OBSERVATIONAL COSMOLOGY

Problem sheet 4 - 22/06/2017

1. The X-ray and SZE observables (X-ray surface brightness and SZE temperature decrement/increment) for a cluster are computed from the following line of sight integrals:

$$
\begin{aligned}
S_X(\theta) &= \frac{1}{4\pi(1+z)^4} \int n_e^2 \frac{\mu_e}{\mu_H} \Lambda_H(T_e)\, dl \\
\Delta T(\theta) &= f(x, T_e) T_{\rm CMB} \int n_e \sigma_T \frac{k_B T_e}{m_e c^2}\, dl
\end{aligned}
$$

where $x = h\nu / kT_{\rm CMB}$ is the dimensionless frequency. If we neglect relativistic corrections, then the spectral function $f(x, T_e)$ does not depend on gas temperature, and is given by: $f(x, T_e) = \left(x \frac{\exp(x)+1}{\exp(x)-1} - 4 \right)$. The integration is along the path length $dl = D_A\, d\theta$. Assuming the intra-cluster gas density distribution can be approximated by the isothermal β-model (temperature T_e is constant):

$$
n_e(\theta) = n_{e0} \left(1 + \frac{\theta^2}{\theta_c^2} \right)^{-3\beta/2}
$$

solve for the central electron density from both the X-ray and the SZE equations, and show that the angular diameter distance to the cluster is related to the central values as following:

$$
D_A \propto \frac{(\Delta T(0))^2}{S_X(0)} \frac{1}{T_e^2\, (1+z)^4\, \theta_c}
$$

Derive the full expression for the above formula. *(Hint: the integrals are to be evaluated from $-\infty$ to $+\infty$, which give ratios of Gamma functions).*

Use the above expression for some typical values for a cluster to get its distance. Here are the numbers for the well known massive cluster Abell 2163: $\beta = 0.674$, $\theta_c = 87.5$ arcsec, $S_X(0) = 1.36 \times 10^{-12}$ ergs s^{-1} cm^{-2} arcmin^{-2}, $\Delta T_{\rm SZ}(0) = -1900\ \mu$K at 30 GHz, $\Lambda_{eH0} = 6.135 \times 10^{-24}$ ergs s^{-1} cm^3, $T_e = 12$ keV and $\mu_e/\mu_H = 0.85$ ($n_H = n_e \mu_e / \mu_H$). What is the value of D_A you get? If the redshift of the cluster is $z = 0.202$, what will be the resulting Hubble parameter for the standard ΛCDM cosmology? *(Hint: the "arcmin^{-2} factor in the X-ray brightness corresponds to the angular scale in sky, so you need to convert it to steradian, and also use dimensionless θ_c. Also note that in the non-relativistic R-J limit $f(x, T_e) \approx -2$.)*

Try to list at least three problems that might cause an inaccurate estimate of the angular diameter distance (and hence wrong cosmology) using this method.

2. Using the same isothermal β-model (with radial distance r instead of θ), and assuming the intra-cluster gas is spherically symmetric and is in hydrostatic equilibrium with the dark matter potential well of the cluster (see lecture notes for hydrostatic equilibrium equation), show that the total mass inside radius r is given by:

$$M_{\rm tot}(< r) = \frac{3\beta k_B T_e}{G\mu m_p} \frac{r^3}{r_c^2 + r^2}.$$