

OBSERVATIONAL COSMOLOGY

PROBLEM SHEET 6 – DUE 04/06/2019

The Wilkinson Microwave Anisotropy Probe (WMAP) was a spacecraft launched in 2001 that measured the temperature anisotropies in the Cosmic Microwave Background Radiation. WMAP has been able to put constraints on several important cosmological parameters. The new Planck spacecraft was launched in 2009 with the goal to improve upon these parameter constraints with its better sensitivity. In this exercise we compare these two spacecrafts in a realistic setting.

The Fisher information matrix for the CMB power spectrum can be written as:

$$F_{ij} = \sum_{\ell=2}^{\ell_{\max}} \frac{(2\ell + 1)f_{\text{sky}}}{2} \left[C_{\ell} + \frac{4\pi}{N} \sigma_N^2 e^{\ell(\ell+1)\sigma_b^2} \right]^{-2} \frac{\partial C_{\ell}}{\partial s_i} \frac{\partial C_{\ell}}{\partial s_j}, \quad (1)$$

where

- s_i, s_j are the parameters;
- the ℓ 's are the multipole indices;
- C_{ℓ} is the angular power spectrum;
- σ_N is the pixel noise;
- $\frac{4\pi}{N} = 1.13(\text{FWHM})^2$ is the beam made by one pixel (FWHM is the full width at half maximum of the beam);
- $\sigma_b = \frac{\text{FWHM}}{2.35}$ is the width of the Gaussian beam;
- f_{sky} is the fractional sky coverage (assume 0.8 for both)

Below is a table with the WMAP and Planck instrument characteristics, for their channels at 94 GHz and 100 GHz, respectively: Using the online tool of CAMB¹ to obtain

Mission	$\sigma_N(\mu K)$	FWHM (arcmin)
WMAP	49	13
Planck	7.6	9.2

the C_{ℓ} 's, plot the joint probability contours at 68.3% confidence level for a couple of cosmological parameters (e.g. $\Omega_b - H_0$) and compare the WMAP result with the forecast for Planck.

¹https://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm

Hints:

- in CAMB, if you uncheck lensing, you get an output with three scalar C_l 's: the TT, EE and TE power spectra. You need the TT spectrum which is already in the correct units (μK^2). (You can check the CAMB notes if interested in the meaning of various input parameters)
- in equation (1) FWHM has to be expressed in radians
- since you do not have an analytic expression for the C_ℓ 's as a function of the parameters, to make the derivatives for the Fisher matrix you have to generate different spectra with different values of the parameters and approximate

$$\left. \frac{\partial C_\ell}{\partial s_i} \right|_{\bar{s}} \approx \frac{C_\ell(\bar{s} + \Delta s) - C_\ell(\bar{s} - \Delta s)}{2\Delta s}. \quad (2)$$

- The inverse of Fisher matrix is the parameter covariance matrix

$$[F]^{-1} = [C] = \begin{bmatrix} \sigma_x^2 & \sigma_{xy} \\ \sigma_{xy} & \sigma_y^2 \end{bmatrix} \quad (3)$$

- The lengths of the semiaxes (a,b) and the inclination (θ) of the error ellipse in the 2-parameter space are calculated as follows:

$$a^2 = \frac{\sigma_x^2 + \sigma_y^2}{2} + \sqrt{\frac{(\sigma_x^2 - \sigma_y^2)^2}{4} + \sigma_{xy}^2} \quad (4)$$

$$b^2 = \frac{\sigma_x^2 + \sigma_y^2}{2} - \sqrt{\frac{(\sigma_x^2 - \sigma_y^2)^2}{4} + \sigma_{xy}^2} \quad (5)$$

$$\tan 2\theta = \frac{2\sigma_{xy}}{\sigma_x^2 - \sigma_y^2} \quad (6)$$