OBSERVATIONAL COSMOLOGY

Problem sheet 1 – Due 11/04/2019

Using their integral form, plot the luminosity distance $D_\mathrm{L}(z)$ and the angular-diameter distance $D_\mathrm{A}(z)$ versus the redshift for different values of the curvature parameter (i.e. open, close or flat universe) and different choices for Ω_m, Ω_Λ, Ω_r.

$$D_\mathrm{H} = \frac{c}{H_0} \tag{1}$$

$$E(z)^2 = \Omega_\mathrm{r}(1+z)^4 + \Omega_\mathrm{m}(1+z)^3 + \Omega_\mathrm{k}(1+z)^2 + \Omega_\lambda \tag{2}$$

$$D_\mathrm{C}(z) = D_\mathrm{H} \cdot \int_0^z \frac{\mathrm{d}z'}{E(z')} \tag{3}$$

$$D_\mathrm{M}(z) = \begin{cases} \frac{D_\mathrm{H}}{\sqrt{\Omega_\mathrm{k}}} \sinh\left(\sqrt{\Omega_\mathrm{k}} \frac{D_\mathrm{C}(z)}{D_\mathrm{H}}\right) & \Omega_\mathrm{k} > 0 \\ D_\mathrm{C}(z) & \Omega_\mathrm{k} = 0 \\ \frac{D_\mathrm{H}}{\sqrt{|\Omega_\mathrm{k}|}} \sin\left(\sqrt{|\Omega_\mathrm{k}|} \frac{D_\mathrm{C}(z)}{D_\mathrm{H}}\right) & \Omega_\mathrm{k} < 0 \end{cases} \tag{4}$$

$$D_\mathrm{L}(z) = (1+z) \cdot D_\mathrm{M}(z) \tag{5}$$

$$D_\mathrm{A}(z) = \frac{D_\mathrm{M}(z)}{1+z} \tag{6}$$

Remember that $\Omega_\mathrm{r} + \Omega_\mathrm{m} + \Omega_\mathrm{k} + \Omega_\Lambda = 1$; note that for an open universe ($k = -1$) $\Omega_\mathrm{k} > 0$, for a closed one ($k = 1$) $\Omega_\mathrm{k} < 0$ and for a flat one ($k = 0$) $\Omega_\mathrm{k} = 0$. You can also try and use different values for H_0 and see what happens.