OBSERVATIONAL COSMOLOGY

PROBLEM SHEET 3 – DUE 30/04/2019

1) Consider N independent data points sampled from a Gaussian distribution of unknown parameters μ and σ^2, where μ is the mean and σ^2 denotes the variance. After building the likelihood function for the model parameters μ and σ^2, derive the expressions for their Maximum Likelihood Estimators $\hat{\mu}$ and $\hat{\sigma}^2$. Prove that $\hat{\sigma}^2$ is a biased estimator. Then show that both MLEs are asymptotically unbiased. Finally, calculate the Fisher information matrix and demonstrate that the variance of $\hat{\mu}$ satisfies the Cramér-Rao bound.

2) Supernovae Ia are used to derive the Hubble diagram and provide strong evidence for the acceleration of cosmic expansion. The key fact is that they are *standardizable candles*. In fact, observing their light curves (i.e. the evolution of light intensity as a function of time) an empirical relationship was found between the maximum luminosity and the width of the curve itself. It can be expressed as

$$M_{max} = a \cdot m_{15} + b \,,$$

where M_{max} is the peak magnitude and m_{15} is the decline of the magnitude 15 days after it has reached its peak.

We want to design a new experiment to measure a and b with great accuracy. For simplicity, let us assume that we can compress the supernova data in a set of bins which are spaced by $\Delta m_{15} = 1$ (using some specific units). We expect statistical errors on M_{max} of $\sigma_M = 0.1$, which are Gaussian distributed and independent. Using the Fisher matrix method, determine the **number of bins** we need to consider to get marginal errors $\sigma_a = 0.01$ and $\sigma_b = 0.01$ on the linear fit parameters a and b.

Hints:

$$\sum_{n=1}^{k} n = \frac{k(k+1)}{2}$$

$$\sum_{n=1}^{k} n^2 = \frac{k(k+1)(2k+1)}{6}$$