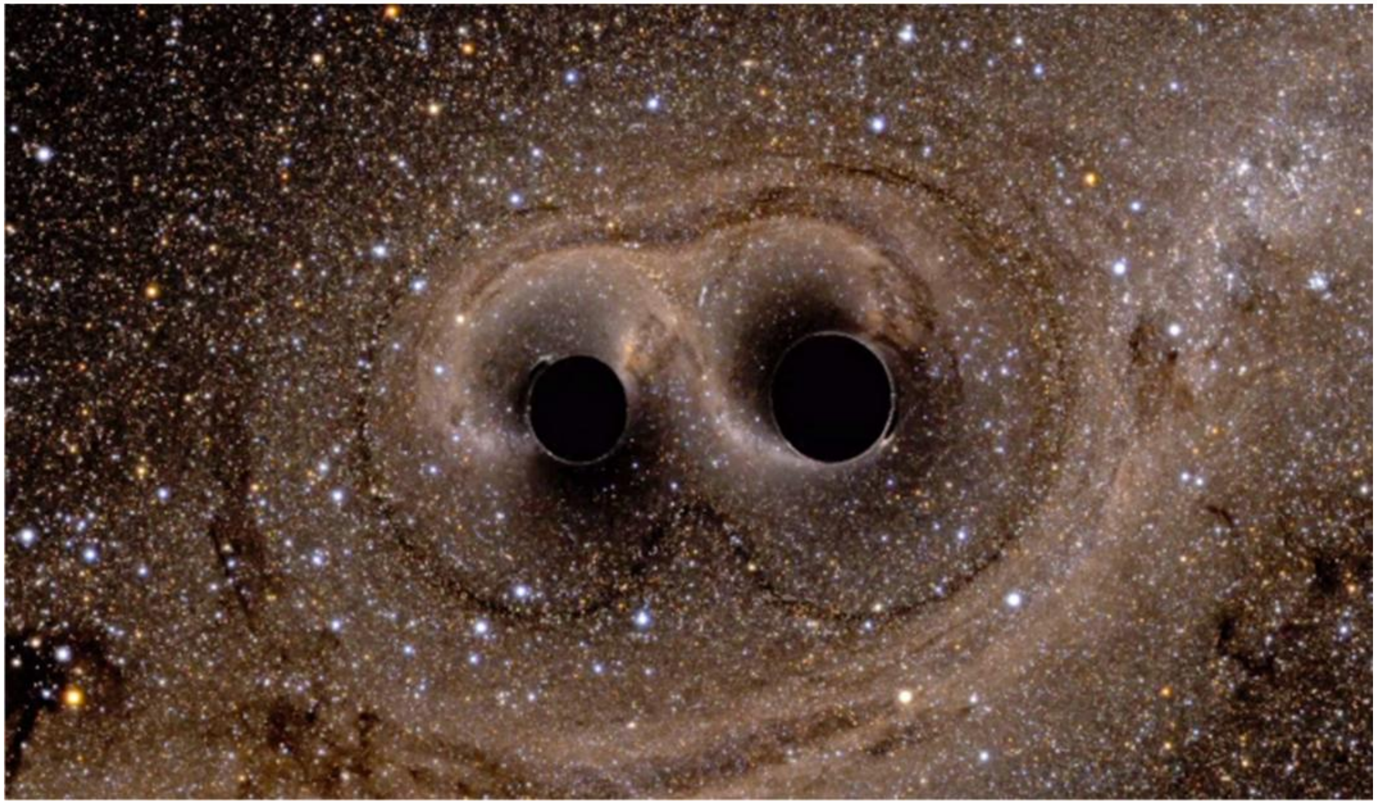
Coalescing Binary Black Holes Originating from Globular Clusters



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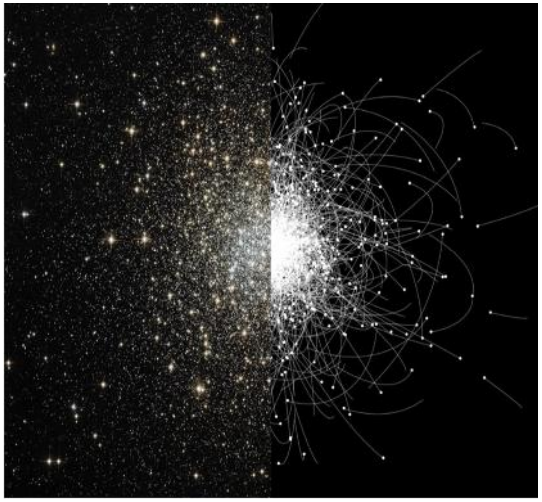
Globular Clusters & Stellar Dynamics

- Spherical collection of stars that orbits a galactic core as a satellite.
- Comprise 100,000 to millions of stars.
- Globular clusters in the Milky Way are estimated to be at least 10 billion years old.
- Stars are clumped closely together, especially near the centre of the cluster.
- Star clusters are excellent laboratories for theories of stellar dynamics.
- In collisional systems like star clusters, 2-body relaxation is important in driving the dynamical evolution of the system.
- Many other physical processes are involved.

47 Tuc



Stellar Dynamics & Numerical Simulations

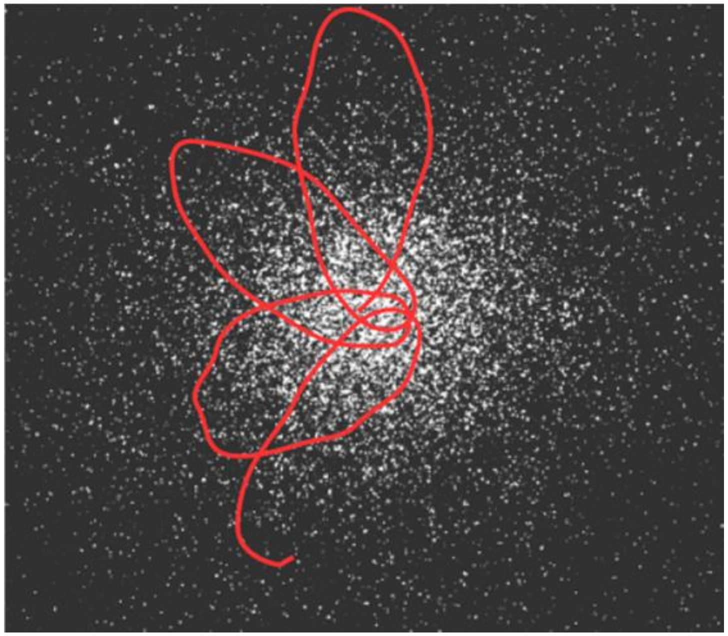
- Theory of stellar dynamics can be modelled using numerical simulations.
- Direct *N*-body Methods
  - Numerical integration of equations of motion
  - Pros: Accurate
  - Cons: Computationally Expensive, $t_{comp} \propto N^3$
- Statistical description given by a distribution function $f(\vec{r}, \vec{v}, m, t)$
  - Evolution of f with time is given by Fokker-Planck equation
  - Too complicated for a numerical solution in the general case (number of simplifications need to be made to reduce it a manageable form).
  - Pros: Fast
  - Cons: Too many underlying assumptions
- In late 1960s and early 70s, Michel Hénon devised a method to compute star cluster evolution based on a Monte Carlo approach.

Hénon’s Monte-Carlo Method

- Statistical way of solving the Fokker-Planck equation.
- Star cluster is treated as a set of spherical shells which represent an individual object (star, binary). Each shell is characterized by: mass (m), energy (*E*) and angular momentum (*J*).
- 3 fundamental assumptions:
  - System is spherically symmetrical.
  - Large $N \ (\gtrsim 1000)$
  - Long term evolution (over a period of time comparable to or larger than the relaxation time t_r).
- Gravitational field can be divided into 2 parts:
  - Smoothed out field or mean field
  - Smaller irregular fluctuating field
- Consider motion during the interval of time Δt such that $t_c \ll \Delta t \leq t_r$
  - On first approximation, we can neglect the fluctuating field.

Hénon’s Monte-Carlo Method

- Star has a rosette orbit described by simple analytical formula (detailed integration along the orbit becomes unnecessary).



- Fluctuating field is not entirely negligible and becomes significant over a time of the order t_r
- A single perturbation is computed at a randomly selected point on the orbit for a randomly selected star.
- Computed single perturbation is multiplied by an appropriate factor in order to account for the cumulative effect of all small individual encounters with the rest of the stars in the system and for other points on the orbit.
- Essentially, the algorithm repeatedly alters *E*, *J* to mimic the effects of gravitational encounters → fixed number of operations → $t_{comp} \propto N$

MOCCA

- **MOnte Carlo Cluster simulAtor**: Code to evolve real size star clusters in time (Hypki & Giersz 2013).
- Based on the application of the Monte Carlo method to star clusters, known as Hénon's Method → Hénon's method was further improved by Jerzy Stodółkiewicz (1980s) → Improved version of the Monte Carlo code developed by Mirek Giersz and collaborators (1998 to 2011).
- Evolution of single stars and binaries (SSE and BSE codes Hurley et al 2000; 2002)
- Direct integration procedures for binary interactions using the *Fewbody* code (Fregeau et al. 2004).
- Realistic treatment of escape processes in tidally limited clusters based on Fukushige & Heggie (2000).
- MOCCA can reproduce N-body results with reasonable precision and accuracy (Giersz et al. 2013; Wang et al. 2016).

What can we do with MOCCA?

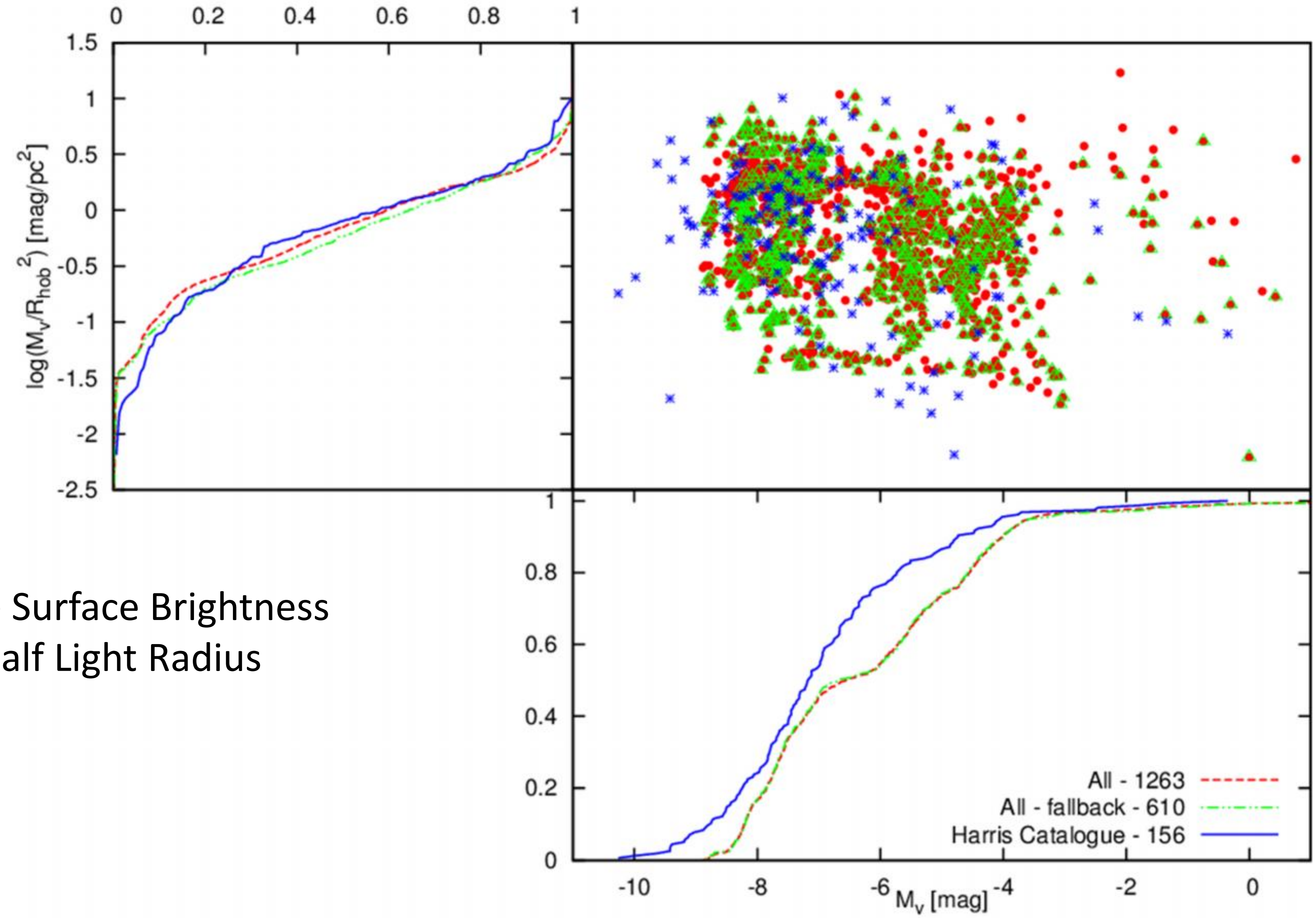
- MOCCA can be used to:
  - simulate evolution of real star clusters (two million body star cluster can be simulated up to a Hubble time within a few days);
  - get full dynamical history of the evolution of all stars in the system; and
  - simulate sizeable surveys of thousands of model star clusters.

MOCCA-Survey Database I

Database of 1948 Star Cluster Models:

- N = 4×10^4 , 1×10^5 , 4×10^5 , 7×10^5 and 1.2×10^6
- Tidal Radius (pc) = 30, 60, 120
- Tidal Radius/Half mass radius = 25,50, Tidally Filling
- King Concentration Parameter (W_0) = 3.0, 6.0,9.0
- Metallicity (Z) = 0.0002, 0.001, 0.005, 0.006, 0.02
- Binary Fraction = 5%, 10%, 30%, 95% (for 95% initial binary parameter distribution given by Kroupa 1995)
- BH and NS Natal Kicks: σ = 265 km/s (Hobbs et al. 2005) or Mass Fallback for BHs (Belczynski, Kalogera & Bulik 2002)
- 2 segment IMF (Kroupa 2001)
  - $0.08 \leq M_\odot \leq 100.0$
- Galactocentric distances span from 1 to 50 kpc

MOCCA-Survey I Models at 12 Gyrs and Galactic GCs



Average Surface Brightness
inside Half Light Radius

Absolute Magnitude

Merging Black Holes: The Detection

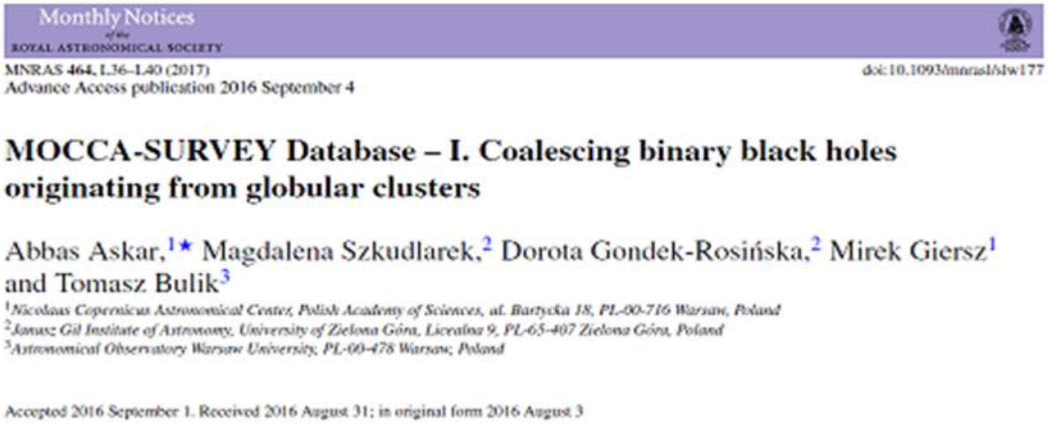
- GW150914: Direct detection of the first gravitational waves (GWs) from a binary black hole (BBH) merger by aLIGO (Abbot et al. 2016 a,b).
- GW151226 and LVT151012 (Abbott et al. 2016c)
- Significant Astrophysical Implications:
  - Confirm the existence of coalescing BBHs.
  - Inferred masses of the coalescing BBHs from GW150914 of about $M{\sim}30\ M_{\odot}$ confirm the presence of massive stellar black holes. ($M < 100\ M_{\odot}$).
- Formation scenarios and origin of the detected coalescing BBHs?
  - Field formation via binary stellar evolution (Belczynski et al. 2016)
  - Formation in dense stellar environments like GCs or galactic nuclei (Antonini & Rasio 2016).
  - Coalescing primordial BHs (Sasaki et al. 2016)

Binary Black Holes & GCs

- Dynamical processes in dense stellar environments are conducive to forming BBHs (Sigurdsson & Hernquist 1993).
- GCs have very low metallicities (typically $Z \lesssim 10^{-3}$) which can result in the formation of massive stellar BHs (Belczynski et al. 2010). → Can form BBHs due to mass segregation during cluster evolution.
- Significant work has been done in trying to use numerical simulations of GCs to predict the detection rates for GW events.
  - MC Simulations: Downing et al. (2011), Rodriguez et al. (2015) & Rodriguez, Chatterjee & Rasio (2016).
  - *N*-body Simulations: Banerjee, Baumgardt & Kroupa (2010), Tanikawa (2013), Bae, Kim & Lee (2014) and Mapelli (2016).

BBHs & MOCCA-Survey Database I

- Collaborators: Magdalena Szkudlarek, Dorota Gondek-Rosinska, Mirek Giersz & Tomek Bulik
- Paper: Askar et al. 2017 (MNRASL Vol.464, Issue 1, pp. L36-L40)



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MOCCA-SURVEY Database – I. Coalescing binary black holes originating from globular clusters

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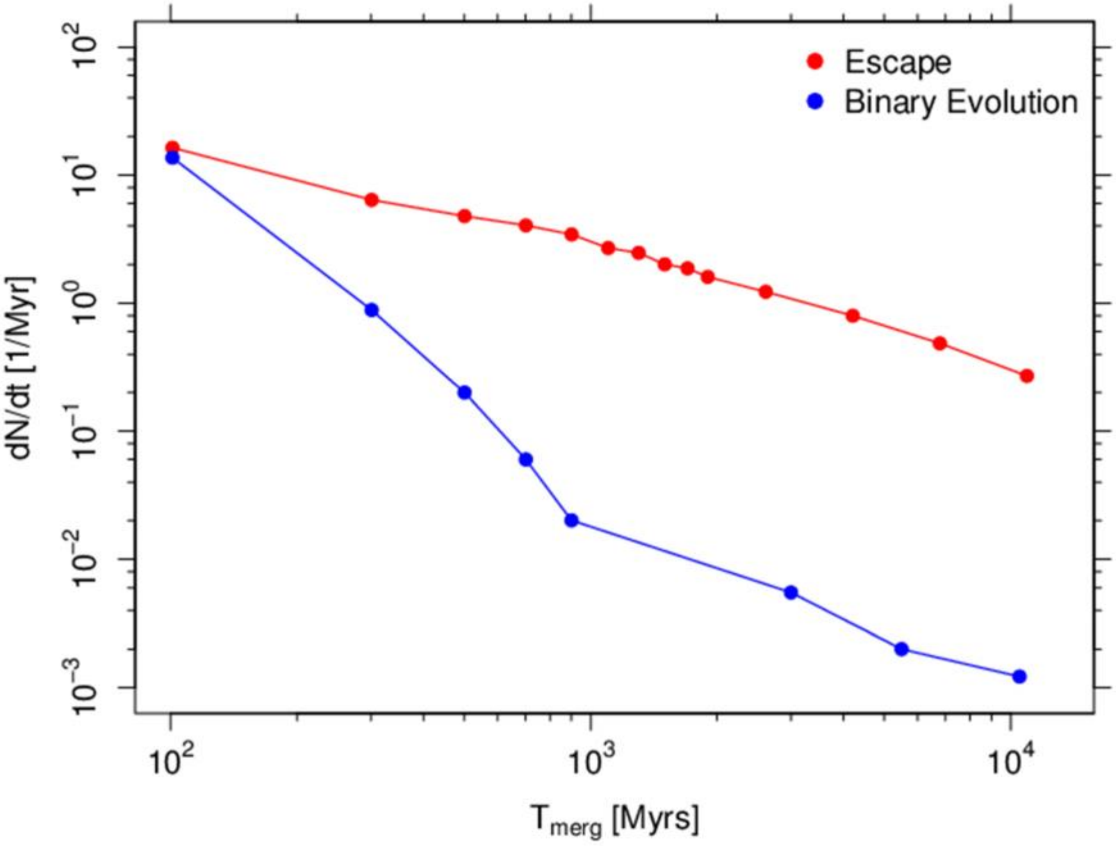
- Goal:
  - Analyze MOCCA-Survey Database I models to find all BBH merger events occurring inside cluster models + All escaping BBHs that will merge within Hubble Time.
  - Calculate local merger rate densities for BBHs originating from GCs.

The Analysis

- Star cluster models in which BH natal kicks were computed according to the mass fallback prescription given by Belczynski et al. (2002). → 985 models
- For escaping BBHs: we know the time at which the binary escaped, BH masses, semi-major axis and eccentricity → calculate proper coalescence times using the formulae derived by Peters (1964).
- BBHs merging inside the cluster via gravitational waves: Binary stellar evolution code
- Masses of both BHs in the BBH before merger were limited to $M < 100\ M_{\odot}$.
- 15,134 coalescing BBHs that escape the cluster and 3000 BBHs that merged inside the cluster within a Hubble time.

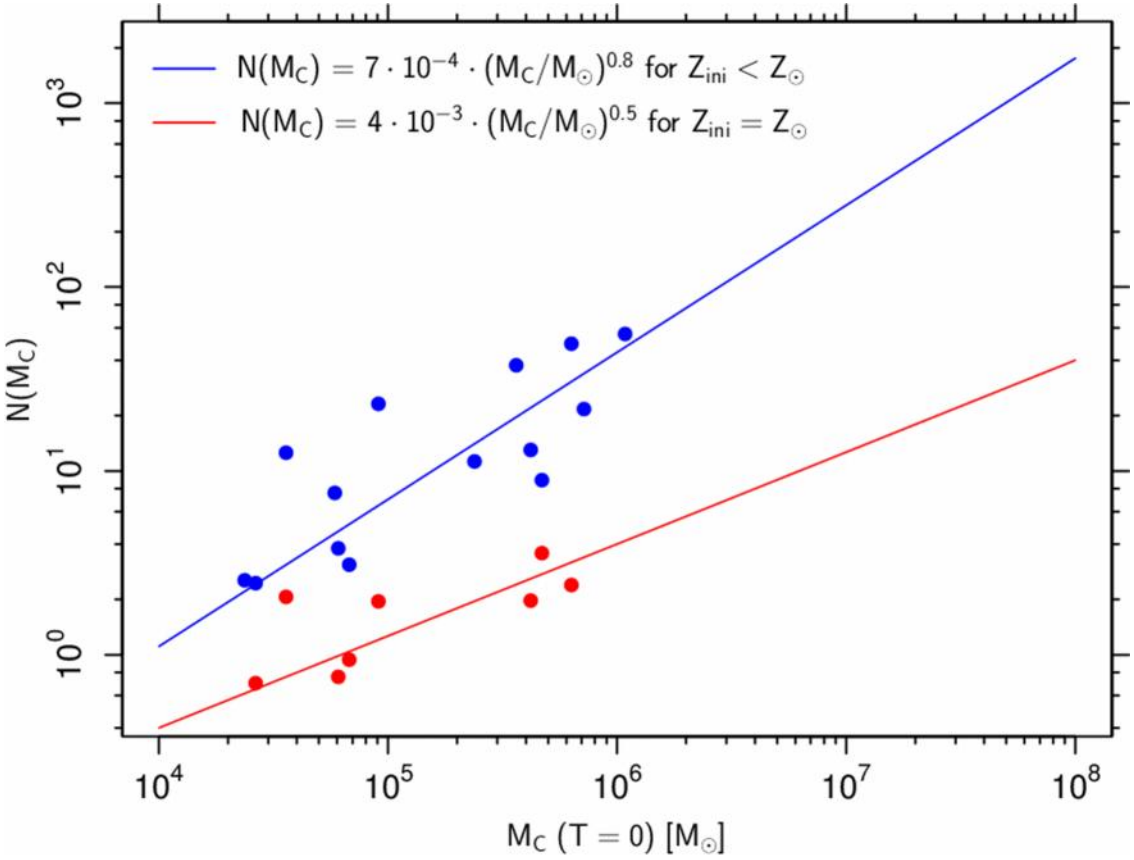
Number & Merger Times



Number of merging BBHs per unit time (1Myr) as a function of merger time.

- The highest rate of mergers is during the early evolution of the GC model; for both escapers and GW mergers inside the cluster, the rate is approximately the same.
- Difference gets larger during the evolution of the cluster. BBHs inside the cluster merge faster than escapers.
- At later times, rate of merging BBHs decreases and is dominated by escapers.

BBH Mergers & Initial Cluster Mass



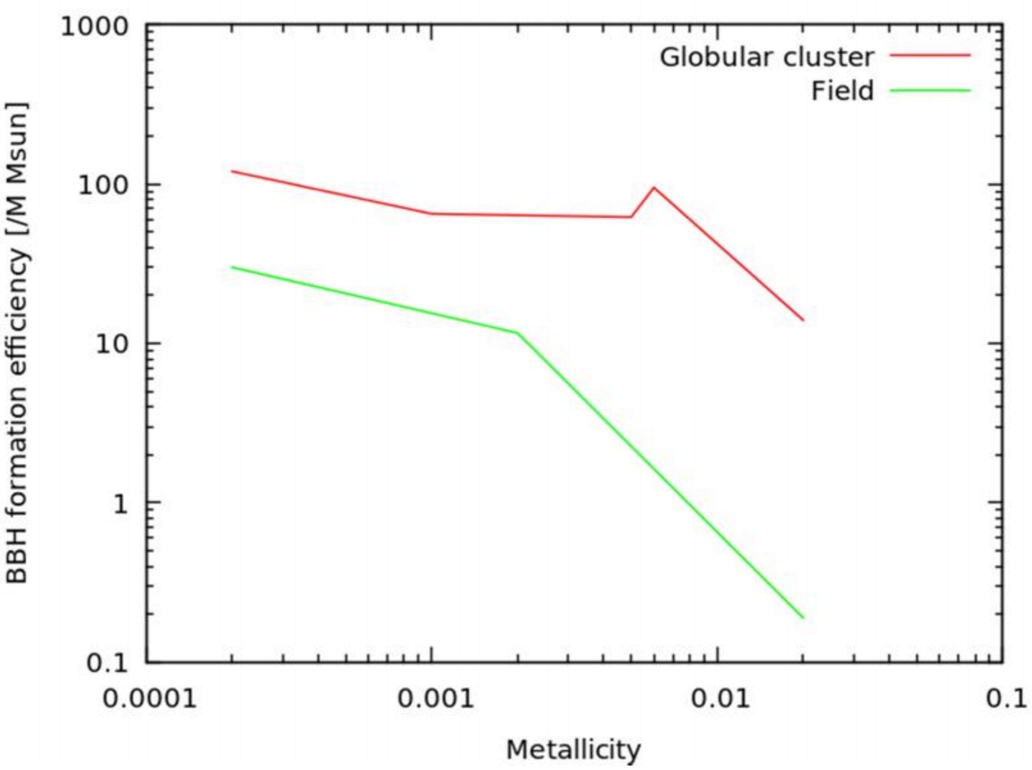
Normalized number of BBHs as a function of initial cluster mass M_c with fitted function $N(M_c)$ (BBH production efficiency).

- Normalization function:

$$N(M_c) = \frac{n}{n_s \cdot M_c / 10^6 \, \mathrm{M}_\odot}$$

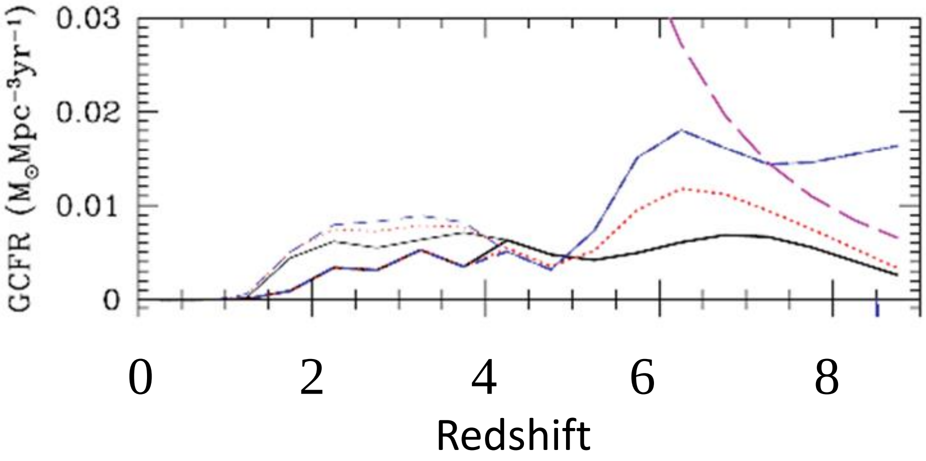
- $Z < 0.02$ – 17 269 merger events
  $Z = 0.02$ – 865 mergers
- Regardless of the metallicity, if the mass of a GC model is large, then the number of merging BBHs is higher.
- Low-metallicity clusters have a greater ratio of producing merging BBHs compared to higher metallicity cluster models.
- If clusters have larger initial masses then they will produce more merging BBHs.

BBH Production Efficiency: GCs vs Field



Efficiency: Number of merging BBH binaries per 10^6 $M_\odot$ of stars .

Field Data from Belczynski et al. 2016.



Upper limits on the GC formation rate as a function of redshift from LF and colour constraints for systems of GCs (Katz & Ricotti 2013)

Local Merger Rate Density of BBH Mergers

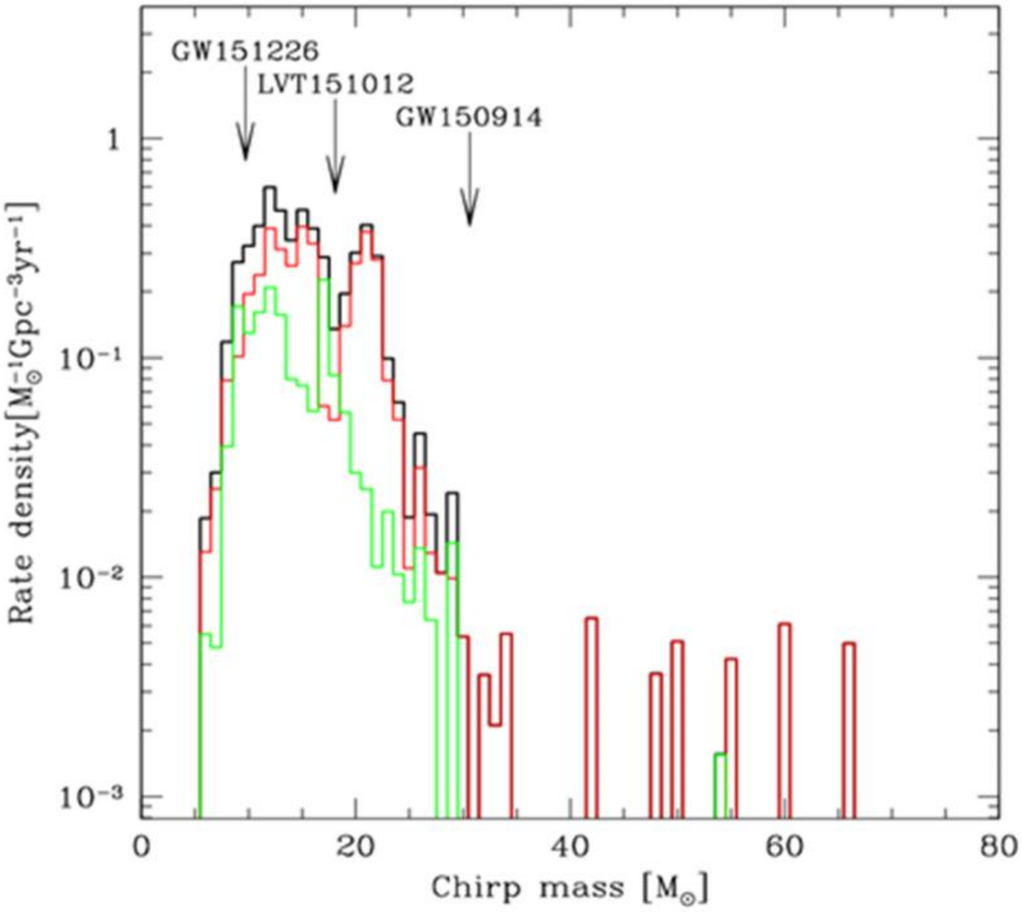
- Follows the same formalism as used in calculation of the local merger rate for field BBHs (Bulik, Belczynski & Rudak 2004).
- GC star formation rate as a function of redshift $\mathrm{SFR}_{\mathrm{GC}}(z)$ (Katz & Ricotti 2013).
- BBHs have different properties such as the delay time (time between formation and merger) and chirp mass.
- Probability of formation of BBHs as a differential probability per unit delay time and per unit chirp mass $\frac{\mathrm{d}P_{\mathrm{BBH}}}{\mathrm{d}t_{\mathrm{del}}\mathrm{d}\mathcal{M}}$
- Local merger rate density can be expressed as:

$$\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}\mathcal{M}} = \int_0^{T_{\mathrm{Hubble}}} \mathrm{d}t_{\mathrm{del}} \frac{\mathrm{SFR}_{\mathrm{GC}}(z(t_{\mathrm{del}}))}{M_{\mathrm{av}}} \frac{\mathrm{d}P_{\mathrm{BBH}}}{\mathrm{d}t_{\mathrm{del}}\mathrm{d}\mathcal{M}}$$

- For simulated models: $\frac{\mathrm{d}P_{\mathrm{BBH}}}{\mathrm{d}t_{\mathrm{del}}\mathrm{d}\mathcal{M}} = M_{\mathrm{sim}}^{-1}\sum_{i=1}^{N}\delta(t_{\mathrm{del}} - t_{\mathrm{del}}^{i})(\delta\mathcal{M} - \mathcal{M}^{i}).$

Local Merger Rate Density of BBH Mergers

- Local merger rate density can now be expressed as $\frac{d\mathcal{R}}{d\mathcal{M}} = M_{\rm sim}^{-1} \sum_{i=1}^{N} \frac{{\rm SFR}_{\rm GC}(z(t^{i}_{\rm del}))}{M_{\rm av}}$
- We find a local merger rate density of at least $5.4\ {\rm Gpc}^{-3} yr^{-1}$



- Figure: The differential rate density per unit chirp mass of coalescing BBHs.
- BH masses strongly depend on the initial mass and the stellar evolution (approximate prescriptions provided by SSE/BSE code) of BH progenitors.
- Number of merging BBHs depends on cluster metallicity and initial cluster mass.

Local Merger Rate Density of BBH Mergers

- Calculated local merger rate density can be 3 to 5 times higher:
  - Uncertainties in initial cluster mass: In order to reproduce the more massive and bright observed GCs, we will need to have initial cluster masses larger than what were simulated in the survey models (up to $10^7\ M_\odot$).
  - Recall the production efficiency:

$$Z = 0.02 \rightarrow N(M_c) = 4 \times 10^{-3} \cdot (M_c/M_\odot)^{0.5} \qquad Z < 0.02 \rightarrow N(M_c) = 7 \times 10^{-4} \cdot (M_c/M_\odot)^{0.8}$$

  - Additionally, the uncertainty in the metallicity composition of GCs in early galaxies and the uncertainties connected with stellar IMF and the maximum stellar mass may also introduce an additional increase in the merger rate.
- Expected rate of events in the first LIGO O1 run (09/2015 to Jan/2016):
  - 0.36 to 1.8 detections
  - In agreement with Rodriguez, Chatterjee & Rasio (2016).

Summary & Conclusion

- Speed of MOCCA code allows us to simulate a large number of models covering the initial parameter space.
- Escapers dominate the local merger rate density. Rate of BBHs merging inside the cluster drops significantly within 1 Gyr.
- BBH production efficiency depends on initial GC mass and metallicity.
- Merger rate density of BBHs with stellar mass and massive stellar BH components originating from GCs would not be more than $\sim 30\ \mathrm{Gpc}^{-3} yr^{-1}$
  - Compatible with the lower bound value of the estimated LIGO BBH merger rate density of $9 - 240\ \mathrm{Gpc}^{-3} yr^{-1}$ (The LIGO Scientific Collaboration 2016)
- Observed events likely to have different formation histories.