Core Collapse Times in Anisotropic Plummer Models

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Outline

- We investigate the dynamical evolution of isolated equal mass star cluster models by means of N-body simulations, primarily focusing on the effects of the presence of primordial anisotropy in the velocity space.
- We show that equilbria characterised by the same initial structural properties (Plummer density profile) and with different degrees of tangentially - biased (radially - biased) anisotropy, reach core collapse earlier (later) than isotropic models
- We interpret this result in light of an accelerated (delayed) phase of the early evolution of collisional stellar systems, which we characterise in terms of the evolution of the velocity moments

Method and initial conditions

- The initial conditions of the models presented in this study are realisations of Dejonghe's (1987) anisotropic Plummer models. The distribution function is:

$$F_q(E,L) = \frac{3\Gamma(6-q)}{2(2\pi)^{\frac{5}{2}}\Gamma(\frac{1}{2}q)} E^{\frac{7}{2}-q} H\left(0, \frac{1}{2}q, \frac{9}{2}-q, 1; \frac{L^2}{2E}\right)$$

where E is energy, L angular momentum, q is the parameter which controls the amount of anisotropy (and lies in the range $-\infty < q < 2$), Γ is the Gamma function and H is a function, which is expressible in terms of the hypergeometric function ${}_2F_1$:

$$H(a,b,c,d;x) = \begin{cases} \frac{\Gamma(a+b)}{\Gamma(c-a)\Gamma(a+d)} x^a {}_2F_1(a+b, 1+a-c; a+d; x) & x \leq 1 \\ \frac{\Gamma(a+b)}{\Gamma(d-b)\Gamma(b+c)} x^{-b} {}_2F_1(a+b, 1+b-d; b+c; x^{-1}) & x > 1 \end{cases}$$

- All models have 8K particles and for a given q value different random seeds were used to create different realisations
- All models have the same mass density profile regardless of the value of q.
- All N-body simulations were run using NBODY6 (Aarseth, 2003)
- In all figures and in the table Hénon units are used

Properties of the N-body simulations

q	N	#	t_{cc}	$2\frac{T_r}{T_\perp}$	Figs
2	8K	4	2133 ± 64	1.96	1-4
0	8K	4	1908 ± 53	1.00	1-4
-2	8K	4	1678 ± 56	0.66	1-4
-6	8K	1	1480	0.40	2-4
$-\infty$	8K	1	700	0.00	2

q parameter for the Dejonghe anisotropic Plummer model
N number of particles
t_{cc} is the average core collapse time (with the standard deviation)
$\#$ is the number of realisation
T_r total kinetic energy of radial motions
$T_\perp$ total kinetic energy of transverse motions
Figs Figures displaying the results from N-body simulations

Note that none of the models show signs of the radial orbit instability, even though q=2 is at the upper limit of the critical value (1.7 ± 0.25) found by Polyachenko and Shukhman (1981)

Radial anisotropy slows down core collapse and tangential speeds it up

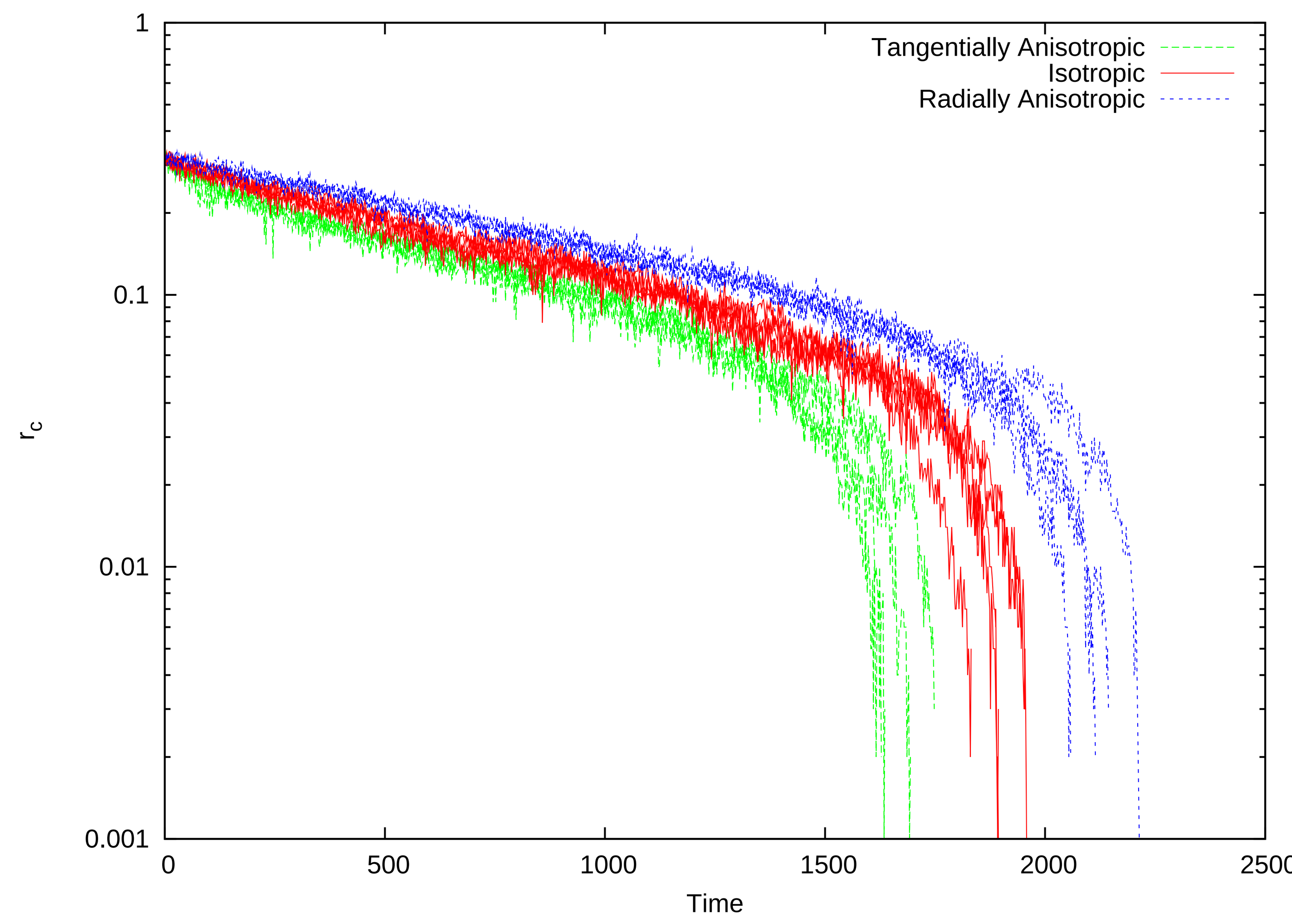

Fig. 1: Time evolution of the core radii of anisotropic Plummer models, blue lines represent radially anisotropic models (q=2), red lines represent isotropic Plummer models (q=0) and green lines represent tangentially anisotropic models (q=-2). Tangential anisotropy accelerates the core collapse while radial anisotropy slows it down. We interpreted this behaviour as a tendency for the inner part of the system to contract (or expand for the radial case) as the system relaxes towards a more isotropic velocity distribution (see box "Jeans Equations").

The effect of increasing tangential anisotropy

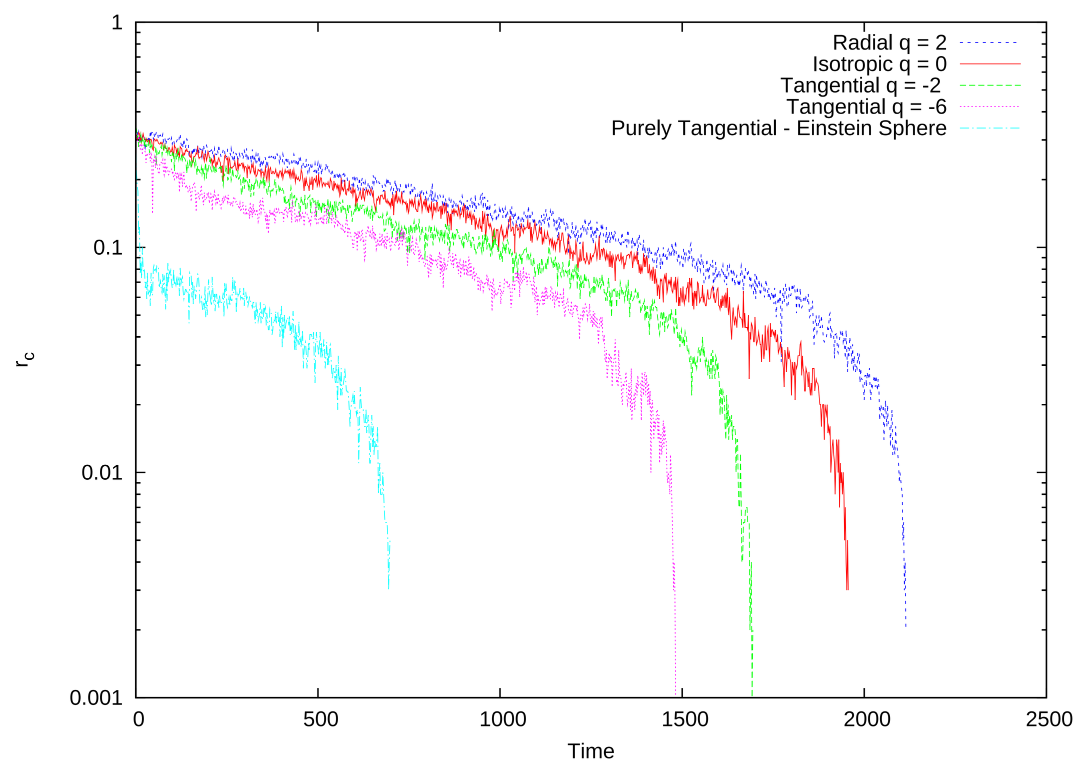

Fig. 2: Time evolution of the core radius of anisotropic models. One realisation from each group in Fig. 1 is included as well as two more tangentially anisotropic models, a q=-6 model and a model consisting entirely of circular orbits ($q \to -\infty$, also referred to as the Einstein Sphere). All models start with approximately the same value of r_c (at $t = 0$). The more tangentially anisotropic the model, the greater the acceleration of core collapse. For the case of the Einstein Sphere, core collapse is faster by almost a factor 3.

Spatial distribution of the anisotropy

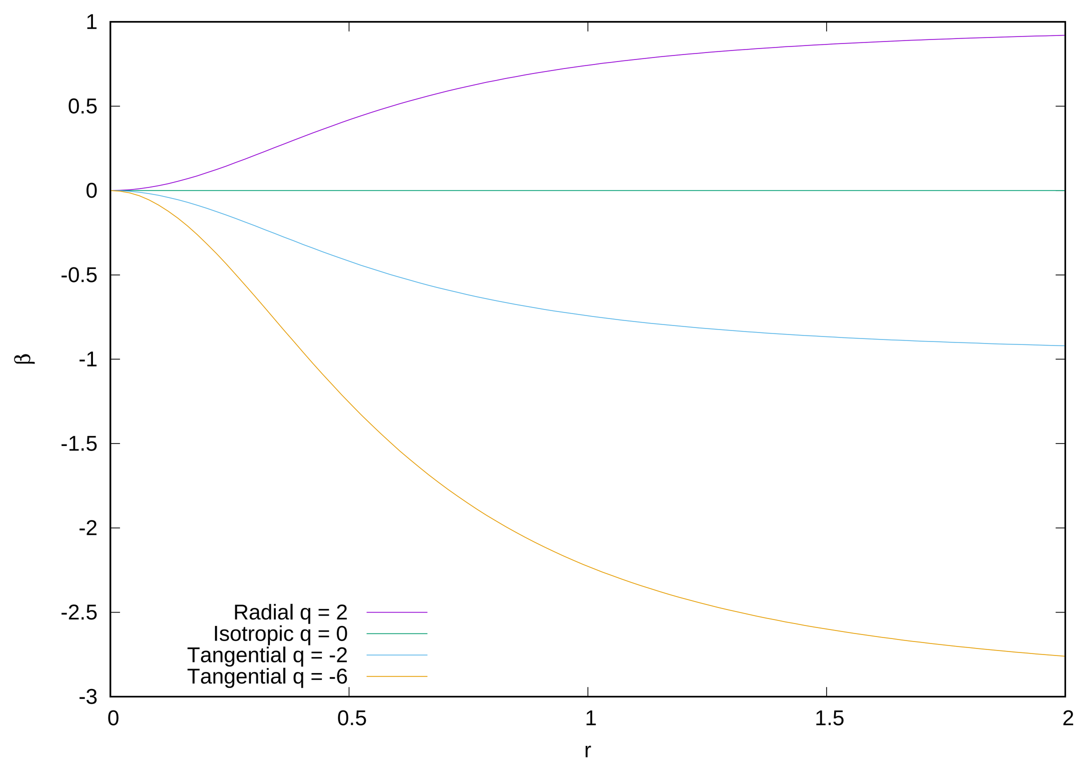

Fig. 3: Anisotropy profiles ($\beta = 1 - \frac{\sigma_t^2}{\sigma_r^2} = \frac{q}{2}\frac{\tilde{r}^2}{1+\tilde{r}^2}$, where $\tilde{r}$ is r divided by the scaled radius of the Plummer model) for four values of q. Note that curves with the same absolute value of q are symmetrical by reflection across the x-axis, therefore the spatial distribution of their anisotropy is equivalent (with opposite sign).

Jeans equations

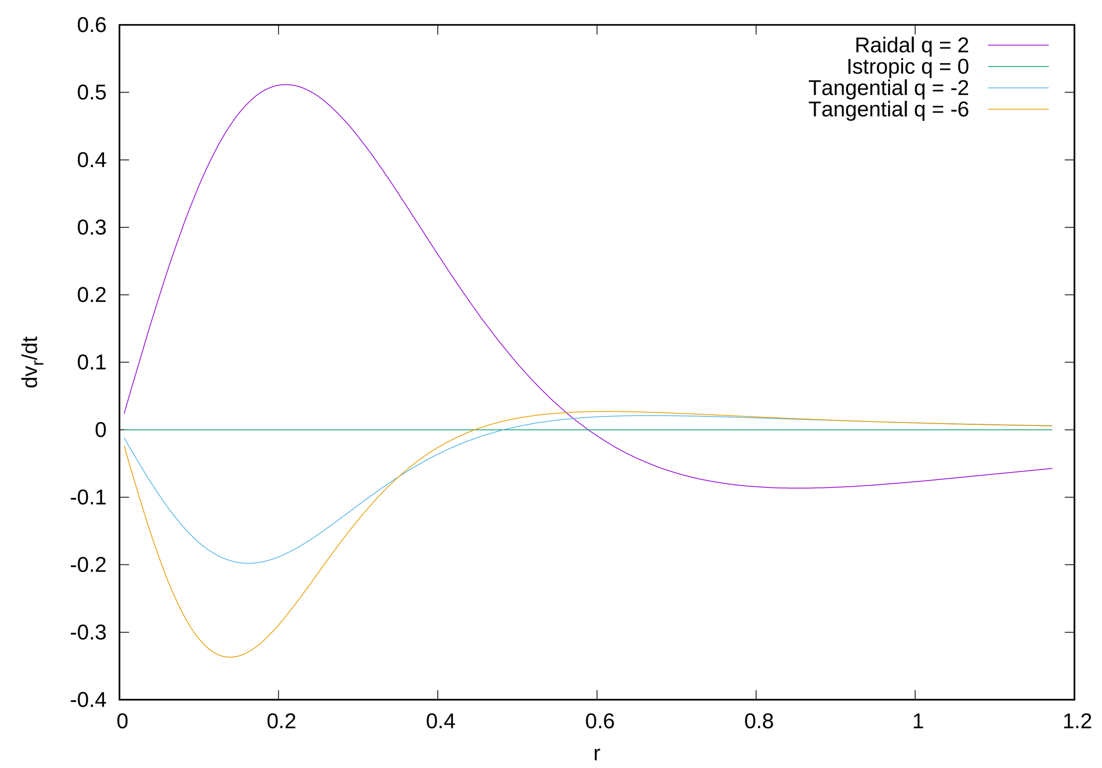

Fig. 4: The predicted response of anisotropic Plummer models, as they evolve from their initial conditions towards velocity isotropy. The curves in the above figure are calculated by using the radial Jeans equation. Assuming spherical symmetry, the radial Jeans equation in spherical coordinates can be written as

$$\frac{\partial \bar{v}_r}{\partial t} = -\frac{1}{\nu}\frac{\partial(\nu\sigma_r^2)}{\partial r} - 2\frac{(\sigma_r^2 - \sigma_t^2)}{r} - \frac{\partial \Phi}{\partial r}.$$

We evaluate this by assuming that ρ and σ_{tot}^2 (where $\sigma_{tot}^2 = \sigma_r^2 + 2\sigma_t^2$) remain constant and that, after a central relaxation time $t_r(0)$, σ_r^2 becomes $\sigma_r^2 + \frac{t_r(0)}{t_r(r)}(\frac{1}{3}\sigma_{tot}^2 - \sigma_r^2)$. For radial (tangential) anisotropy the acceleration is outward (inward).

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