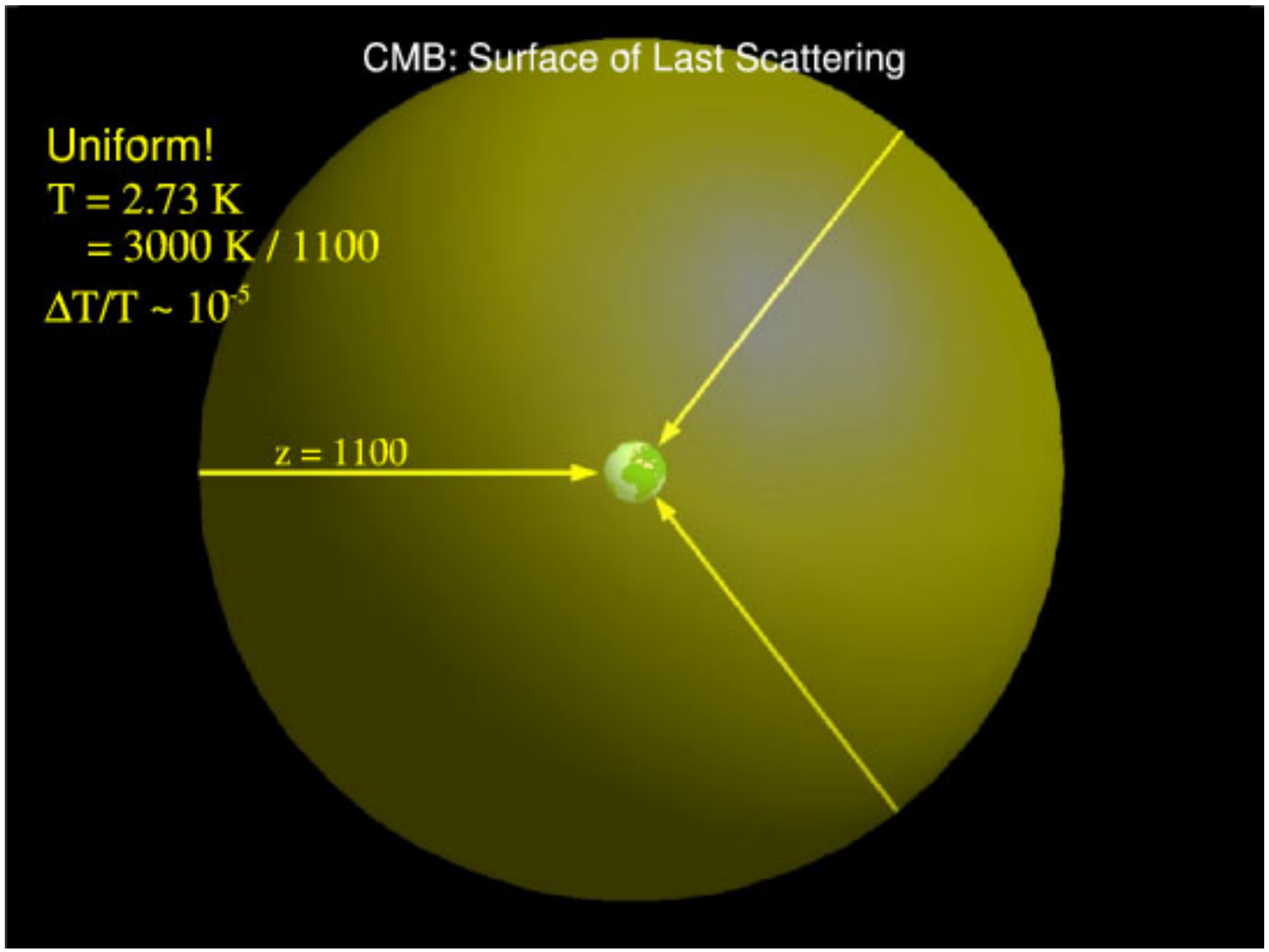
Observational Cosmology

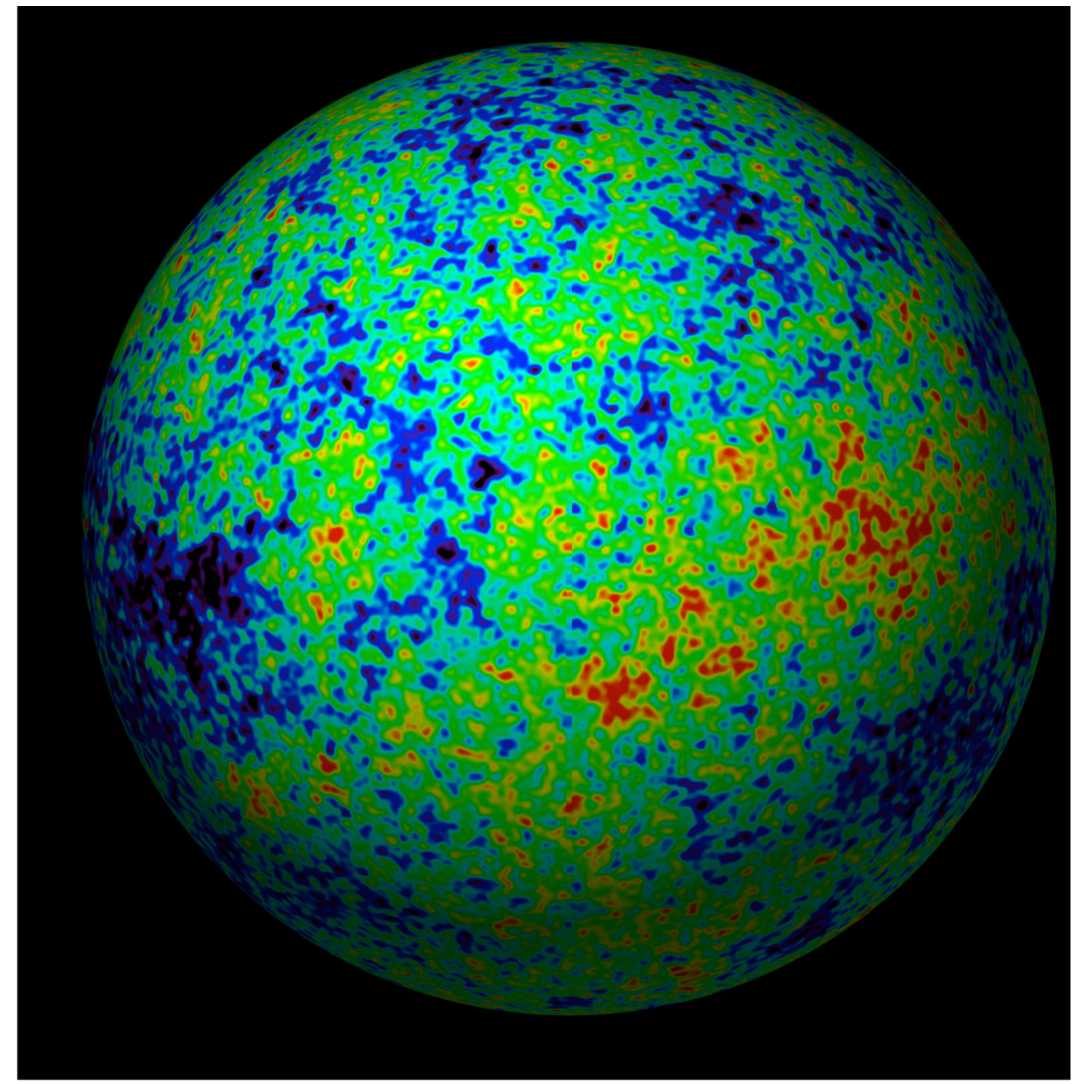
The Cosmic Microwave Background
Part II: Temperature Anisotropies

Kaustuv Basu

Course website:
https://www.astro.uni-bonn.de/~kbasu/ObsCosmo

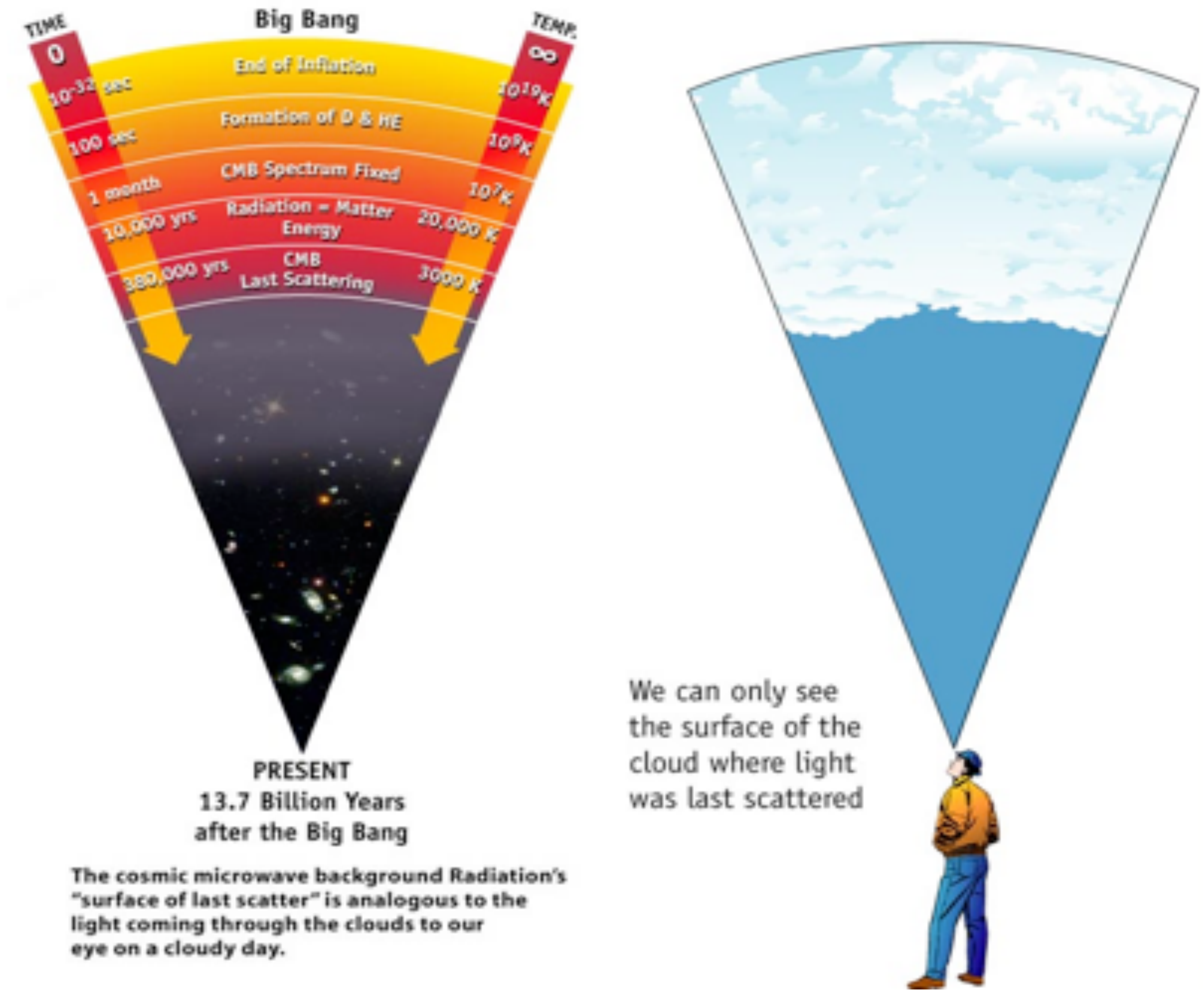
The last scattering "sphere"





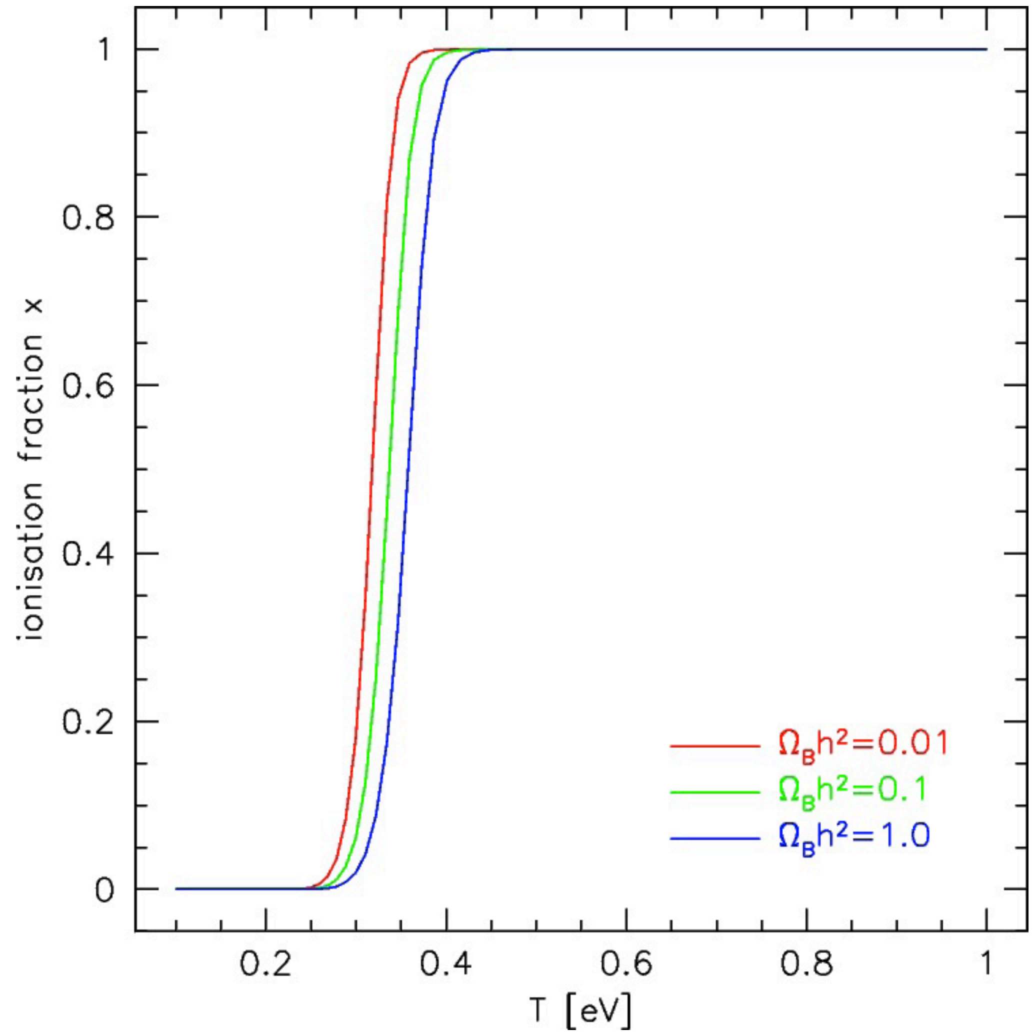
anisotropy amplitude $\Delta T/T \sim 10^{-5}$

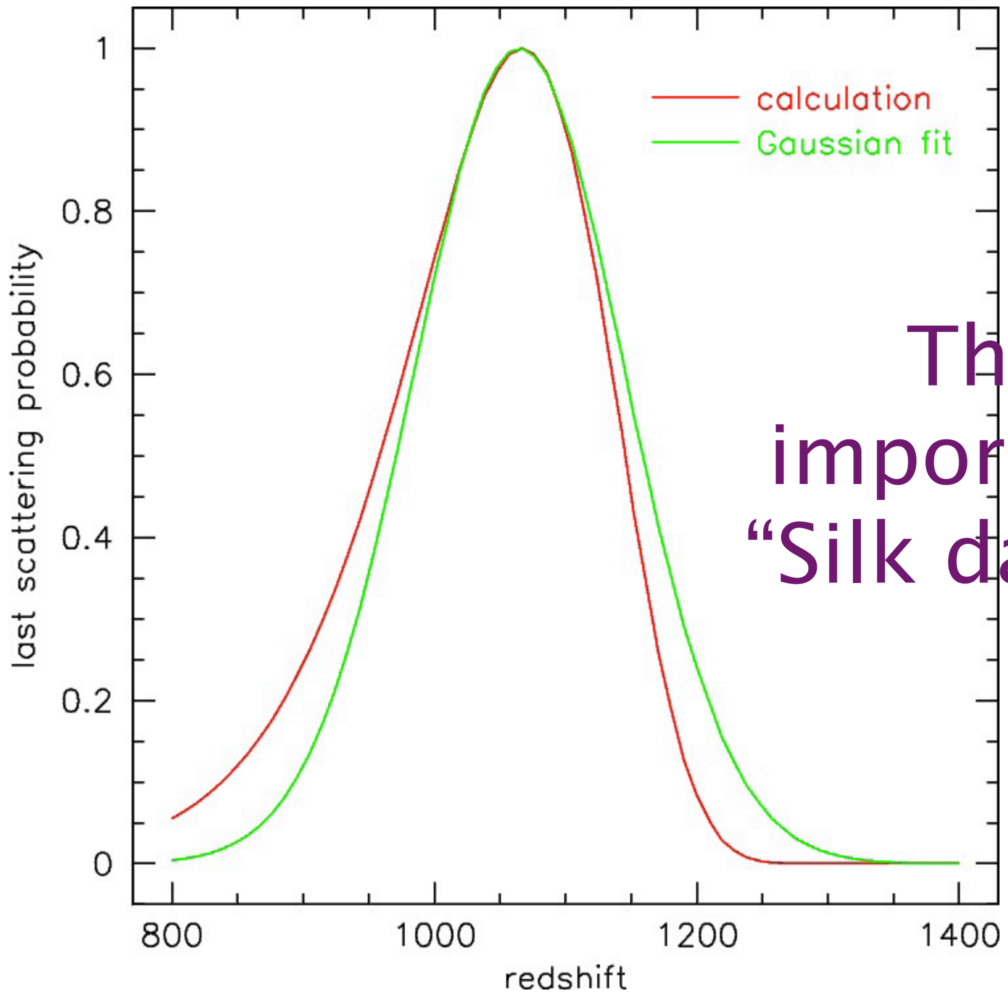
The Last Scattering Surface



All photons have travelled roughly the same distance since recombination. We can think of the CMB being emitted from inside of a spherical surface, we're at the center. (This surface has a thickness)

Thickness of the last scattering surface



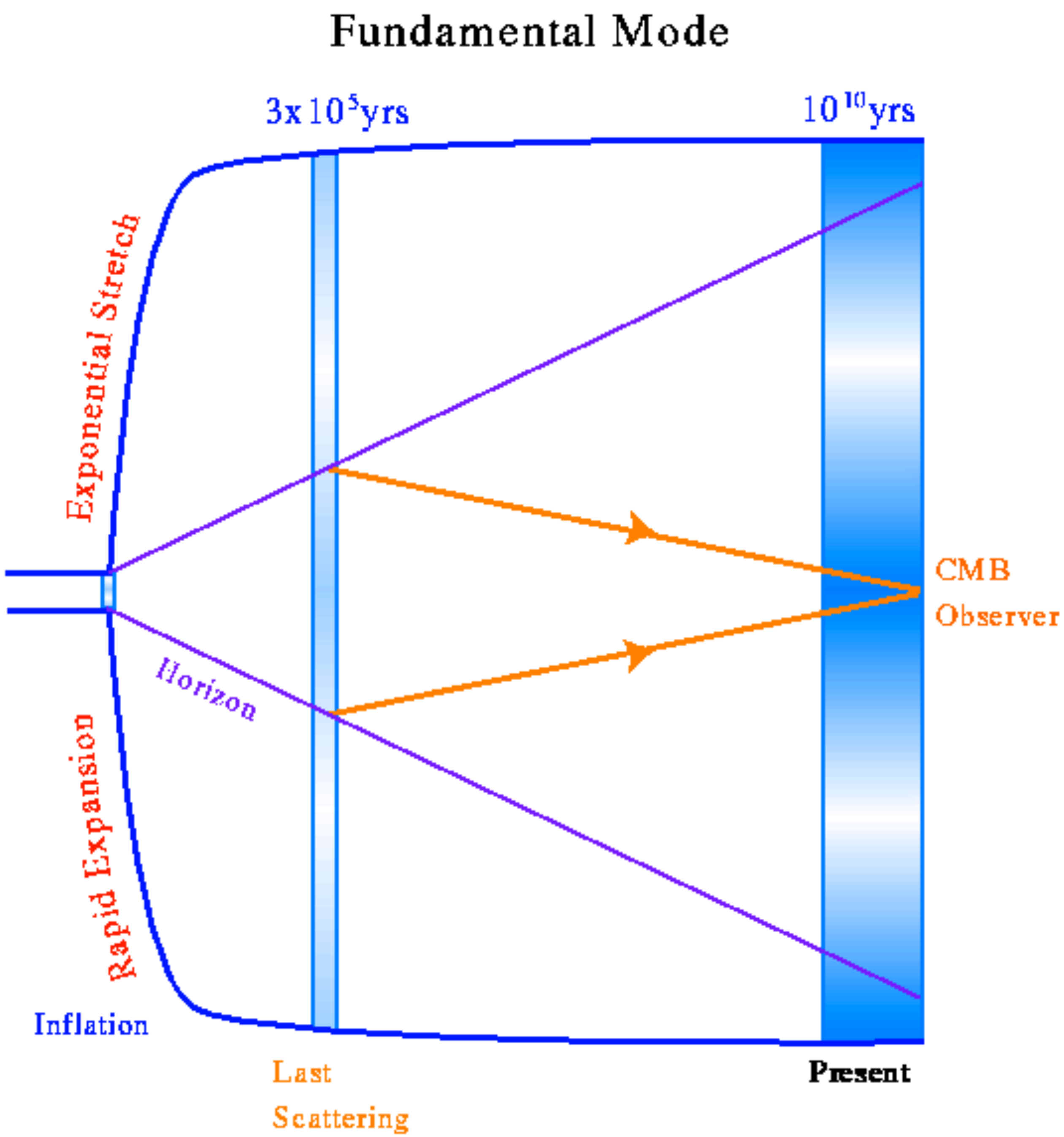


This is important for "Silk damping"

The visibility function is defined as the probability density that a photon is last scattered at redshift z: $g(z) \sim \exp(-\tau)\, d\tau/dz$

Probability distribution is well described by Gaussian with mean $z \sim 1100$ and standard deviation $\delta z \sim 80$.

Horizon scale at Last Scattering



Distance to the last scattering surface (assume Ω_m=1 EdS universe)

$$r_{LS} = \frac{c}{H_0}\int_0^{z_{LS}} (1+z)^{-3/2} dz$$
$$= \frac{2c}{H_0}(1-(1+z_{LS})^{-1/2})$$

Thus, the factor $2c/H_0$ is approximately the comoving distance to the LSS ($z_{LS}>>1$).

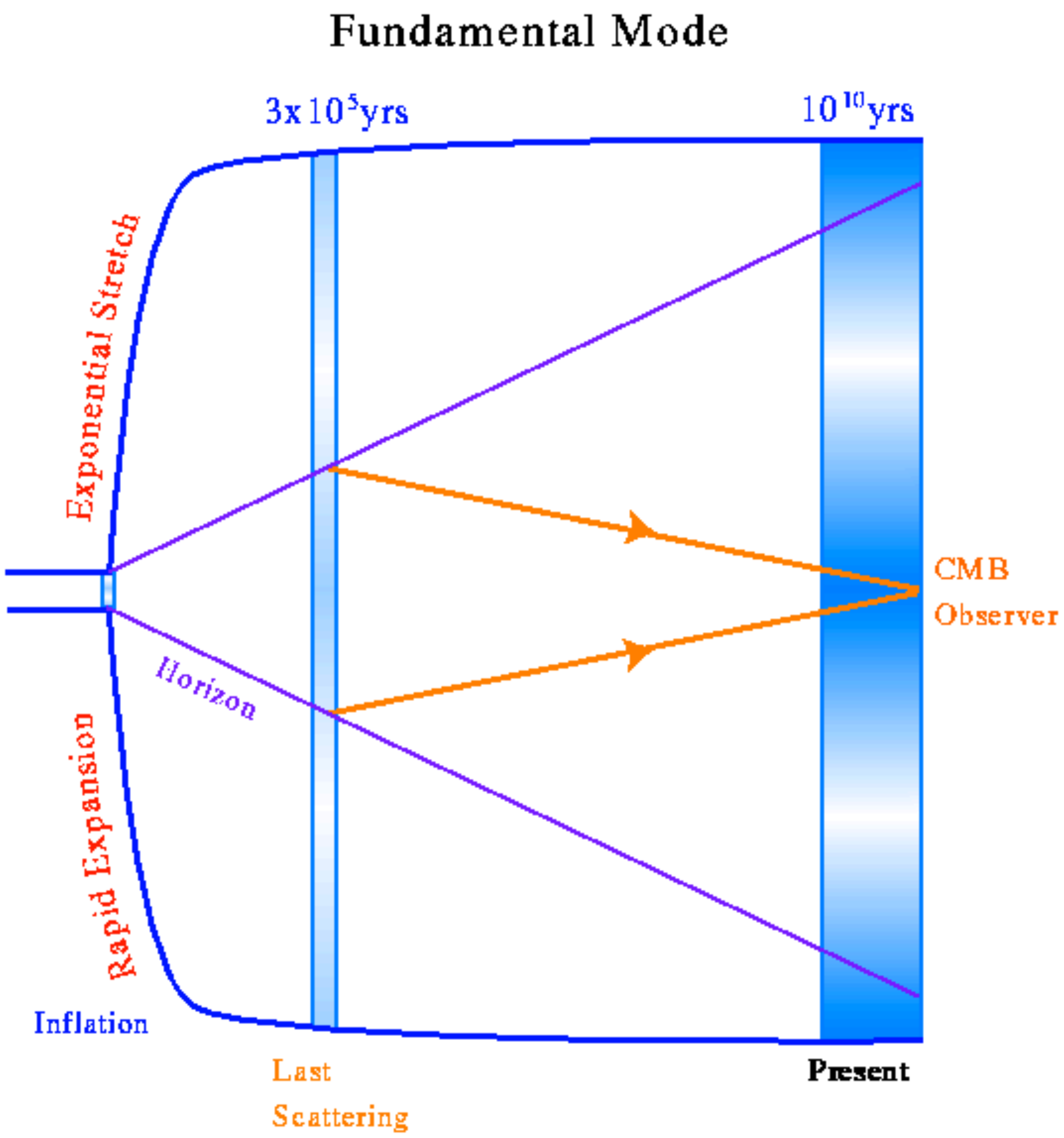
The particle horizon length at the time of last scattering (i.e. the distance light could travel since big bang) is give by

$$d_H(z=z_{LS}) = \int_{z_{LS}}^{\infty} \frac{dz}{H(z)} = \frac{2c}{H_0}(1+z_{LS})^{-1/2}$$

for $z_{LS} \approx 1100$, means that

$$\theta_H^{LS} = (1+z_{LS})^{-1/2} \approx 1.7^\circ$$

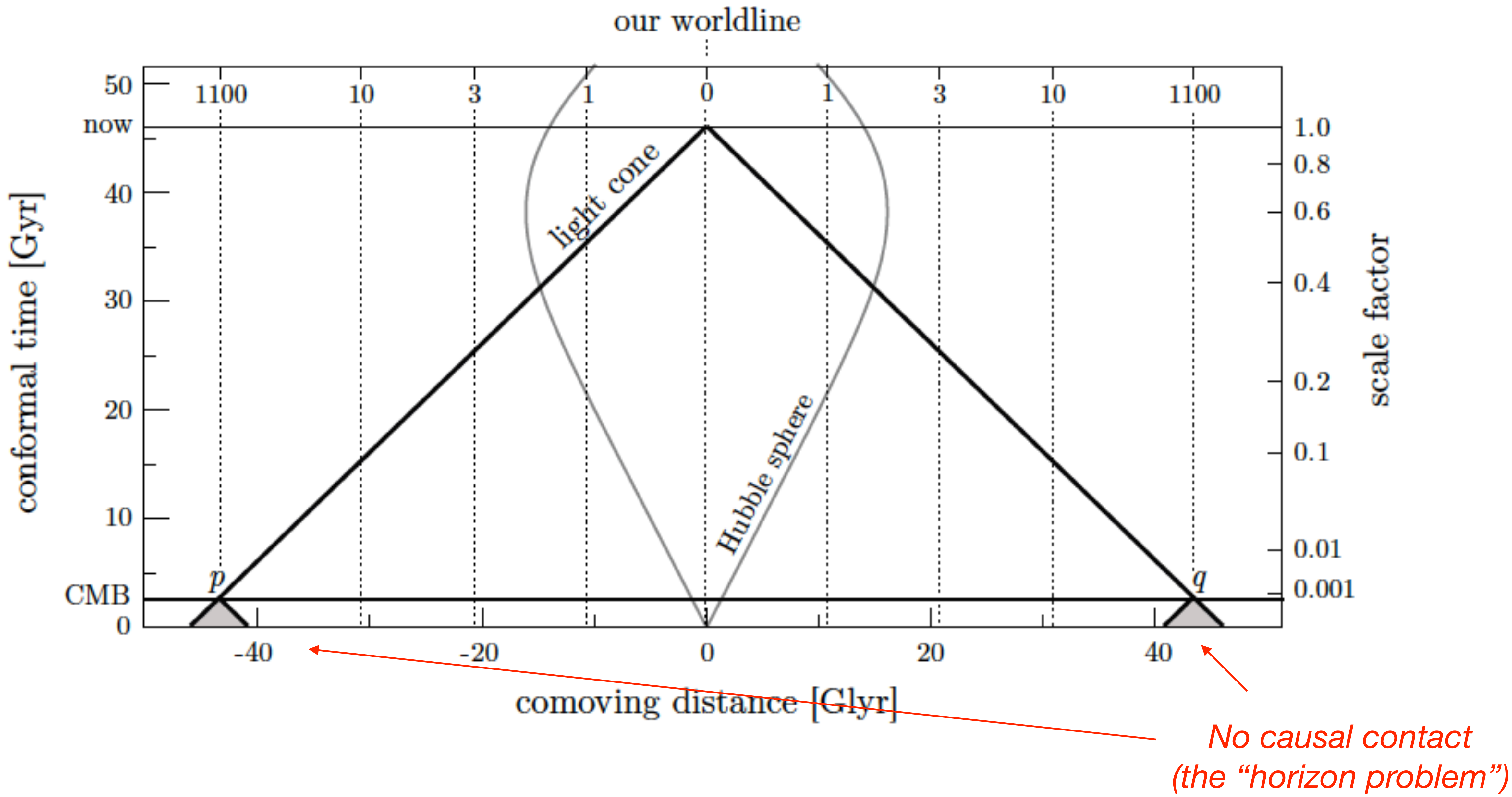
Horizon scale at Last Scattering



This tells us that scales larger than ~1.7° in the sky were not in causal contact at the time of last scattering. However, the fact that we measure the same mean temperature across the entire sky suggests that all scales were once in causal contact – this led to the idea of Inflation.

Inflationary theories suggests that the Universe went through a period of very fast expansion, which would have stretched a small, causally connected patch of the Universe into a region of size comparable to the size of the observable Universe today.

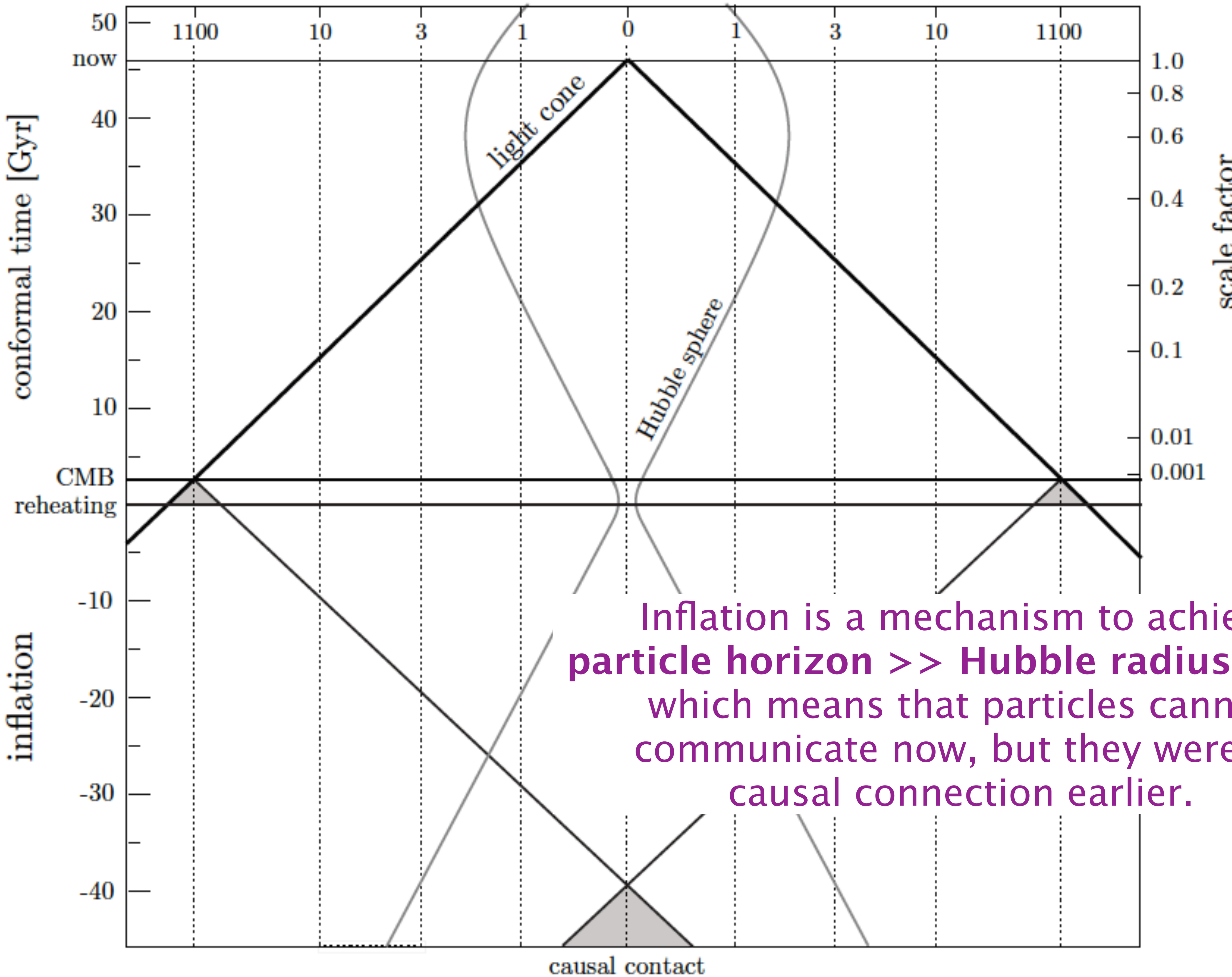
Inflation & the horizon problem



Inflationary solution to the horizon problem: The comoving Hubble sphere shrinks during inflation and expands during the conventional Big Bang evolution (at least until dark energy takes over at $a \approx 0.5$). **Conformal time during inflation is negative (i.e. there is time before and after $\tau=0$).** The spacelike singularity of the standard Big Bang ($\tau=0$) is replaced by the reheating surface. All points in the CMB have overlapping past light cones and therefore originated from a causally connected region of space.

Image credit: Daniel Baumann
(see www.damtp.cam.ac.uk/user/db275/Cosmology/Lectures.pdf)

Inflation & the horizon problem



Inflation is a mechanism to achieve
particle horizon >> Hubble radius $(aH)^{-1}$
which means that particles cannot communicate now, but they were in causal connection earlier.

Image credit: Daniel Baumann
(see www.damtp.cam.ac.uk/user/db275/Cosmology/Lectures.pdf)

Predictions of inflation

Inflationary models make **specific set of predictions that can be verified with CMB data:**

- Small spacial curvature
- Nearly scale-invariant spectrum of density perturbations
- CMB temperature anisotropies from large to small angular scales (Sachs-Wolfe effect and acoustic peaks)
- Gaussian perturbations
- Existence of primordial gravity waves!

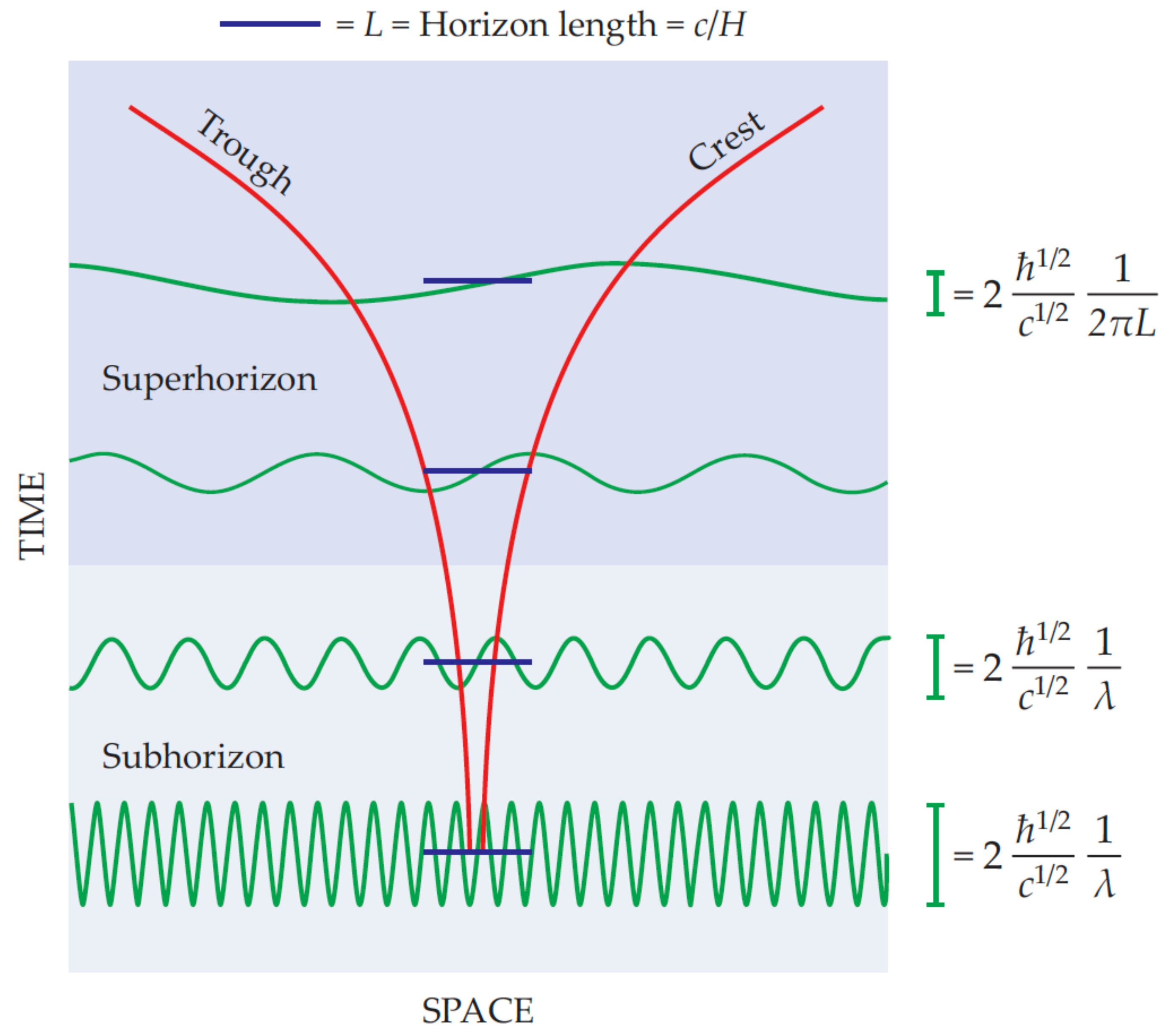
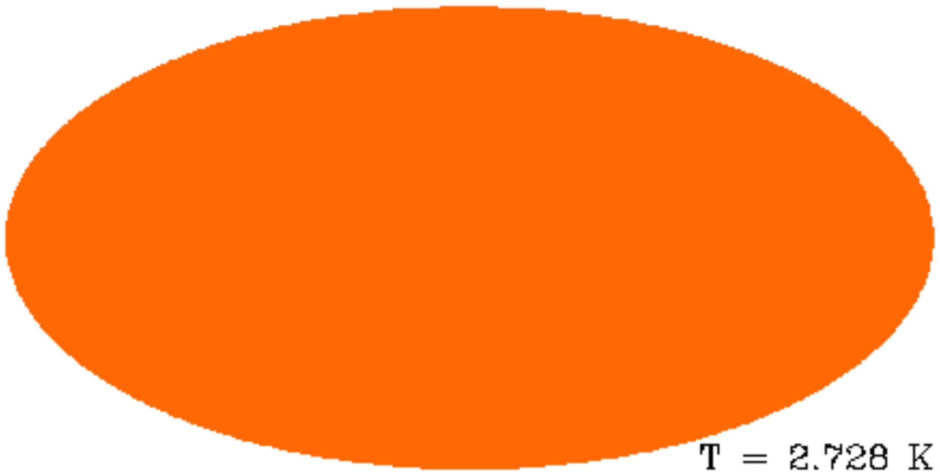
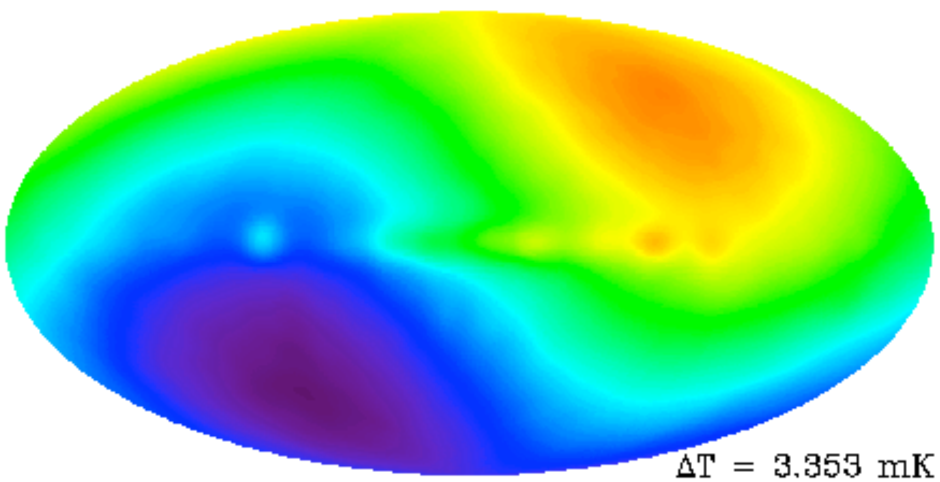


Figure from Carlstrom, Crawford & Knox (2015), Physics Today

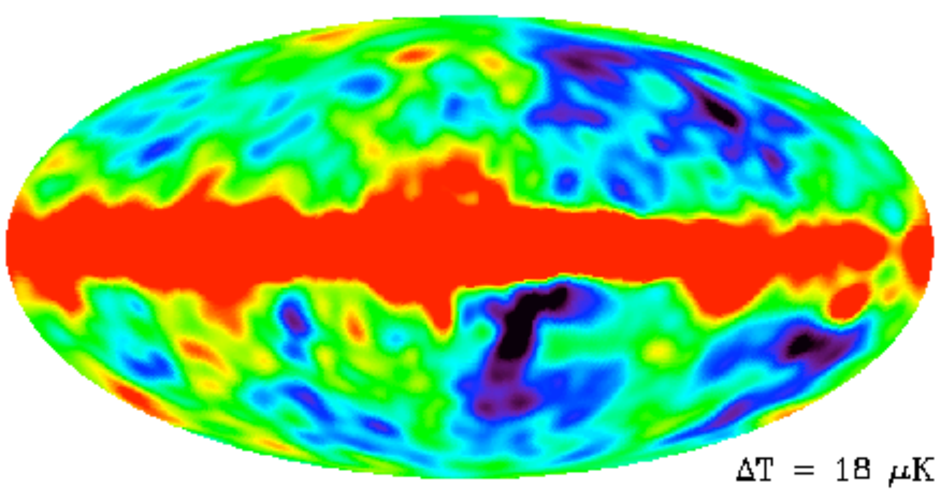
Amplitude of temp. anisotropies



CMB is primarily a uniform glow across the sky!



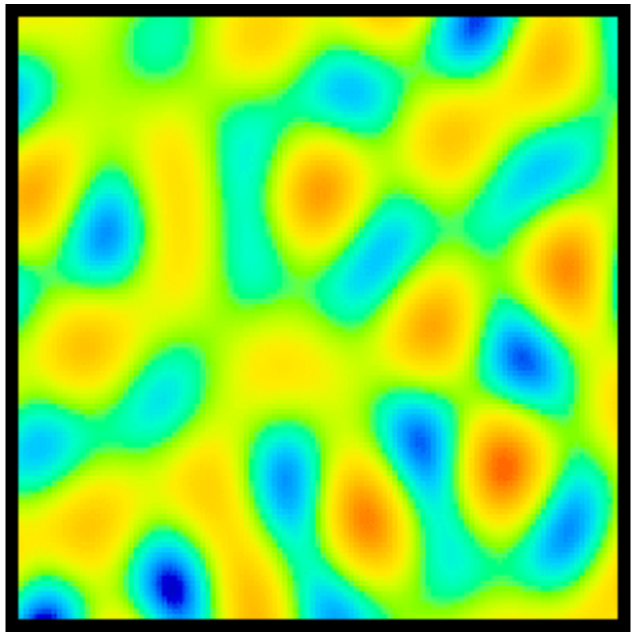
Turning up the contrast, dipole pattern becomes prominent at a level of 10^{-3}. This is from the motion of the Sun relative to the CMB.



Enhancing the contrast further (at the level of 10^{-5}, and after subtracting the dipole, temperature anisotropies appear.

Anisotropy amplitude

From the fact that non-linear structures exist today in the Universe, the linear growth theory predicts that density perturbations at z = 1100 (the time of CMB release) must have been of the order of



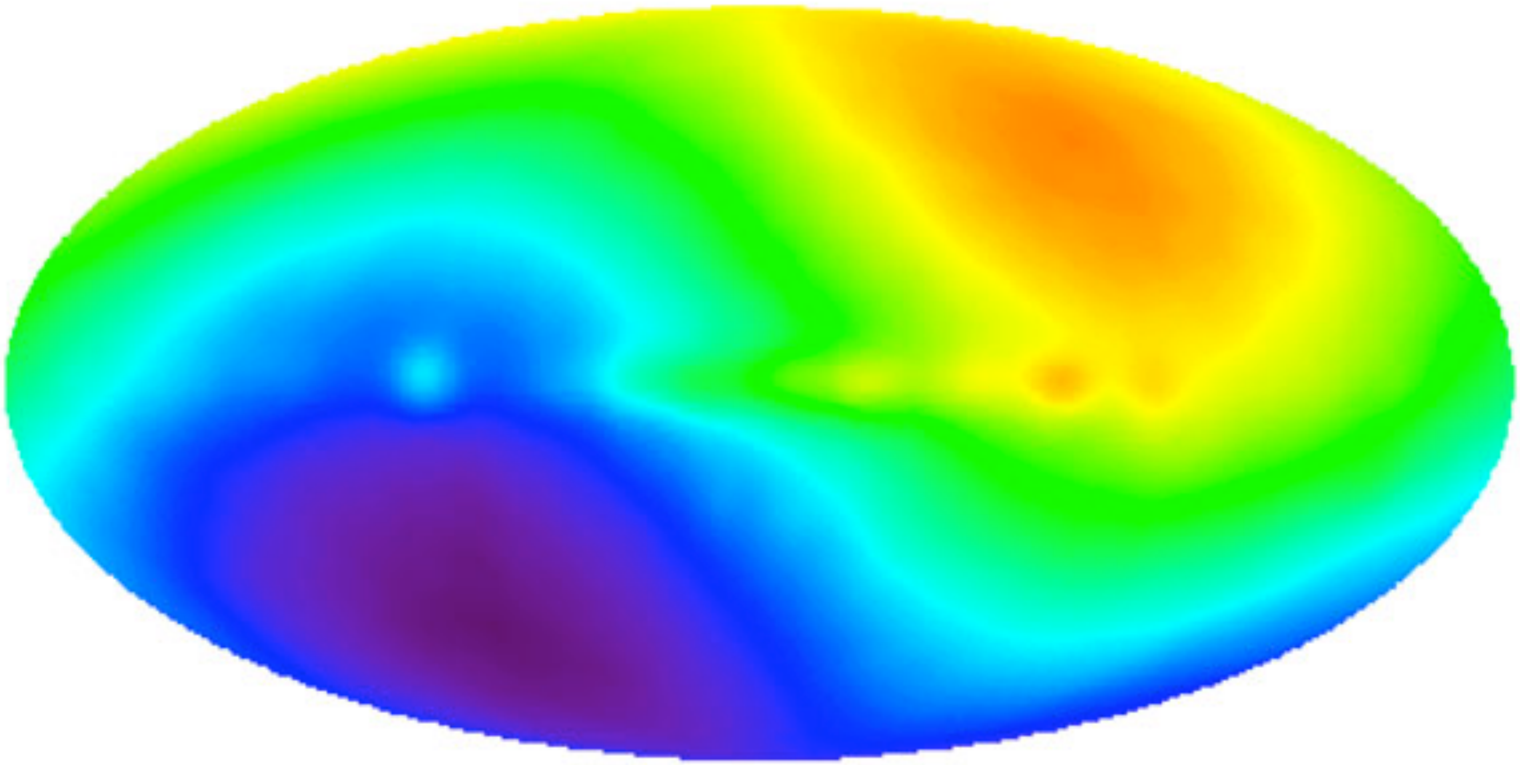
$$\delta(a_{\rm CMB}) = \frac{\delta(a=1)}{D_+(a_{\rm CMB})} \gtrsim 10^{-3}$$

After the CMB was found in 1965, fluctuations were sought at the relative level of 10^{-3}, but they were not found. Eventually they were found at a level of 10^{-5}.

The reason is that density contrast we see *today* is dominated by dark matter, while the CMB temperature differences are couple to baryons. DM perturbations grow independently of the baryons. While the radiation-Baryon fluid oscillated and therefore didn't grow in amplitude, the DM perturbations continued to grow. Since DM has no coupling to the electromagnetic spectrum, nor to the baryons, this growth happened without pumping the perturbations in the CMB to equal levels.

In fact, this can be seen as a proof that a *baryonic universe* can not form the present structures, and a non-interacting form of matter must dominate!

The CMB dipole



$$I'(\nu')=(1+(v/c)\cos\theta)^3 I(\nu)$$
$$\nu'=(1+(v/c)\cos\theta)\,\nu$$
$$T(\theta)=T\,(1+(v/c)\cos\theta)$$

- Measured velocity: 390±30 km/s
- After subtracting out the rotation and revolution of the Earth, the velocity of the Sun in the Galaxy and the motion of the Milky Way in the Local Group one finds: v = 627 ± 22 km/s
- Towards Hydra-Centaurus, l=276±3° b=30±3°

Can we measure an intrinsic CMB dipole ?

Modulation and aberration of anisotropies

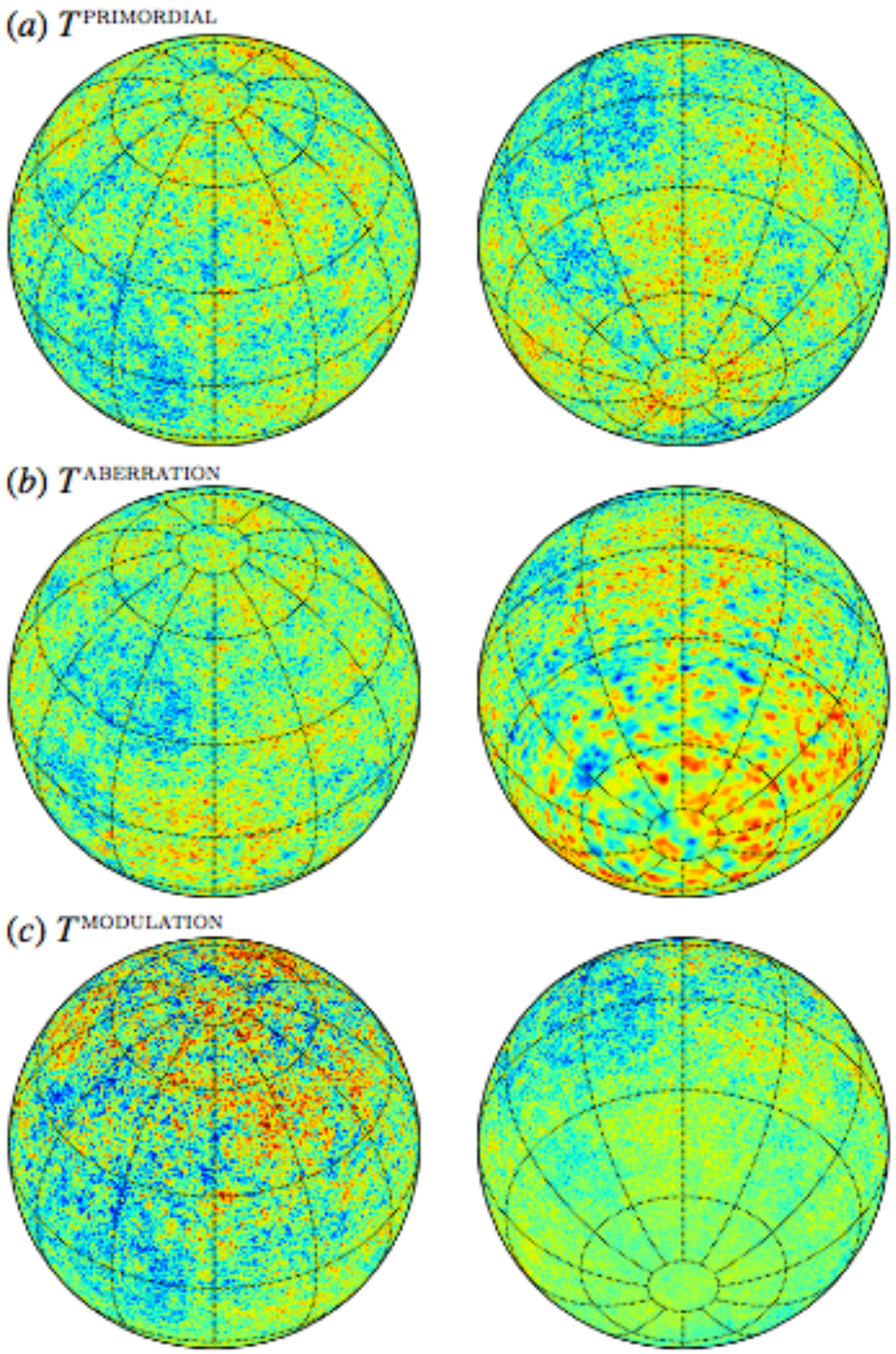
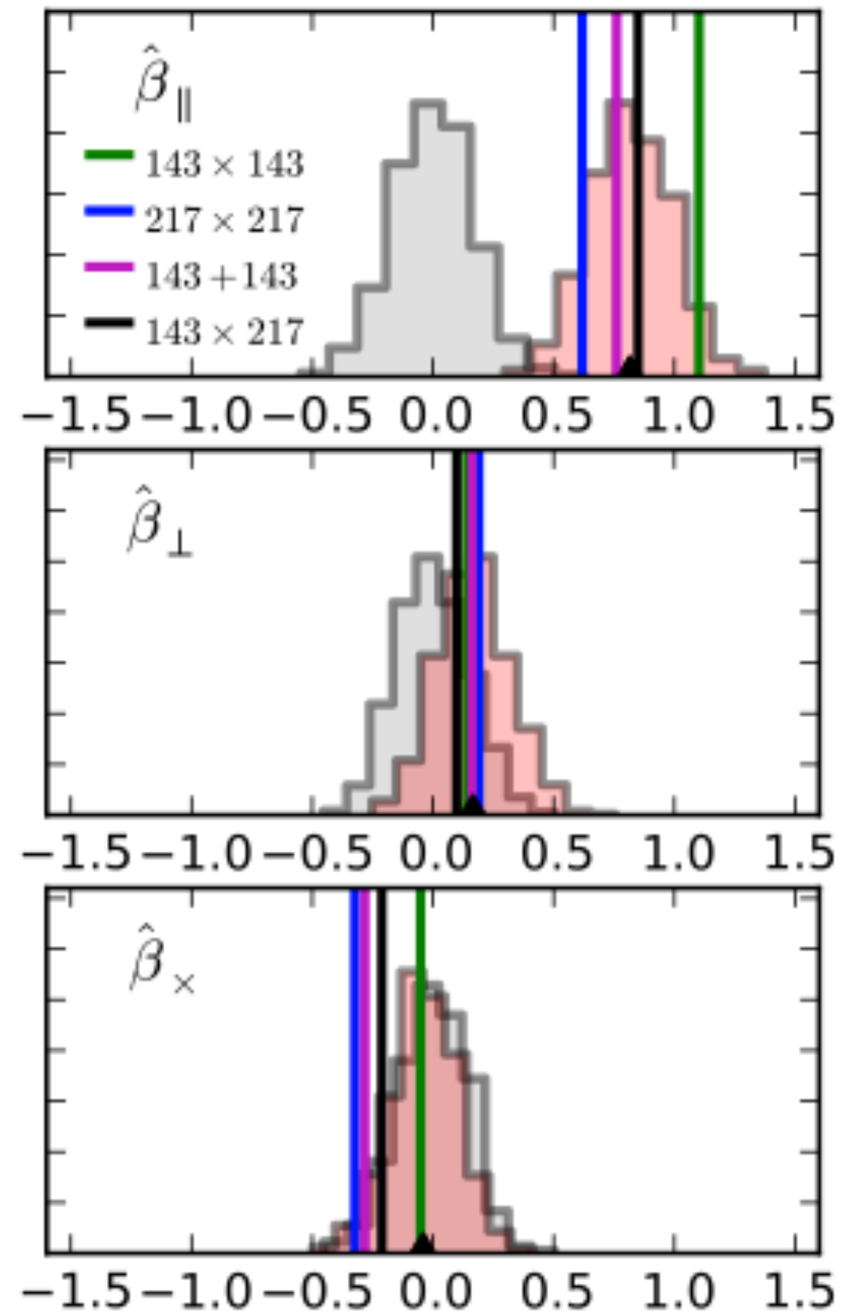


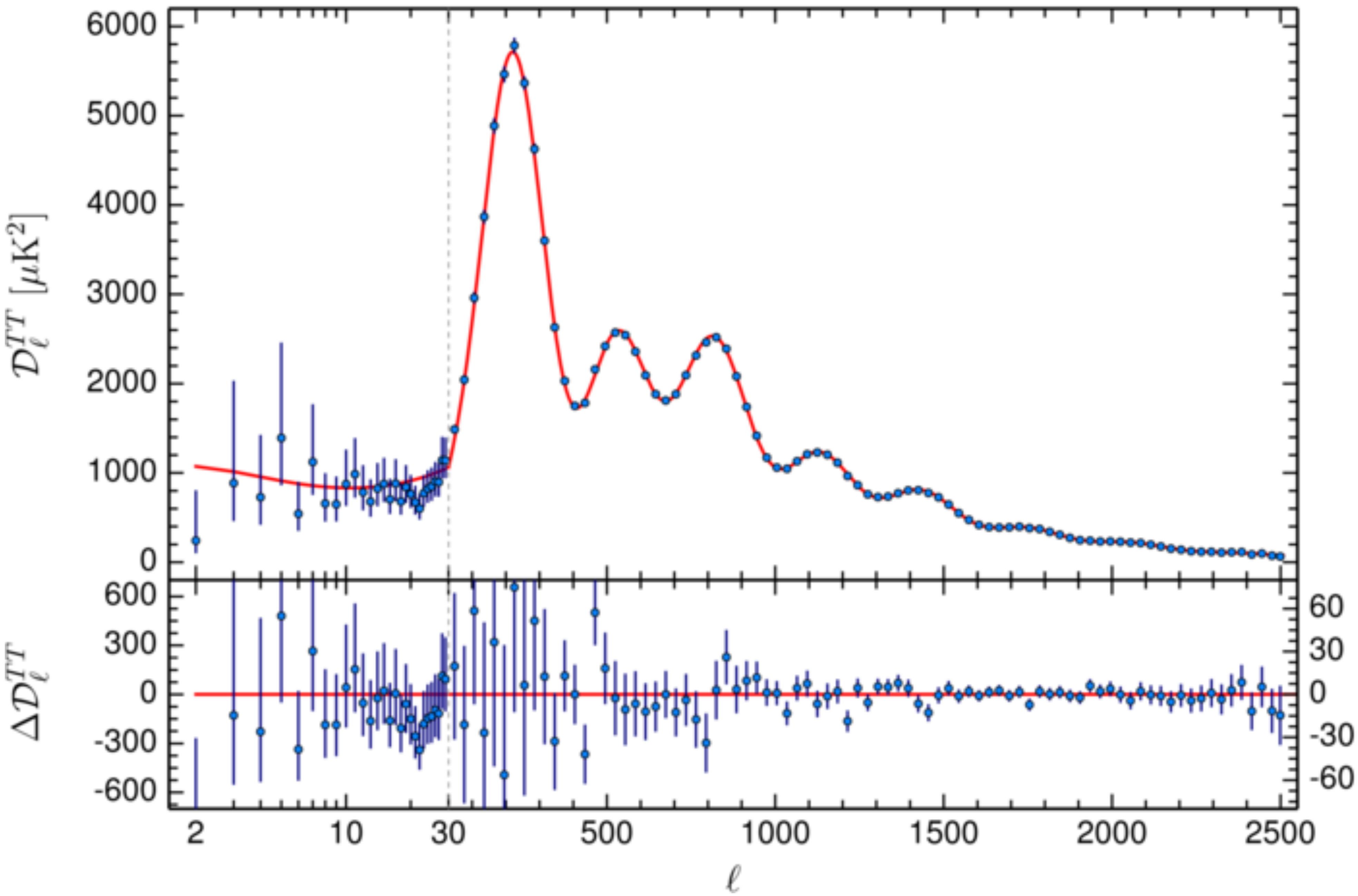
Fig. 1. Exaggerated illustration of the Doppler aberration and modulation effects, in orthographic projection, for a velocity $v = 260\,000\,\mathrm{km\,s^{-1}} = 0.85c$ (approximately 700 times larger than the expected magnitude) toward the northern pole (indicated by meridians in the upper half of each image on the left). The aberration component of the effect shifts the apparent position of fluctuations toward the velocity direction, while the modulation component enhances the fluctuations in the velocity direction and suppresses them in the anti-velocity direction.

Both these are level ~β effects. Modulation is the same as dipole, but working on ΔT instead of T_0. Aberration is a relativistic shift in position.



Planck collaboration (2013)

Temperature anisotropies: The CMB power spectrum



Planck 2015 result

CMB temperature anisotropies

- The basic observable is the CMB intensity as a function of frequency and direction on the sky. Since the CMB spectrum is an extremely good black body with a fairly constant temperature across the sky, we generally describe this observable in terms of a temperature fluctuation

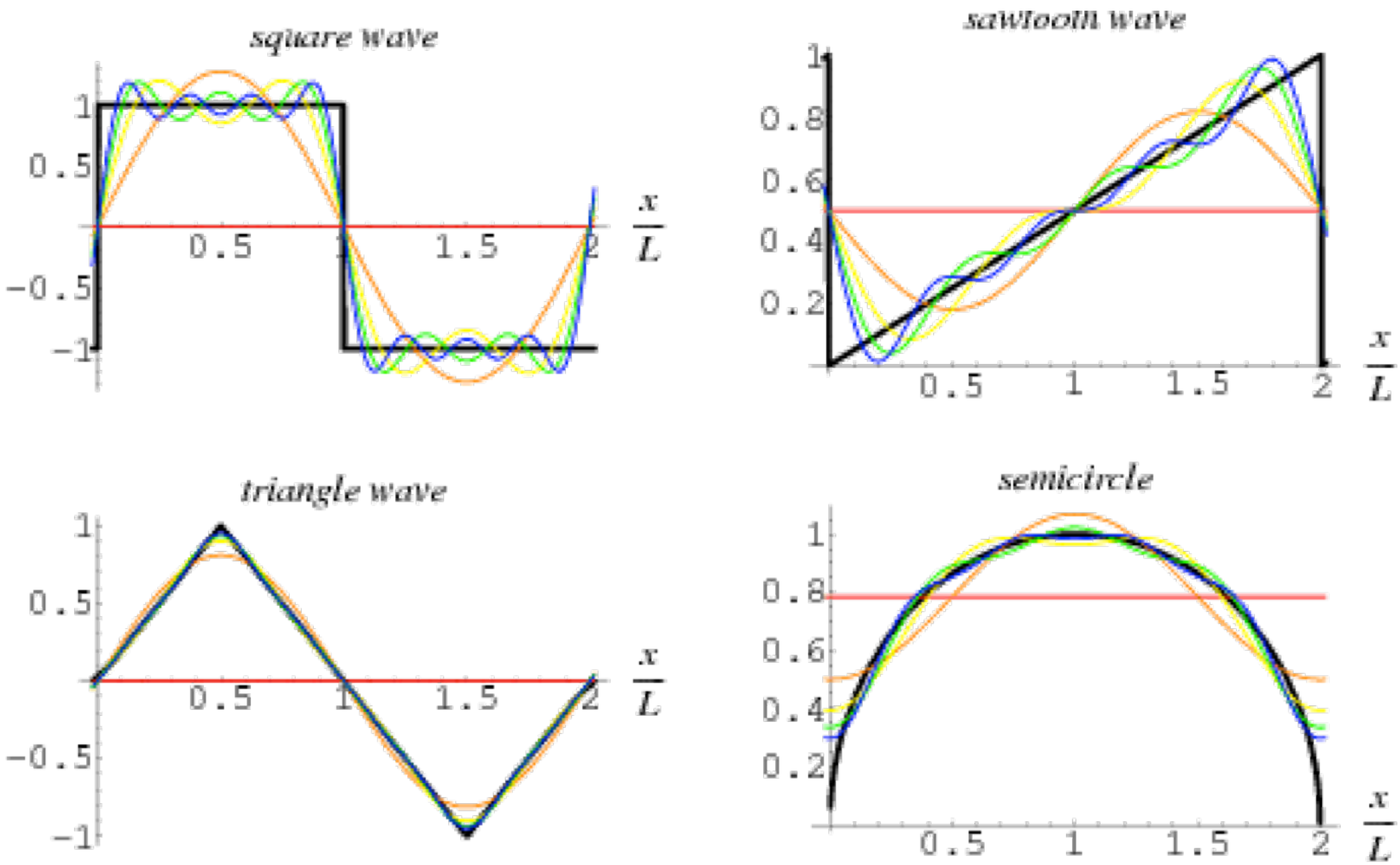
$$\frac{\Delta T}{T}(\theta, \phi) = \frac{T(\theta, \phi) - \bar{T}}{\bar{T}}$$

- The equivalent of the Fourier expansion on a sphere is achieved by expanding the temperature fluctuations in spherical harmonics

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{\ell=1}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y^{\ell}_{m}(\theta, \phi)$$

$$\ell = 0, 1, \ldots\,, \quad m = -\ell, -\ell+1, \ldots, \ell$$

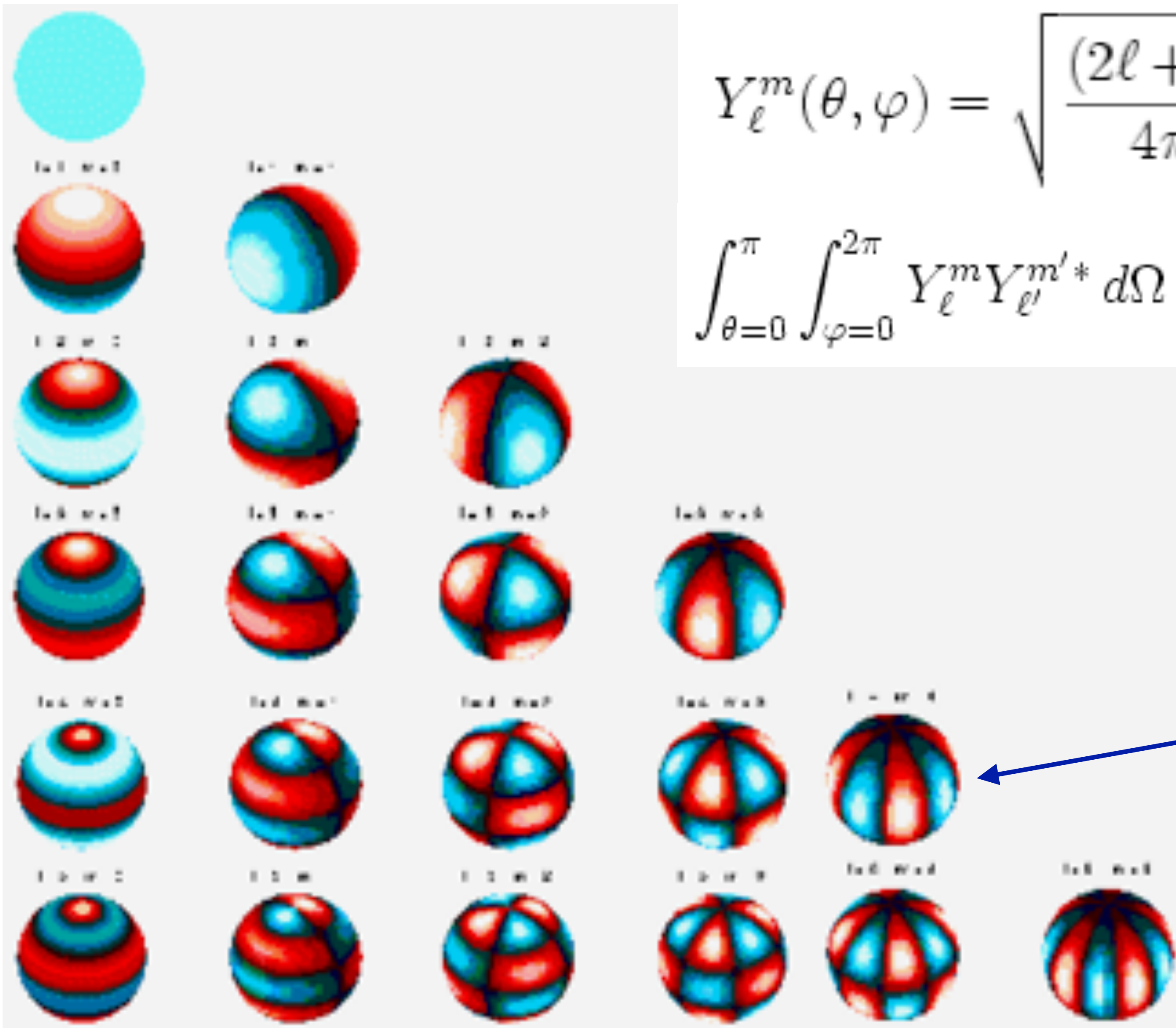
Analogy: Fourier series



Sum sine waves of different frequencies to approximate any function.

Each has a coefficient, or amplitude.

Spherical harmonics functions



$$Y_\ell^m(\theta,\varphi) = \sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}} \cdot e^{im\varphi} \cdot P_\ell^m(\cos\theta)$$

$$\int_{\theta=0}^{\pi}\int_{\varphi=0}^{2\pi} Y_\ell^m Y_{\ell'}^{m'*}\, d\Omega = \delta_{\ell\ell'}\delta_{mm'} \qquad d\Omega = \sin\theta\, d\varphi\, d\theta$$

Very roughly:

$\vartheta \approx 360°/(2\ell+1) \approx 180°/\ell$

(imagine cutting up a sphere into $2\ell+1$ stripes!)

But an exact relation between multipole ℓ and 1D angle θ cannot be given.

Visualizing the multipoles

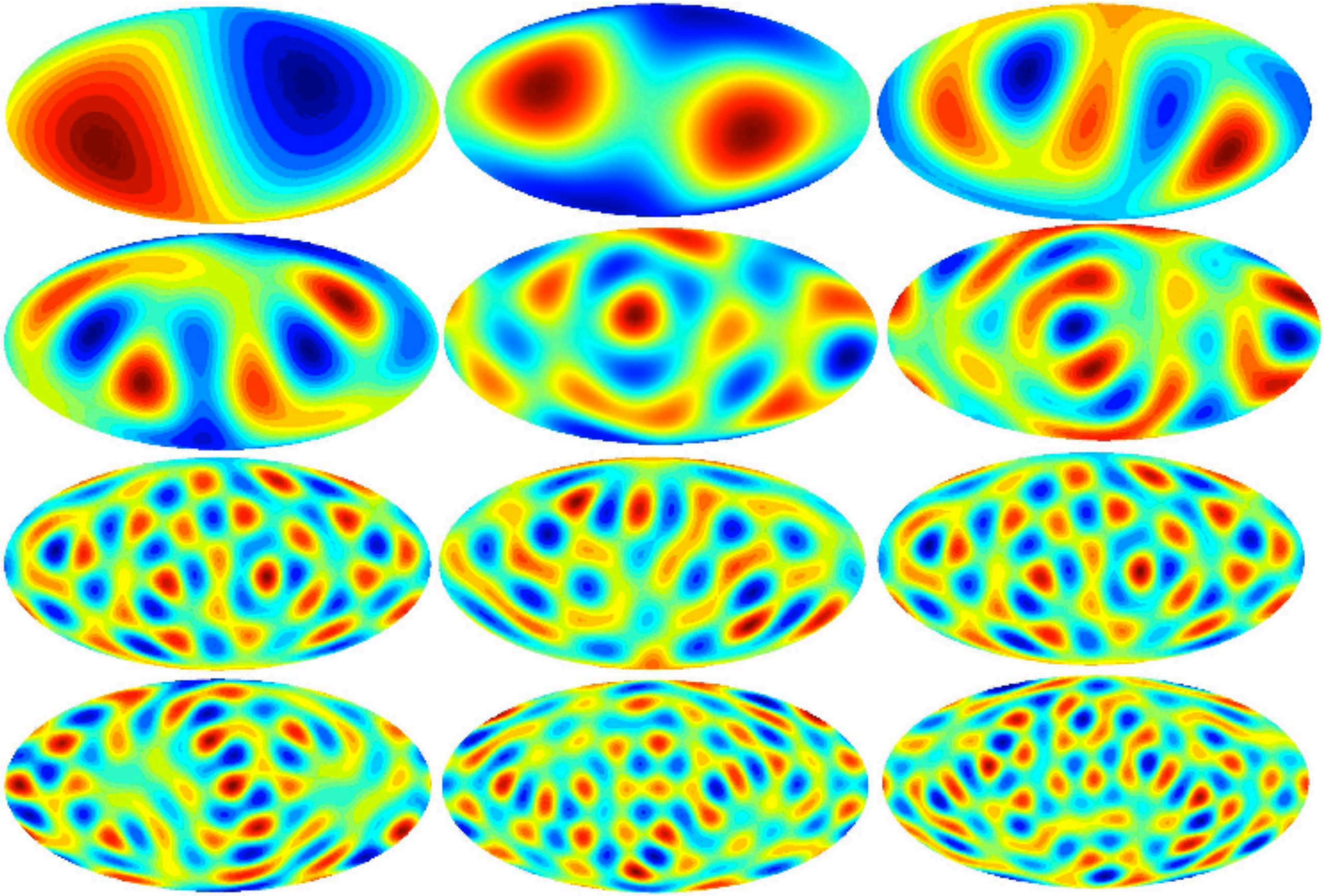
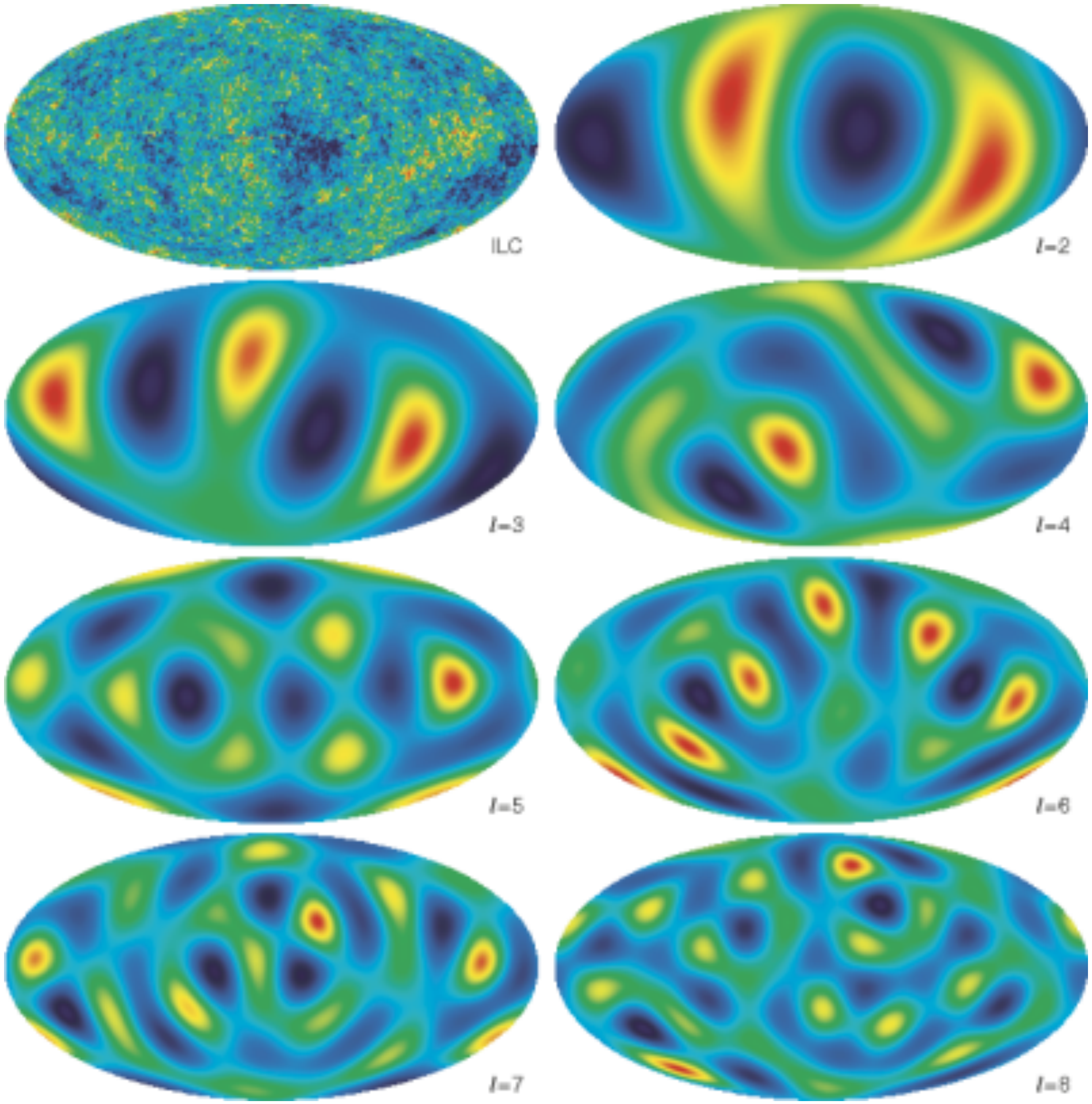
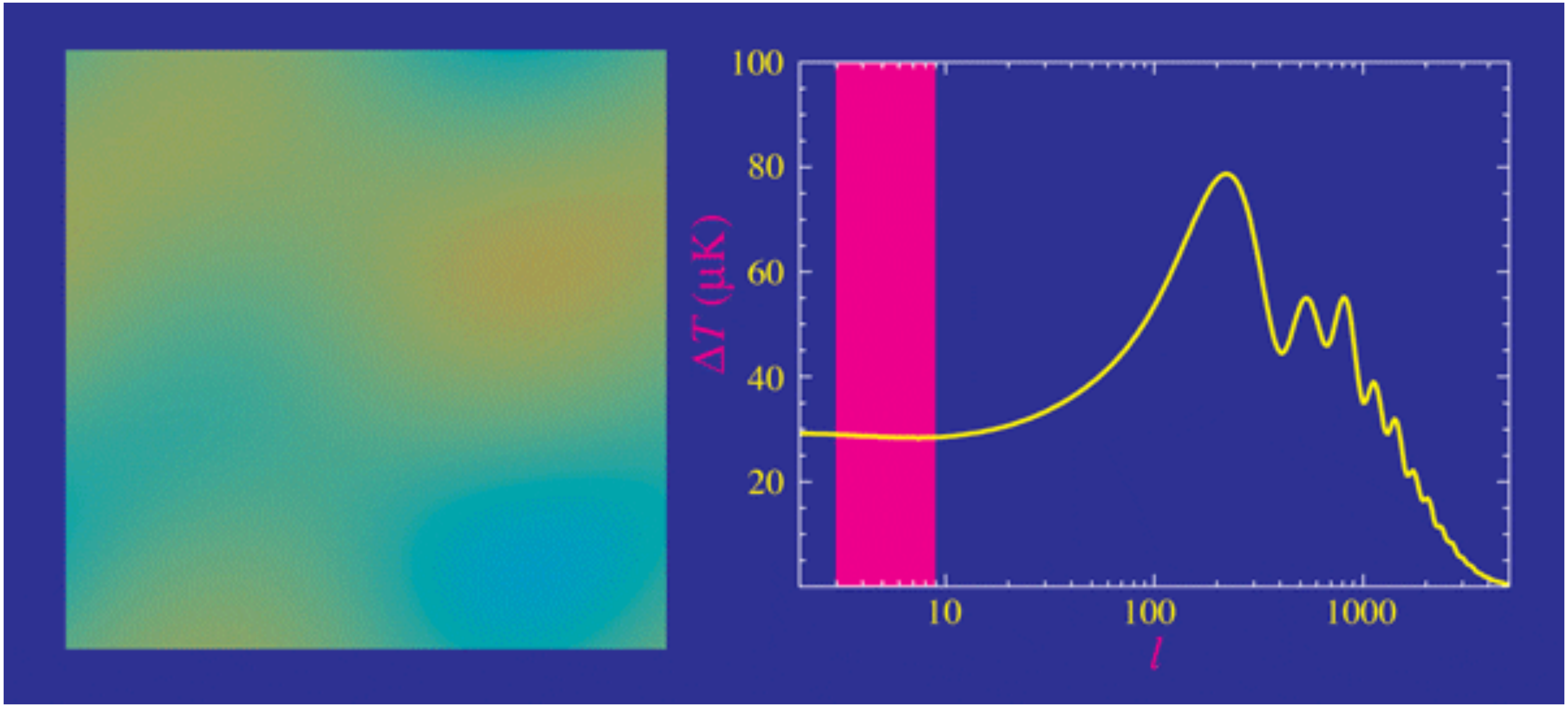


Figure 6: Randomly generated skies containing only a single multipole ℓ. Staring from top left: $\ell = 1$ (dipole only), 2 (quadrupole only), 3 (octupole only), 4, 5, 6, 7, 8, 9, 10, 11, 12. Figure by Ville Heikkilä.

Visualizing the multipoles (real sky!)



Power at different scales



Credit: Wayne Hu
http://background.uchicago.edu/~whu/metaanim.html

Power spectrum of temperature anisotropies

$$C_\ell \equiv \langle |a_{\ell m}|^2 \rangle = \frac{1}{2\ell+1} \sum_m \langle |a_{\ell m}|^2 \rangle \,,$$

$$a_{\ell m} = \int d\Omega Y^*_{\ell m}(\theta,\phi) \frac{\delta T}{T}(\theta,\phi)$$

$$\langle a_{\ell m} \rangle = 0 \qquad \langle a_{\ell m} a^*_{\ell' m'} \rangle = 0 \qquad \text{if } \ell \neq \ell' \text{ or } m \neq m'$$

For a **Gaussian random field**, the C_l contains all the statistical information of the temperature anisotropies. Thus the analysis of CMB temperature anisotropies (and likewise for its polarization anisotropies, see later) consists of computing the C_l-s from the real sky and comparing those with the theoretical predictions.

Spherical harmonics properties

$$C_\ell \equiv \langle |a_{\ell m}|^2 \rangle = \frac{1}{2\ell+1} \sum_m \langle |a_{\ell m}|^2 \rangle \,.$$

l~180°/θ relates to the angular size of the pattern, whereas m relates to the direction on the sky. Thus $\langle |a_{lm}|^2 \rangle$ is independent of m (since CMB is isotropic).

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell \,.$$

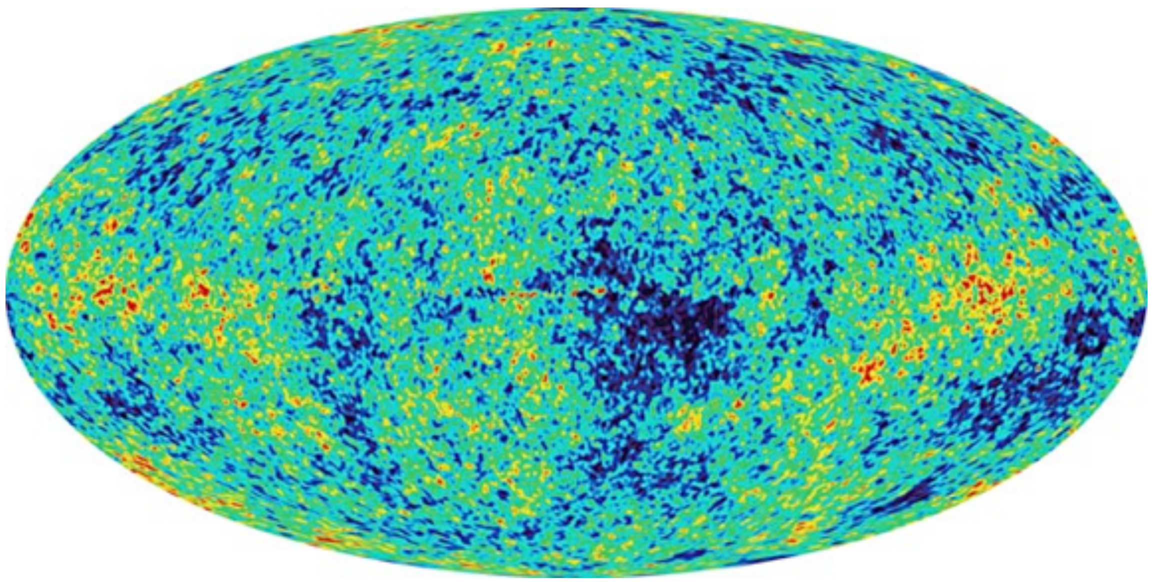
The mean $\langle a_{lm} \rangle$ is zero (Gaussian random field), but we can calculate its variance:

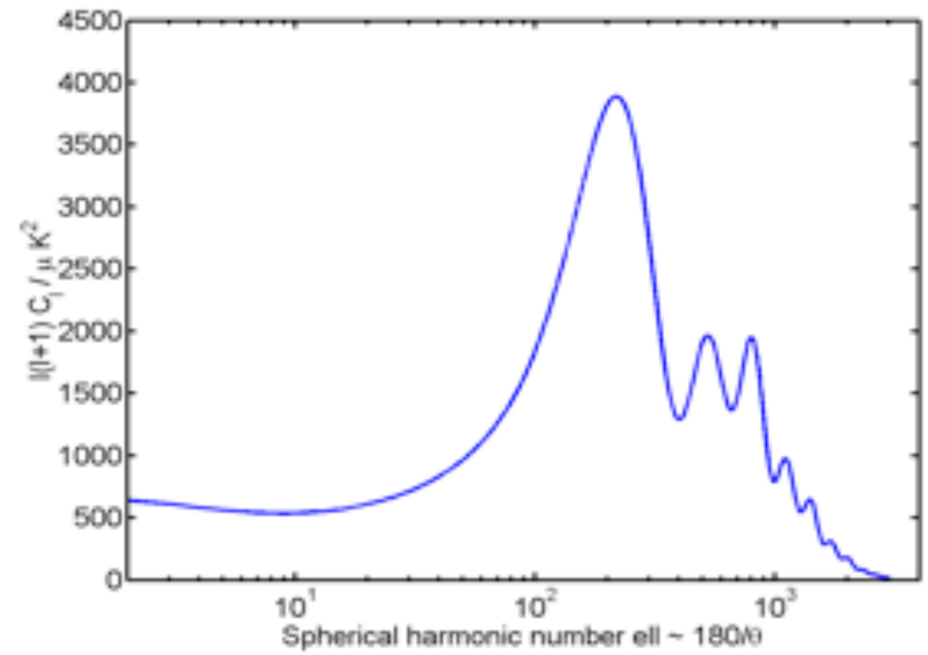
$$\begin{aligned}
\left\langle \left(\frac{\delta T(\theta,\phi)}{T} \right)^2 \right\rangle &= \left\langle \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta,\phi) \sum_{\ell' m'} a^*_{\ell' m'} Y^*_{\ell' m'}(\theta,\phi) \right\rangle \\
&= \sum_{\ell\ell'} \sum_{mm'} Y_{\ell m}(\theta,\phi) Y^*_{\ell' m'}(\theta,\phi) \langle a_{\ell m} a^*_{\ell' m'} \rangle \\
&= \sum_\ell C_\ell \sum_m |Y_{\ell m}(\theta,\phi)|^2 = \sum_\ell \frac{2\ell+1}{4\pi} C_\ell \,,
\end{aligned}$$

Thus, if we plot $(2\ell+1)C_\ell/4\pi$ on a linear ℓ scale, or $\ell(2\ell+1)C_\ell/4\pi$ on a logarithmic ℓ scale, the area under the curve gives the temperature variance, i.e., the expectation value for the squared deviation from the average temperature. It has become customary to plot the angular power spectrum as $\ell(\ell+1)C_\ell/2\pi$, which is neither of these, but for large ℓ approximates the second case. (Reason is the S-W effect)

CMB power spectrum

How to constrain cosmology from this measurement?





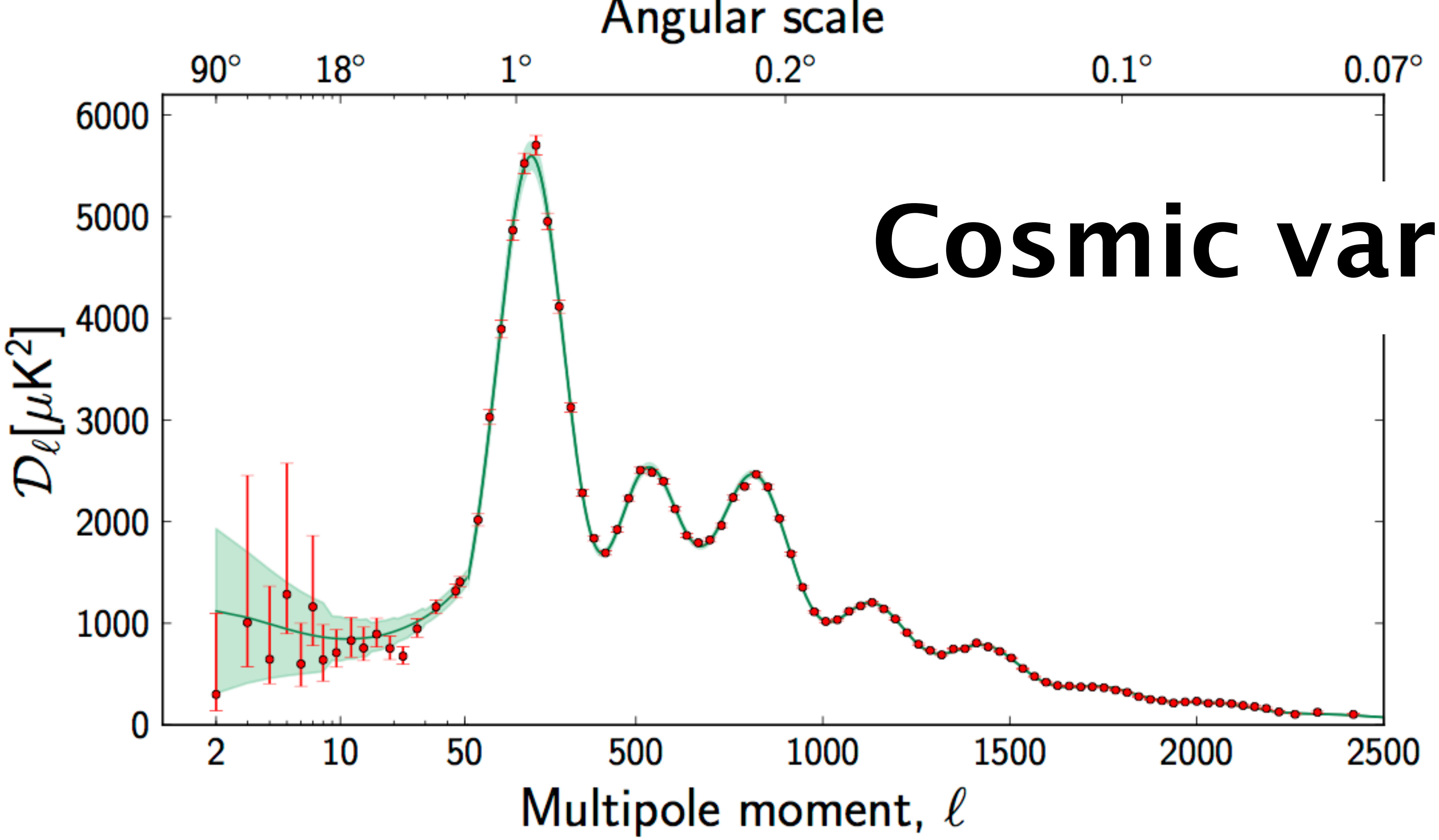
Use spherical harmonics in place of sine waves: $$\frac{\Delta T}{T}(\theta,\phi)=\sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{\ell}a_{\ell m}Y_m^\ell(\theta,\phi)$$

Calculate coefficients, a_{lm}, and then the statistical average: $$c_\ell=\langle|a_{\ell m}|\rangle^2$$

Amplitude of fluctuations on each scale - that's what we plot.

(TT power spectrum)

Cosmic variance



We only have one Universe, so we are intrinsically limited to the number of independent m−modes we can measure – there are only (2l + 1) of these for each multipole.

We obtained the following expression for the power spectrum:

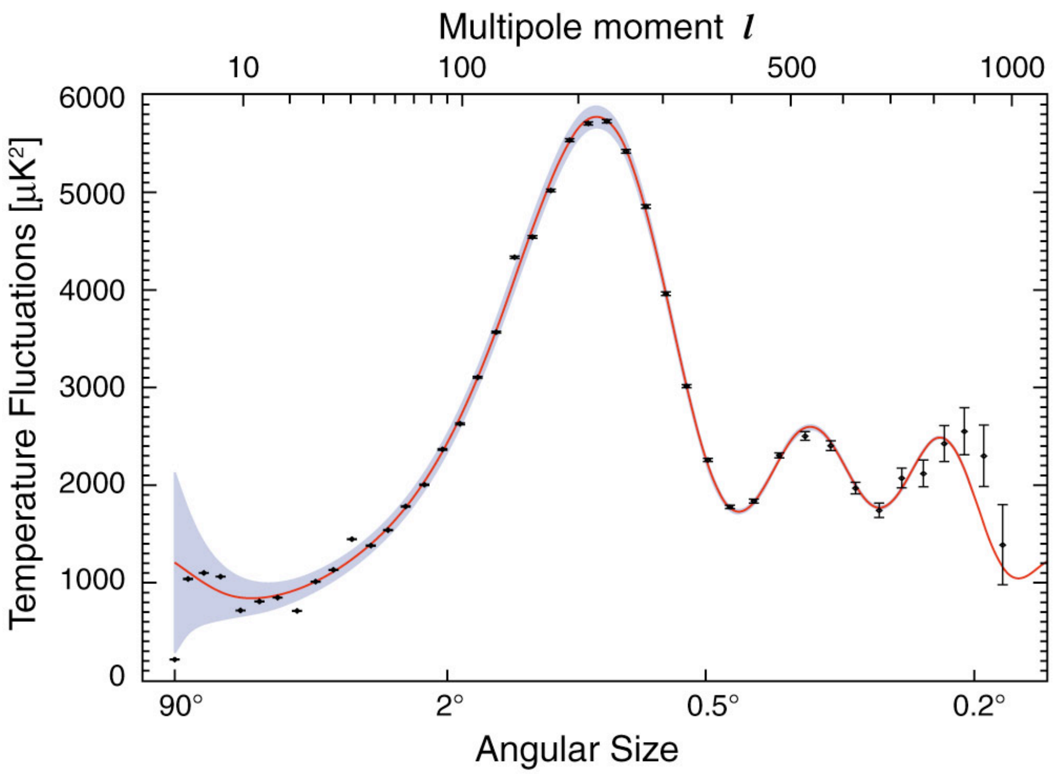
$$C_\ell = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell}\langle|a_{lm}|^2\rangle$$

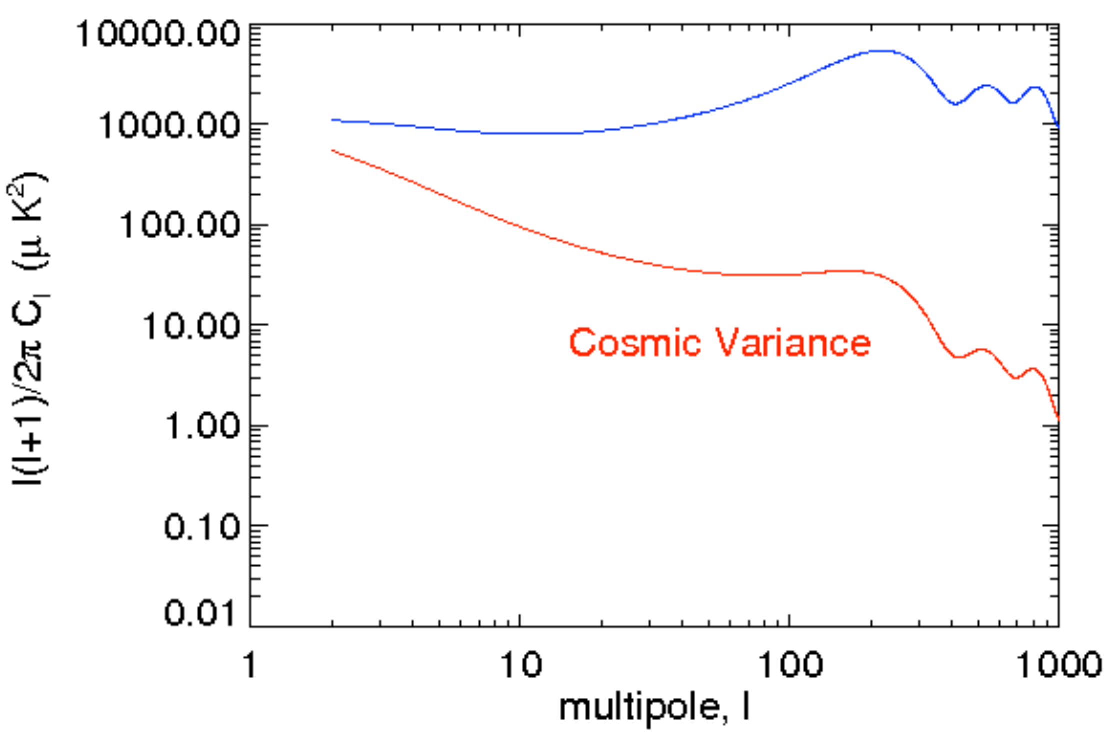
We can see $(2l+1)C_l$ follows a χ^2 distribution. It has (2l+1) degrees of freedom, thus its variance is simply 2(2l+1). Therefore we get:

$$\Delta C_l / C_l = \sqrt{2/(2\ell+1)}$$

How well we can estimate an average value from a sample depends on how many points we have on the sample. This cosmic variance is an **unavoidable** source of uncertainty when constraining models!

Cosmic variance & sample variance





- **Cosmic variance**: on scale l, there are only 2l+1 independent modes

$$\Delta C_\ell = \sqrt{\frac{2}{2\ell+1}} C_\ell$$

- If the fraction of sky covered is f, then the errors are increased by a factor $1/\sqrt{f_{sky}}$ and the resulting variance is called **sample variance** (f ~0.65 for the PLANCK satellite)

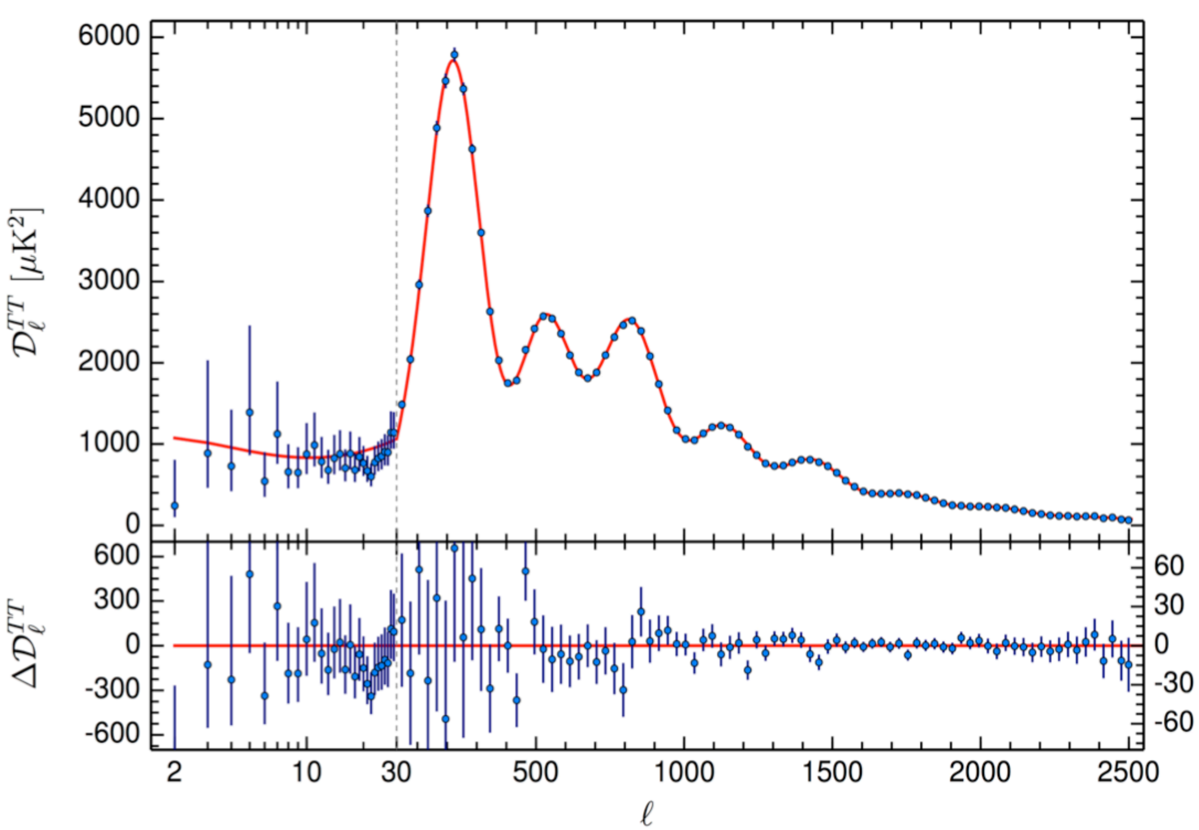
$$(\Delta\hat{C}_l)^2 = \frac{2}{(2l+1)f_{\rm sky}^2(l)}(C_l + N_l)^2$$

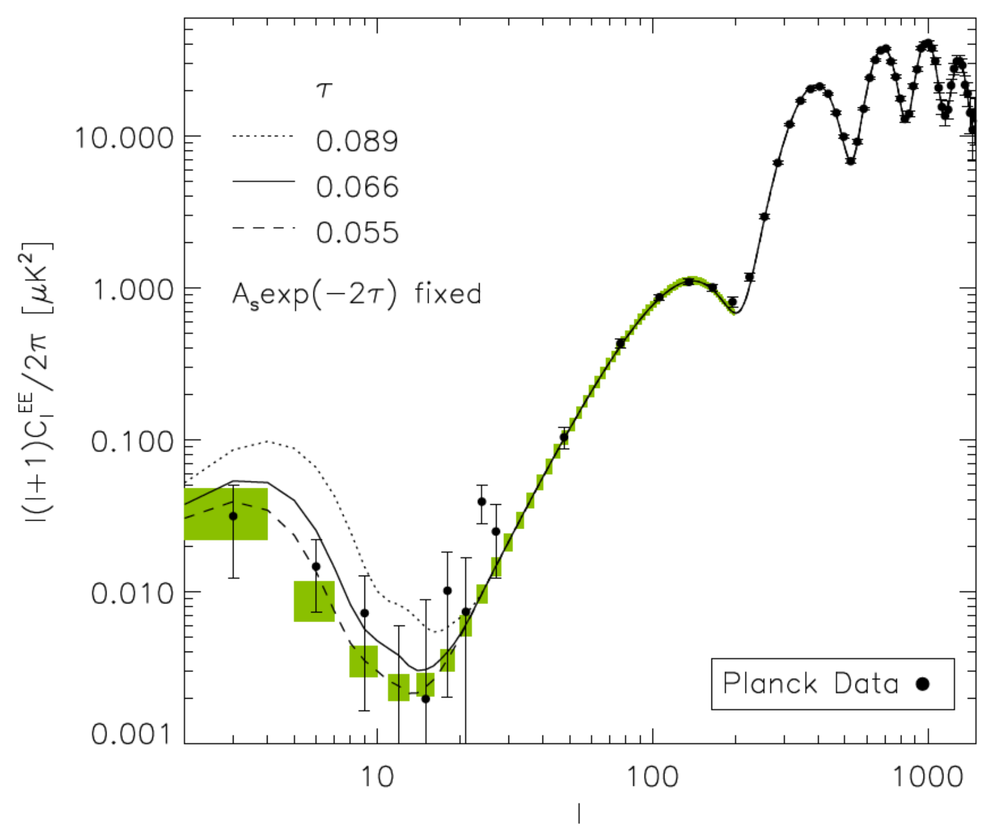
Cosmic variance limits on CMB data

Current Planck temperature power spectrum (TT) is already cosmic variance limited, out to the angular scales where primordial effects are dominant.

Within the next decade, LiteBIRD satellite will make cosmic variance-limited measurement of the EE mode of polarization power (highly useful for measuring the universe's optical depth, for example).

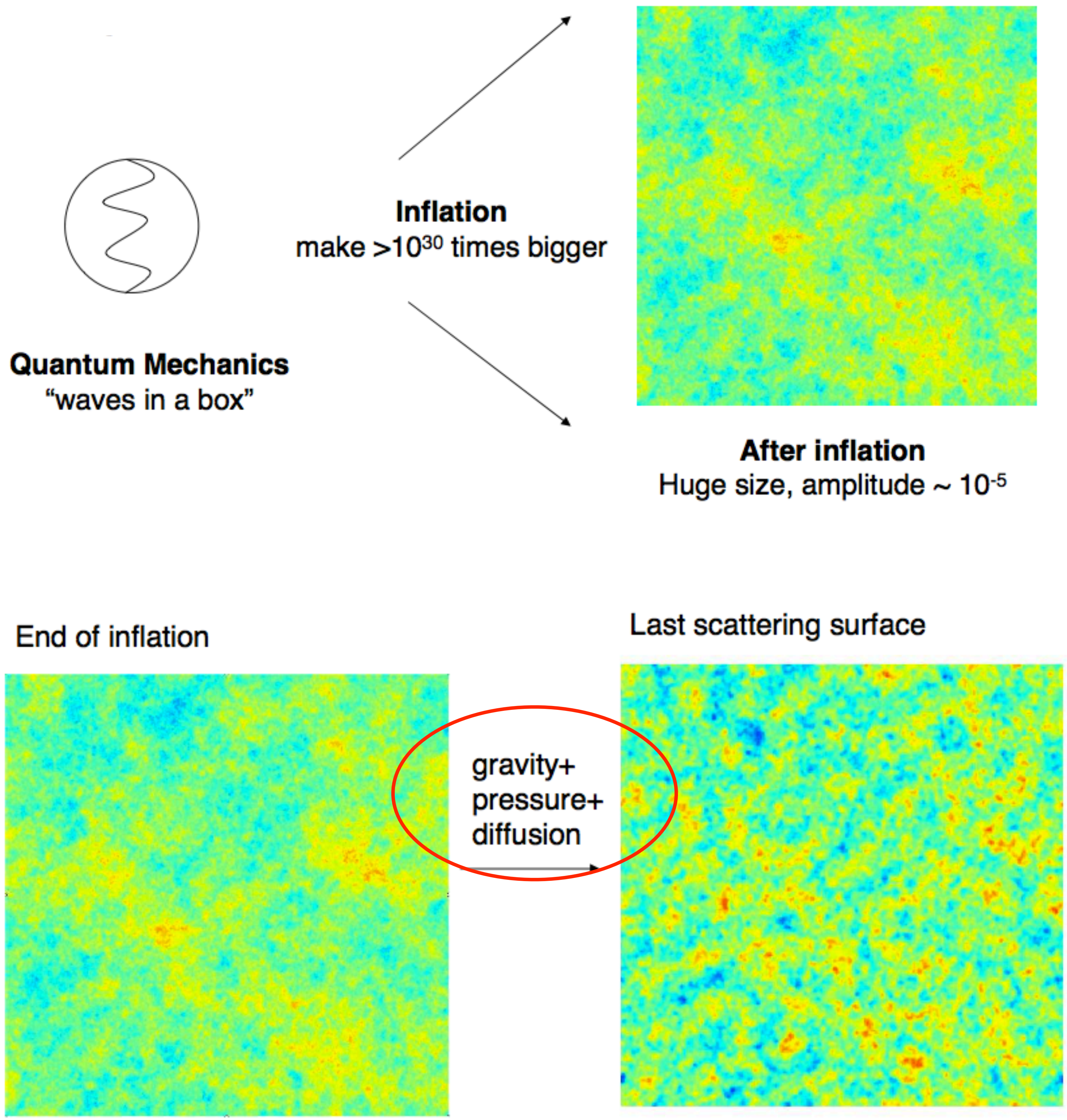
Cosmic variance-limited BB polarization measurement is still long way into the future!



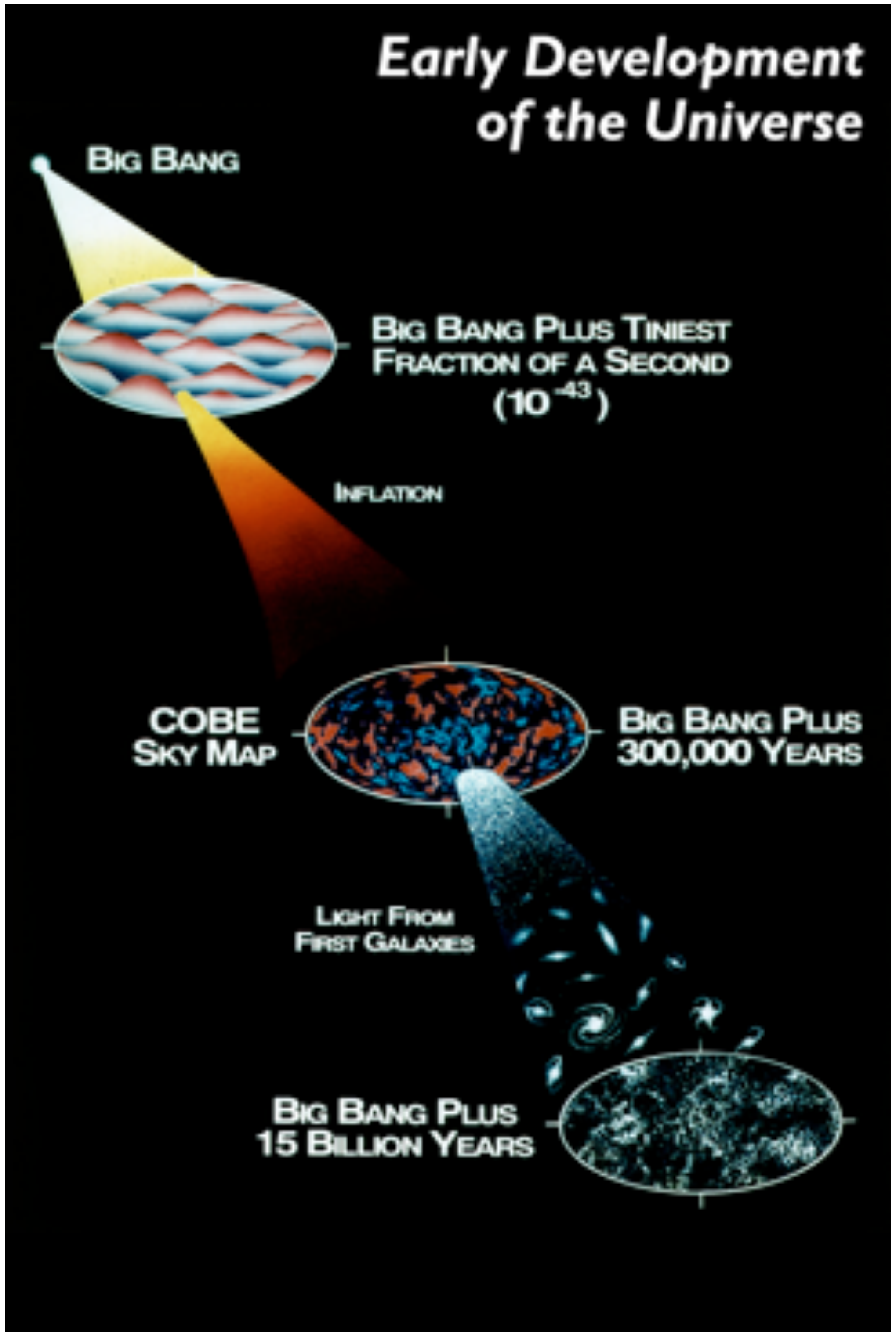


The origin of the temperature anisotropies

Primordial temperature anisotropies



Primordial temperature anisotropies



At recombination, when CMB started to stream freely, structure had already started to form

This created the "hot" and "cold" spots in the CMB

These were the seeds of structure we see today

Should not confuse between the "creation" of the CMB photons before, at the blackbody surface (z >10^6), and their "release" from the last scattering surface (z~300,000).

Most of the CMB photons that we receive were created at much earlier epoch through matter/anti-matter annihilation, and thus, were formed as gamma rays (now cooled down to microwave).

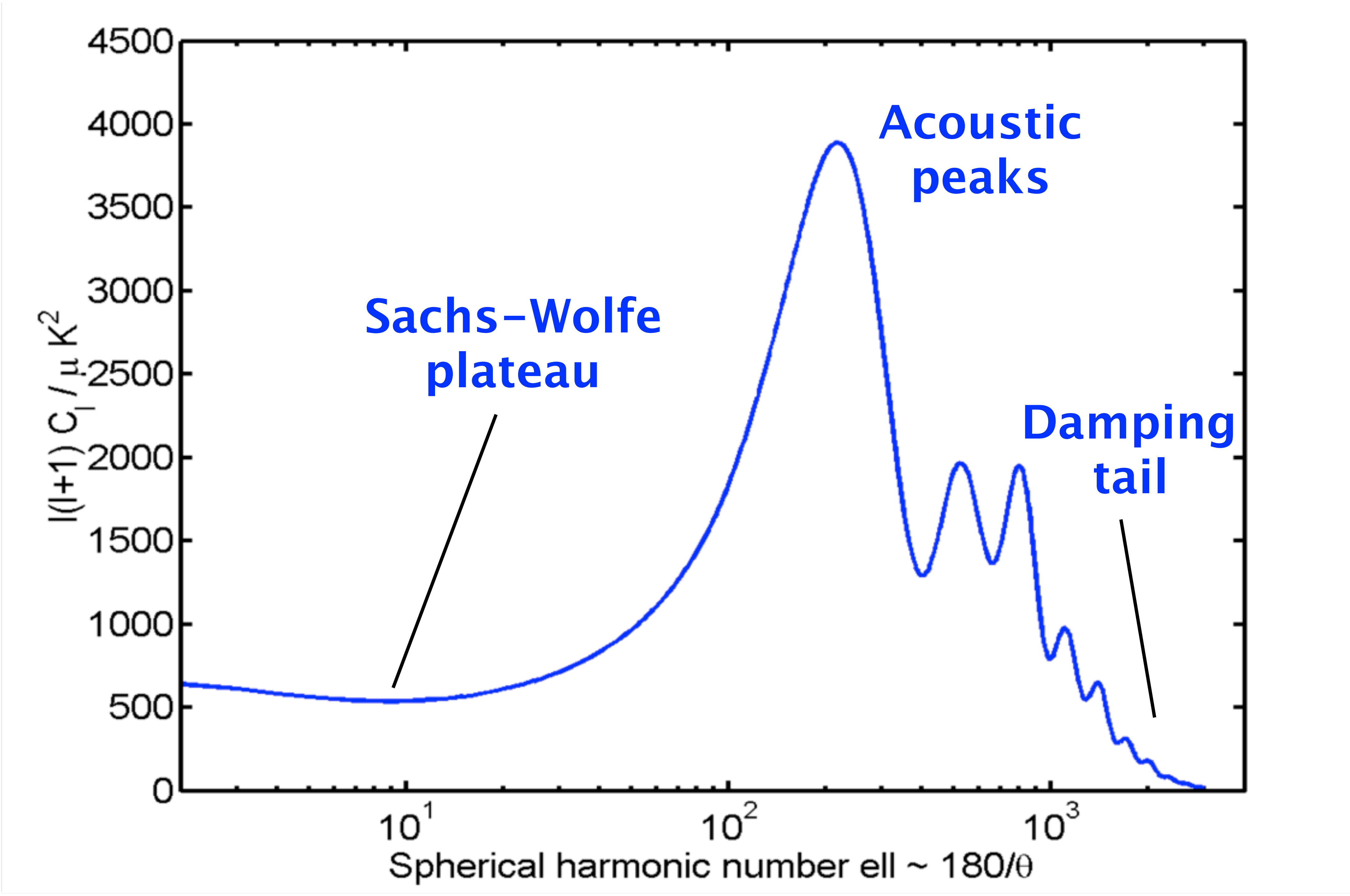
Sources of ΔT

Table 1. Sources of temperature fluctuations.

PRIMARY	Gravity	
	Doppler	
	Density fluctuations	
	Damping	
	Defects	Strings
		Textures
SECONDARY	Gravity	Early ISW
		Late ISW
		Rees-Sciama
		Lensing
	Local reionization	Thermal SZ
		Kinematic SZ
	Global reionization	Suppression
		New Doppler
		Vishniac
"TERTIARY" (foregrounds & headaches)	Extragalactic	Radio point sources
		IR point sources
	Galactic	Dust
		Free-free
		Synchrotron
	Local	Solar system
		Atmosphere
		Noise, *etc.*

Max Tegmark (astro-ph/9511148)

Power spectrum: primary anisotropies



Sources of primary anisotropies

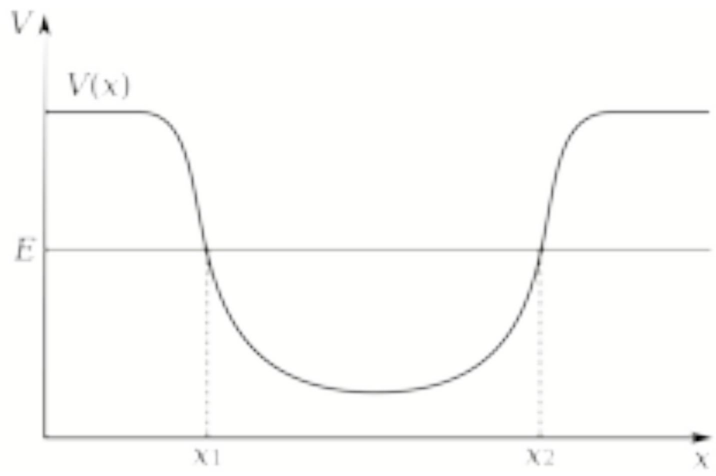
Quantum density fluctuations in the dark matter were amplified by inflation. Gravitational potential wells (or “hills”) developed, baryons fell in (or moved away).

Various related physical processes affected the CMB photons:

- **Perturbations in the gravitational potential (Sachs-Wolfe effect):** photons that last scattered within high-density regions have to climb out of potential wells and are thus redshifted
- **Intrinsic adiabatic perturbations (the acoustic peaks):** in high-density regions, the coupling of matter and radiation will also compress the radiation, giving a higher temperature. This creates sound-like waves.
- **Velocity perturbations (the Doppler troughs):** photons last-scattered by matter with a non-zero velocity along the line-of-sight will receive a Doppler shift

Sachs–Wolfe effect

$$\Delta\nu/\nu \sim \Delta T/T \sim \Phi/c^2$$



Additional effect of time dilation (see next slide) while the potential evolves leads to a factor of 1/3 (e.g. White & Hu 1997):

$$\frac{\Delta T}{T} \sim \frac{1}{3}\frac{\Delta\Phi}{c^2}$$

The temperature fluctuations due to the so-called Sachs–Wolfe effect are due to two competing effects: (1) the redshift experienced by the photon as it climbs out of the potential well toward us and (2) the delay in the release of the radiation, leading to less cosmological redshift compared to the average CMB radiation.

The first contribution leads to a redshift of the order of:

$$\frac{\delta T_1}{T} = \frac{\delta\Phi}{c^2}$$

Sachs–Wolfe effect

The second contribution (redshift due to time delay) can be computed roughly as the following. Because of general relativity, the proper time goes slower inside the potential well than outside. The cooling of the gas in this potential well thus also goes slower, and it therefore reaches 3000 K at a later time relative to the average Universe.

The time delay (in terms of global time t) is:

$$\frac{\delta t}{t} = -\frac{\delta\Phi}{c^2} \tag{8.7}$$

This means that 3000 K is reached at a slightly larger (global) scale parameter $a + \delta a > a$. Since in the Einstein-de-Sitter Universe we have $a \propto t^{2/3}$ we can write

$$\frac{\delta a}{a} = \frac{2}{3}\frac{\delta t}{t} = -\frac{2}{3}\frac{\delta\Phi}{c^2} \tag{8.8}$$

Now, from that point $a = (a_{\rm cmb} + \delta a)$ until today $a = 1$ the redshift due to expansion is less by:

$$\frac{\delta z}{z} = -\frac{\delta a}{a} \tag{8.9}$$

which leads to a positive contribution to the temperature fluctuation δT that we observe today:

$$\frac{\delta T_2}{T} = -\frac{\delta z}{z} = \frac{\delta a}{a} = -\frac{2}{3}\frac{\delta\Phi}{c^2} \tag{8.10}$$

The total is the sum of both contributions:

$$\frac{\delta T}{T} = \frac{\delta T_1}{T} + \frac{\delta T_2}{T} = \frac{1}{3}\frac{\delta\Phi}{c^2} \tag{8.11}$$

Sachs-Wolfe effect

For power-law index of unity primary density perturbations, n_s=1 (Zel'dovich, Harrison ~1970), the Sachs-Wolfe effect produces a flat power spectrum: $C_l^{SW} \sim const / l(l+1)$.

For an independent random variable $\mathcal{R}_\mathbf{k}$ and its power-spectrum $\mathcal{P}_\mathcal{R}(k)$:

$$C_\ell \equiv \frac{1}{2\ell+1}\sum_m \langle |a_{\ell m}|^2 \rangle$$
$$= \frac{4\pi}{25}\int_0^\infty \frac{dk}{k}\mathcal{P}_\mathcal{R}(k) j_\ell(kx)^2 \,,$$

the final result for an arbitrary primordial power spectrum $\mathcal{P}_\mathcal{R}(k)$.

The integral can be done for a power-law power spectrum, $\mathcal{P}(k) = A^2 k^{n-1}$. In particular, for a scale-invariant ($n = 1$) primordial power spectrum,

$$\mathcal{P}_\mathcal{R}(k) = \text{const.} = A^2 \,,$$

we have

$$C_\ell = A^2 \frac{4\pi}{25}\int_0^\infty \frac{dk}{k} j_\ell(kx)^2 = \frac{A^2}{25}\frac{2\pi}{\ell(\ell+1)} \,,$$

since

$$\int_0^\infty \frac{dk}{k} j_\ell(kx)^2 = \frac{1}{2\ell(\ell+1)} \,.$$

We can write this as

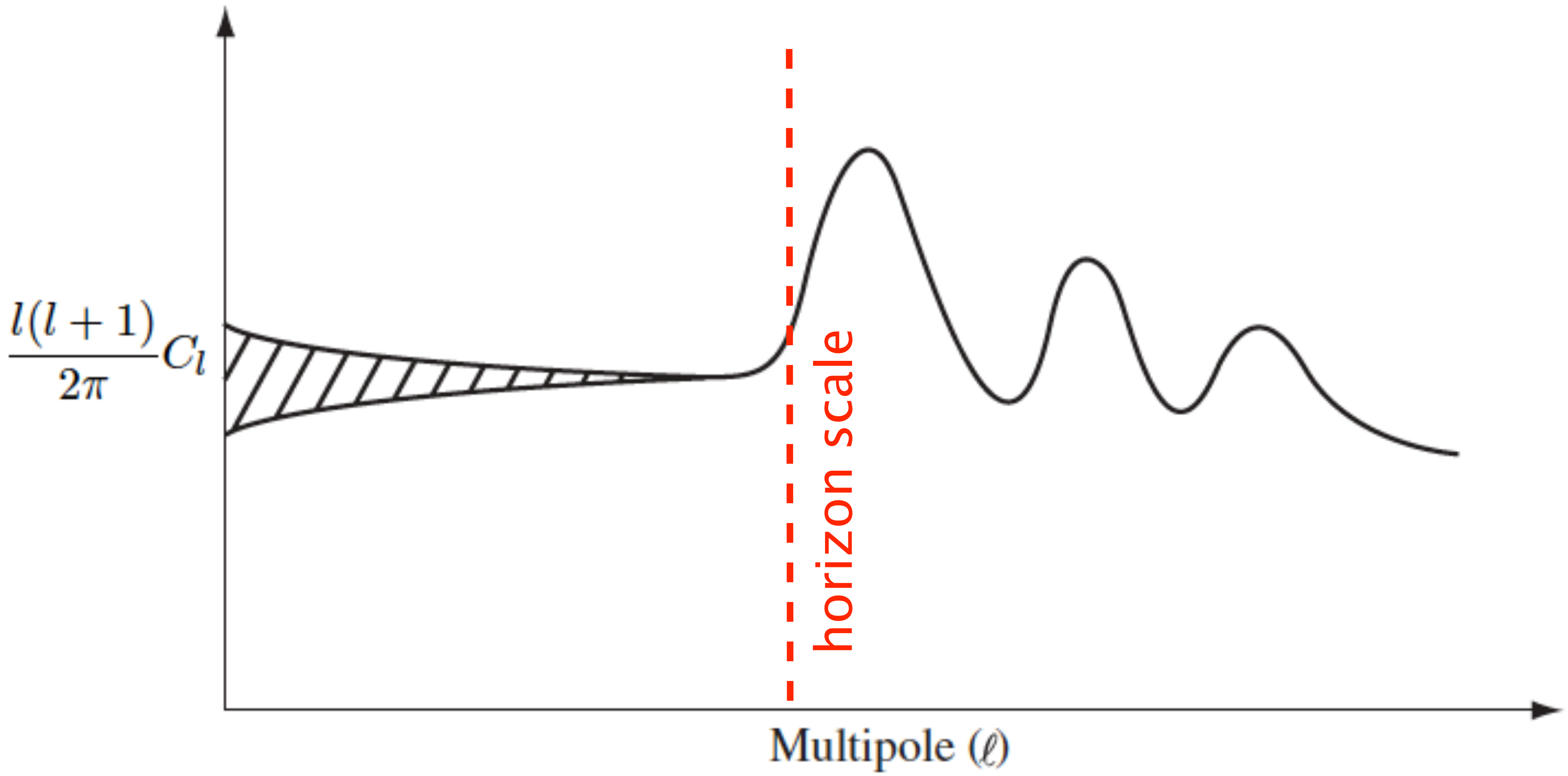
$$\frac{\ell(\ell+1)}{2\pi}C_\ell = \frac{A^2}{25} = \text{const.} \quad \text{(independent of } \ell\text{)}$$

(Lecture notes by Hannu Kurki_Suonnio, Uni Helsinki)

Sachs–Wolfe effect

For power-law index of unity primary density perturbations, n_s=1 (Zel'dovich, Harrison ~1970), the Sachs-Wolfe effect produces a flat power spectrum: $C_l^{SW} \sim$ const./ l (l+1).

This is the main effect creating temperature fluctuations at θ ≳ 2° scales. At smaller angles, perturbations enter the horizon and they have time to evolve from their initial state. This evolution is driven by the baryons!

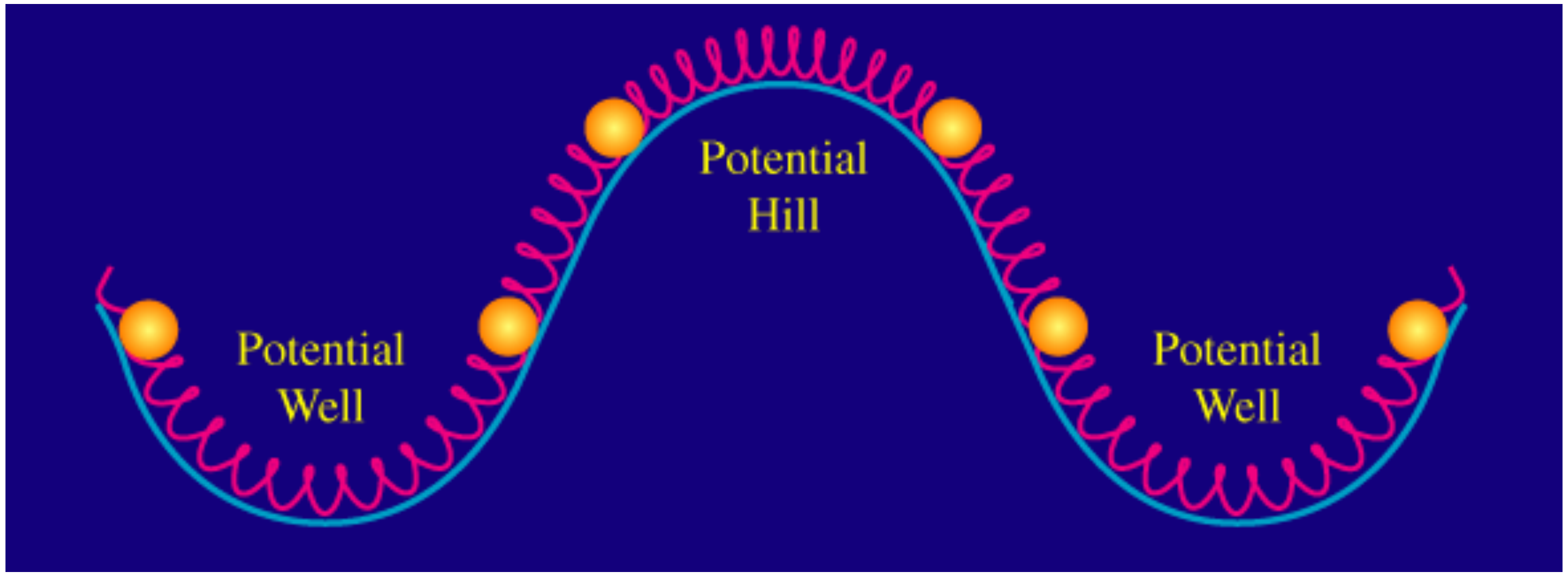


Acoustic oscillations

- Dark matter already created the perturbations in gravitational potential
- Baryons fall into these potential wells: Photon baryon fluid heats up
- Radiation pressure from photons resists collapse, overcomes gravity, expands: Photon-baryon fluid cools down
- Oscillating cycles exist on all scales. All of these standing waves stop oscillating at recombination — when photons and baryons decouple.

Credit: Wayne Hu

Springs: photon pressure



Balls: baryon mass

Acoustic peaks

Oscillations took place on all scales. We see temperature features from modes which had reached the extrema

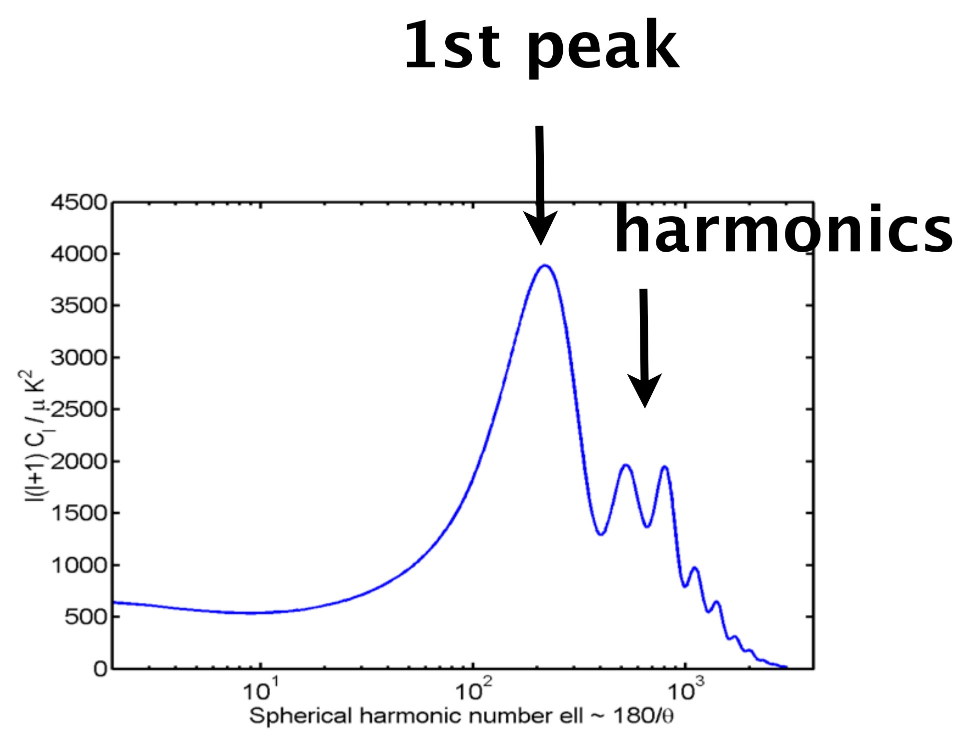
- Maximally compressed regions were hotter than the average
  Recombination happened later, corresponding photons experience less red-shifting by Hubble expansion: HOT SPOT

- Maximally rarified regions were cooler than the average
  Recombination happened earlier, corresponding photons experience more red-shifting by Hubble expansion: COLD SPOT

Harmonic sequence, like waves in pipes or strings:

2nd harmonic: mode compresses and rarifies by recombination
3rd harmonic: mode compresses, rarifies, compresses

➡ 2nd, 3rd, .. peaks



Acoustic oscillations theory

Equations of motion for self-gravitating non-relativistic gas
(here photon-baryon fluid):

Continuity eqn.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0$$

Euler eqn.

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{\nabla p}{\rho} - \nabla \Phi$$

Poisson eqn.

$$\nabla^2 \Phi = 4\pi G \rho$$

From these three we get the perturbation equation for the photon gas

For radiation-dominated era, set $p=(1/3)\rho c^2$

$$\begin{aligned} \dot{\delta} + \tfrac{4}{3}\nabla \cdot \vec{u} &= 0 \\ \dot{\vec{u}} + 2H\vec{u} &= -\frac{\frac{1}{4}c^2\nabla\delta + \nabla\delta\Phi}{a^2} \\ \nabla^2 \delta\Phi &= 8\pi G \rho_0 a^2 \delta \end{aligned}$$

$$\frac{3}{4}\ddot{\delta} + \frac{3}{2}H\dot{\delta} = \frac{c^2\nabla^2\delta}{4a^2} + 8\pi G \rho_0 \delta$$

We want to solve for this δ.

Acoustic oscillations theory

(perturbation equation from previous slide)

$$\frac{d^2\delta}{dt^2} + 2H\frac{d\delta}{dt} = \frac{c^2}{3a^2}\nabla^2\delta + \frac{32}{3}\pi G\rho_0\delta \qquad (8.12)$$

Now let us introduce the *conformal time* τ, defined by

$$\tau = \int_0^t \frac{dt'}{a(t')}$$

We can now write the second derivative $d^2\delta/dt^2$ as

$$\frac{d^2\delta}{dt^2} = \frac{1}{a^2}\frac{d^2\delta}{d\tau^2} + H\frac{d\delta}{dt}$$

so Eq. (8.12) becomes

$$\frac{d^2\delta}{d\tau^2} + 3Ha\frac{d\delta}{d\tau} = \frac{c^2}{3}\nabla^2\delta + \frac{32}{3}\pi a^2 G\rho_0\delta$$

Acoustic oscillations theory

In Fourier space, where ω this time belongs to the *conformal* time τ, we thus obtain the dispersion relation:

$$\omega^2 - 3Ha\dot{\omega} = \frac{c^2}{3}k^2 - \frac{32}{3}\pi a^2 G\rho_0 \qquad (8.16)$$

If we assume that we have large enough k and ω that we can ignore both the gravitational term on the right and the term proportional to H on the left, then we arrive at:

$$\omega^2 = \frac{c^2}{3}k^2 \qquad (8.17)$$

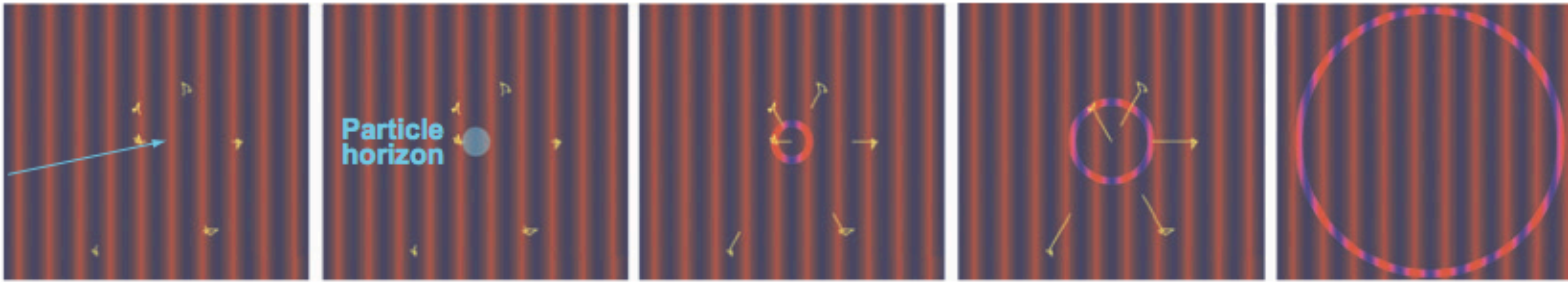
This means that we have solutions of the form

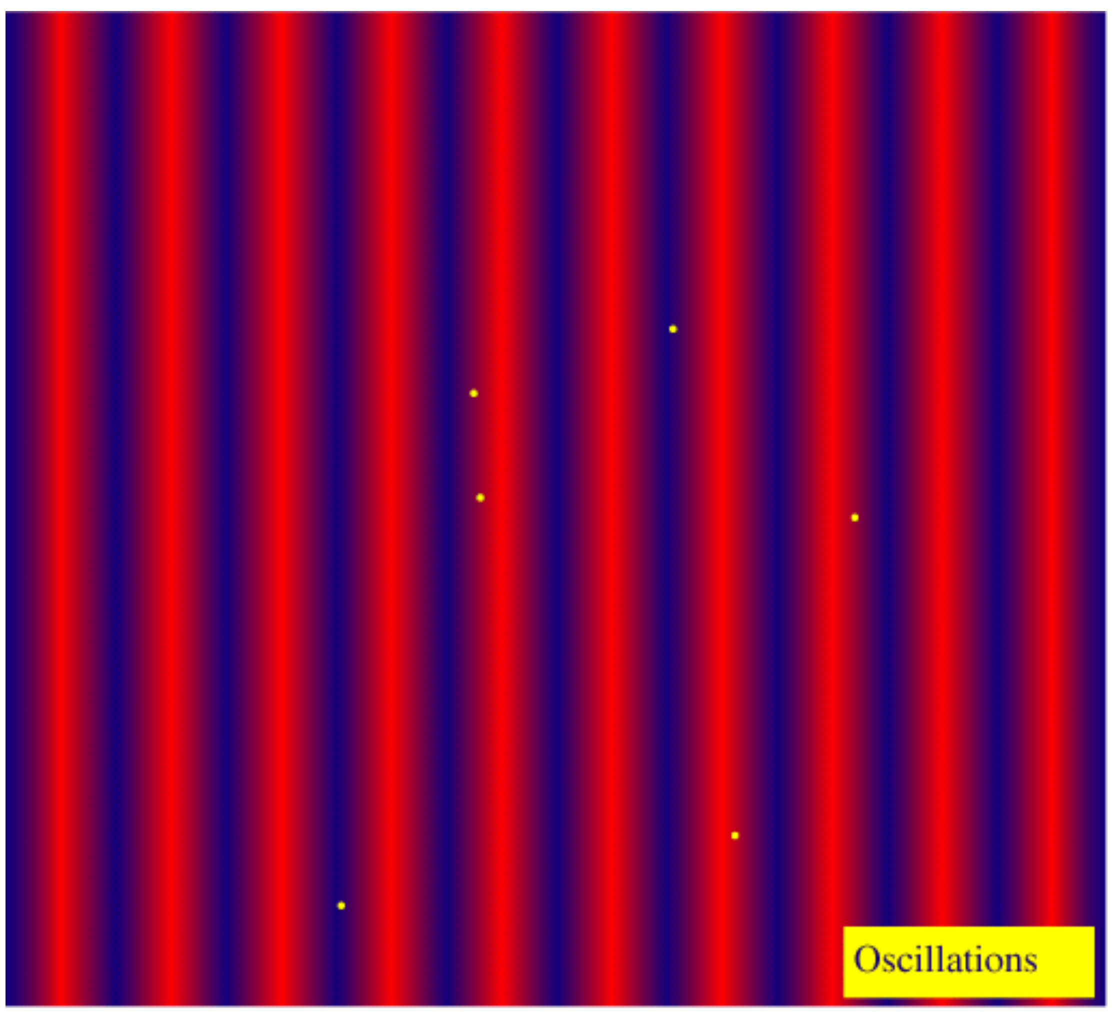
$$\delta(x,\tau) = \delta_0 \cos(kx+\varphi)\cos\left(\frac{c}{\sqrt{3}}k[\tau - \tau_{\rm start}(k)]\right) \qquad (8.18)$$

The last equation is a standing wave with an interesting property: its phase is fixed by the factor in parenthesis. This means that for every wave number k we know precisely what the phase of the oscillating standing wave at the time of the CMB release is. For some modes this phase may be π/2, in which case the density fluctuation has disappeared by the time of CMB release, but the motion is maximum. For others the density fluctuation may be near maximum (phase 0 or π). This gives the distinct wavy pattern in the power spectrum of the CMB.

Angular variations

Density fluctuation on the sky from a single k mode, and how it appears to an observer at different times:

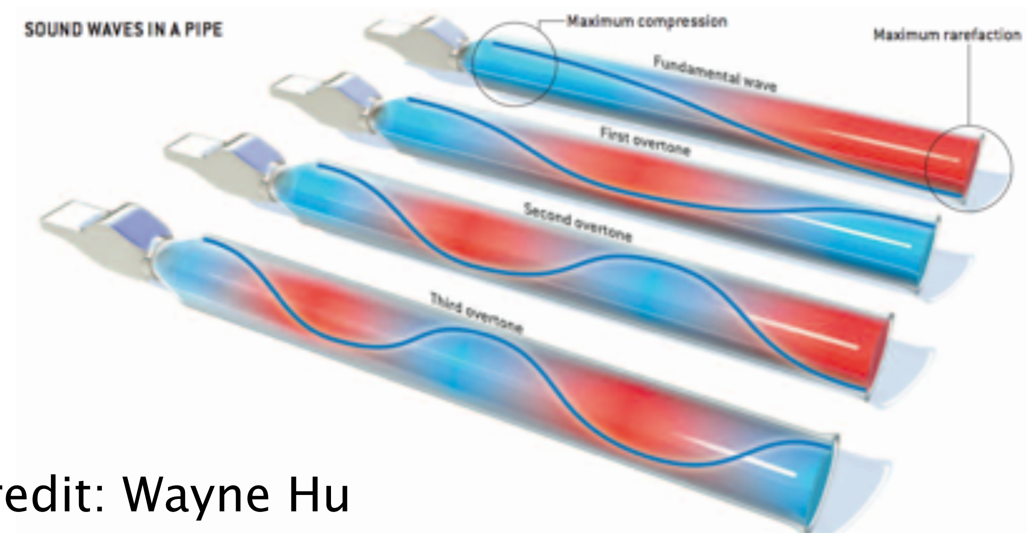




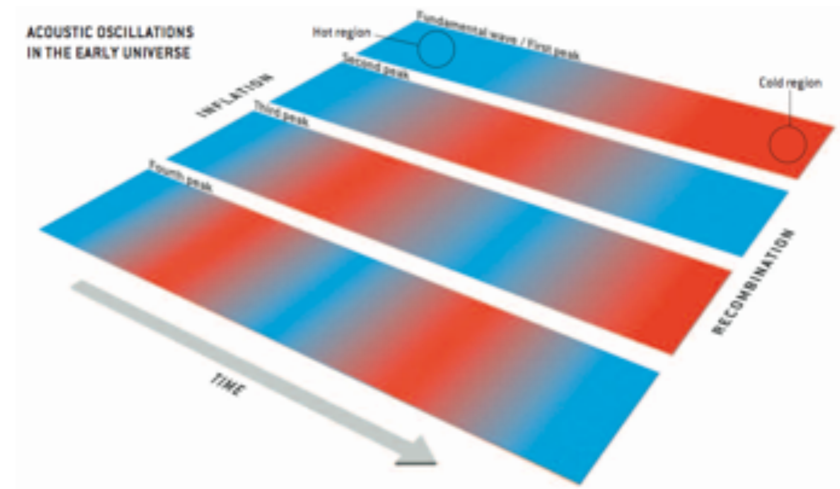
Animation by Wayne Hu

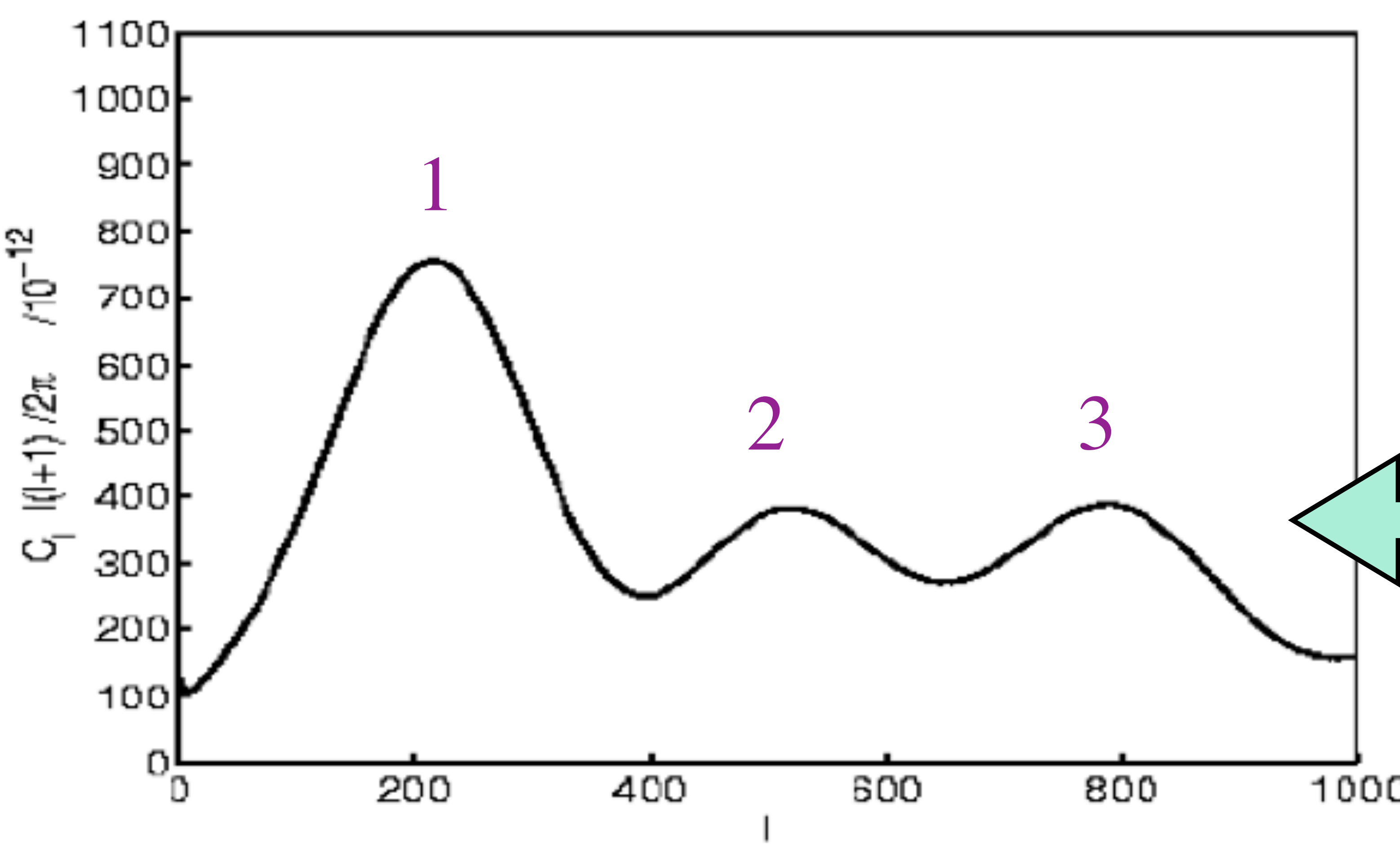
These frames show one super-horizon temperature mode just after decoupling, with representative photons last scattering and heading toward the observer at the center. (From left to right) Just after decoupling; the observer's particle horizon when only the temperature monopole can be detected; some time later when the quadrupole is detected; later still when the 12-pole is detected; and today, a very high, well-aligned multipole from just this single mode in k space is detected.

Harmonic sequence



Credit: Wayne Hu





Modes with half the wavelengths oscillate twice as fast ($\nu = c/\lambda$).

Peaks are equally spaced in ℓ

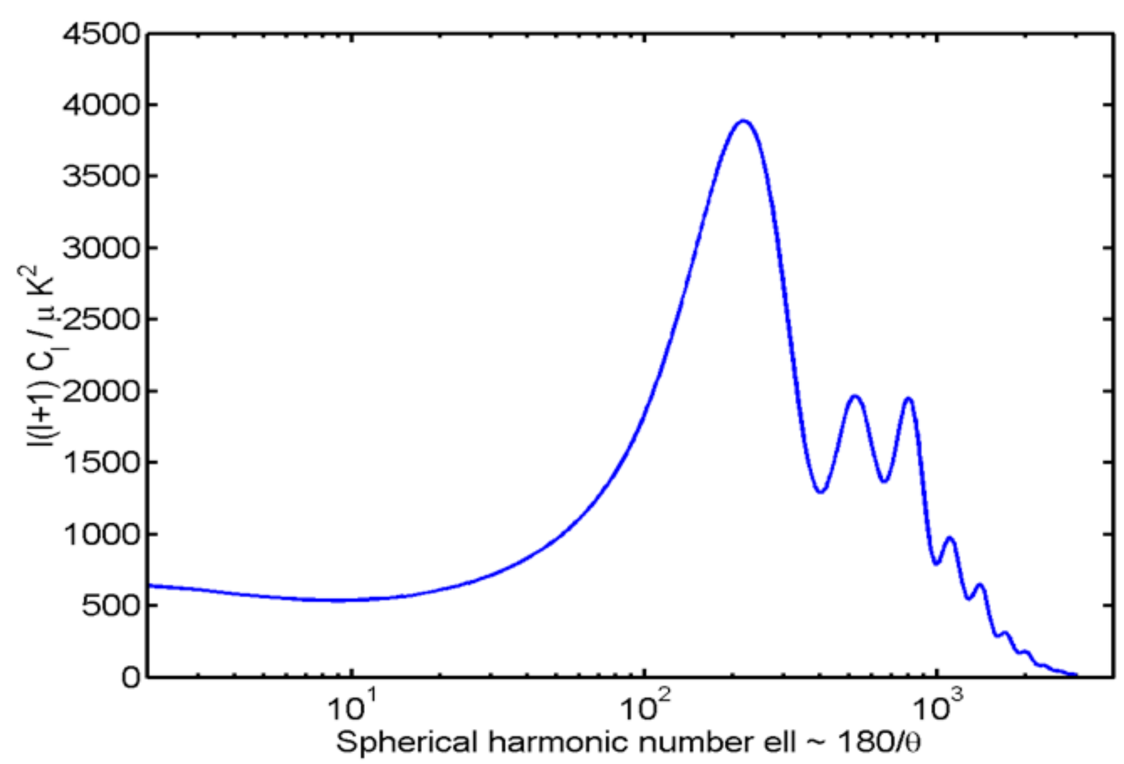
Doppler (velocity) effect

Times in between maximum compression/rarefaction, modes reached maximum velocity

This produced temperature enhancements via the Doppler effect (non-zero velocity along the line of sight)

<u>This contributes power in between the peaks</u>

➡ **Power spectrum does not go to zero** (it does at very high l-s)

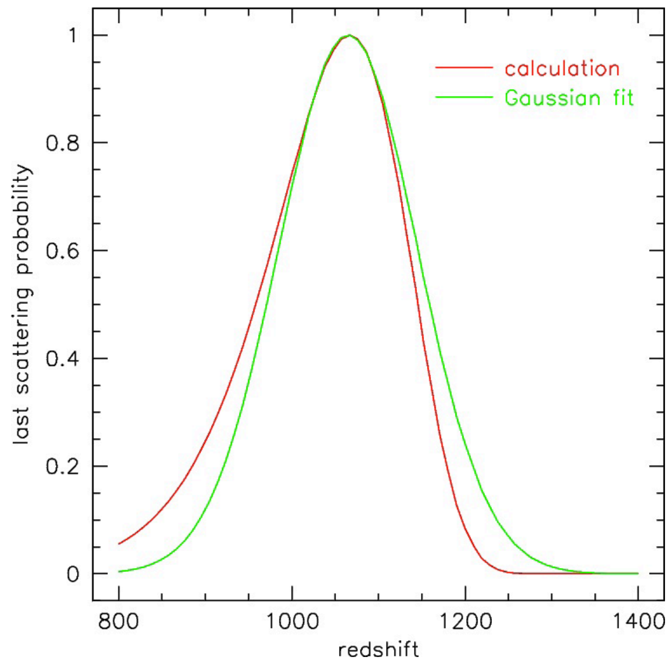


Damping and diffusion

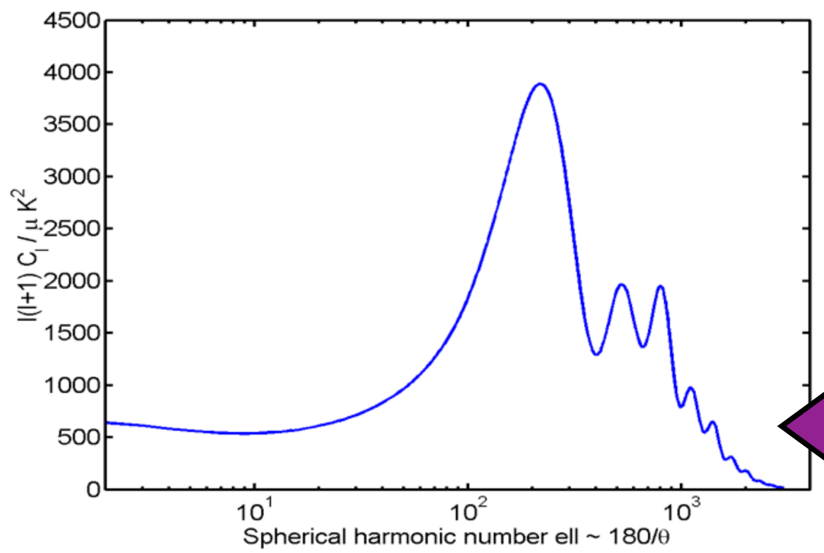
- Photon diffusion suppresses fluctuations in the baryon-photon plasma (Silk damping)

- Recombination does not happen instantaneously and photons execute a random walk during it. Perturbations with wavelengths which are shorter than the photon mean free path are damped (the hot and cold parts mix together)

In the power spectrum scales below ~10 arcmin ($l > \sim 10^3$) are damped.

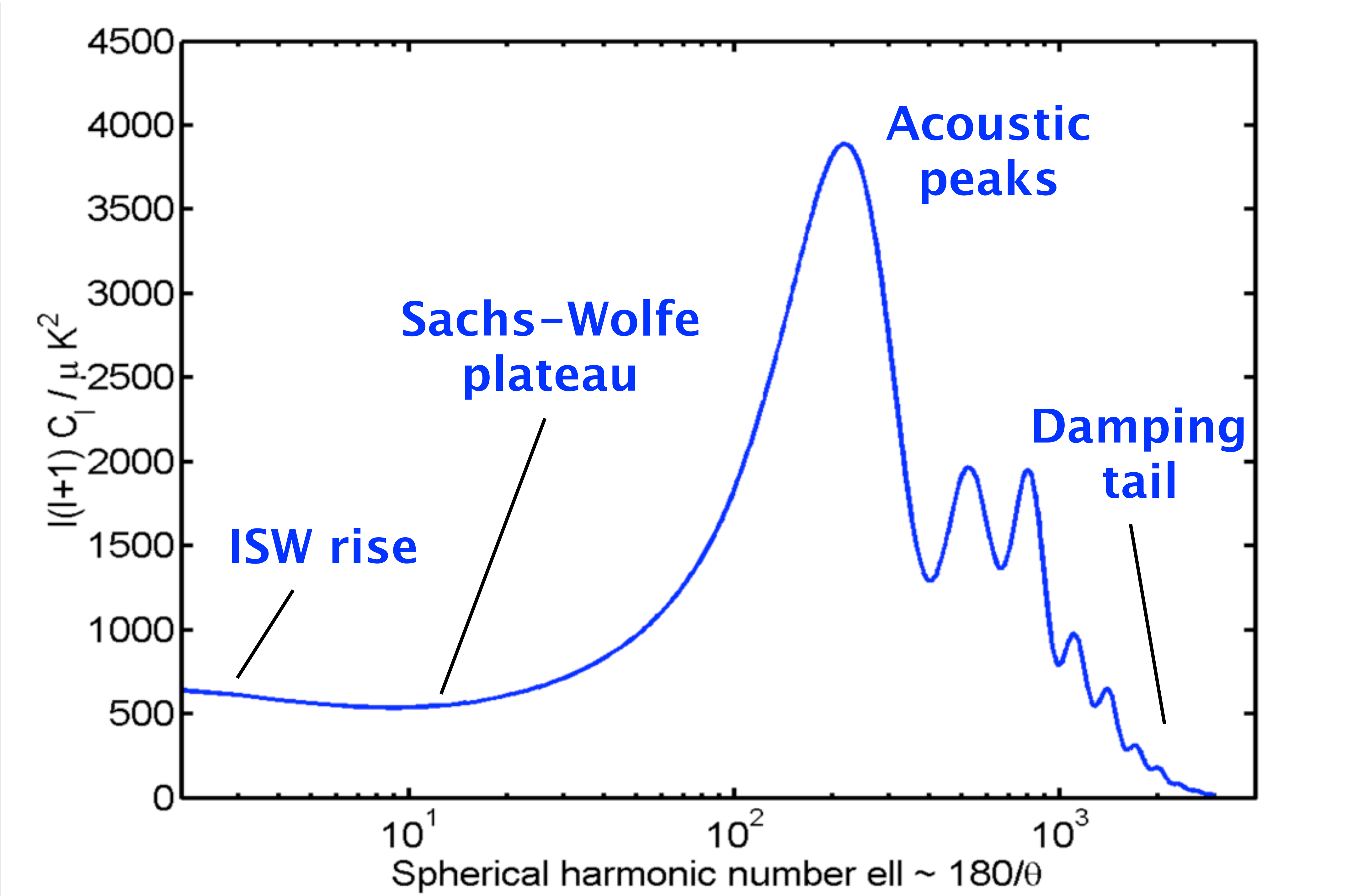


Thickness of the LSS is comparable to the oscillation scales



Power falls off

Power spectrum summary



SECONDARY temperature anisotropies

Table 1. Sources of temperature fluctuations.

PRIMARY	Gravity	
	Doppler	
	Density fluctuations	
	Damping	
	Defects	Strings
		Textures
SECONDARY	Gravity	Early ISW
		Late ISW
		Rees-Sciama
		Lensing
	Local reionization	Thermal SZ
		Kinematic SZ
	Global reionization	Suppression
		New Doppler
		Vishniac
"TERTIARY"	Extragalactic	Radio point sources
		IR point sources
(foregrounds & headaches)	Galactic	Dust
		Free-free
		Synchrotron
	Local	Solar system
		Atmosphere
		Noise, *etc.*

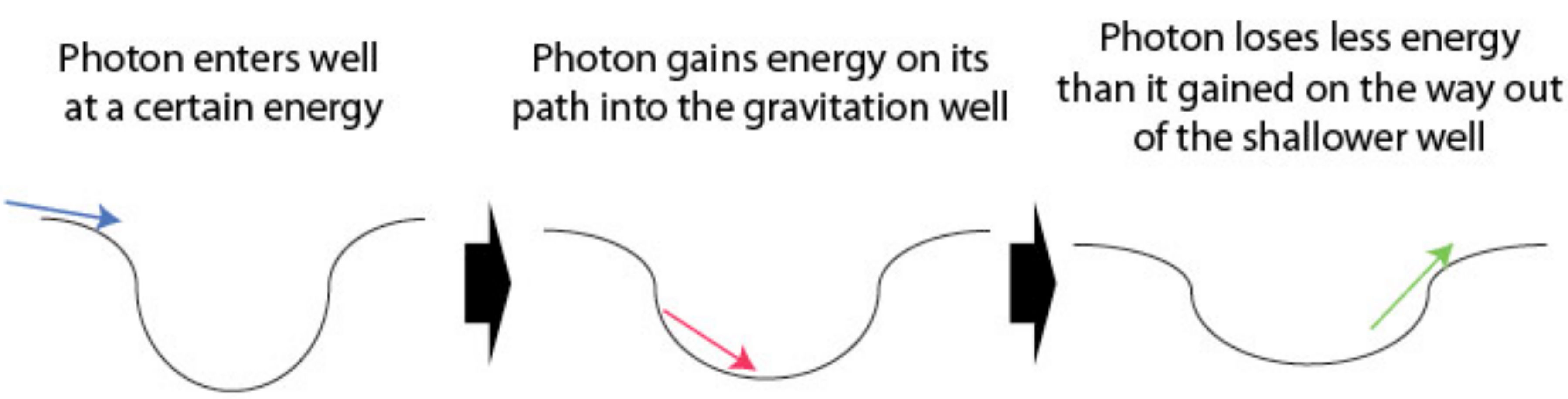
Integrated Sachs–Wolfe effect

Temperature anisotropies due to density change and associated gravitational potential (scaler perturbations) at a given point **x** along the direction **n**

Large-scale (linear) anisotropies:

$$\frac{\Delta T}{T}(\vec{n}) \approx \phi_e(\vec{n}) + \int_e^o \frac{\partial \phi}{\partial t} dt + \vec{n}\cdot(\vec{v}_o - \vec{v}_e) + \left(\frac{\Delta T}{T}(\vec{n})\right)_e$$

SW	ISW	Doppler	Acoustic Oscillations
(grav.pot @ decoupling)	(grav.pot evolution)	(motion emitters @ dec)	(photon-baryon @ dec)



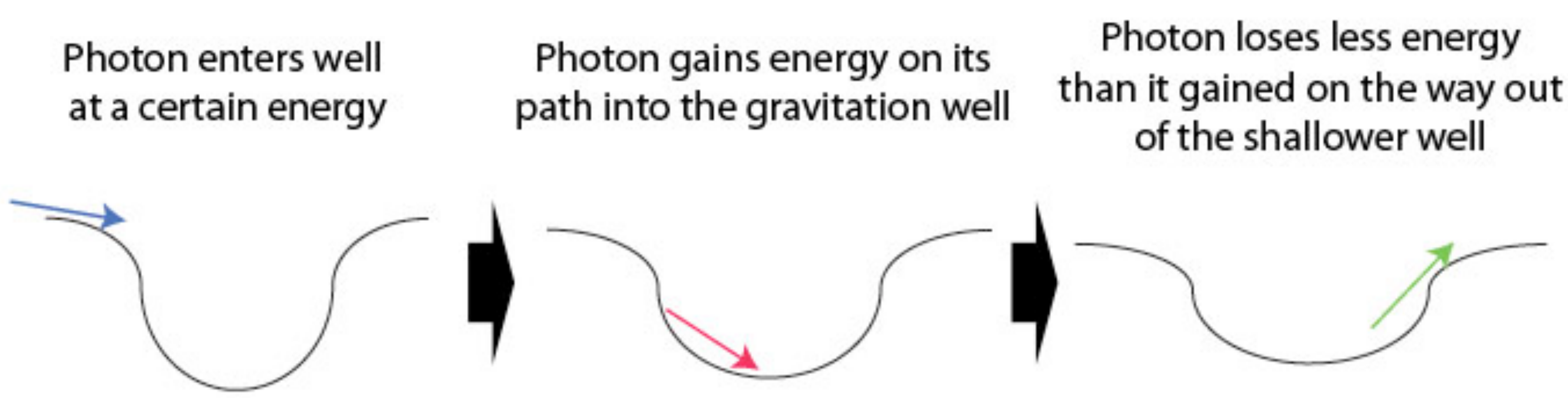
Gravitational well of galaxy supercluster - the depth shrinks as the universe (and cluster) expands

Integrated Sachs-Wolfe effect

• The early ISW effect is caused by the small but non-negligible contribution of photons to the density of the universe at the epoch of last scattering.

• The late ISW effect is caused by interaction of the photons with the large-scale structure after their decoupling.

- Gravitational blueshift on infall does not cancel redshift on climb-out
- Contraction of spatial metric doubles the effect: $\Delta T/T \sim 2\Delta\Phi$
- Effect of potential hills and wells cancel out on small scales



Gravitational well of galaxy supercluster - the depth shrinks as the universe (and cluster) expands

Integrated Sachs–Wolfe effect

$$\frac{\delta T}{T_0}(\hat{\boldsymbol{n}}) = -\frac{2}{c^2}\int_0^{r_{\rm LSS}} \mathrm{d}r\, \frac{\partial \phi(r, \hat{\boldsymbol{n}})}{\partial r}$$

- From Poisson's equation: $-k^2\, a^2\, \phi_k = 4\pi G \rho_b(z)\, \delta_k$, with a the scale factor, so

$$\phi_k = -\frac{4\pi G}{k^2}\rho_b(z=0)\,\frac{\delta_k}{a}$$

- In a critical Universe, $\delta_k \propto a$ and $\phi_k = const$
- In a Λ Universe, the growth of potentials is *suppressed* compared to the critical case:

$\Rightarrow$ **positive correlation between** ϕ_k **and** $\delta T/T_0$

ISW effect as Dark Energy probe

Cosmic evolution of dark energy is parametrized by $w(a) \equiv p_{DE}/\rho_{DE}$
For a cosmological constant, w=−1. In general, $\rho_{DE} \sim a^{-3(1+w)}$

The ISW effect constraints the dynamics of acceleration, be it from dark energy, non-flat geometry, or non-linear growth.

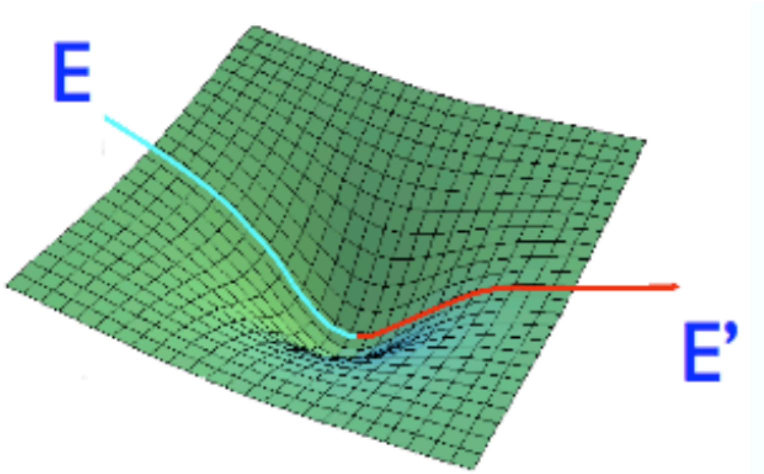
In the absence of curvature, measurement of ISW is measurement of DE.

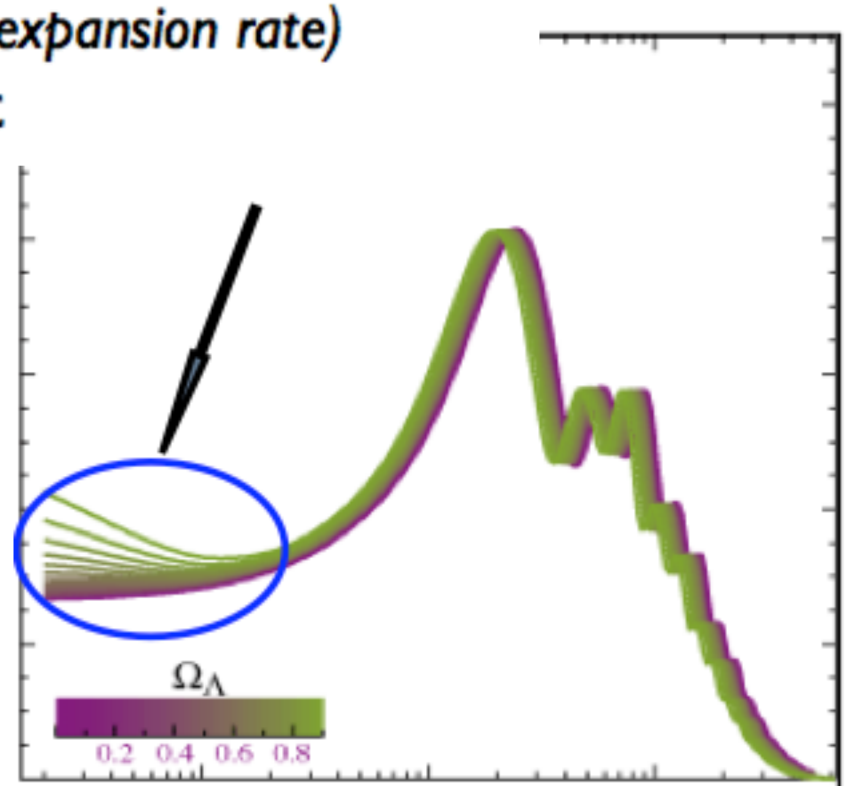
EdS universe : $\delta \propto t^{2/3} \propto a$ for $\delta << 1$

$\Omega_m = 1, \Omega_\Lambda = 0, \Omega_k = 0$

ϕ=const *(linear growth = expansion rate)*

E'=E ⟹ No ISW effect

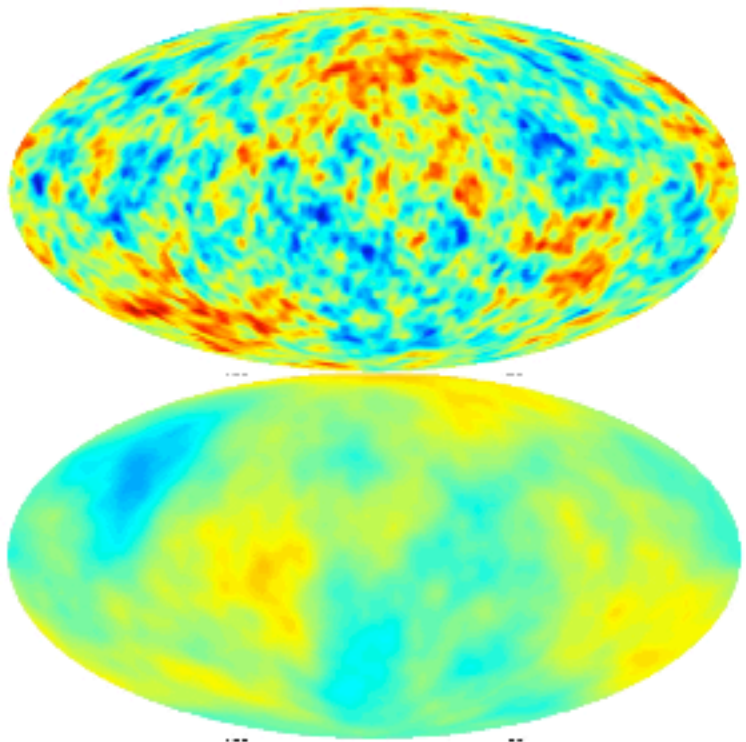




- Linear regime - Integrated Sachs-Wolfe effect (ISW) (Sachs & Wolfe 1967)
- Non-linear regime - Rees-Sciama effect (RS) (Rees & Sciama 1968)

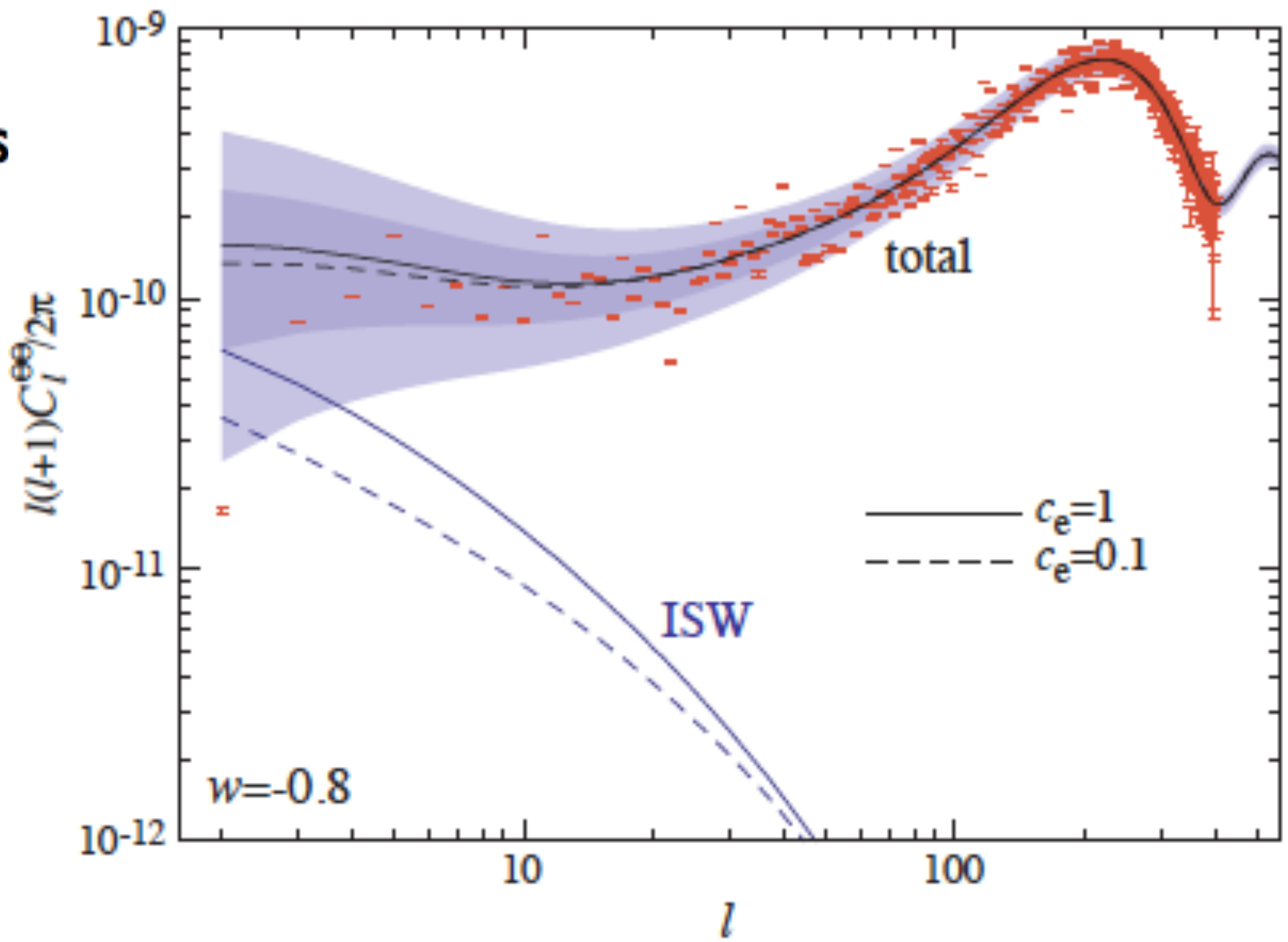
But, we have cosmic variance problem

Primary anisotropies (SW+acoustic peaks largely dominate the ISW signal



primary (z~1000)

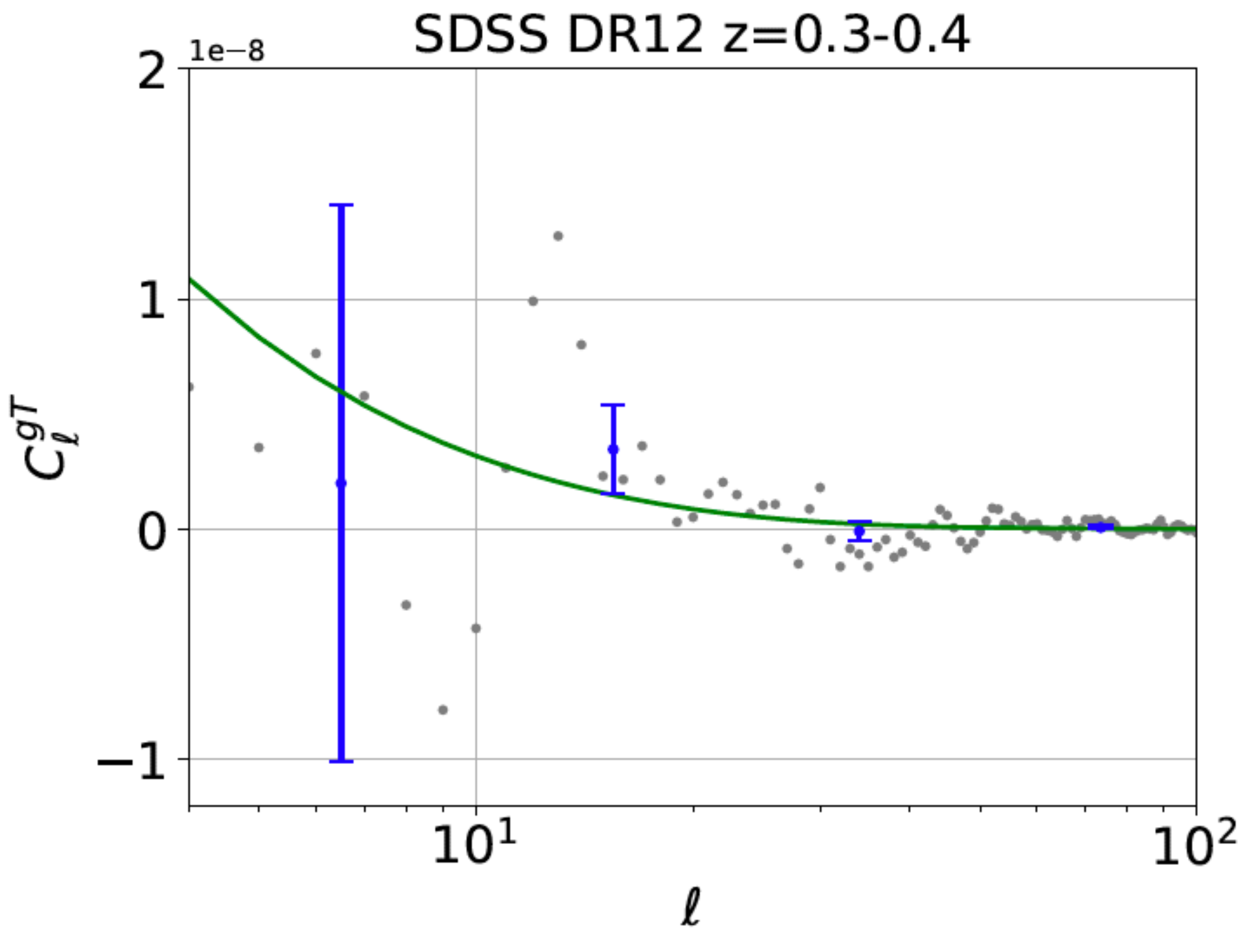
ISW (z<3)



Large-scale anisotropies are dominated by cosmic variance (impossible to extract the ISW signal from CMB maps alone)

Solution: Cross-correlate with other probes of dark energy, which has large sky coverage (e.g. optical, X-ray or radio surveys of galaxies, tSZ signal)

Solution: cross-correlate with others



Two examples. The plot above shows cross power spectrum of SDSS galaxies and CMB temperature (from Stölzner et al. 2018).

thermal SZ effect-ISW cross-correlation (Creque-Serbinowski et al. 2016)

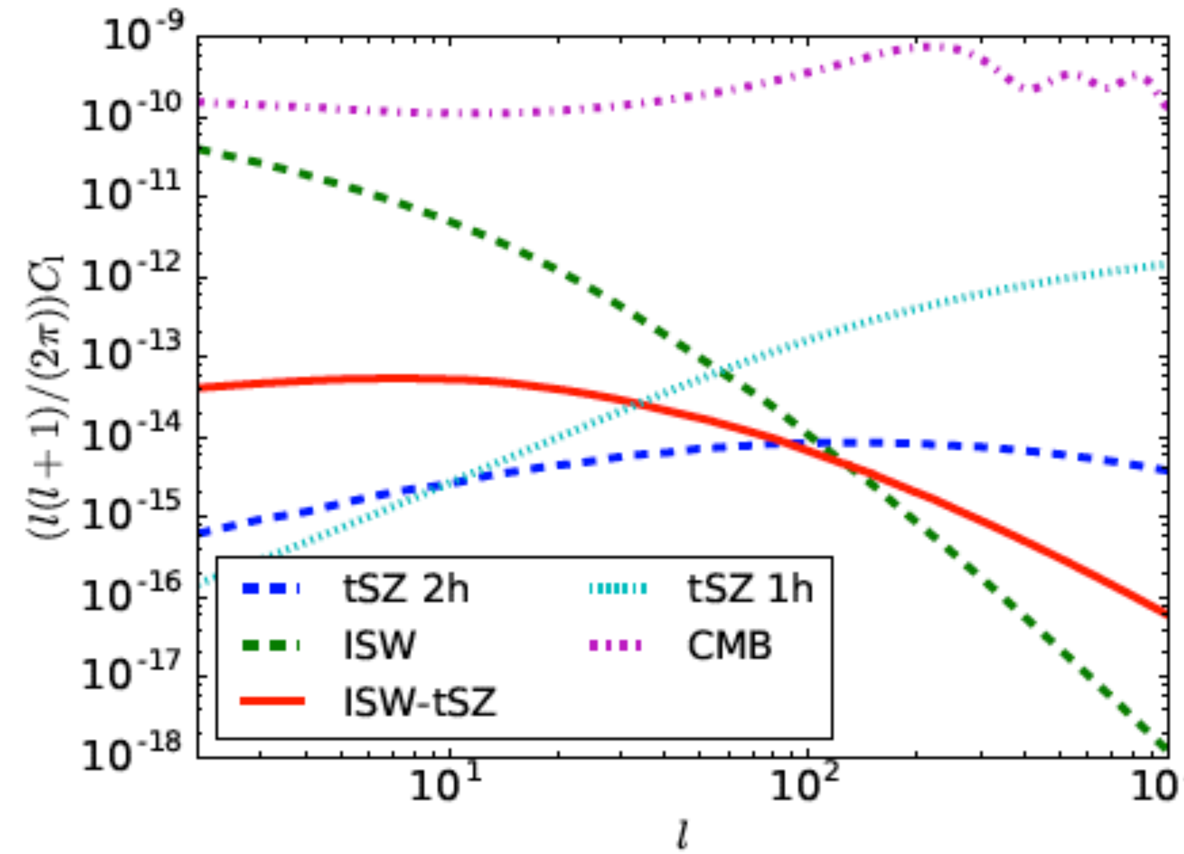
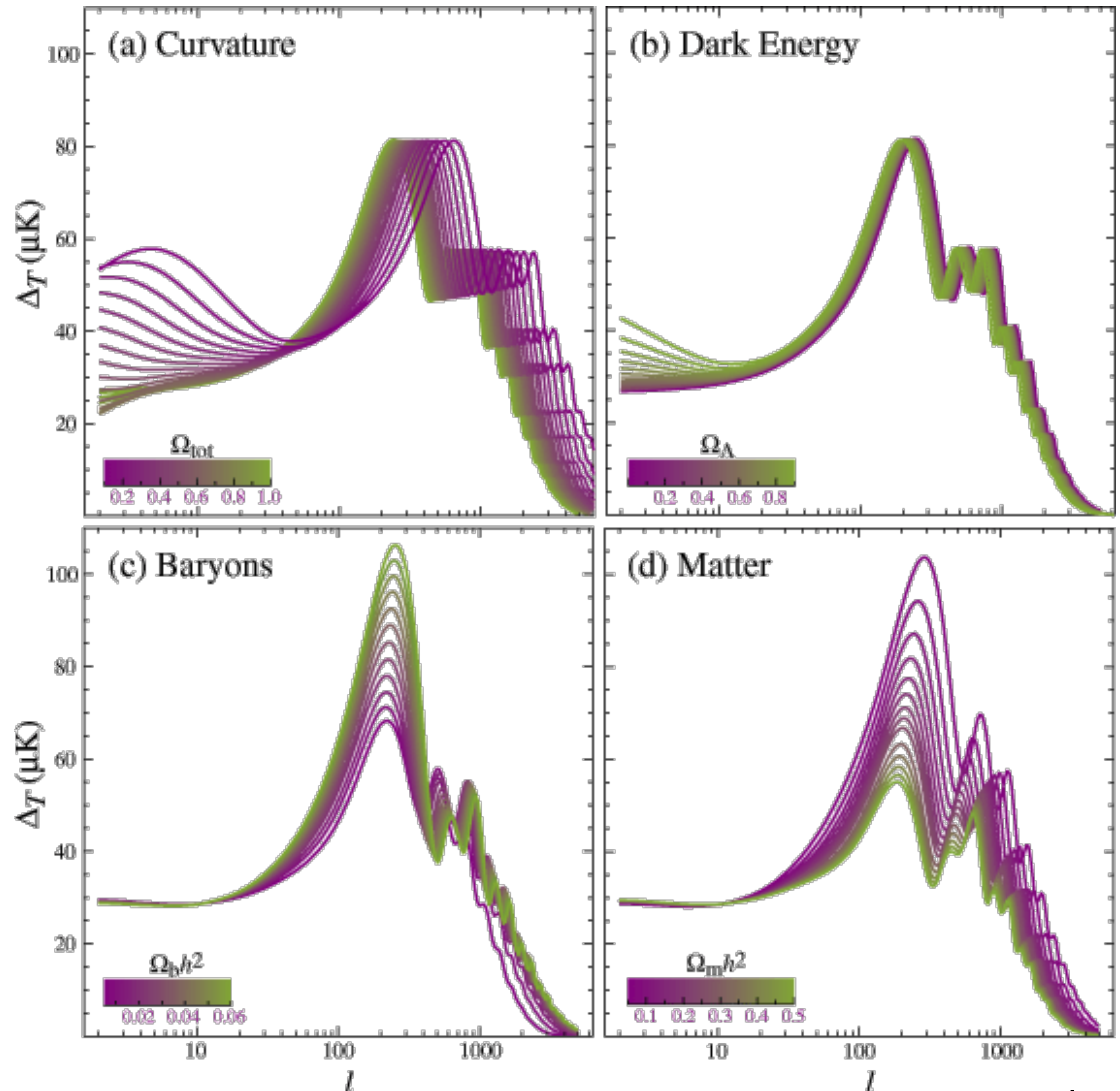


FIG. 1. The ISW power spectrum $C_l^{\rm isw}$ (green) and the two-halo contribution $(y, 2h)$ to C_l^{yy} (blue) are shown in dashed lines, while C_l^{yT} is shown in solid red. The one-halo contribution $(y, 1h)$ to C_l^{yy} is dotted. The CMB power spectrum is shown dot-dashed for comparison.

Which way the peaks move?



Credit: Wayne Hu

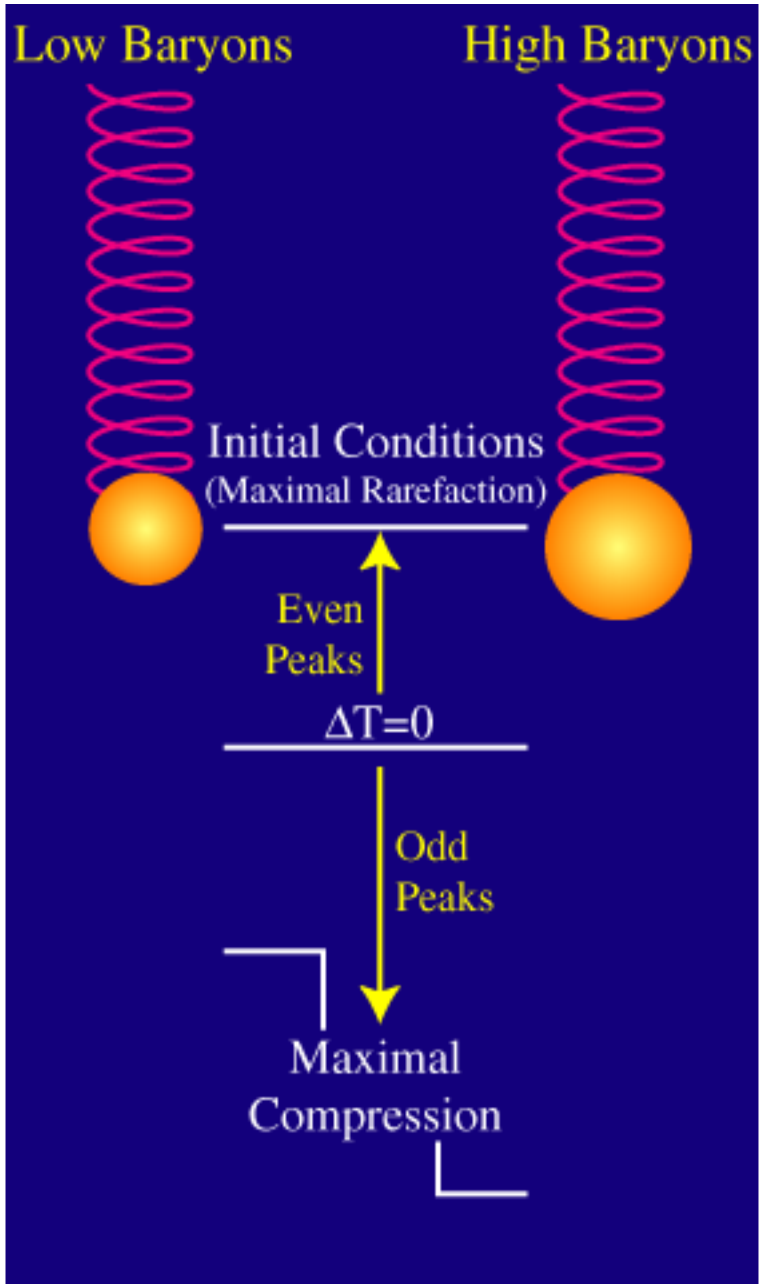
Baryon loading

The presence of more baryons **add inertia**, and increases the amplitude of the oscillations (baryons drag the fluid into potential wells).

Perturbations are then compressed more before radiation pressure can revert the motion.

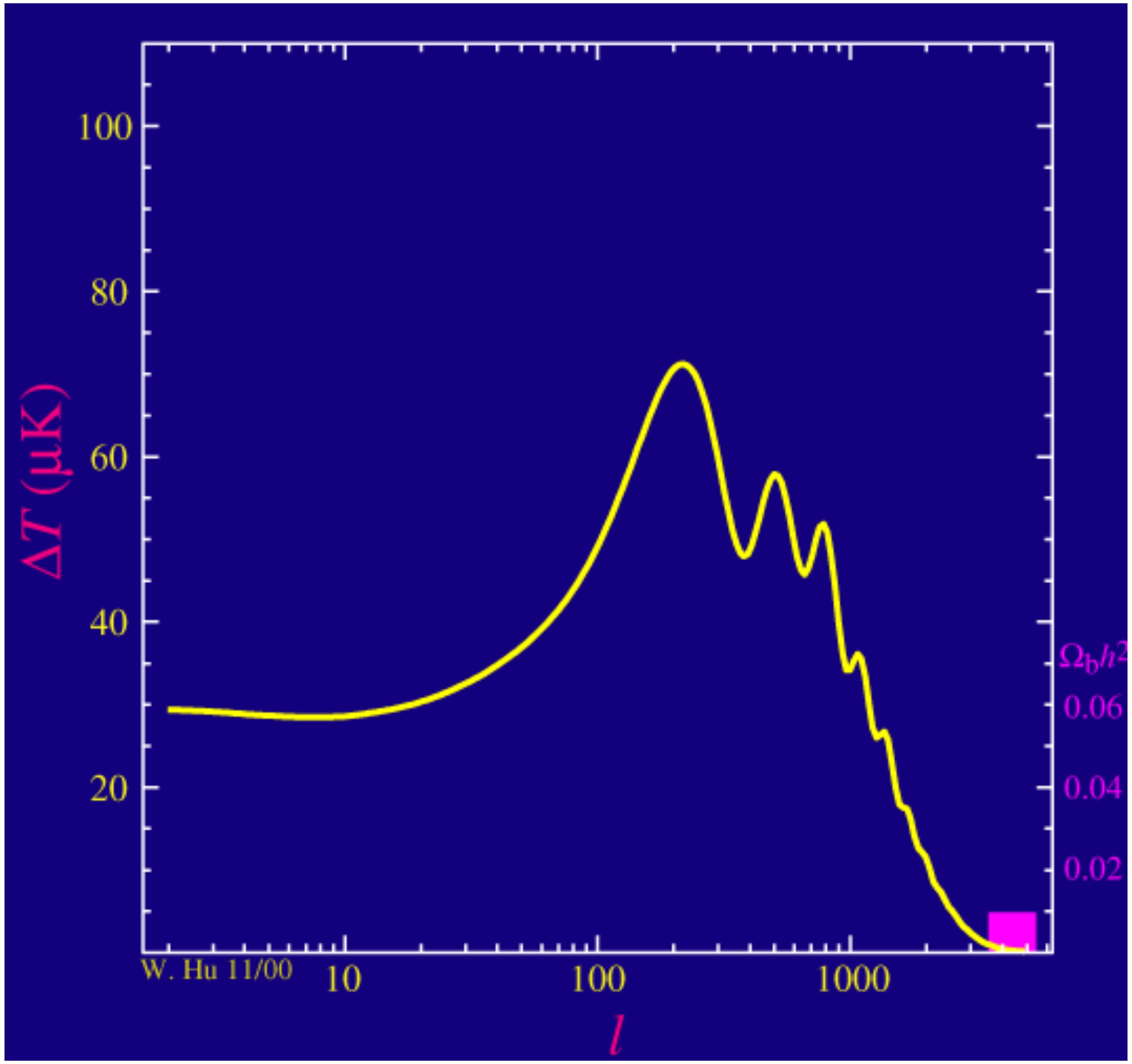
This causes a **breaking of symmetry in the oscillations**, enhancing only the compressional phase (i.e. every odd-numbered peak).

This can be used to measure the abundance of cosmic baryons.



Credit: Wayne Hu

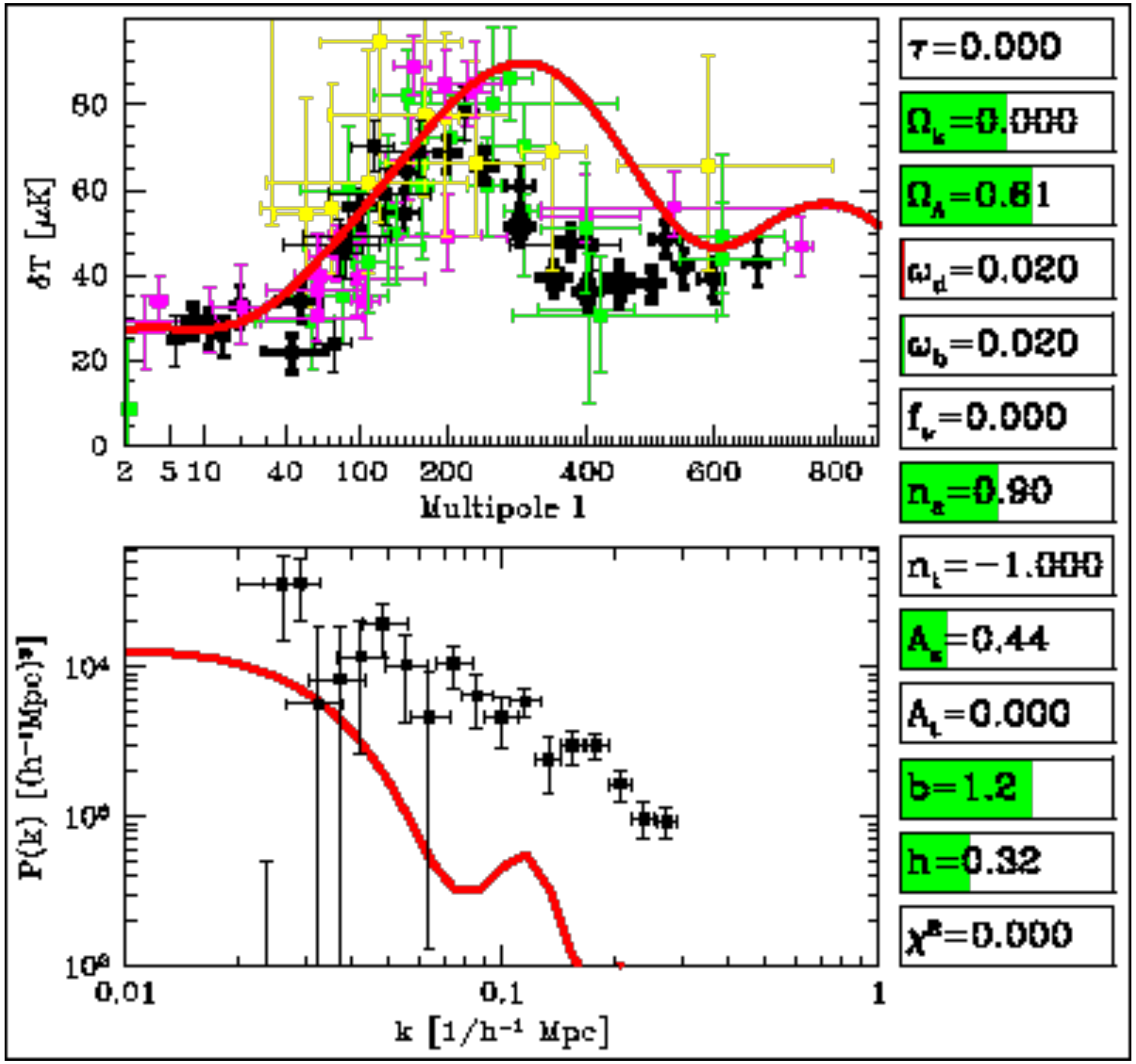
Baryons in the power spectrum



Credit: Wayne Hu

Power spectrum shows baryon enhance every other peak, which helps to distinguish baryons from cold dark matter.
Baryons also change the damping scale at the tail!

DM in the power spectrum



Credit: Max Tegmark

As the Dark Matter density decreases, while keeping other parameters fixed, two things happen:

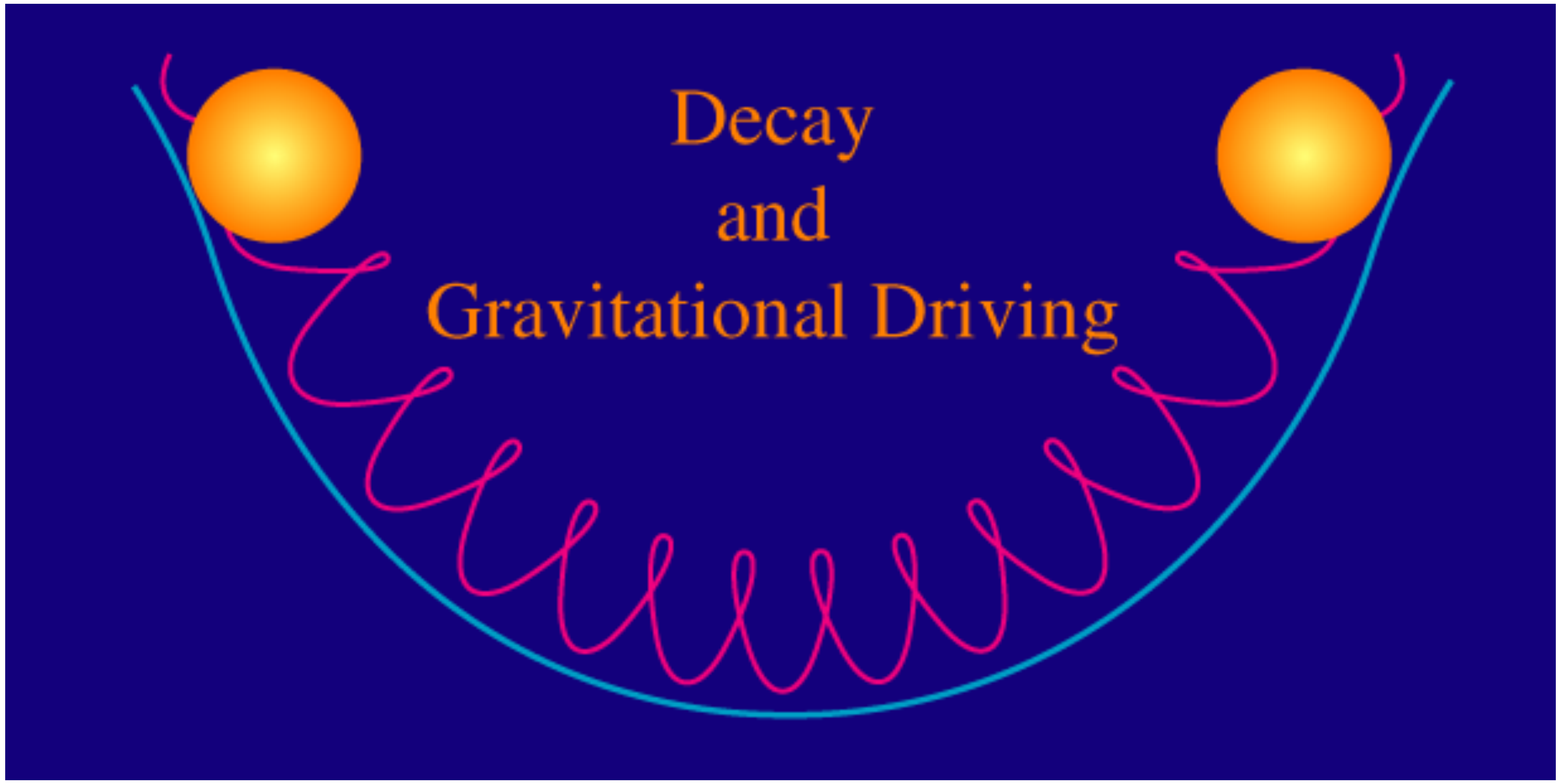
(1) The baryon fraction increases, creating an effect similar to raising the baryon density.

(2) The matter-to-radiation ratio changes, making the contribution of radiation in the total matter-energy density of the universe more significant. The enhanced radiation pressure causes the gravitation potential to decay, a phenomenon known as **radiation driving**. This decay in turn drives the oscillations stronger by eliminating the force that otherwise would oppose it.

Radiation driving force

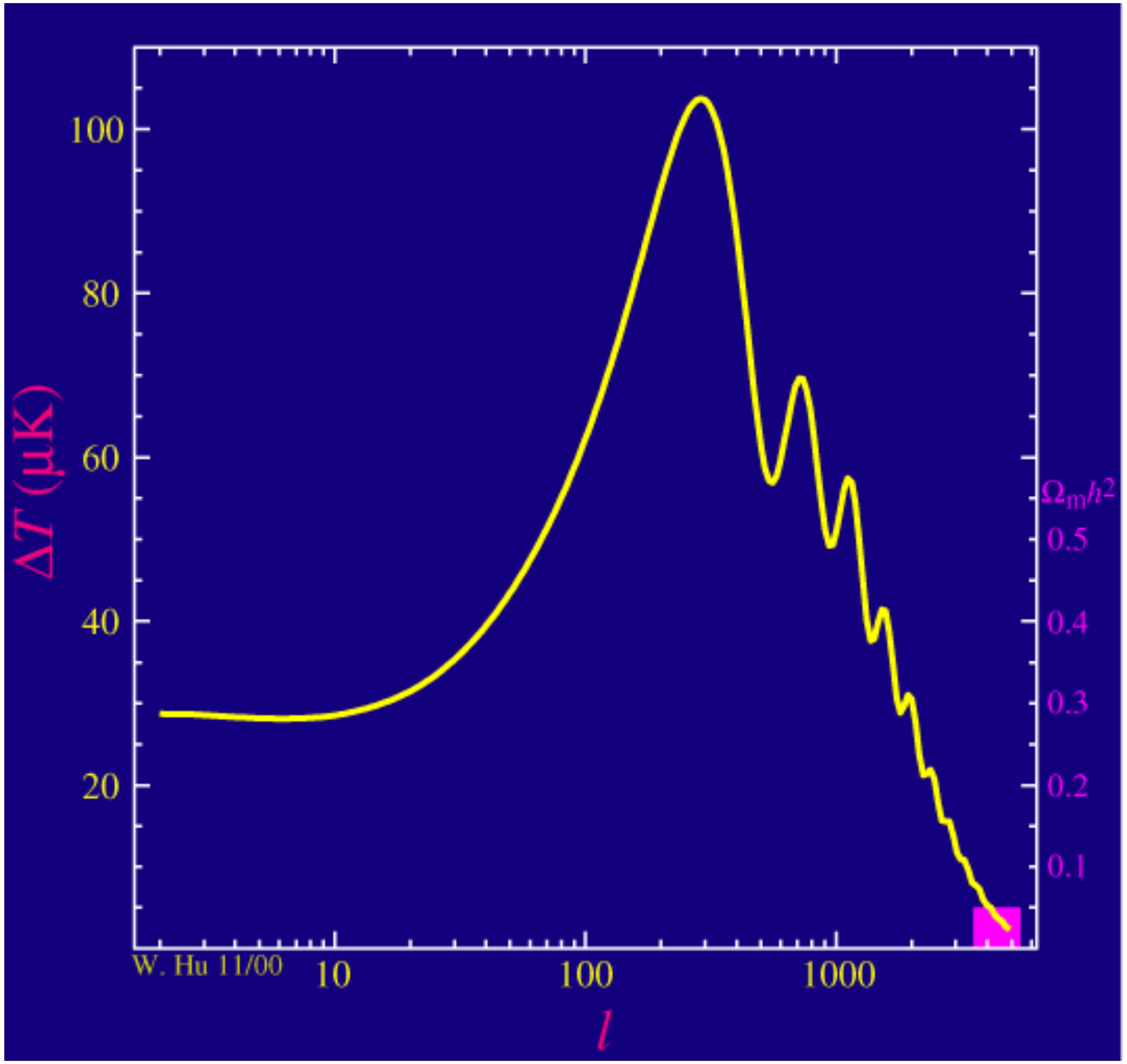
Decreasing the matter density correspondingly increases the contribution of radiation in determining the gravitational potentials, and brings the epoch of matter-radiation equality close to the epoch of recombination.

When the radiation (CMB photons) energy density starts to dominate over the matter energy density, the gravitational potential in which the photon-baryon fluid oscillates can not be taken as a constant. The potential decays to drive the amplitude of the oscillation up.



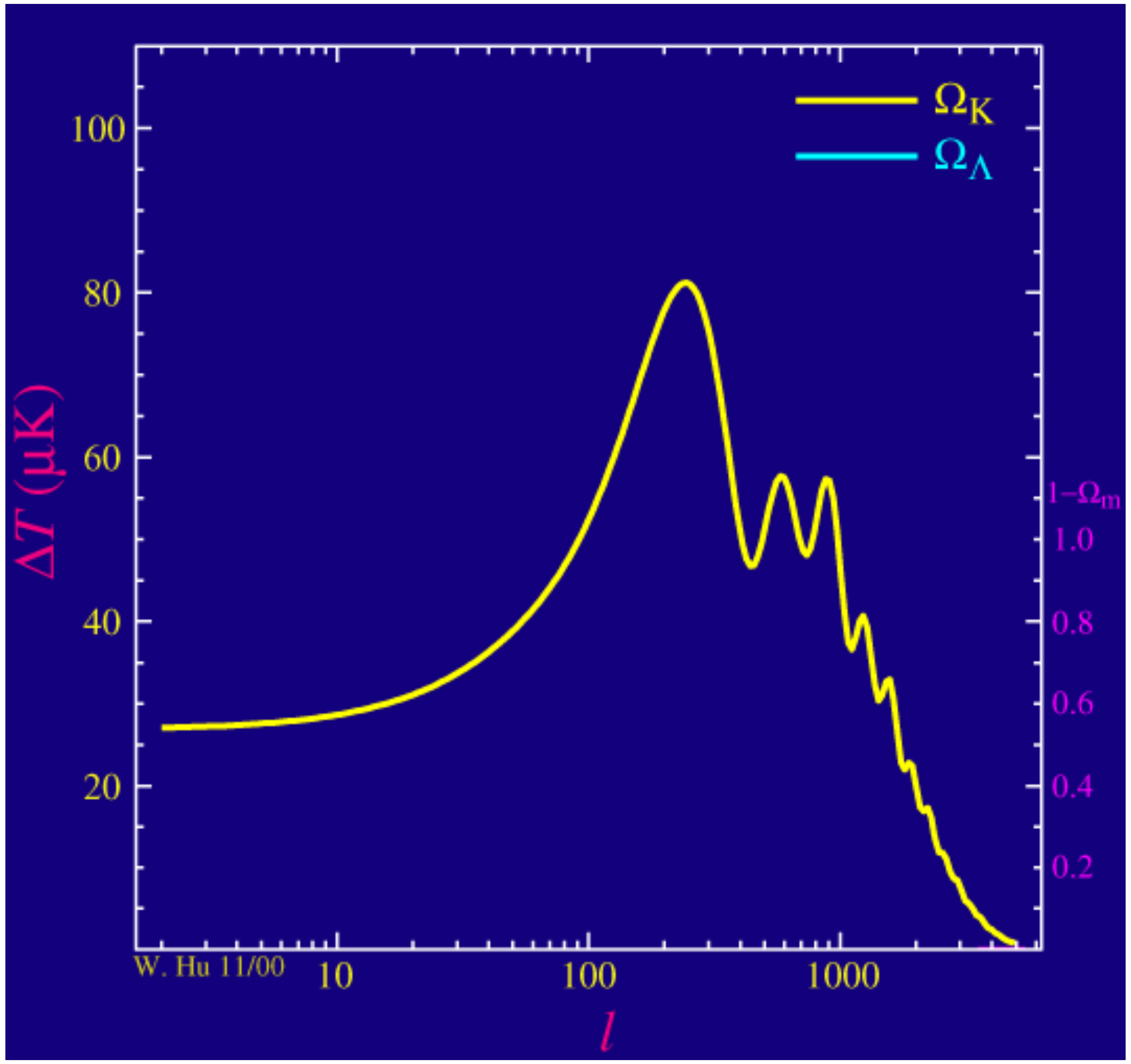
Credit: Wayne Hu (see “Radiation Driving Force”)

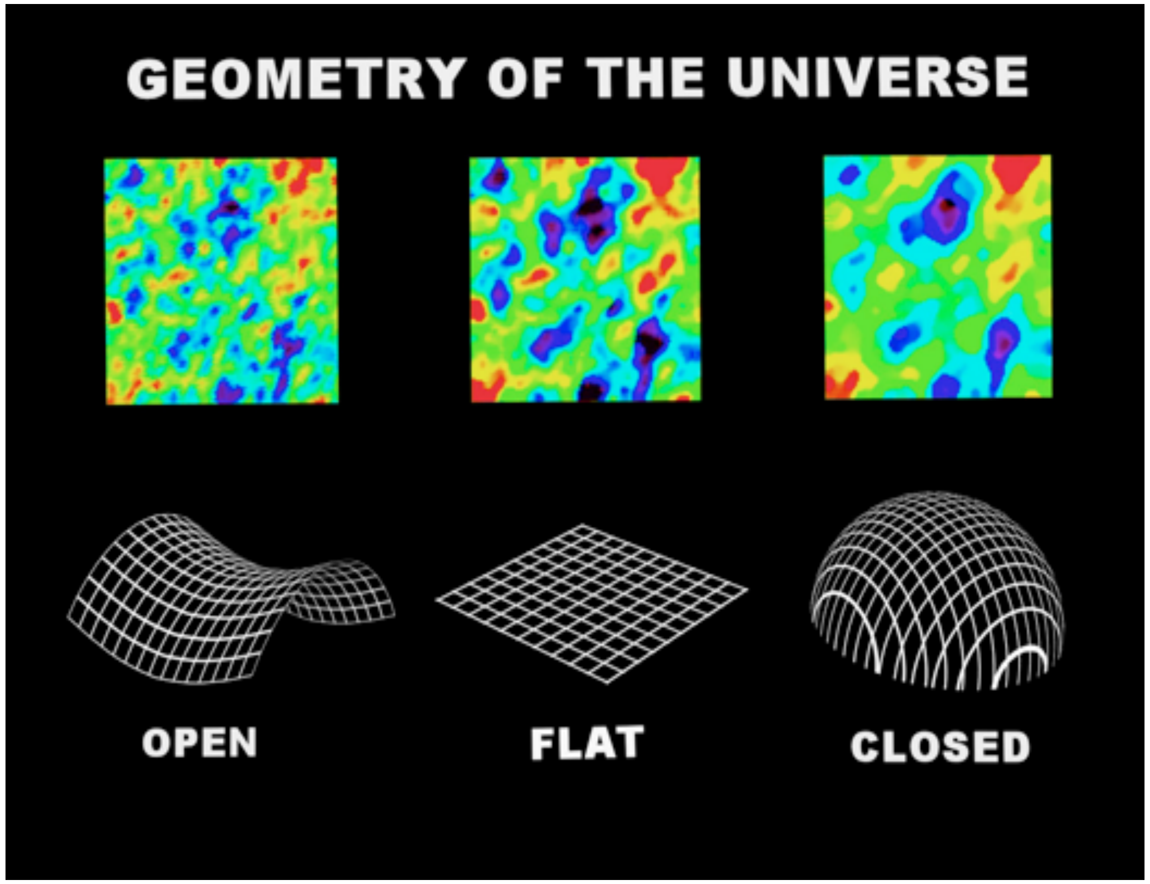
Dark matter in the power spectrum



Credit: Wayne Hu

Effect of curvature



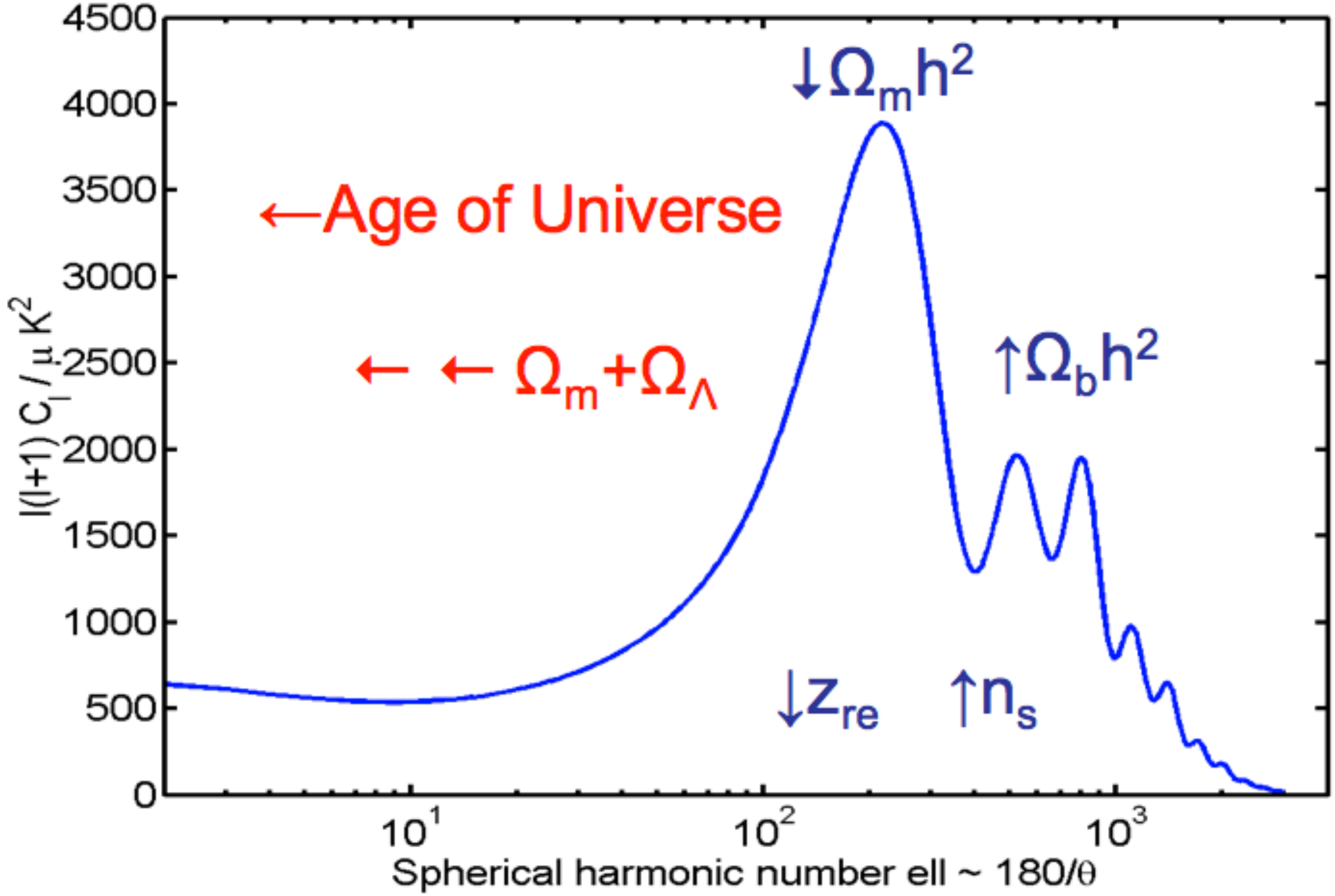


Credit: Wayne Hu

Ω_K does not change the amplitude of the power spectrum, rather it shifts the peaks sideways. This follows from the conversion of the physical scales (on the LSS) to angular scales (that we observe), which depends on the geometry.

Curvature (cosmological constant, Ω_Λ) also causes the ISW effect on large scales, by altering the growth of structures in the path of CMB photons.

CMB parameter cheat sheet



Only six parameters for the CMB

The TT power spectrum is adequately described by six independent parameters:

- Ω_0 total density parameter
- A amplitude of primordial scalar perturbations (at some pivot scale k_p)
- n spectral index of primordial scalar perturbations
- τ optical depth due to reionization (discussed in Sec. 12.9.6)
- $\omega_b \equiv \Omega_b h^2$ "physical" baryon density parameter
- $\omega_m \equiv \Omega_m h^2$ "physical" matter density parameter

Adding extra parameters like neutrino mass or DE equation of state does not improve the goodness of the fit, so those are taken at their fiducial values.
The other cosmological parameters, for example H_0, are derived from these six:

$$\Omega_0 = \Omega_m + \Omega_\Lambda \quad \Rightarrow \quad \Omega_m = \Omega_0 - \Omega_\Lambda$$

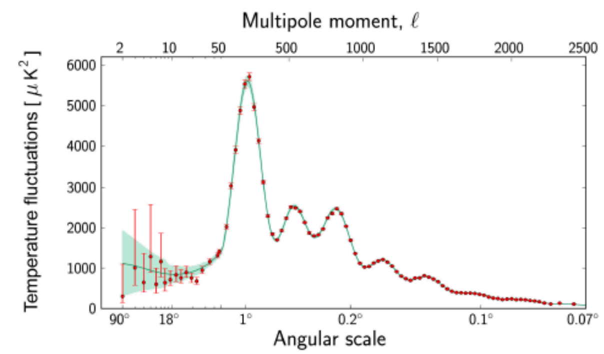
$$h = \sqrt{\frac{\omega_m}{\Omega_m}} = \sqrt{\frac{\omega_m}{\Omega_0 - \Omega_\Lambda}}$$

$$\Omega_b = \frac{\omega_b}{h^2} = \frac{\omega_b}{\omega_m}(\Omega_0 - \Omega_\Lambda)$$

Parameter constraints from *Planck*

- Angular scale of first acoustic peak

$$\theta_* = \frac{s_*}{D_A} = (1.04148 \pm 0.00066) \times 10^{-2} \quad \text{[Planck 2013]}$$

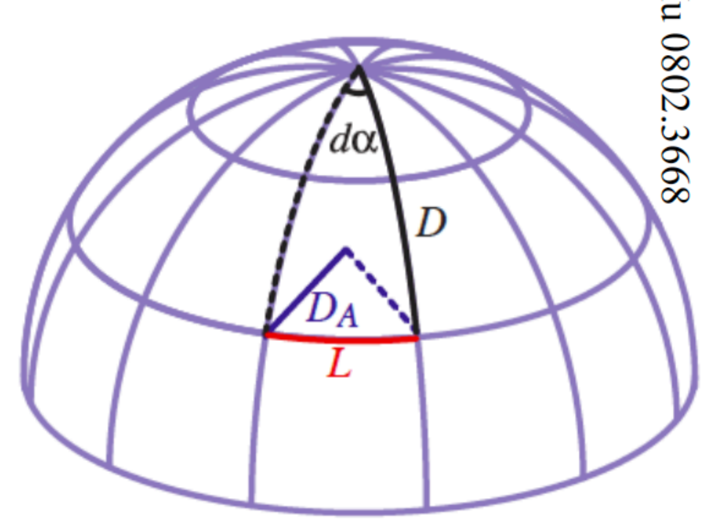


  - Sound horizon at recombination

$$s_* = \int_0^* c_s d\eta \propto (\Omega_m h^2)$$

  - Horizon distance to recombination

$$D_* = \eta_0 - \eta_* = \int_0^* \frac{dz}{H(z)} \propto h^{-1}$$



Hu 0802.3668

- Curved space

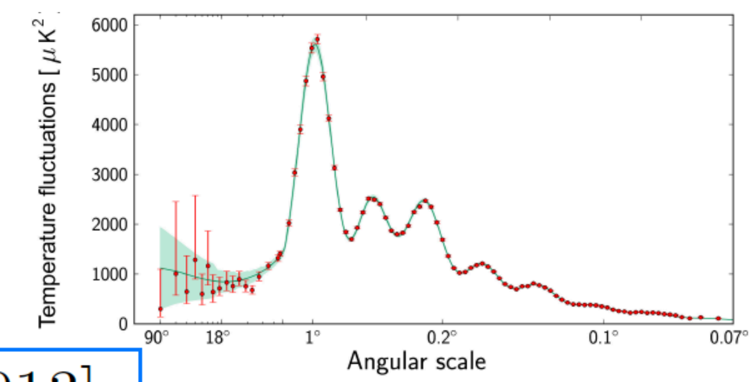
$$D_A = R\sin\left(\frac{D_*}{R}\right) \approx D_* \left(1 + \frac{\Omega_\kappa H_0^2 D_*^2}{6}\right)$$

$$\Rightarrow \Omega_\kappa = -0.042^{+0.043}_{-0.048} \quad \text{[Planck 2013]}$$

- Flat space

$$\theta_* = \frac{s_*}{D_*} \propto \Omega_m h^3 = 0.0959 \pm 0.0006 \quad \text{[Planck 2013]}$$

Parameter constraints from *Planck*



- Second peak relative height
  - Baryon loading suppresses even peaks (rarefaction)

$$\Omega_b h^2 = 0.02207 \pm 0.00033 \quad \text{[Planck 2013]}$$

- Third and higher peaks
  - Decay of potential in radiation era enhances higher peaks, suppressed by matter density

$$\Omega_m h^2 = 0.1423 \pm 0.0029 \quad \text{[Planck 2013]}$$

- Damping tail
  - Fixed in basic Lambda-CDM cosmology
  - Requires spectral tilt

$$n = 0.9603 \pm 0.0073 \quad \text{[Planck 2013]}$$

- Reionisation $\tau = 0.097 \pm 0.038 \quad \text{[Planck 2013]}$
  - Suppresses all small angle anisotropies below l=20
  - Approximately degenerate with spectral tilt – *can be broken by polarisation*

Parameter constraints from *Planck*

All columns assume the ΛCDM cosmology with a power-law initial spectrum, no tensors, **spatial flatness**, A cosmological constant as dark energy, and sum of neutrino masses to be 0.06 eV.

Planck CMB results 2013

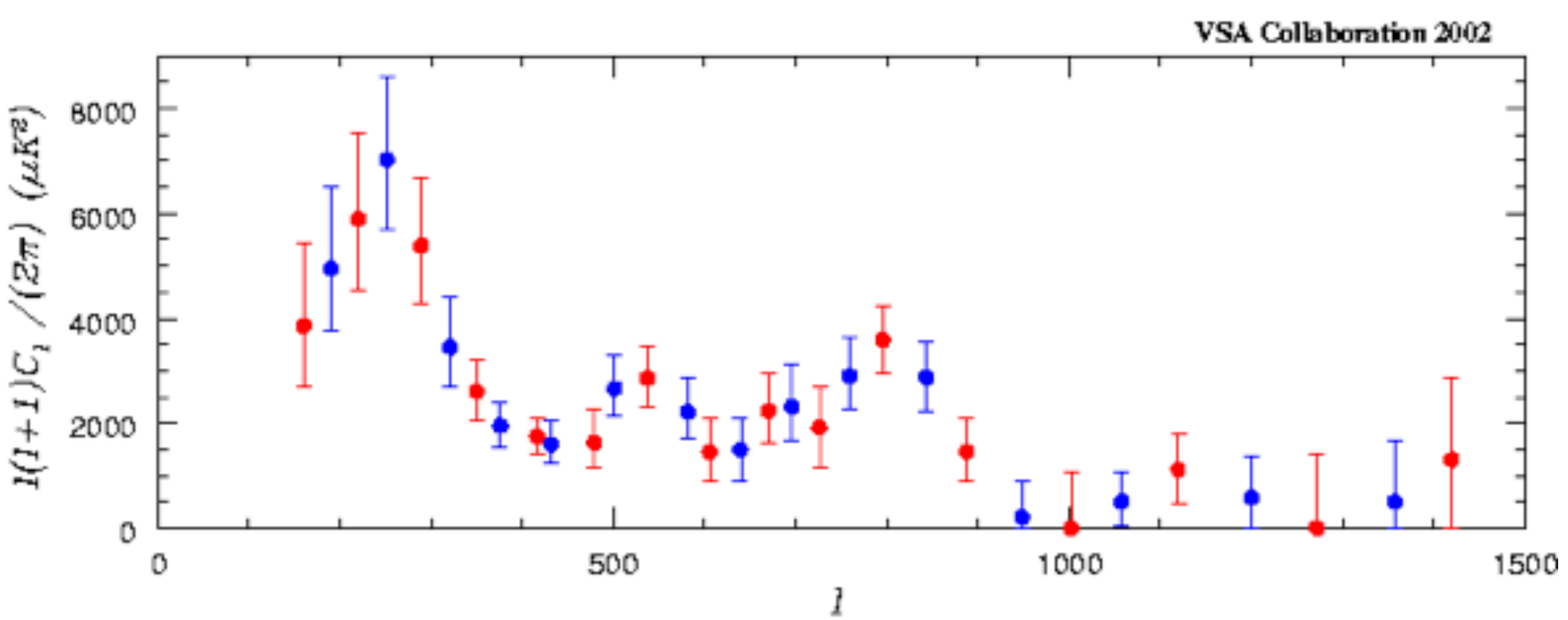
		Planck+WP +highL	*Planck*+WP +highL+BAO	*WMAP*9+eCMB +BAO
6-parameter combination to fit the TT data	$\Omega_b h^2$	0.02207 ± 0.00027	0.02214 ± 0.00024	0.02211 ± 0.00034
	$\Omega_c h^2$	0.1198 ± 0.0026	0.1187 ± 0.0017	0.1162 ± 0.0020
	$100\,\theta_{MC}$	1.0413 ± 0.0006	1.0415 ± 0.0006	–
	n_s	0.958 ± 0.007	0.961 ± 0.005	0.958 ± 0.008
	τ	$0.091^{+0.013}_{-0.014}$	0.092 ± 0.013	$0.079^{+0.011}_{-0.012}$
	$\ln(10^{10}\Delta^2_{\mathcal{R}})$	3.090 ± 0.025	3.091 ± 0.025	3.212 ± 0.029
Derived from the above	h	0.673 ± 0.012	0.678 ± 0.008	0.688 ± 0.008
	σ_8	0.828 ± 0.012	0.826 ± 0.012	$0.822^{+0.013}_{-0.014}$
	Ω_m	$0.315^{+0.016}_{-0.017}$	0.308 ± 0.010	0.293 ± 0.010
	Ω_Λ	$0.685^{+0.017}_{-0.016}$	0.692 ± 0.010	0.707 ± 0.010

Make your own CMB experiment!

- Design experiment to measure $\dfrac{\Delta T}{T}(\theta, \phi)$

- Find component amplitudes $a_{\ell m} = \int_\Omega \dfrac{\Delta T}{T}(\theta, \phi) Y^*_{\ell m}(\theta, \phi) d\Omega$

- Plot $c_\ell = \langle |a_{\ell m}| \rangle^2$ against l (where l is inverse of angular scale, $l \sim \pi / \theta$)



Generating theoretical C_l

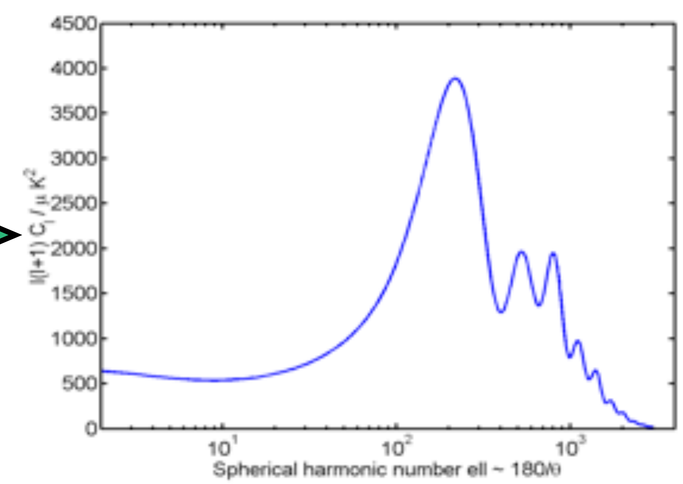
INPUT

Favorite cosmological model:
Ω_m, Ω_Λ, σ_8, H_0, ..

Boltzmann solvers

powerful cosmological codes (CMBFAST, CAMB)

OUTPUT



Fit to data

Codes like CMBFAST or CAMB evolve the perturbations in different species (CDM, baryons, photons, neutrinos) independently and then add them up. The perturbations are small so linear theory suffices.

Calculation of the C_l-s (codes like CMBFast & CAMB)

Boltzmann transport equation describes the evolution of the photon distribution function

$$\delta f_T(\hat{\mathbf{n}}, \mathbf{x}, \eta) = \left(T\frac{\partial f}{\partial T}\right)_{\rm CMB} \frac{\Delta T}{T}$$

$$\frac{\partial}{\partial \eta}\frac{\Delta T}{T}(\hat{\mathbf{n}}, \mathbf{x}, \eta) = {\rm Coll.} + {\rm Grav.}$$

Collisional part describes the scattering of the photons with electrons

Gravitational part describes the motion of the photons in the perturbed background

Differential form in Fourier space

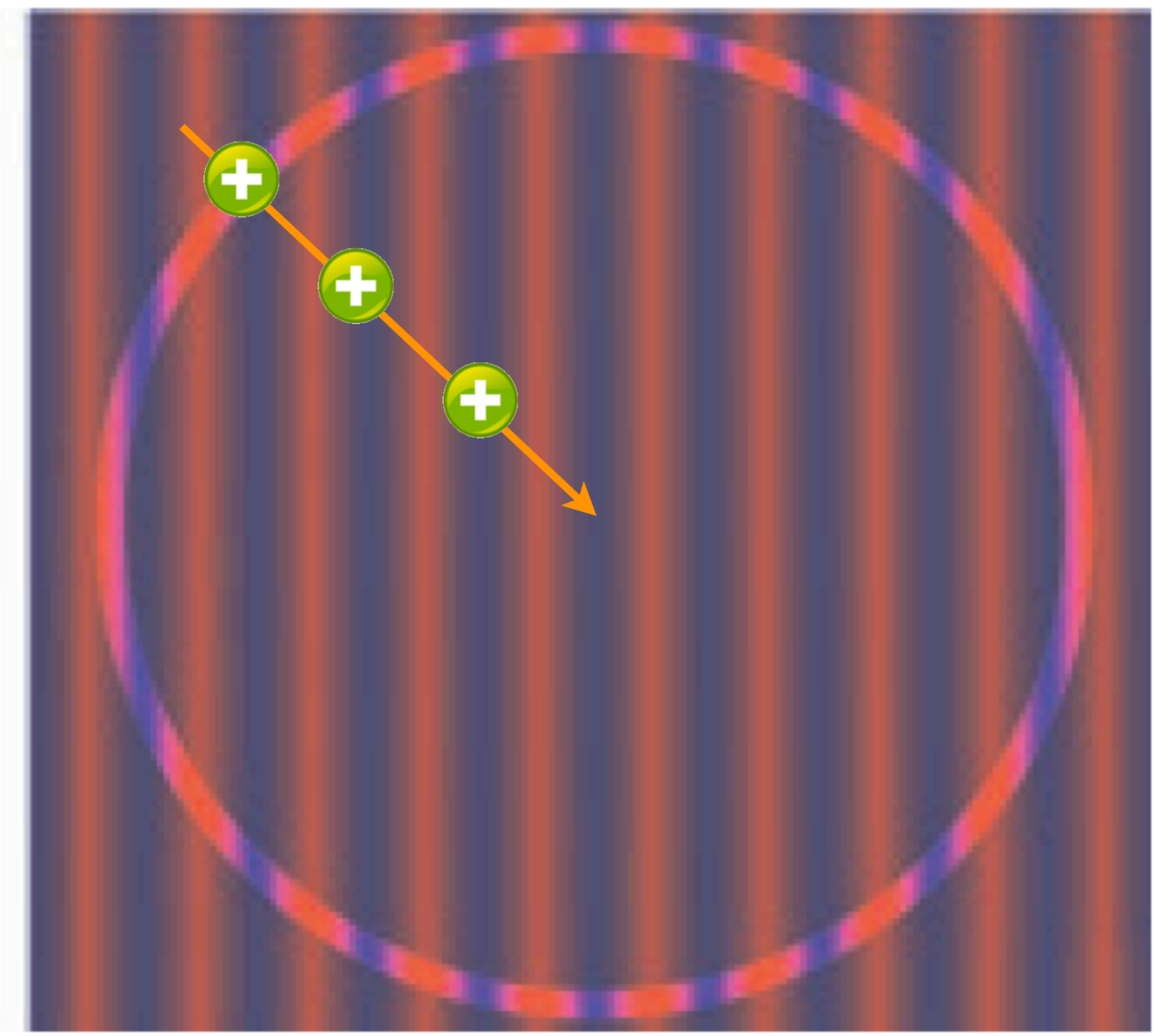
Scalar perturbations

$$\begin{aligned}
&\dot{\Delta}_T + ik\mu\Delta_T = \dot{\Phi} - ik\mu\Psi \\
&+\dot{\tau}\left[-\Delta_T + \Delta_{T_0} + i\mu v_B + \frac{1}{2}P_2(\mu)\Pi\right] \\
&\dot{\Delta}_P + ik\mu\Delta_P = \dot{\tau}\left[-\Delta_P + \frac{1}{2}\{1 - P_2(\mu)\}\,\Pi\right]
\end{aligned}$$

$$C_\ell = (4\pi)^2 \int k^2 dk P(k) |\Delta_{T\ell}(k, \eta_0)|^2$$

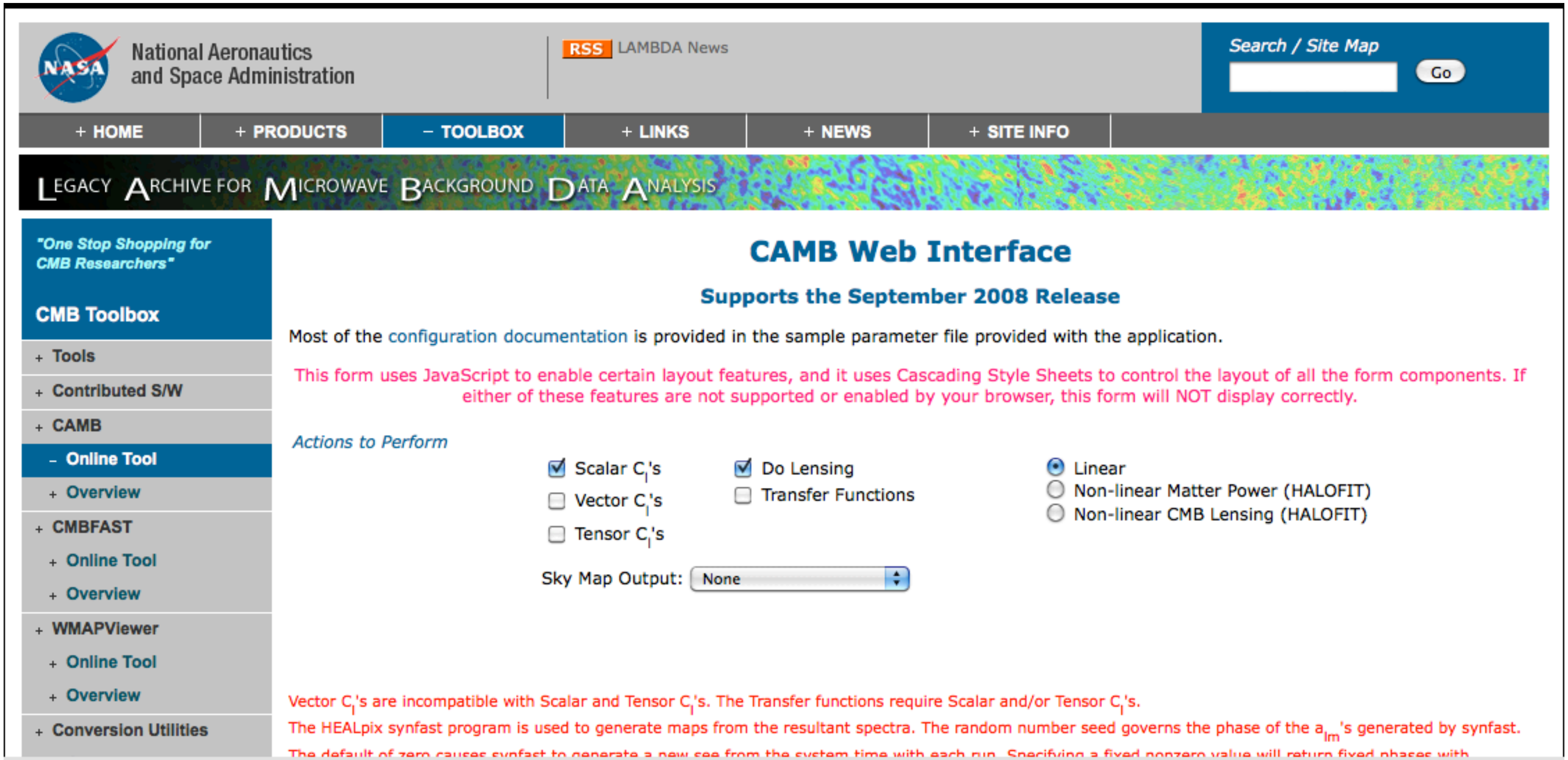
Reference: Seljak & Zaldarriaga (1996)

Calculation of the C_l–s (codes like CMBFast & CAMB)



- We know how the intensity distribution for a single k-mode looks like for any given instant
- Choose one single k–mode and evolve that from before the recombination until today (coupled & linearized Boltzmann and Einstein equations)
- Compute the contribution of that k–mode to the power spectrum (C_l–s) by line of sight integration
- Average over all possible phases, and sum up the contributions from all the k–modes!

Online C_l calculators



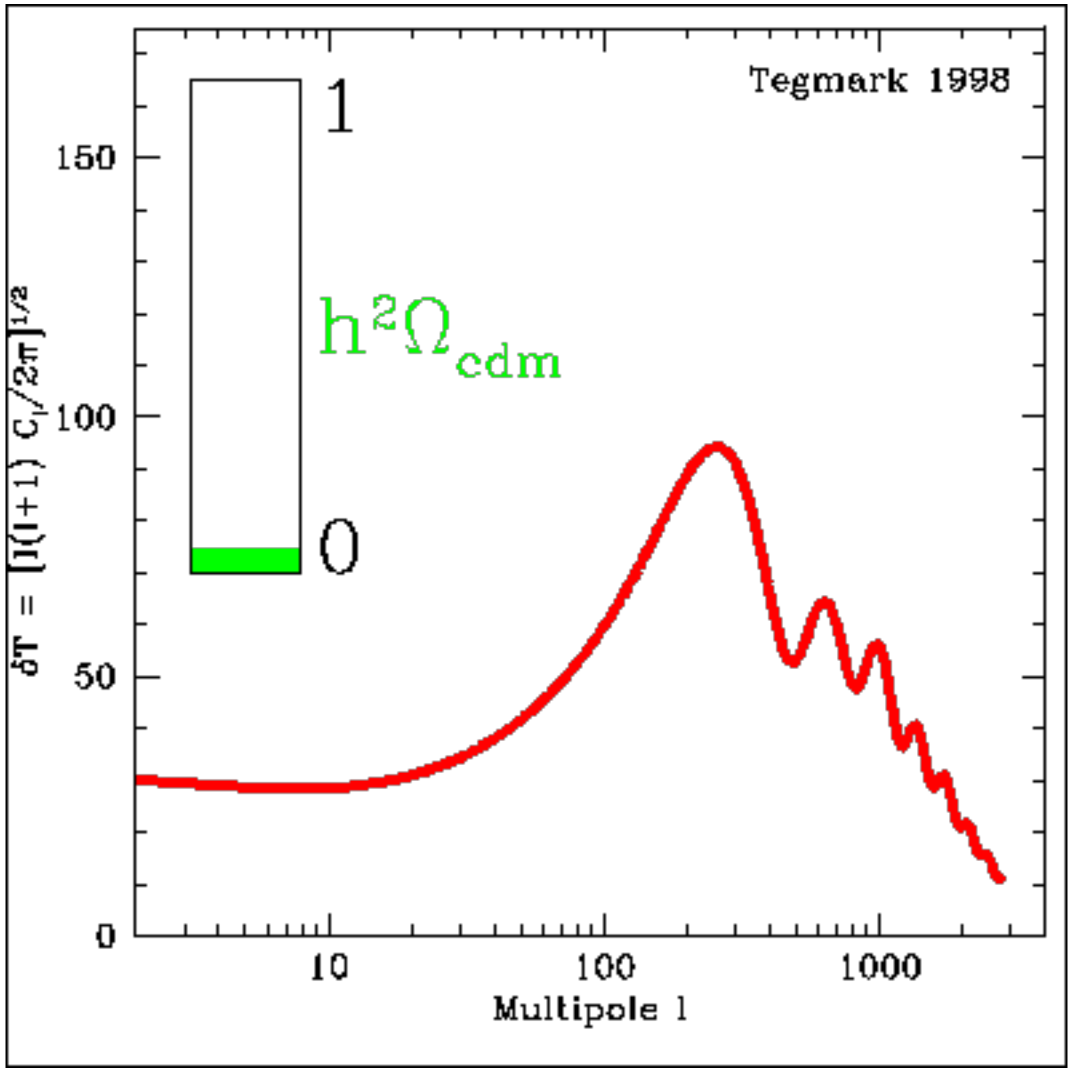
CMB Toolbox: https://lambda.gsfc.nasa.gov/toolbox/

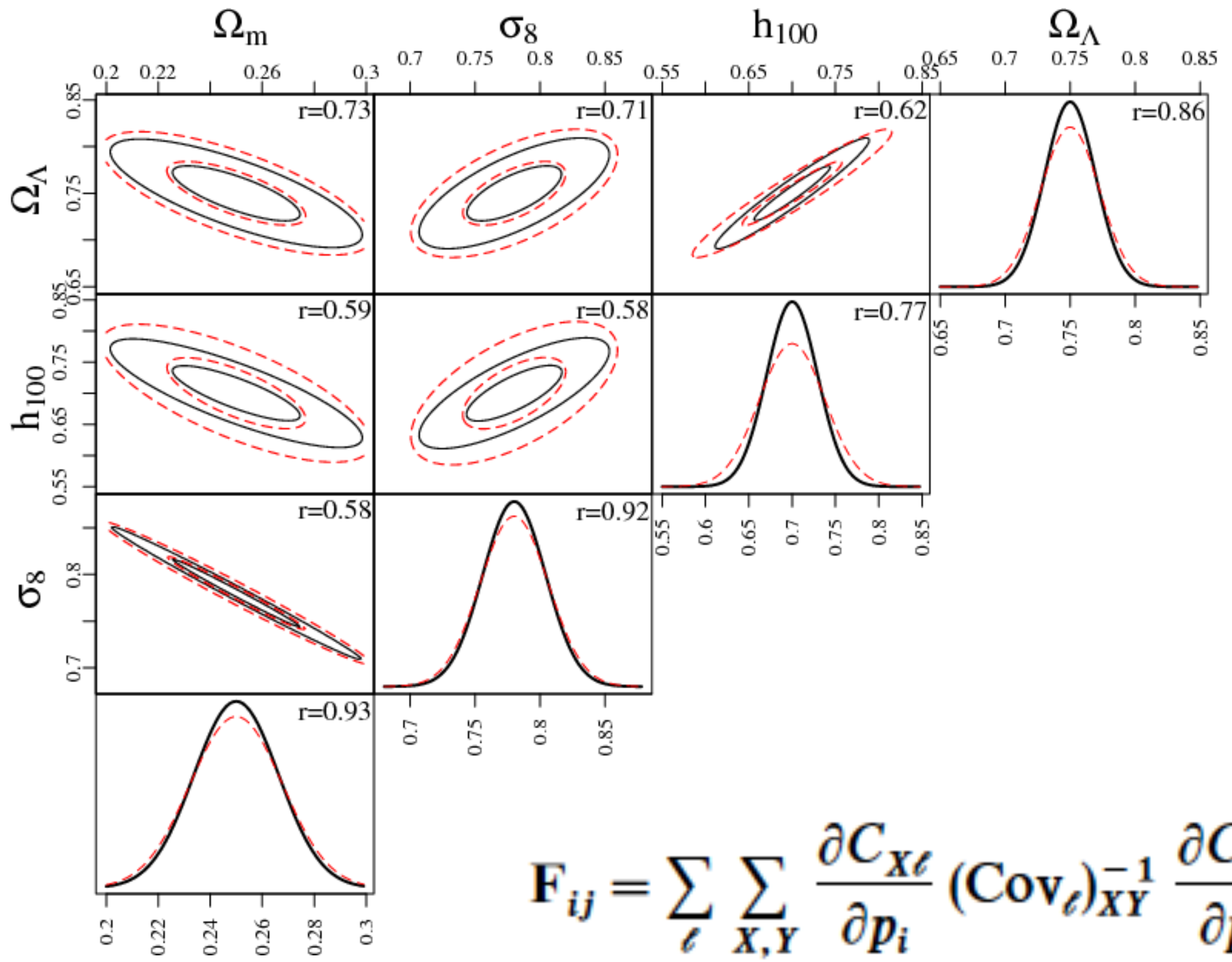
CAMB website: http://camb.info/

CAMB Python page: http://camb.readthedocs.io/en/latest/

(try the online example notebook!)

Outline of the CMB exercise





$$\mathbf{F}_{ij} = \sum_{\ell} \sum_{X,Y} \frac{\partial C_{X\ell}}{\partial p_i} (\mathrm{Cov}_\ell)^{-1}_{XY} \frac{\partial C_{X\ell}}{\partial p_j}$$

$$(\mathrm{Cov}_\ell)_{TT} = \frac{2}{(2\ell+1) f_{\mathrm{sky}}} (C_{T\ell} + w_T^{-1} B_\ell^{-2})^2$$

We will use online CMB tools, e.g.
https://lambda.gsfc.nasa.gov/toolbox/tb_camb_form.cfm

parameter constraints

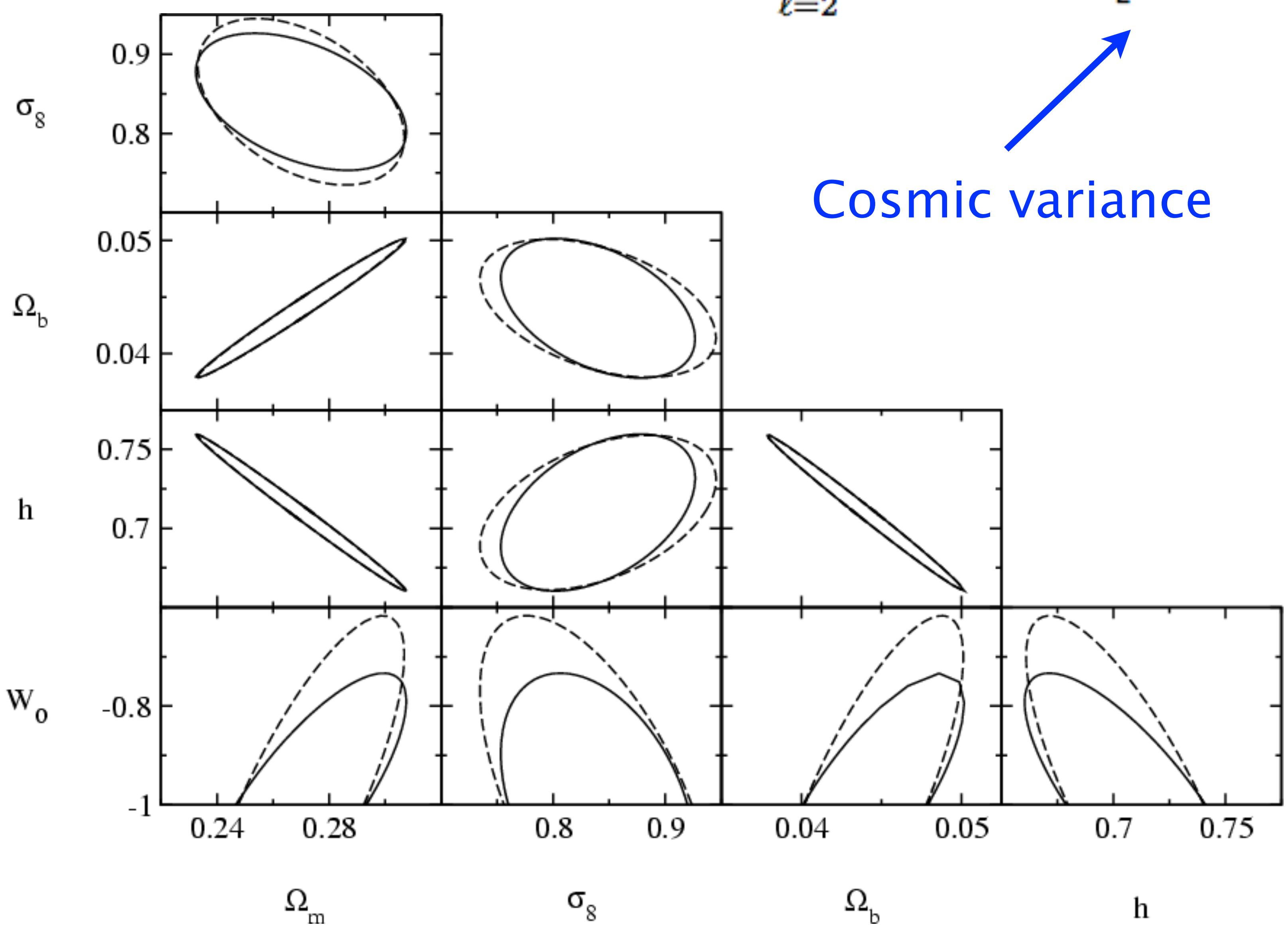
$$\sigma_{\alpha_i} \geq \sqrt{\frac{1}{F_{ii}}}$$

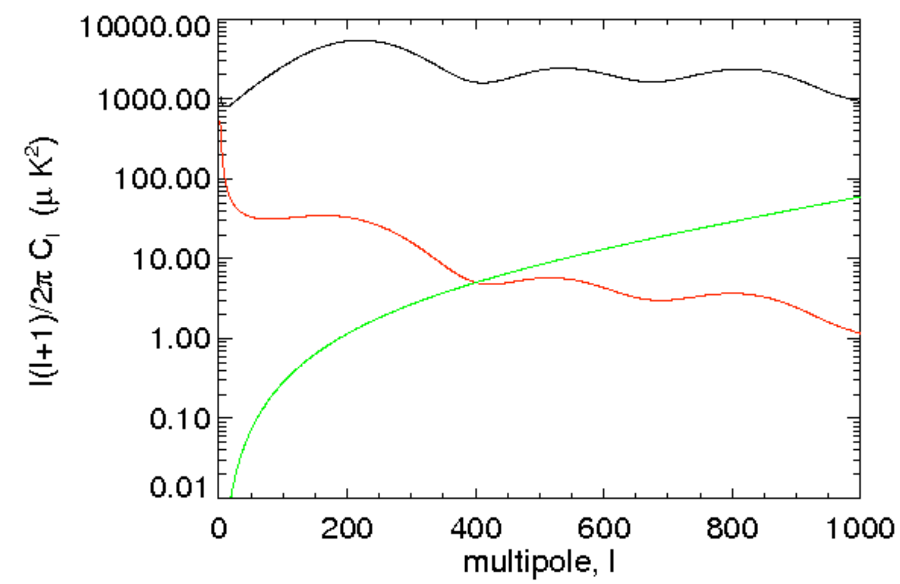
Parameter estimation (Exercise)

$$F_{ij} = \sum_{\ell=2}^{\ell_{max}} \frac{(2\ell+1)f_{sky}}{2} \left[C_\ell + \frac{4\pi}{N}\sigma_N^2 e^{\ell(\ell+1)\sigma_b^2}\right]^{-2} \frac{\partial C_\ell}{\partial s_i}\frac{\partial C_\ell}{\partial s_j}$$

Cosmic variance

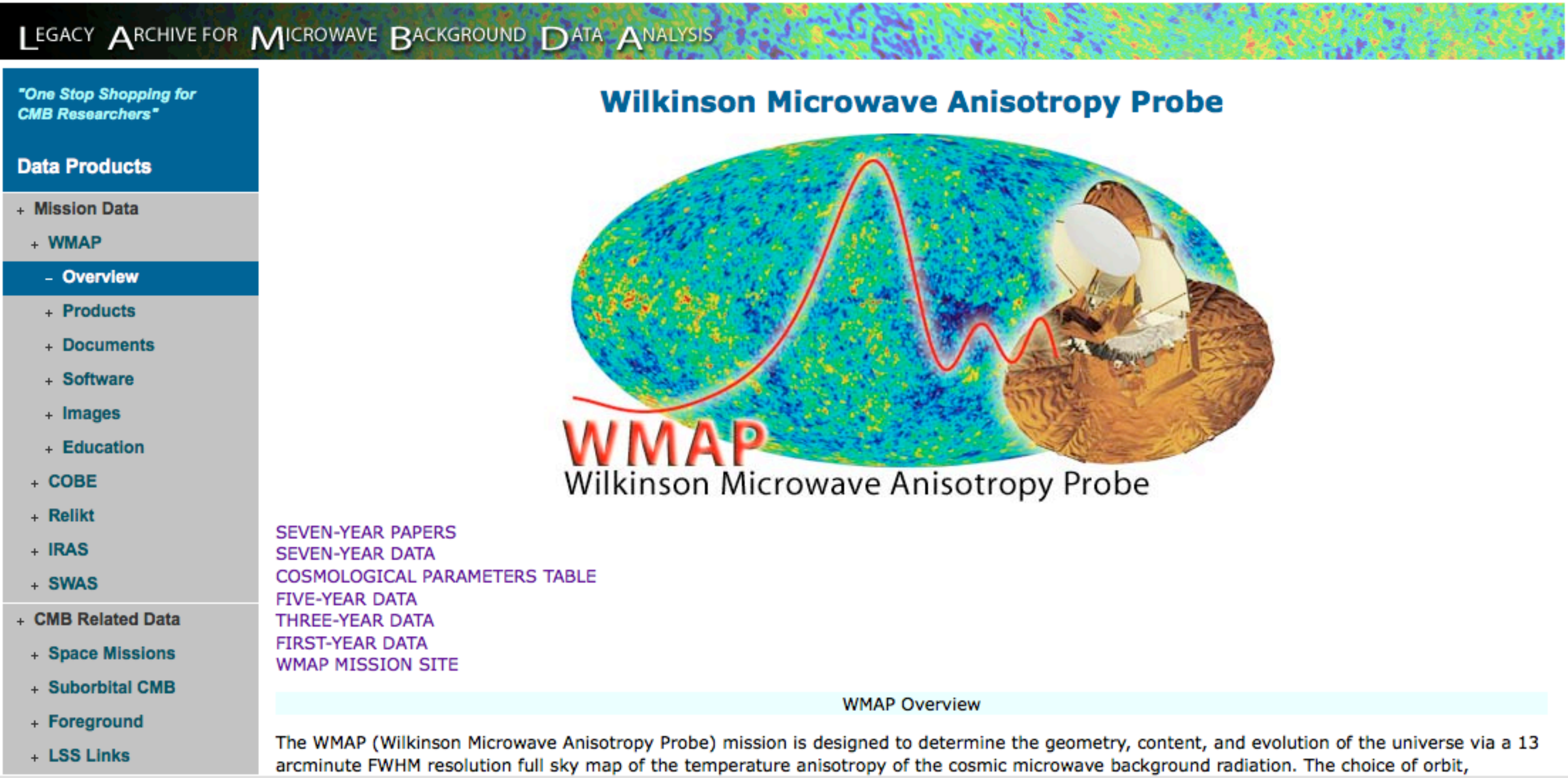
Noise per beam





Plot your own power spectra (two for each parameter and the fiducial). Then add the noise term and sum up!

Check the LAMBDA website



LEGACY ARCHIVE FOR MICROWAVE BACKGROUND DATA ANALYSIS

"One Stop Shopping for CMB Researchers"

Data Products

- Mission Data
  - WMAP
    - Overview
    - Products
    - Documents
    - Software
    - Images
    - Education
  - COBE
  - Relikt
  - IRAS
  - SWAS
- CMB Related Data
  - Space Missions
  - Suborbital CMB
  - Foreground
  - LSS Links

Wilkinson Microwave Anisotropy Probe

SEVEN-YEAR PAPERS
SEVEN-YEAR DATA
COSMOLOGICAL PARAMETERS TABLE
FIVE-YEAR DATA
THREE-YEAR DATA
FIRST-YEAR DATA
WMAP MISSION SITE

WMAP Overview

The WMAP (Wilkinson Microwave Anisotropy Probe) mission is designed to determine the geometry, content, and evolution of the universe via a 13 arcminute FWHM resolution full sky map of the temperature anisotropy of the cosmic microwave background radiation. The choice of orbit,

How to go further with CMB?

Cosmic Variance

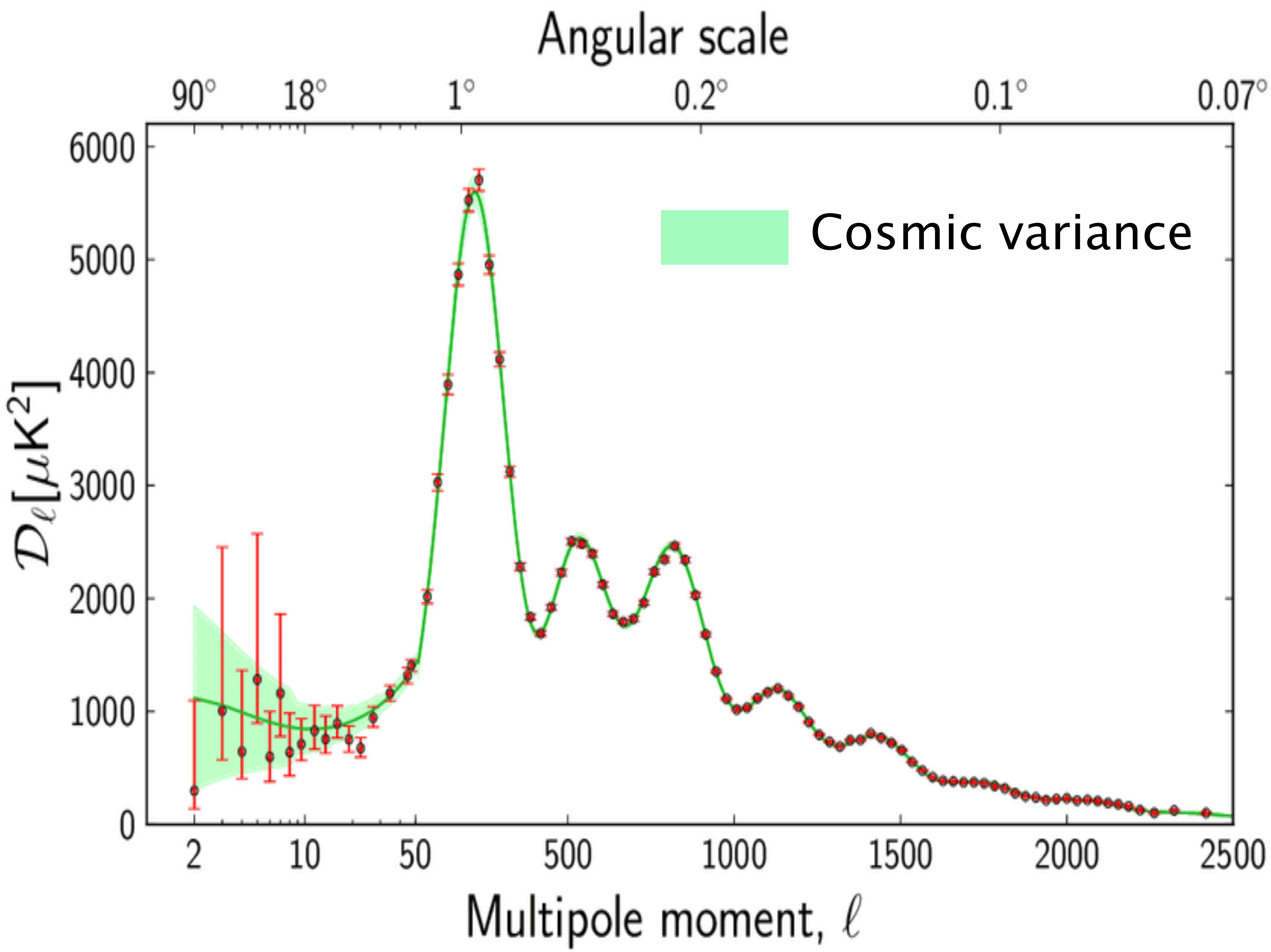
› We only have one realization (our sky), i.e., one event.

› TT **at all scales** is now cosmic variance limited.

To go further:

› TT at very large l (secondary effects)

› **Polarization**



Questions?