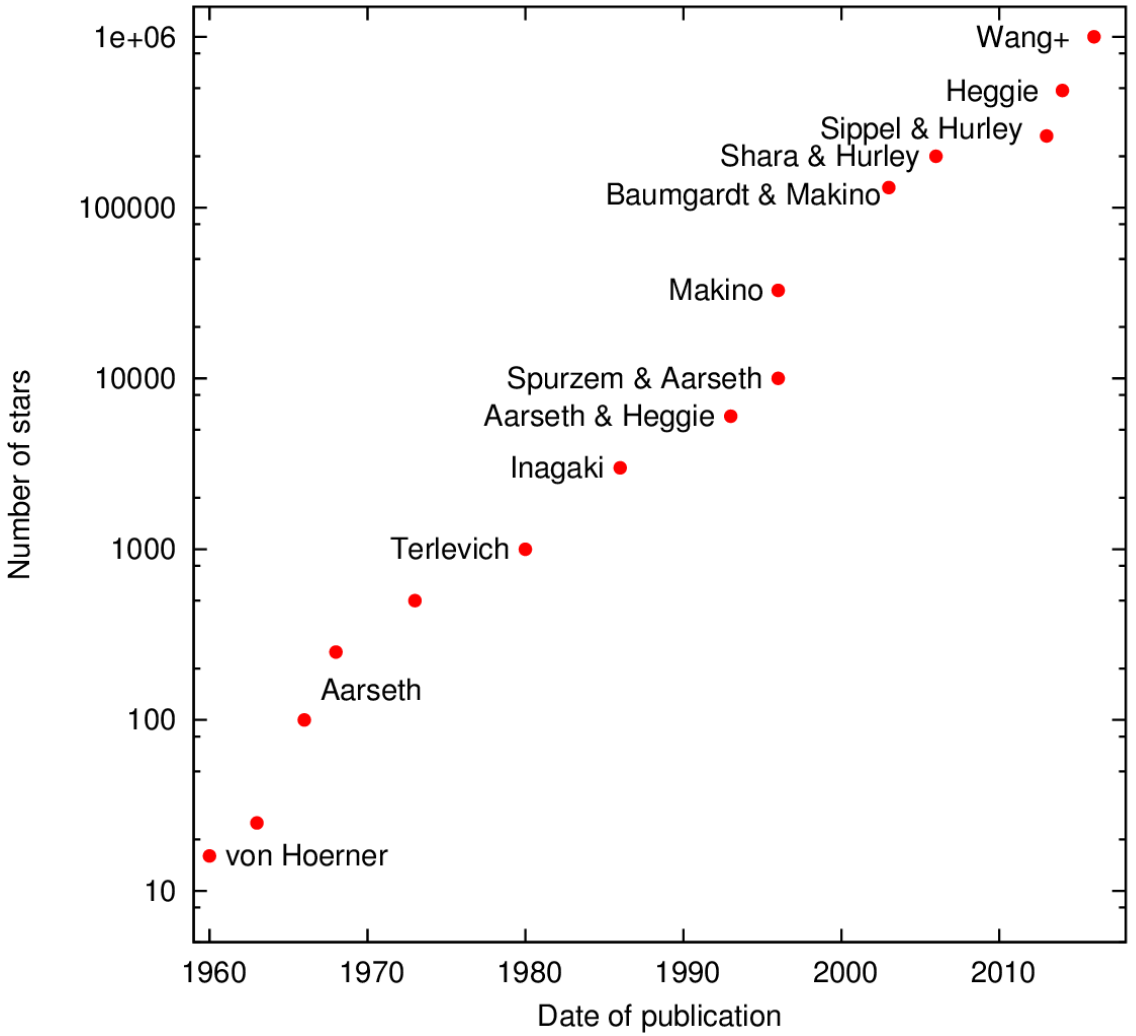
    Focus: *N*-body codes

2) Models for fitting to observations

    Focus: Potential escapers

Context: globular star clusters

Progress in *N*-body modelling



Thanks to Anna Sippel for some corrections

- These models take year(s)
- Typical models are smaller,
  - generally need scaling
- Don't forget Monte Carlo!
  - almost as good and takes day(s)
  - Problems: tidal effects, rotation

CODES

- NBODY6
- Starlab
- NBODY6++
- AMUSE
- several others (see later)

HARDWARE

- Laptops
- Desktops
- Clusters
- Desktops+GPU
- Clusters+GPU

What was published in the last 12 months?

Category	Number	ADS query	% real	Real Number
Observational papers	1524	star+cluster+observation	26%	400
Simulation papers	449	star+cluster+simulation	22%	100

Of the simulation papers....

N-body simulations	118	...+body	57
Monte Carlo	43	...+(Monte Carlo|MOCCA)	17

Of the N-body simulation papers (after further pruning, e.g. abstracts, papers on tails, etc)

NBODY6	17	φGPU	1
STARLAB	6	NBODY6tt	1
AMUSE	4	GANDALF	1
NBODY6++	4	Hermite order 6	1
HiGPUs	3	phiGRAPEch	1
NBODY4	1		
Hermite+GRAPE9	1	Total	41

Always quote your code! *See astro-ph/1611.06232*

Making N-body codes work for all

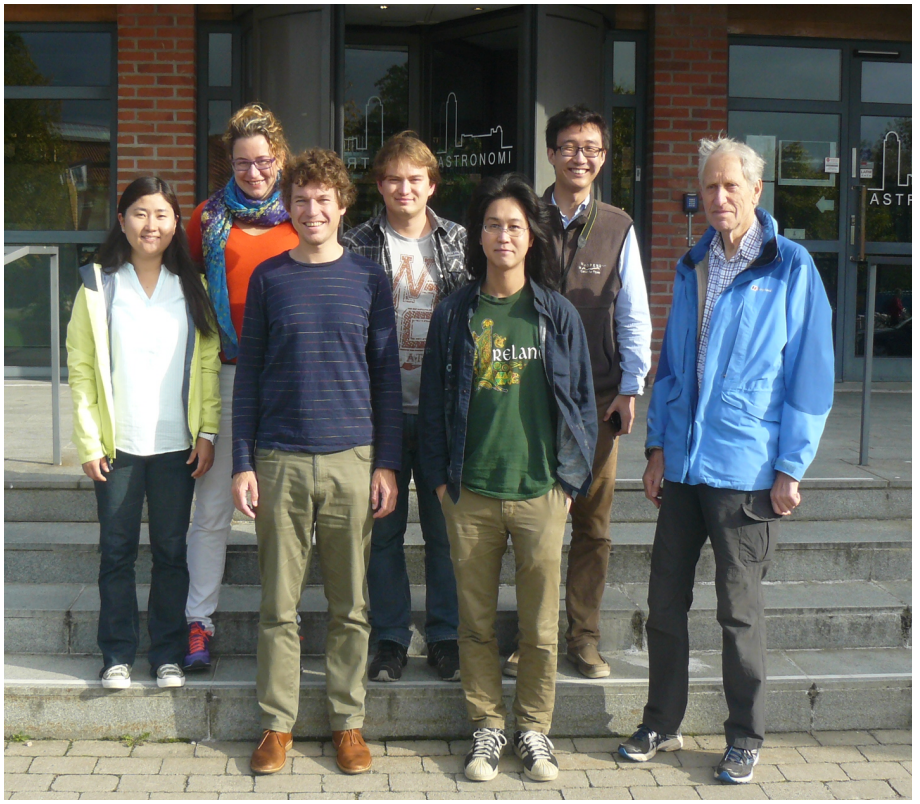
- “My code has bugs in it” *(Teuben+ 2016)*
- Wide variety of scientific problems, operating systems, hardware
- Software developers can't cater for everyone
- Many individual users have experienced problems, and devised solutions, modifications, workarounds, personalisations, etc
  - These circulate by gossip, if at all
- Community efforts
  - Early MODEST was devoted to software
  - MODEST goes underground: occasional meetings of N-body users (Cambridge 2009, Kobe 2012, Prague 2015, Lund 2015, …)
  - No publications, but....

The NBODYx Wiki

- Idea emerged at Lund, September 2015
- https://github.com/nbodyx/Nbody6/wiki

(Anna Sippel, Long Wang)

  - Share your personal bug fixes
  - Post solutions for problems you encounter
  - You recently emailed Sverre about a problem and there was a great solution? Post a summary here!



- Over a year later (22/11/16) it was empty
- Now it's not

Snapshot modelling

Finding a model to fit the observations now

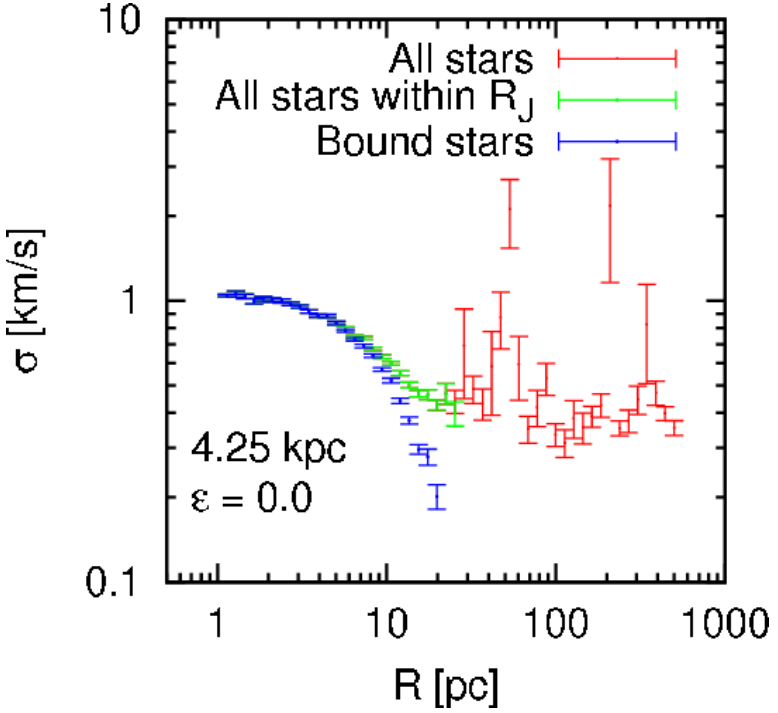
- King models
  - Multimass variants
- Woolley, Wilson, LIMEPY , $f^{(v)}$ models (de Vita+ 2016)
- Rotating and anisotropic models

http://www.cosmic-lab.eu/Cosmic-Lab/Talks_PDF_files/Varri.pdf

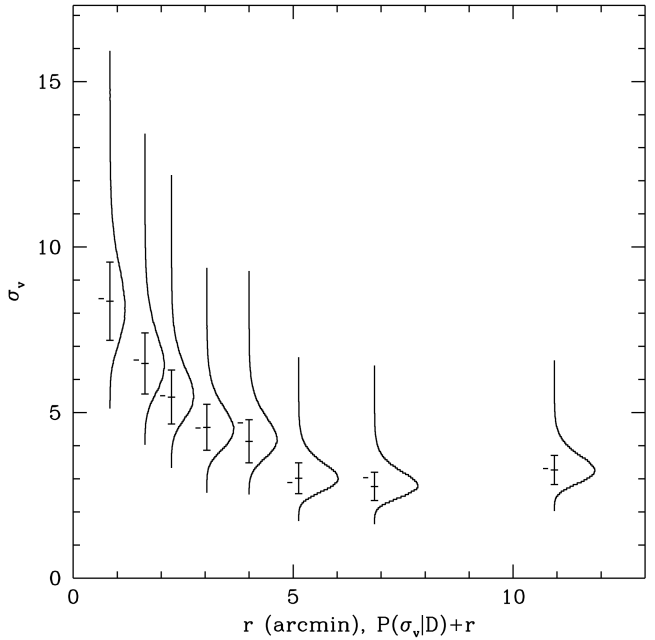
- Jeans models
- Schwarzschild models
- Tidally self-consistent models
  - Only these models include the tidal field inside the cluster
  - These models are still deficient

Potential escapers

- Stars *inside* the tidal radius but *above* the energy of escape
- ~5-10% of stars in full-size star clusters (based on scaling of *N*-body models)
- Excluded in all the usual models
- Large effect on velocity dispersion profile



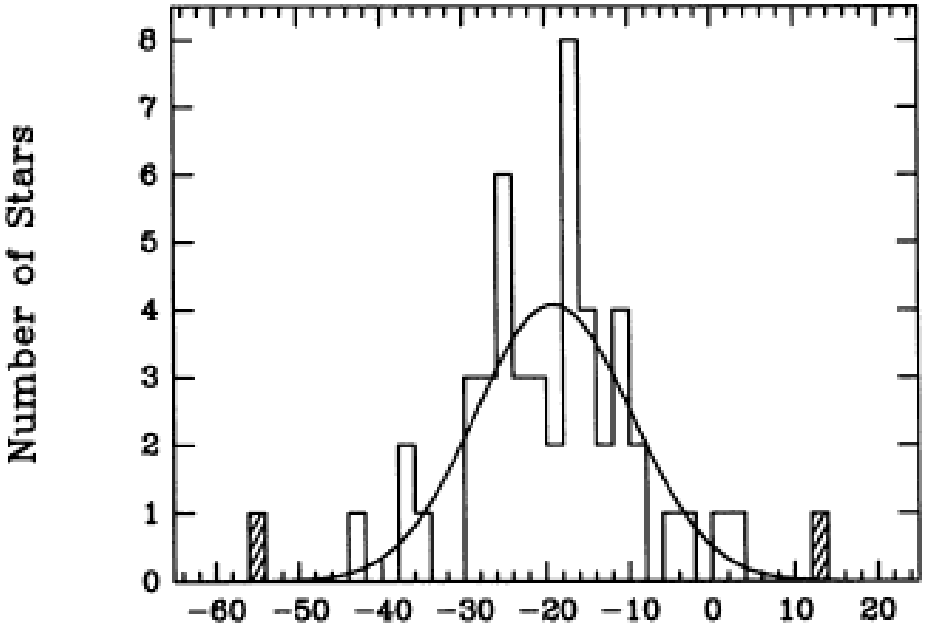
N-body model (Küpper+ 2010)

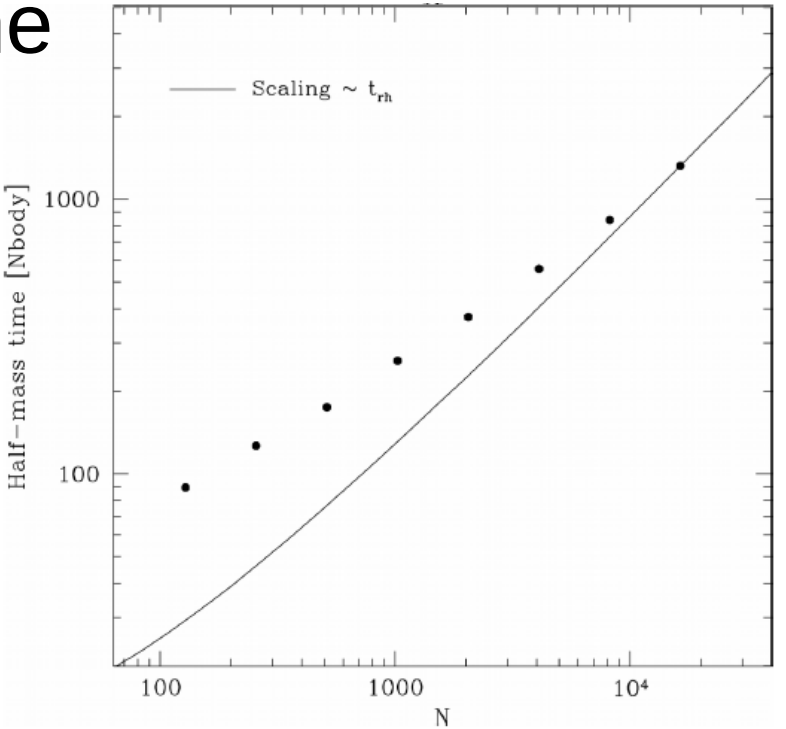


M15 (Drukier+ 1998)

Other effects of potential escapers

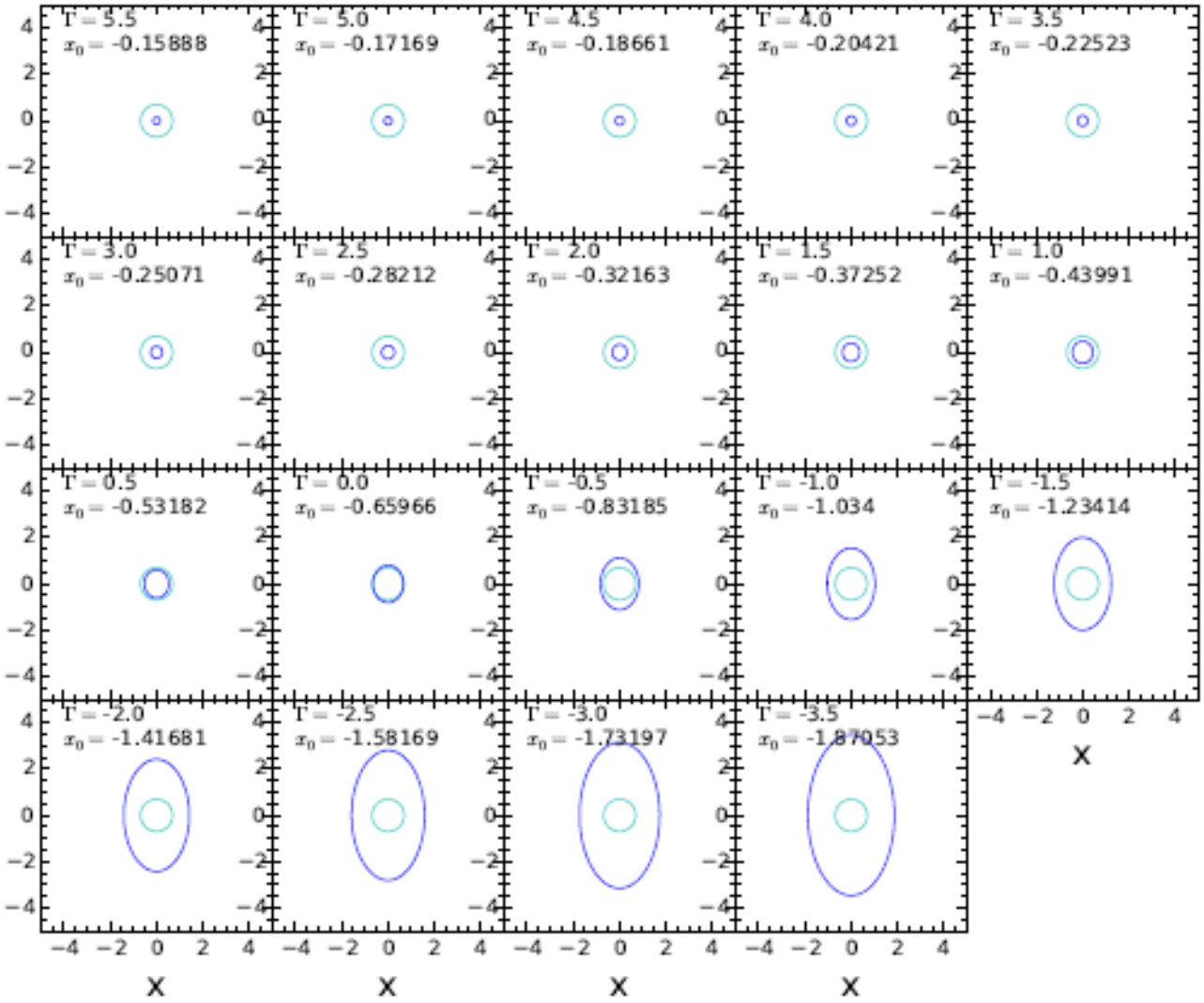
- Canonballs
  - Individual l.o.s. velocities in 47 Tuc
  - Up to 4σ
  - (Meylan+1991)
- Scaling of the lifetime
  - (Baumgardt 2001)





f-orbits (Hénon 1970)

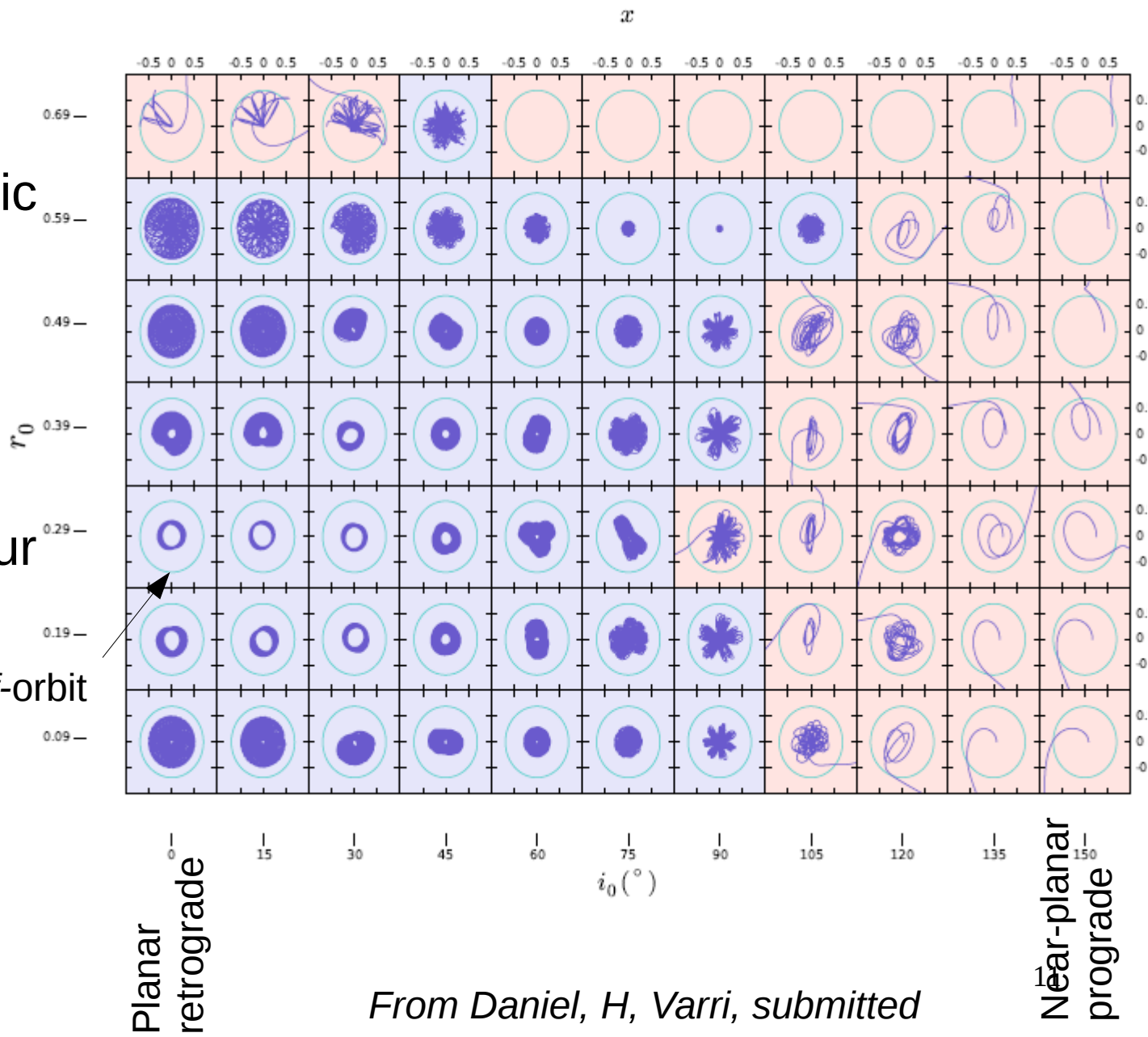
- Use a rotating frame to study motions of stars in a point-mass cluster potential
- Energy E_R (like Jacobi integral Γ) is conserved
- There are stable periodic orbits
  - At low E_R these are small near Keplerian orbits, inside tidal radius (top left)
  - At large E_R these are nearly epicycles (bottom right)
  - Between Jacobi energy and ~(these remain inside the tidal radius
- Examples of (stable) potential escapers



KateDaniel

Some 3-dimensional orbits at $\Gamma = 3$

- Many orbits remain inside tidal radius for at least 16 galactic orbits (blue)
- Mostly retrograde
- High-inclination orbits show behaviour like Lidov-Kozai oscillations



From Daniel, H, Varri, submitted

The distribution of non-escapers

- Aim to construct a model which includes a population of non-escapers
- By Jeans' Theorem, we attempt to specify this in terms of integrals (constants, invariants) of the motion, to give an equilibrium distribution
- Γ is an integral
- Equations of motion are exactly those of

  Lidov-Kozai theory in quadrupole approximation

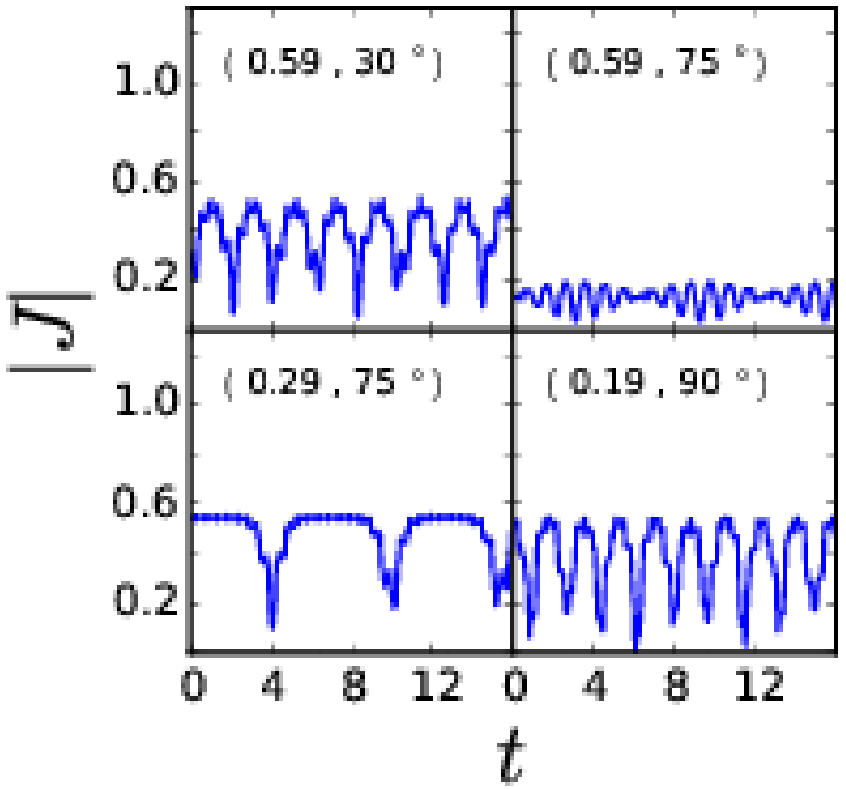
$$\begin{aligned}
\ddot{x}_R &= 2\dot{y}_R + 3x_R - \frac{x_R}{r_R^3}, \\
\ddot{y}_R &= -2\dot{x}_R - \frac{y_R}{r_R^3} \\
\ddot{z}_R &= -z_R - \frac{z_R}{r_R^3}.
\end{aligned}$$

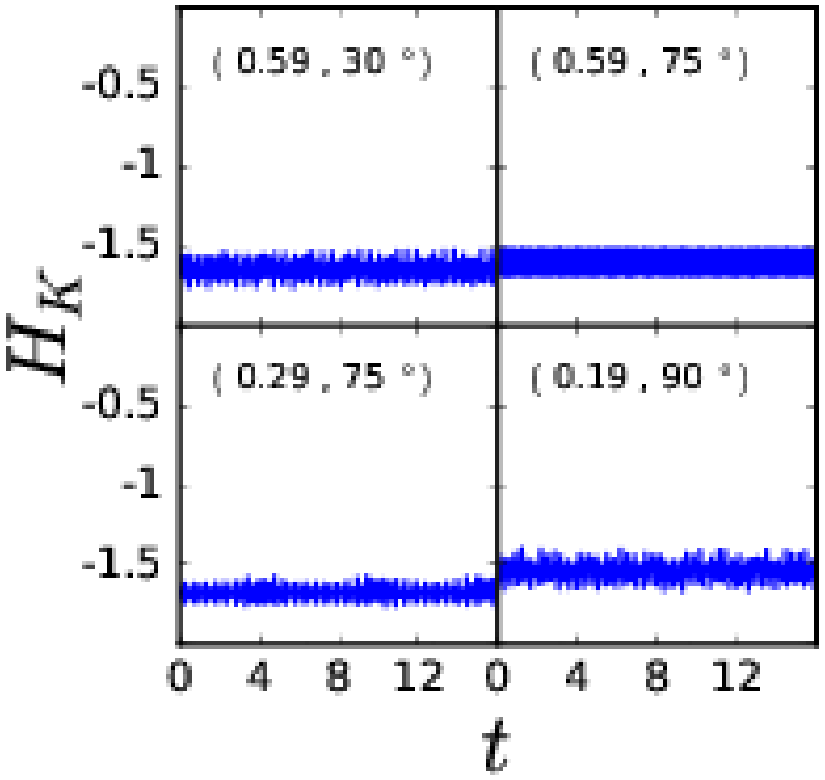
- Approximate integrals of LK theory:
  - $\langle H_K \rangle$, i.e. Kepler energy averaged

    over the Kepler motion
  - $\langle J_z \rangle$, i.e. *z*-component of angular momentum,

    averaged over Kepler and Galactic motions
  - To evaluate these from initial conditions we correct for oscillatory terms using first-order perturbation theory

Numerical illustration



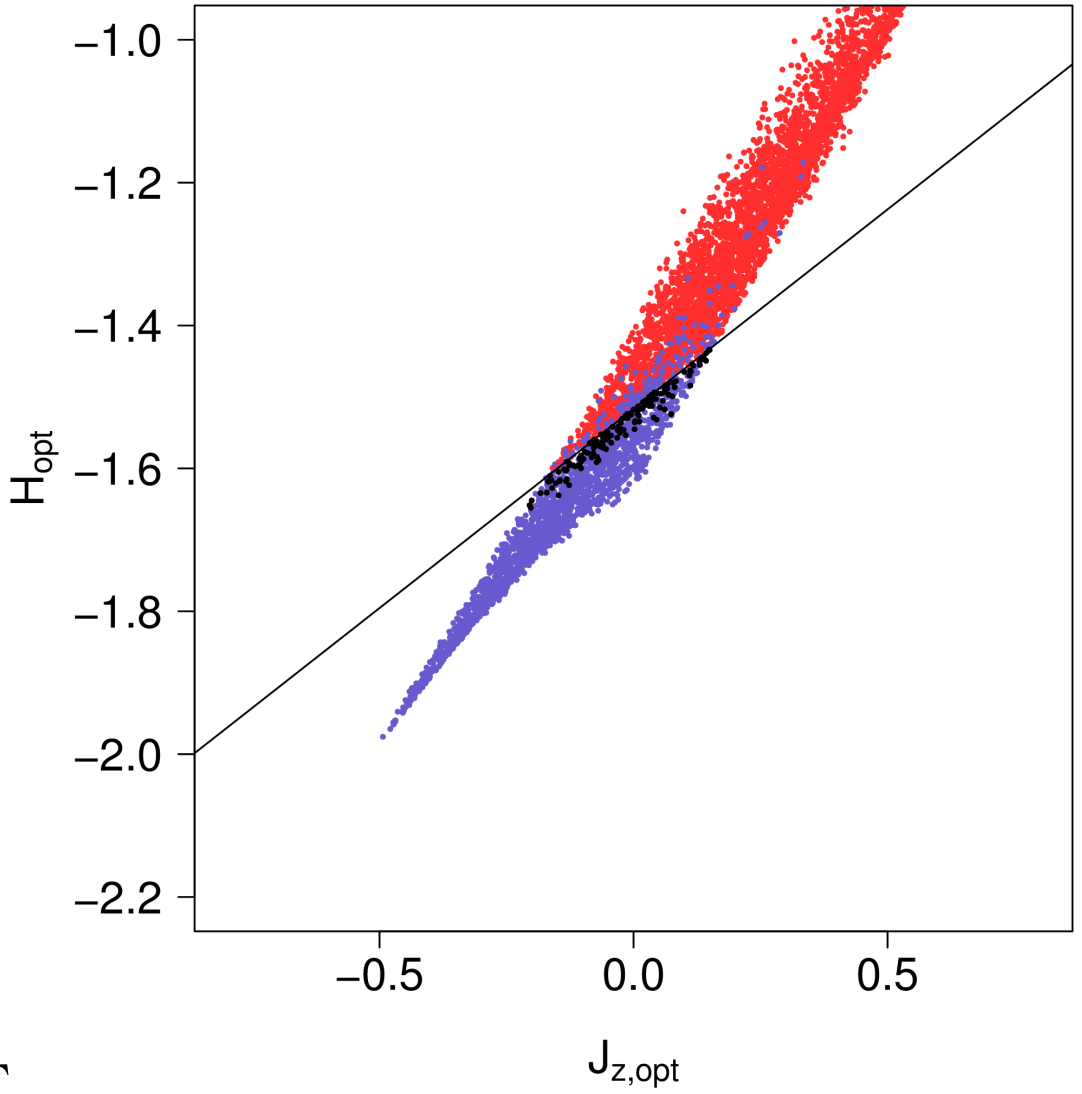


- Not an integral of LK theory

- AverageH_K is an integral
- Mainly high-frequency oscillations

Escapers and non-escapers

- Example $\Gamma = 3$
- “Generalised Lindblad diagram”
- Red/black: escapers
- Blue: non-escapers
- Choose a straight line to minimimise number of misclassified points
- repeat for several discrete values of Γ
- interpolate for other values of Γ



Combined model

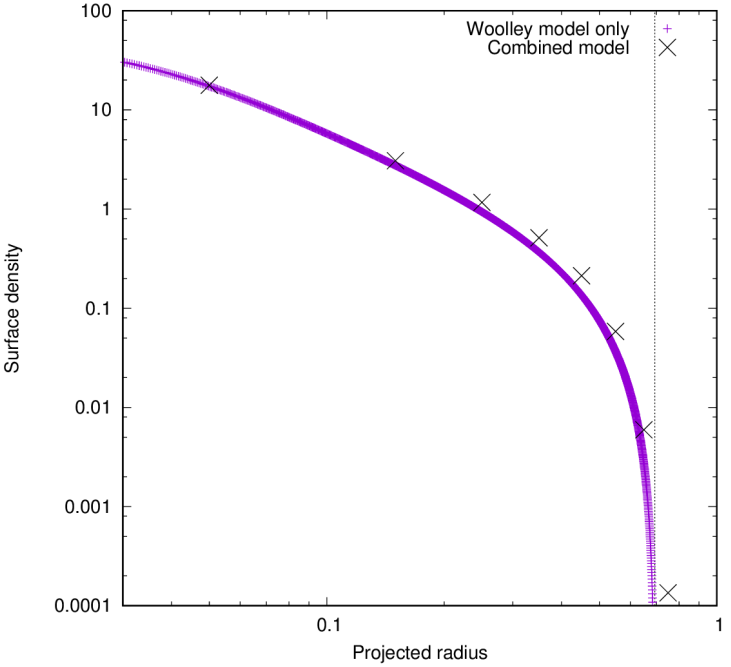
- Woolley model, $W_0 = 7$
  - Distribution function

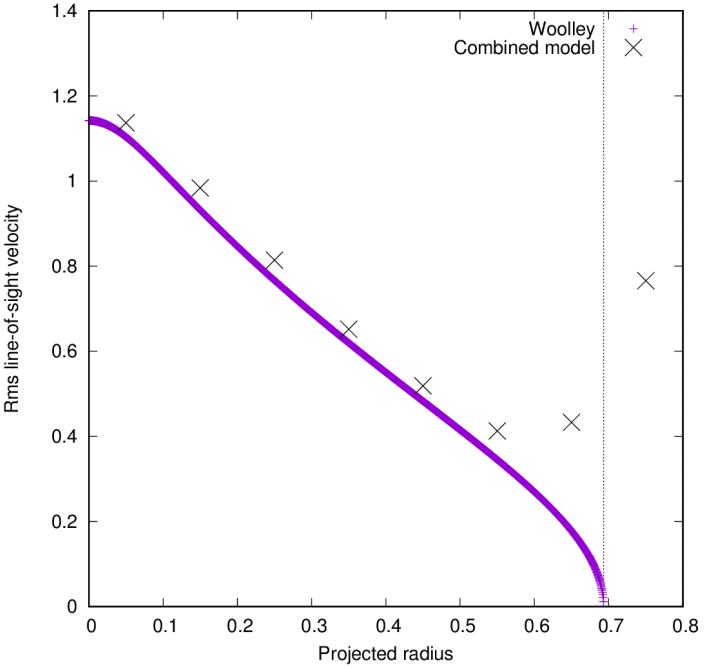
    Aexp(-j^2E) below Jacobi energy

- Non-escaper population
  - Distribution function

    Aexp(-j^2E_R) above Jacobi energy

    with removal of escapers
  - About 14% of total





Take-home messages

- Don't forget the wiki
- Don't forget potential escapers

With thanks to

- *coauthors Kate Daniel and Anna Lisa Varri*
- *Pascale Garaud and ISIMA*
- *authors of all N-body codes*