N-Body Treatments of Black Holes

Sverre Aarseth

Institute of Astronomy, Cambridge

Introduction

N-Body Interface

Post-Newtonian Implementations

Decision-Making

Case Study

Tidal Disruption

Methods Summary

Hermite	Single stars and binary c.m.
KS	Binaries & close encounters
Hierarchies	Generalized binaries
ARC	Compact subsystems
Post-Newtonian	Dominant BH binary
TRIPLE & QUAD	Unperturbed 3/4-body

N-Body Interface

Initialization Compact subsystem

Reference body As singles + force correction

Perturbers $\gamma = \dfrac{2m_p}{m_k + m_{k+1}} \left(\dfrac{R_k}{R_p} \right)^3 > 10^{-7}$

Prediction Every function call

Perturber list Frequent updates

Membership Gain or loss

Decision-making Chain vectors

Termination Two-body and/or singles

Selection of Chain Members

1. Binary apocentre $\quad \Delta t_{\rm cm} < \Delta t_{\rm close}$

   Encounter search $\quad d_{\rm min} < 2a,\ \dot{d} < 0,$

2. Perturbed KS $\quad a < 5R_{\rm min},\ \gamma > 0.05$

   Add perturbers $\quad j < N$ or $j > N$

3. Switch to PN $\quad K^* = 14,\ \Delta w > 0.001$

   Add perturber $\quad j < N$ or $j > N$

4. Dormant KS $\quad$ Activate if $\gamma > 10^{-5}$

   Add perturber $\quad j < N$ or $j > N$

Post-Newtonian Terms

Equation of motion $$\frac{d^2\mathbf{r}}{dt^2} = \frac{M}{r^2}\left[(-1+A)\frac{\mathbf{r}}{r} + B\mathbf{v}\right]$$

First-order precession $$M = m_1 + m_2, \quad \eta = \frac{m_1 m_2}{M^2}$$

$$A_1 = 2(2+\eta)\frac{M}{r} - (1+3\eta)v^2 + \frac{3}{2}\eta\dot{r}^2$$

$$B_1 = 2(2-\eta)\dot{r}$$

Higher-order precession $$A_2 = ..., \quad B_2 = ..., \quad A_3 = ..., \quad B_3 = ...$$

Gravitational radiation $$A_{5/2} = \frac{8}{5}\eta\frac{M}{r}\dot{r}\left(\frac{17M}{3r} + 3v^2\right)$$

$$B_{5/2} = -\frac{8}{5}\eta\frac{M}{r}\left(3\frac{M}{r} + v^2\right)$$

Total GR perturbation

$$\mathbf{P}_{GR} = \frac{M}{c^2 r^2}\left[(A_1 + \frac{A_2}{c^2} + \frac{A_{5/2}}{c^3})\frac{\mathbf{r}}{r} + (B_1 + \frac{B_2}{c^2} + \frac{B_{5/2}}{c^3})\mathbf{v}\right]$$

Radiation energy loss $$\Delta E_{GR} = \frac{m_1 m_2}{M}\int \mathbf{P}_{GR}\cdot\mathbf{v}\,dt$$

GR radiation time-scale $$t_{\rm GR} = \frac{5}{64}\frac{c^5\, g(e)\, a^4}{X(1+X)\, m_x^3}, \quad c = \frac{3\times 10^5}{V^*}$$

Decision-making graduated PN terms from Δw or $t_{\rm GR}$

Decision-Making

Increasing GR effect $t_{\mathrm{GR}} < 500,\ \ 50,\ \ 1$

$\mathrm{IPN} = 1,\ \ 2,\ \ 3$

Einstein shift $\Delta w = \dfrac{6\pi M}{ac^2(1-e^2)}$

Graduated scale $\Delta w > (1, 10, 100) \times 10^{-4}$

Coalescence $R_{\mathrm{co}} = 4R_{\mathrm{Sch}} = \dfrac{8\,M}{c^2}$

Alternative merging $\mathrm{IPN} = 3,\ \ N = 2,\ \ N_{\mathrm{p}} = 0$

$\mathrm{IPN} \geq 2,\ \ a\,(1-e) < R_{\mathrm{co}}$

$\mathrm{IPN} = 3,\ N = 3,\ a_1\,(1-e_1) > 100\,a$

Unperturbed KS $t_{\mathrm{GR}} < 500\ \&\ \Delta w > 1 \times 10^{-4}$

Integration $\dot{a},\ \dot{e}\ +$ rotation $\Rightarrow \mathbf{u},\ \mathbf{u}'$

Simulations

Initial conditions $N = 1 \times 10^5,$ Kroupa IMF in $0.1 - 50 M_\odot$

Plummer & tides $R_h = 2$ pc, $V^* = 10\, km/s,$ $T^* = 0.2\,$Myr

N-body units $G = 1,\ \bar{m} = 1/N,\quad E = -\dfrac{1}{4},\ \bar{v}^2 = \dfrac{1}{2}$

Hard binary $\dfrac{m^2}{2a} \simeq \dfrac{1}{2}\bar{m}\bar{v}^2, \quad \Rightarrow a_{\rm hard} \simeq 2/N$

Super-hard $-\dfrac{m^2}{2a} = E, \quad \Rightarrow a_0 = 2/N^2, \quad$ or $\ 2 \times 10^{-10}$

Dynamics $m_{\rm bh} \simeq 20\ M_\odot, \quad E_{\rm b} = 0.01E, \quad a \simeq 3 \times 10^{-5}$

Coalescence $R_{\rm co} = \dfrac{8(m_1 + m_2)}{c^2}, \quad R_{\rm co} \simeq 7 \times 10^{-12}$

PN condition $a(1-e) \simeq 10^4 R_{\rm co}, \quad \Rightarrow e > 0.999$

Time-scale $a = 3 \times 10^{-5},\ e = 0.999,\ m = 20\,{\rm M}_\odot,\ \tau_{\rm co} \simeq 200\,$Myr

Einstein shift $\Delta w = \dfrac{6\pi M}{ac^2(1-e^2)} = 3 \times 10^{-4}$

Unperturbed GR Orbit

Compact objects $\max\,(K_1^*, K_2^*) \;>\; 12$

Derivatives $\dot{a} = -\dfrac{64 M_1 M_2 (M_1 + M_2)}{5 c^5 a^3 (1-e^2)^{7/2}}$

$\dot{e} = -\dfrac{304 e M_1 M_2 (M_1 + M_2)}{15 c^5 a^4 (1-e^2)^{5/2}} g(e)$

Einstein shift $\Delta\omega = \dfrac{6\pi (M_1 + M_2)}{a c^2 (1-e^2)}$

Safety condition $\Delta t = \min\,\left(\dfrac{0.01 a}{\dot{a}}, \Delta t_0 \right)$

KS rotation $\theta = \dfrac{\Delta\omega \Delta t}{T_{\rm K}}, \quad$ rotate by $\theta/2$

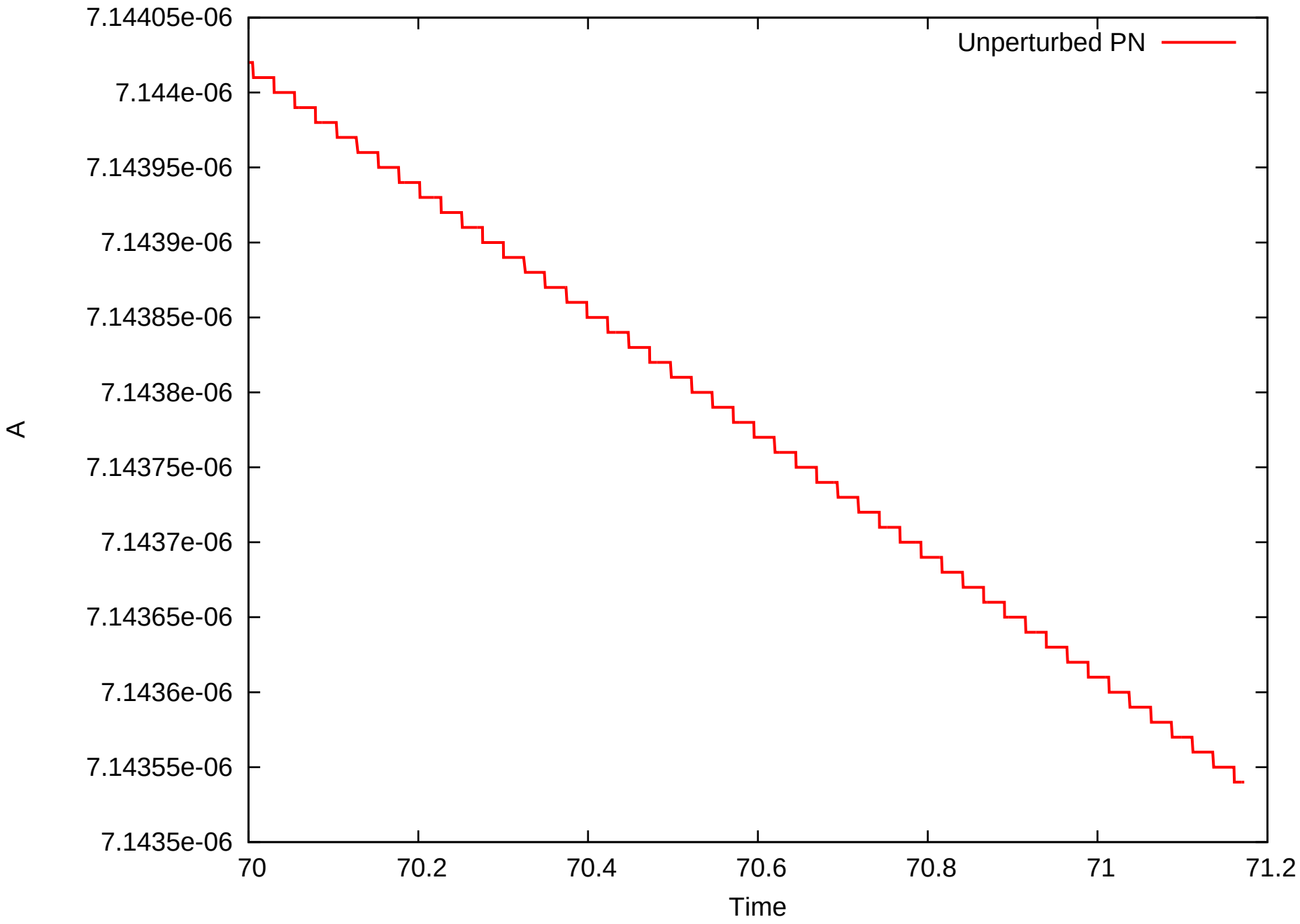
New elements $a_1 = a + \dot{a}\Delta t, \quad e_1 = \max\,(e + \dot{e}\Delta t, 0)$

Energy updating $\Delta E = \mu (H_{\rm old} - H_{\rm new})$

Modify KS orbit Shrink, $e = \text{const}$; decrease, $H = \text{const}$

Validity $1 \times 10^{-4} < \Delta\omega < 1 \times 10^{-3}$

Unperturbed KS $\gamma < 1 \times 10^{-6}$


Unperturbed PN
A
Time
7.14405e-06
7.144e-06
7.14395e-06
7.1439e-06
7.14385e-06
7.1438e-06
7.14375e-06
7.1437e-06
7.14365e-06
7.1436e-06
7.14355e-06
7.1435e-06
70
70.2
70.4
70.6
70.8
71
71.2

Binary Ejections

t/Myr	m_1	m_2	P	$v_{\rm esc}$	e
230	18.0	14.8	600	21.0	0.19
370	17.7	16.0	700	14.0	0.84
550	15.9	17.9	110	14.0	0.72
860	18.2	17.3	390	5.0	0.96
1780	19.4	16.6	260	2.0	0.97

Coalescence

t/Myr	m_1	m_2	a/au	e	$a(1-e)/R_{\rm co}$
150	21.3	17.5	7	0.9998	1×10^{-3}
270	18.2	13.3	10	0.9999	4×10^{-4}
330	18.2	16.4	500	0.99999	5×10^{-4}

Tidal Didsruption

Initial conditions	Plummer model $N = 10^5$, IMF $5 - 0.2\,M_\odot$
Scaling factors	$R^* = 4 \times 10^5$ au, $T^* = 0.16$ Myr, $V^* = 12$ km/s
Central black hole	$M_{\rm BH} = 20$ *or* $10\,M_\odot$ at rest
Magorrian & Tremaine	$R_{\rm dis} = \left(\frac{M_{\rm BH}}{m^*}\right)^{1/3} r^*, \quad r\odot = 1.2 \times 10^{-8}$
Outcomes	Accretion by WD & NS, otherwise by $m/10$
KS events	Velocity kick after BH accretion of $0.1m$
ARC events	Instant removal after BH accretion of $0.1m$

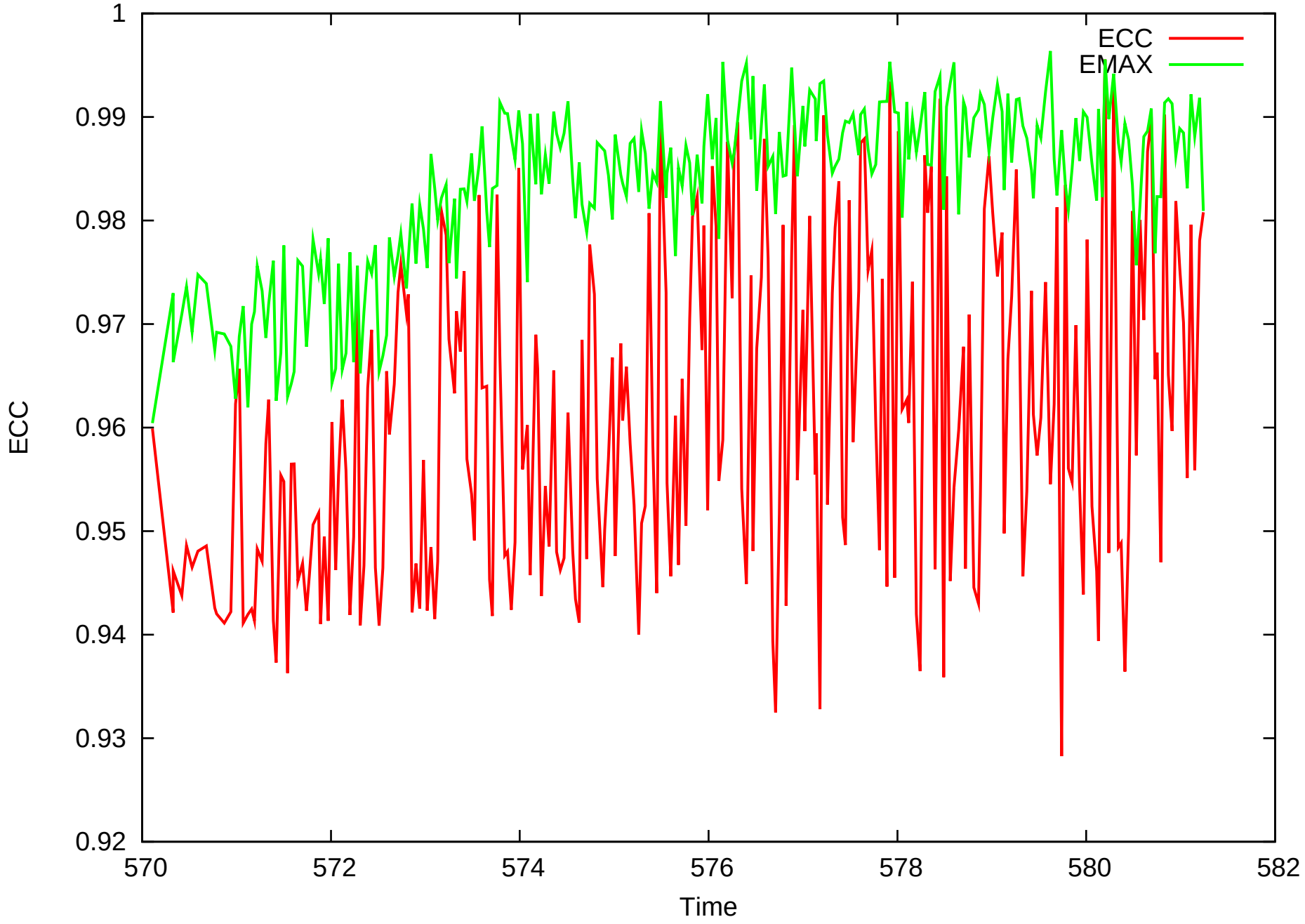
Disruption Events

Model	t/Gyr	$M_{\rm bh}$	Counts
D7	3.3	10	74*
D10	3.8	20	108*
D12	3.3	20	51
D12b	3.8	20	67
D13	3.8	10	33
D14	3.2	10	23

*: Accretion $dm = 0.5\ m$

Eccentricity D13: > 0.99 (17) > 0.999 (10) > 1 (2)

D14: > 0.99 (10) > 0.999 (7) > 1 (0)


ECC
EMAX
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
ECC
570
572
574
576
578
580
582
Time