

OBSERVATIONAL COSMOLOGY

PROBLEM SHEET 1 - 20/05/2017

Using their integral form, plot the luminosity distance $D_L(z)$ and the angular-diameter distance $D_A(z)$ versus the redshift for different values of the curvature parameter (i.e. open, close or flat universe) and different choices for Ω_m , Ω_Λ , Ω_r .

$$D_H = \frac{c}{H_0} \quad (0.1)$$

$$E(z)^2 = \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_\lambda \quad (0.2)$$

$$D_C(z) = D_H \cdot \int_0^z \frac{dz'}{E(z')} \quad (0.3)$$

$$D_M(z) = \begin{cases} \frac{D_H}{\sqrt{\Omega_k}} \sinh\left(\sqrt{\Omega_k} \frac{D_C(z)}{D_H}\right) & \Omega_k > 0 \\ D_C(z) & \Omega_k = 0 \\ \frac{D_H}{\sqrt{|\Omega_k|}} \sin\left(\sqrt{|\Omega_k|} \frac{D_C(z)}{D_H}\right) & \Omega_k < 0 \end{cases} \quad (0.4)$$

$$D_L(z) = (1+z) \cdot D_M(z) \quad (0.5)$$

$$D_A(z) = \frac{D_M(z)}{1+z} \quad (0.6)$$

Remember that $\Omega_r + \Omega_m + \Omega_k + \Omega_\Lambda = 1$; note that for an open universe ($k = -1$) $\Omega_k > 0$, for a closed one ($k = 1$) $\Omega_k < 0$ and for a flat one ($k = 0$) $\Omega_k = 0$. You can also try and use different values for H_0 and see what happens.